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Safety state evaluation method of the highway tunnel structure

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ABSTRACT

This study proposes an evaluation method for the structural safety of expressway tunnels utilizing possibility and prospect theories to address the influence of multiple indicators on the structural safety of expressway tunnels and the imprecision of human-bounded rationality in assessing results. It constructs the probability distribution of safety level by determining the safety level of the highway tunnel structure. The reference distribution function of each monitoring index is then derived using the expected value of experts. Based on the possibility theory, the possibility distribution of the monitoring results of indicators is obtained, and the mapping relationship between the monitoring indicators and the possibility distribution function of safety status grade is developed. Finally, the prospect theory evaluates the highway tunnel structure's safety status. This method is applied to assess the structural safety of a highway tunnel, which verifies its effectiveness and practicability, and provides a new method for evaluating the structural safety of a highway tunnel.

1. Introduction

Highway tunnels are vital in transportation and regional economy as essential infrastructure. However, the environment of highway tunnels is highly complex, making them vulnerable to the comprehensive impact of construction conditions, natural environment, and human factors, which poses a severe threat to the service safety of tunnel structures. At the same time, the number and mileage of highway tunnels are developing rapidly. Using China as an example, the number of highway tunnels was 23,268, and the total length of tunnels reached 24.6989 million meters in 2021. The number of super-long highway tunnels increased to 1175, and the length of super-long highway tunnels reached 5.2175 million meters. As a result of facing such a large-scale operating tunnel, it is necessary to adopt a scientific and reasonable tunnel safety state evaluation method to determine the safety state of the tunnel structure and direct maintenance efforts. Therefore, it is crucial to investigate the safety state evaluation method of the highway tunnel structure.

The safe state of tunnel structure operation is the prerequisite to ensuring tunnel benefits' sustainable development. Japan proposes to use the concept of soundness to assess the remaining life of railway tunnels and hydraulic tunnel structures in the safety state evaluation of tunnel structures during operation. The United States uses the concept of structural damage degree to evaluate the durability of structures [1]. Some scholars, such as Wu et al. [2], have proposed tunnel safety state evaluation methods. The safety state of the tunnel structure has been evaluated utilizing an analysis of the fractal dimension of cracks. Arends et al. [3] suggested a tunnel safety evaluation method using probabilistic risk assessment and economic optimization. Maleki et al. [4] discussed two related safety factors based on the short-term and long-term behavior of shallow buried tunnels combined with the elastic viscoplastic constitutive model and evaluated the safety of shallow buried tunnels. Zhou et al. [5] presented a tunnel structure safety state evaluation method by

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torsional wave velocity. The principle is to obtain the elastic modulus of the lining structure through the torsional wave velocity and use the calculated stiffness to assess the safety state of the tunnel structure. Lai et al. [6] revealed the distribution characteristics and development law of the above issues in the lining structure by monitoring the lining cracks, tunnel leakage, and cavities behind the lining, as well as analyzing the corresponding monitoring data to determine lining structure's damage level of the based on this. Huang et al. [7] introduced a visual evaluation method for tunnel damage based on cloud theory, which effectively considered the fuzziness and randomness of the evaluation system and improved the accuracy of structural damage evaluation results. Rao et al. [8] established a fuzzy comprehensive evaluation model for the safety of highway tunnel structures in karst areas during the operation period. They used the principle of maximum membership to evaluate the safety of highway tunnel structures in a karst area. In addition, data mining technology [9], grey theory [10], reliability theory [11], cluster analysis [12] and other methods are also used to evaluate the safety status of tunnel structures.

Most of the above methods for evaluating tunnel structure safety status determine the safety status based on the maximum contribution of indicators and are incapable of effectively considering the impact of other indicators and assume that people are entirely rational. However, in evaluating highway tunnel structure safety, there are usually many problems, such as multiple indicators and complex relationships between quantitative and qualitative processing, leading to deviations in the safety evaluation results.

The possibility theory is adopted on the fuzzy set theory, which interprets the uncertainty problem as a potential difference from the probability. It is an uncertainty treatment method that can be utilized to determine the comprehensive impact of multiple indicators on the structural safety of highway tunnels. Prospect theory predicts people's behavior in the risk decision-making process, which is inconsistent with the traditional expected value and expected utility theories. This method can effectively address the problem of the imprecision of human-bounded rationality in the evaluation results. These studies introduce the possibility and prospect theories into the safety evaluation of highway tunnel structures and propose an evaluation method for highway tunnel structure safety. This approach is applied to evaluate the structural safety of a highway tunnel, which verifies its effectiveness and practicability, and provides a new technique.

2. Theory

2.1. Possibility theory

Gaines and Kohout [13] first proposed the possibility theory, but the specific concept was unclear. Thereafter, Zadeh [14] proposed the concept of possibility theory based on a large number of sample information. The possibility theory is based on the fuzzy set, which interprets the uncertainty problem as a possible difference from the probability. Hence, it can be considered an uncertainty treatment method.

Definition 1 Let x be a variable on the universe U, $\widetilde{A}$ is the fuzzy subset on U, the membership function $u_{\widetilde{A}}(x)$ represents the compatibility with element x, and $\widetilde{A}$ is the fuzzy constraint R(X) associated with x, $R(X) = \widetilde{A}$, and the probability distribution associated with variable x is Π_x, then

$$\Pi_x = R(X) \tag{1}$$

In Eq. (1), let π_x be the probability distribution function of Π_x, which is numerically equal to the membership function $u_{\widetilde{A}}(x)$, then $\forall x \in U$ has Eq. (2)

$$\pi_x = u_{\widetilde{A}}(x) \tag{2}$$

Definition 2: Let $\widetilde{A}$ be a fuzzy subset on universe U, then the cut set of $\widetilde{A}$ is

$$A_\lambda = \{x | u_{\widetilde{A}}(x) \geq \lambda\} = [a_1(\lambda), a_2(\lambda)] \tag{3}$$

In Eq. (3), where $\lambda \in [0, 1]$, and $a_1(\lambda)$ and $a_2(\lambda)$ are two endpoints of the fuzzy subset, respectively.

Definition 3: Let (X, P) and (Y, R) be two fuzzy fields, let $\dot{y} = \{y | y \in r \in R\}$, $\dot{Y} = \{\dot{y} | y \in Y\}$, and $\widetilde{R}$ be the backup field generated by $\dot{Y}$. If the set-valued mapping $\xi : X \rightarrow \rho(Y)$ satisfies

$$r \in R \Rightarrow \xi^{-1}(R) \in P \tag{4}$$

In Eq. (4), ξ is the possibility set-valued mapping represented by $E(X, P; Y, R) = \{\xi | \xi \text{ is the possibility set} - \text{valued mapping from } (X, P) \text{ to } (Y, R)\}$.

Definition 4: Let (Ω, A, Π) be a fuzzy measurable space, (X, B) be a fuzzy field, and $\xi \in E(\Omega, A; X, B)$, then

$$u_\xi(x) = \Pi(\{w \in \Omega | x \in \xi(w)\}) = \Pi(\xi^{-1}(x)) \tag{5}$$

In Eq. (5), $u_\xi(x)$ is called the falling shadow of the possibility set-valued mapping ξ expressed as Eq. (6):

$$F(X, \Pi) = \{u_\xi | \xi \in E(\Omega, A; X, B)\} \tag{6}$$

2.2. Prospect theory

Prospect theory is a new risk decision-making approach proposed by Kahneman and Tversky [15,16] in 1979 based on the expected value theory and expected utility theory through extensive psychological research. It has been widely used in financial risk and information management. Prospect theory makes scheme decisions through the value of the prospect. The value function and weight function determine the size of the prospect value.

$$V = \sum_{i=1}^{k} v(D_i) w(p_i) \tag{7}$$

In Eq. (7), V is the prospect value, w(p) is the weight function, and v(D) is the value function.

Tversky et al. [16] proposed that the value function is a power function defined as follows:

$$v(D) = \begin{cases} D^{\alpha}, D \geq 0 \\ -\lambda(-D)^{\beta}, D < 0 \end{cases} \tag{8}$$

where D is the income and loss value, α and β are the concave-convex degrees of the regional value function of income and loss, respectively, in which ($0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$). The larger the value of α and β, the more decision-makers tend to take risks; λ is the loss avoidance coefficient that reflects the characteristics for which the loss area is steeper than the income area.

Zeng et al. [17] obtained the value of the risk preference coefficient in the Chinese context through a large number of experiments, thus $\alpha = 1.21$, $\beta = 1.02$, $\lambda = 2.25$.

3. Evaluation model

3.1. Evaluation process

The basic idea of evaluation: on the basis of determining the safety condition grade of highway tunnel structure, construct the probability distribution of safety condition grade, and then determine the reference distribution function corresponding to each monitoring and evaluation index under different safety condition grades. The results of the monitoring indicators are processed to obtain the indicators' possibility distribution, and the relationship between monitoring indicators and the possibility distribution function of safety status grade is constructed. In this study, the profit and loss matrix is constructed, and the value function is calculated

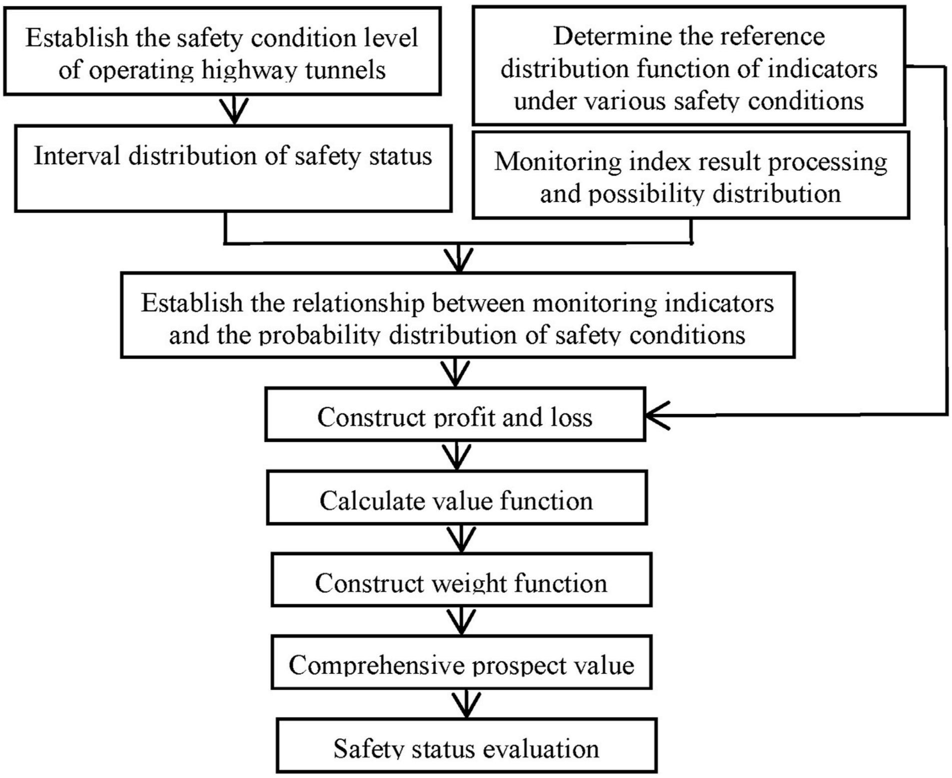

Fig. 1. Safety state evaluation process of the highway tunnel structure.

based on the prospect theory. The trapezoidal fuzzy number is used to determine the comprehensive weight of each index to different safety levels. According to the value function and weight function, the foreground value of the highway tunnel structure safety condition is calculated, and the highway tunnel structure safety condition is evaluated according to the foreground value. The evaluation model process is as follows (Fig. 1).

3.2. Determine the safety status level and interval distribution

The structural safety level of operating highway tunnels is standard for determining the safety conditions in tunnels and the basis for guiding engineering decisions. In engineering, the safety level shown in Table 1 is usually utilized to evaluate the structural safety of highway tunnels. In this paper, the safety condition level is described by the interval number in the classical set; hence the safety condition level 1 [0, 0.2], the safety condition level 2 [0.2, 0.4], the safety condition level 3 [0.4, 0.6], the safety condition level 4 [0.6, 0.8], the safety condition level 5 [0.8, 1], and its probability distribution are shown in Fig. 2.

3.3. Index reference distribution function

The decision-maker has its own reference point that archives certain goals. If the target value exceeds or is lower than the reference point, the decision-maker will be more sensitive [18]. In prospect theory, the selection of reference points is usually based on the expected value of experts on the target. However, the reference point information given is fuzzy to some extent due to the differences in experts' understanding and preferences. Therefore, this paper uses the possibility distribution method to construct the reference distribution function of monitoring indicators under each safety level. The specific steps are as follows.

(1) Based on their knowledge structure and experience, many experts provide the expected values E_{ij}^{q} ($i = 1,2,3,4,5$, $j = 1,2,\ldots,n$) of the corresponding monitoring indicators under different safety levels. E_{ij}^{q} is the expected value of the monitoring indicator j under the safety level i of expert q.
(2) Calculate the average value $\overline{E}_{ij}$ of the expected value of each index and take it as the safety standard value. According to the possibility theory, the probability corresponding to the average value is 1.
(3) Determine the support set $[E_{ij}^{'}, E_{ij}^{''}] = [\min(E_{ij}^{q}), \max(E_{ij}^{q})]$ using the maximum and minimum values of the expected value.
(4) Each index's reference distribution function $\pi_{ij}^{r}(s)$ is constructed using the following equation, and the reference information is obtained based on this Eq. (9) [19], so that $s = E_{ij}^{q}$.

$$\pi_{ij}^{r}(s) = \begin{cases} \left(\dfrac{s - E_{ij}^{'}}{\overline{E}_{ij} - E_{ij}^{'}}\right), E_{ij}^{'} \leq s < \overline{E}_{ij} \\ 1 \ , \overline{E}_{ij} \\ \left(\dfrac{-s + E_{ij}^{''}}{E_{ij}^{''} - \overline{E}_{ij}}\right), \overline{E}_{ij} < s \leq E_{ij}^{''} \end{cases} \tag{9}$$

3.4. Probability distribution of monitoring index results

Due to the influence of monitoring personnel and equipment sensors, there are certain errors in the monitoring result information. Therefore, according to the possibility theory, each monitoring index's possible distribution function can be constructed to represent the fuzzy monitoring information [19]. There are many methods to construct the possibility distribution function. This paper adopts the possibility distribution function of the index monitoring results through the monitoring data results of each index. The specific steps are as follows.

(1) Let $X = \{x_1, x_2, \ldots, x_n\}$ be the data sample of a monitoring index of highway tunnel structure; then the samples' mean value for the monitoring data result can be defined as $m = \frac{1}{n}\sum_{i=1}^{n} x_i$.

Table 1
Highway tunnel structure safety level.

Level	Describe	Safety value	Engineering decision
1	Very safe	[0,0.2]	Normal management and inspection
2	Safe	[0.2,0.4]	Need to strengthen management and inspection
3	Safer	[0.4,0.6]	Strengthen prevention and monitoring, and formulate preventive measures
4	Unsafe	[0.6,0.8]	Control measures and early warning schemes need to be taken
5	Danger	[0.8,1]	Stop operation, avoid risks, and take emergency plans

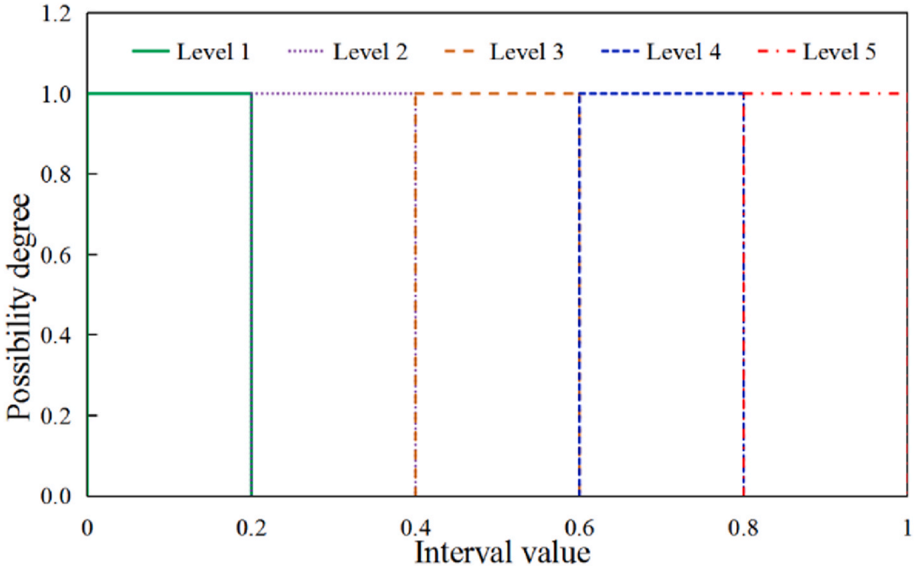

Fig. 2. Probability distribution of safety status level.

(2) Take the samples' mean m as the dividing point, divide the monitoring data into two groups $X_1 = \{x_i | x_i < m, i = 1, 2, ..., n_1\}$ and $X_2 = \{x_i | x_i > m, i = 1, 2, ..., n_2\}$, and calculate the mean values of the two groups of monitoring data as $m_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$, $x_i < m$ and $m_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} x_i$, $x_i > m$ respectively.

(3) The triangular probability distribution function represents the data of each monitoring index $\pi(x_i) = (\mathrm{m}, 3(m - m_1), 3(m_2 - m))$, in which $3(m - m_1)$ and $3(m_2 - m)$ account for the left and right widths of the triangle distribution function, respectively.

3.5. Functional relationship between structural monitoring indicators and safety status

Since deterministic functions are not able to describe complex and variable mapping relationships between the monitoring indicators and the structural safety of highway tunnels, it is easy to obtain inaccurate evaluation results and the possibility that set-valued mapping can make up for the lack of deterministic function to describe the complex mapping relationship between the two types of uncertain sets. Therefore, this paper uses possibility set-valued mapping to construct the functional relationship between the possibility distribution of each monitoring index and the possibility distribution of the safety status level. The steps adopted herein are as follows.

(1) Let (X, P) and (Y, R) be the two fuzzy fields of the monitoring index and the structural safety information of the highway tunnel, respectively, and the corresponding probability distributions are π_p and π_R , respectively.

(2) Determine the shadow of each monitoring index and the structural safety status information of the highway tunnel.

$$\begin{cases} \lambda \mapsto P_\lambda \\ \alpha \mapsto R_\alpha \end{cases} \tag{10}$$

In Eq. (10), P_λ is the shadow of monitoring index information, $\lambda \in [0, 1]$ is the interception level of monitoring index information, R_α is the shadow of highway tunnel structural safety status information, and $\alpha \in [0, 1]$ is the interception level of highway tunnel structural safety status information.

(3) Based on the expansion principle, the mapping of points is extended to the fuzzy set mapping ζ, and the joint shadow $(\lambda\ ,\ \alpha) \mapsto (P_\lambda^C \times Y) \cup (P_\lambda \times R_\beta)$ of monitoring indicators and safety conditions is obtained on the Z : X = Y axis. For $\forall x_0 \in X$, the possibility set value mapping between the highway tunnel structure monitoring index and the safety condition level is [19]

$$\pi_{ij}^s = \pi_{P\rightarrow R}(x_0, y) = (1 - P(x_0)) \vee P(x_0)R(y) \tag{11}$$

where $\vee$ is the larger operation symbol, $P(x_0) = \{\pi_P(x_0) | x_0 \in P(\lambda, \alpha)\} = \{\pi_P(x_0) | x_0 \in P_\lambda\}$ is the possibility distribution of a monitoring index, $R(y) = \{\pi_R(y) | y \in R(\lambda, \alpha)\} = \{\pi_R(y) | y \in R_\alpha\}$ is the probability distribution of safety status level, and x_0 and y are the corresponding values after normalization.

3.6. Construct profit and loss matrix

Before constructing the profit and loss matrix, it is necessary to judge the income or loss. Accordingly, by finding the safety state value $\overline{G}_{ij}$ with π_{ij}^{s} the probability of 1, and comparing it with the safety standard value $\overline{E}_{ij}$, when $\overline{G}_{ij} < \overline{E}_{ij}$, the structure tends to be safe, indicating income. When $\overline{G}_{ij} \geq \overline{E}_{ij}$, the structure tends to be vulnerable, indicating loss. The equations for the income and loss values are defined as follows:

$$D_{ij} = \begin{cases} 1 \Big/ \left(\pi_{ij}^{s} \times \pi_{ij}^{r}\right) \text{—profit} \\ -1 \Big/ \left(\pi_{ij}^{s} \times \pi_{ij}^{r}\right) \text{—loss} \end{cases} \tag{12}$$

where the profit and loss matrix can be constructed as $(D_{ij})_{m \times n}$.

3.6.1. Construct weight function

The weight of risk assessment indicators is usually quantified by using the "1–9″ scale method, but this scale method cannot truly reflect the objective situation. For example, if *i* is slightly more important than *j*, the importance ratio of *i* to *j* is 0.75:0.25 b y using the "1–9″ scale method, which obviously does not conform to the actual judgment thinking. The trapezoidal fuzzy number is improved by introducing the trapezoidal membership function to the "1–9″ scale method, which makes up for the defects of the "1–9″ scale method. In fact, the trapezoidal membership function can reflect the probability and statistical characteristics of the evaluation. Therefore, in order to conform to the actual evaluation strategy, the monitoring index weight determination method based on the trapezoidal fuzzy number [20] is adopted using the trapezoidal fuzzy number scale level assignment, as shown in Table 2.

The calculation process herein is as follows.

(1) Compare K experts on the same index set and give the judgment matrix $R^{(e)} = (r_{ij}^{\bullet(e)})_{n \times n}$ respectively, where $(r_{ij}^{\bullet(e)})_{n \times n} = (l_{ij}^{(e)}, m_{ij}^{(e)}, n_{ij}^{(e)}, s_{ij}^{(e)})$, eth means the e the prospect theory-expert, and e > 1. The trapezoidal fuzzy number judgment matrix is transformed into a general matrix using the trapezoidal center of gravity equation, and the consistency of the judgment matrix is investigated. Once the consistency requirements are met, the preference information of all experts is integrated and calculated as Eq. (13):

$$\begin{aligned} r_{ij}^{\bullet} &= \left(l_{ij}, m_{ij}, n_{ij}, s_{ij}\right) = \\ &\frac{1}{Y} \times \left(r_{ij}^{\bullet 1}, r_{ij}^{\bullet 2}, \ldots, r_{ij}^{\bullet e}\right) = \\ &\frac{1}{Y} \times \left(\sum_{e=1}^{Y} l_{ij}{}^{e}, \sum_{e=1}^{Y} m_{ij}{}^{e}, \sum_{e=1}^{Y} n_{ij}{}^{e}, \sum_{e=1}^{Y} s_{ij}{}^{e}\right) \end{aligned} \tag{13}$$

(2) Calculate the fuzzy evaluation value $v_i^{\bullet}$ as Eq. (14):

$$v_i^{\bullet} = \left(\alpha_i \delta^{-1}, \beta_i \gamma^{-1}, \gamma_i \beta^{-1}, \delta_i \alpha^{-1}\right) \tag{14}$$

where $\alpha_i = (\prod_{j=1}^{n} l_{ij})^{1/n}$, $\alpha = \sum_{i=1}^{n} \alpha_i$, $\beta_i = (\prod_{j=1}^{n} m_{ij})^{1/n}$, $\beta = \sum_{i=1}^{n} \beta_i$; $\gamma_i = (\prod_{j=1}^{n} n_{ij})^{1/n}$, $\gamma = \sum_{i=1}^{n} \gamma_i$, $\delta_i = (\prod_{j=1}^{n} s_{ij})^{1/n}$, $\delta = \sum_{i=1}^{n} \delta_i$.

Table 2
Quantitative table of index impact.

Comparison of the influence of two indicators *i* and *j*	Quantitative results
i has no effect on *j*	$\tilde{5}/\tilde{5} = (1, 1, 1, 1)$
i has less impact on *j*	$\tilde{6}/\tilde{4} = (1, 11/9, 13/7, 7/3)$
i has a general impact of I on *j*	$\tilde{7}/\tilde{3} = (3/2, 13/7, 3, 4)$
i has a great impact on *j*	$\tilde{8}/\tilde{2} = (7/3, 3, 17/3, 9)$
i has a great influence on *j*	$\tilde{9}/\tilde{1} = (4, 17/3, 9, 9)$

Note: the membership function corresponding to trapezoidal fuzzy number $\tilde{1}$ is (1, 1, 3/2, 2), the membership function corresponding to trapezoidal fuzzy number $\tilde{x}$ is (x-1 , x-1/2 , x+1/2 , x+1), x = 2 to 8, and the membership function corresponding to trapezoidal fuzzy number $\tilde{9}$ is (8, 17/2, 9, 9). For any two trapezoidal fuzzy numbers, $A_1^{\bullet} = (l_1, m_1, n_1, s_1)$ and $A_2^{\bullet} = (l_2, m_2, n_2, s_2)$ with $A_1^{\bullet} > 0$, $A_2^{\bullet} > 0$, then $A_1^{\bullet}/A_2^{\bullet} = (l_1/s_2, m_1/n_2, n_1/m_2, s_1/l_2)$.

(3) Calculate the expected value $E(v_i^{\bullet})$ of the fuzzy evaluation value $v_i^{\bullet}$ of each monitoring index. The larger the expected value is, the more important the corresponding index is.

$$E\left(v_i^{\bullet}\right) = \frac{l_i + m_i + n_i + s_i}{4} \tag{15}$$

(4) The weight of each monitoring index is calculated (as Eq. (16)) using the normalization method in Eq. (15).

$$w_{ij} = \frac{E\left(v_i^{\bullet}\right)}{\sum_{i=1}^{n} E\left(v_i^{\bullet}\right)} \tag{16}$$

(5) The influence degree of each index on the same safety level is calculated and combined with the weight calculation method in the prospect theory, as Eq. (17):

$$w'\left(w_{ij}\right) = \begin{cases} \dfrac{\left(w_{ij}\right)^{\gamma}}{\left(\left(w_{ij}\right)^{\gamma} + \left(1 - w_{ij}\right)^{1/\gamma}\right)}, D_{ij} > 0 \\ \dfrac{\left(w_{ij}\right)^{\delta}}{\left(\left(w_{ij}\right)^{\delta} + \left(1 - w_{ij}\right)^{1/\delta}\right)}, D_{ij} \leq 0 \end{cases} \tag{17}$$

where γ and δ are the bending degree parameters of the weight function, respectively.

Zeng et al. [17] highlighted that the values of γ and δ in the Chinese context are 0.55 and 0.49, respectively. The weights of different indicators for the same safety situation are obtained by normalizing over the following Eq. (18):

$$W' = \frac{w'\left(w_{ij}\right)}{\sum_{i=1}^{n} w'\left(w_{ij}\right)} \tag{18}$$

3.7. Calculate the comprehensive prospect value

The value function $v(D_{ij})$ is calculated by combining Eq. (8) and Eq. (12), and the comprehensive prospect value V_i is calculated by using the following equation.

$$V_{\mathrm{i}} = \sum_{j=1}^{n} v\left(D_{ij}\right) W' \tag{19}$$

The calculated comprehensive foreground value under different safety condition levels is sorted, and the safety condition level corresponding to the minimum foreground value is selected as the safety condition level for the highway tunnel structure.

Table 3
Measured data of defects in some sections of the tunnel.

Sample serial number	Crack length (m)	Crack width (mm)	Leakage state	Lining strength ratio	Lining thickness ratio	Cavity depth (mm)	Deformation ratio	Flaking diameter (mm)	Spalling depth (mm)
1	2.12	0.3	0.625	1	0.895	28	0.02	52	6
2	1.54	1.3	0.625	1	0.840	45	0.03	43	5
3	1.64	1.7	0.875	0.98	0.851	321	0.15	121	8
4	1.56	2.2	0.625	0.95	0.786	350	0.26	145	12
5	2.30	2.4	0.625	0.89	0.697	289	0.48	255	14
6	1.07	0.8	0.625	1	0.854	94	0.03	103	10
7	0.95	2.1	0.875	1	0.784	44	0.02	87	8
8	0.83	1.3	0.625	1	0.911	21	0.05	80	6
9	0.53	0.6	0.875	1	0.934	53	0.02	67	7
10	0.34	0.3	0.875	0.95	0.786	57	0.03	58	8
11	0.50	0.8	0.625	1	0.887	78	0.05	61	6

Note: assign a value to the flowing water state, and the infiltration is 0.875. Drip is 0.625. Inrush current is 0.375. Injection is 0.125. No disease is 1. Lining strength ratio = actual lining strength/design strength, lining thickness ratio = actual lining thickness/design thickness, internal limit ratio = lining deformation/internal limit distance.

4. Case analysis

A highway tunnel in Guangxi is used as an example to illustrate the practicability of this method. Based on the on-site investigation, the rock in the tunnel's section is deeply weathered and broken, and joint fissures are developed. The primary issues, such as peeling off the lining surface, the cavity behind, and thickness deterioration, often occur in the on-site investigation section of the tunnel. Currently, the standards or specifications for structural safety assessment of highway tunnels in service have not been formed, and there are no other appropriate specifications for reference. The structural safety indicators of operating highway tunnels mainly include lining thickness, lining strength, structural deformation, water leakage, concrete peeling, etc. Furthermore, these indicators can be directly obtained through corresponding detection technology. Based on the actual situation's comprehensive analysis, nine indices that have a significant impact and are simple to control are selected herein as the evaluation parameters, including crack length (CL), crack width (CW), leakage state (WL), lining strength ratio (LS), lining thickness ratio (LT), cavity depth (CD), deformation ratio (DR), spalling diameter (FD), spalling depth (SD). More than 60 groups of structural defect data of highway tunnels were collected. Table 3 lists the measured data of defects in some tunnel sections.

Fig. 3 depicts the expected and average values of the collected experts for each index under various safety status levels. A total of 9 experts participated in this assessment, including tunnel designers (2 person), scientific researchers (4 person), construction directors (2 person) and technicians (1 person). Combined with the contents of Section 2.3, the reference distribution of each index under different safety status levels is determined, as indicated in Fig. 4(a–i).

The data results for various indicators are sorted out by highway tunnel monitoring, and the contents of Section 2.4 are utilized to construct the distribution of monitoring results of various indicators, as shown in Fig. 5.

Combined with the probability distribution of the safety status grade of highway tunnel structure and the probability distribution of the monitoring results of each index proposed in Section 2.2, the functional relationship between the monitoring index and the safety status is established by using Eq. (11) and π_{ij}^{s} is obtained. The income or loss is determined by comparing π_{ij}^{s} and π_{ij}^{r}. A profit and loss matrix D_{ij} is constructed using Eq. (12), and the results are listed in Table 4.

The value function is calculated by substituting the profit and loss matrix in Table 4 into Eq. (8), as follows:

$$v(D_{ij}) = \begin{bmatrix} 4.4278 & -12.6641 & 7.7493 & 4.1559 & 5.8824 & -2.8606 & -6.0911 & -7.5053 & 9.1956 \\ -12.4633 & 9.6678 & -5.3891 & 9.3655 & -12.3897 & 4.1559 & 8.1253 & 12.9919 & -5.4158 \\ 9.3773 & -20.1865 & 8.1246 & -17.7598 & 15.3725 & -29.2373 & 9.3701 & -29.1608 & -33.7769 \\ -20.3305 & -22.7306 & -15.2841 & -20.3322 & 16.6812 & 20.9472 & -19.6131 & 18.8547 & 20.9466 \\ 11.6075 & 13.6951 & 11.5714 & 13.6981 & 21.3401 & 20.8871 & 12.8594 & 20.8785 & 16.6832 \end{bmatrix}$$

Collect the scores of experts on the weights of various monitoring indicators, and combine the contents in Section 2.7 to obtain the weight matrix of different indicators for the same safety situation. Table 5 provides the results.

The value function matrix and weight matrix are substituted into Eq. (19) to obtain the comprehensive prospect value V_i of each safety condition level of the highway tunnel structure, which are $V_1 = = 0.3508$, $V_2 = 1.6931$, $V_3 =$ -18.0949, $V_4 =$ -3.3395, $V_5 =$ 28.7320, respectively. The safety status is ranked as $V_5 > V_2 > V_1 > V_4 > V_3$ based on the comprehensive foreground value. Accordingly, the principle that the minimum foreground value is the safety status of the highway tunnel structure, which is determined to be level 3. The tunnel's structure is relatively safe, so it is necessary to strengthen prevention and monitoring and formulate preventive or

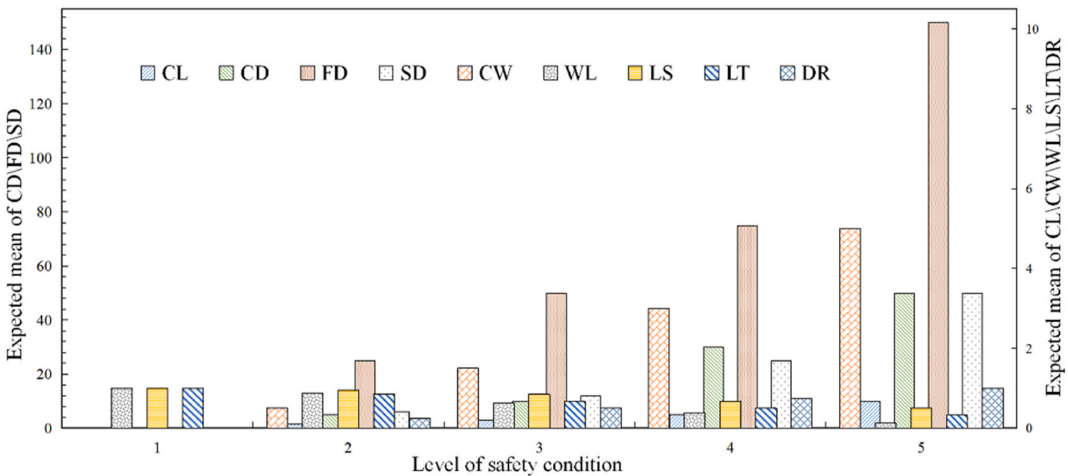

Fig. 3. Expected mean value of each index.

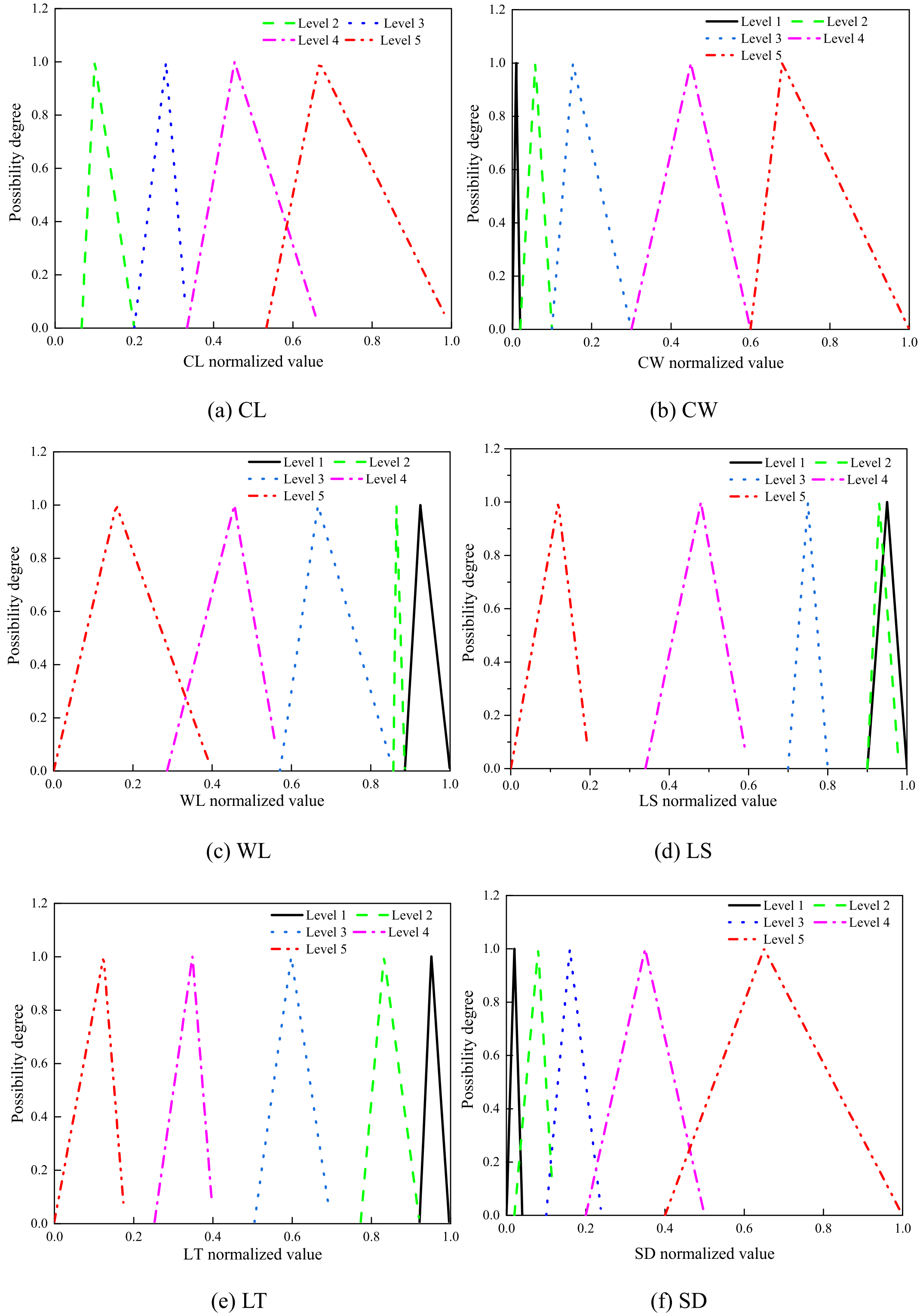

Fig. 4. Reference distribution of each index corresponding to each safety condition level: (a) CL. (b) CW. (c) WL. (d) LS. (e) LT. (f) SD. (g) FD. (h) DR. (i) CD.

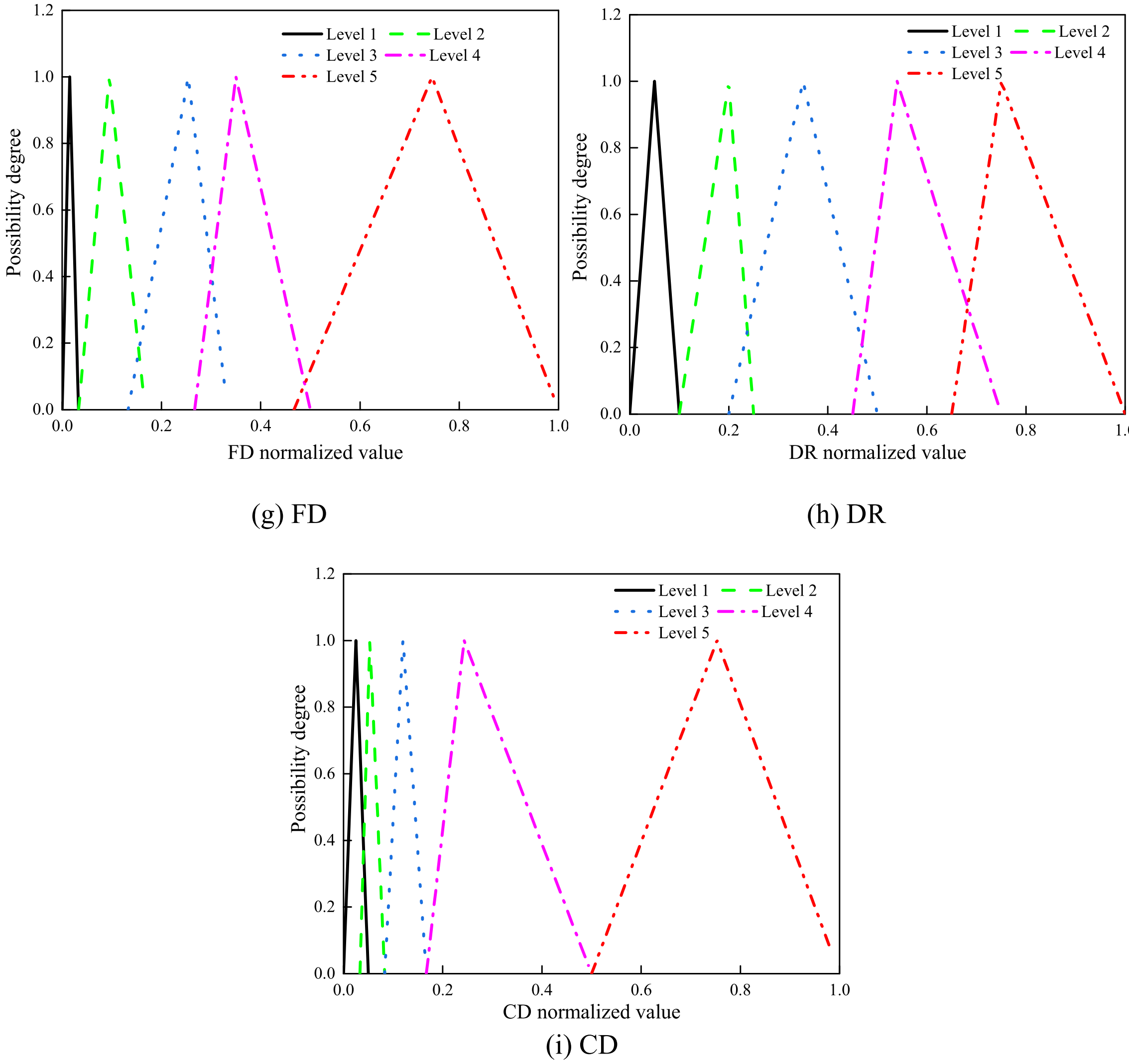

Fig. 4. (*continued*).

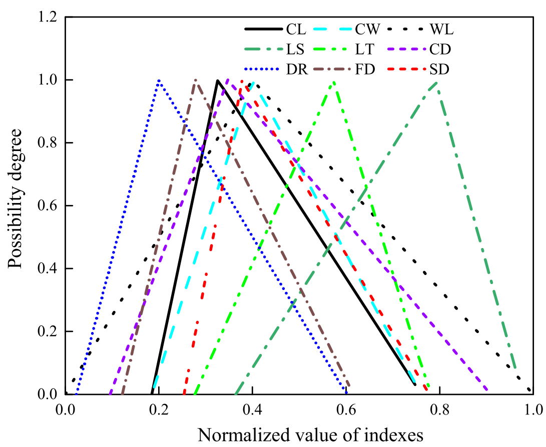

Fig. 5. The probability distribution of the monitoring results of each index.

Table 4
Profit and loss matrix.

Safety condition level	CL	CW	WL	LS	LT	CD	DR	FD	SD
1	3.4201	−5.4410	5.4316	3.2456	4.3251	−1.2654	−2.6548	−3.2578	6.2567
2	−5.3564	6.5211	−2.3545	6.3521	−5.3254	3.2456	5.6485	8.3251	−2.3659
3	6.3587	−8.5940	5.6481	−7.5799	9.5671	−12.3571	6.3547	−12.3254	−14.2354
4	−8.6541	−9.6546	−6.5425	−8.6548	10.2354	12.3549	−8.3546	11.3257	12.3546
5	7.5849	8.6958	7.5654	8.6974	12.5461	12.3256	8.2549	12.3214	10.2364

Table 5
Weight matrix.

Safety condition level	CL	CW	WL	LS	LT	CD	DR	FD	SD
1	0.2022	0.2062	0.2142	0.1894	0.1764	0.1987	0.1925	0.1966	0.1956
2	0.2021	0.1995	0.2018	0.1883	0.1906	0.1883	0.2123	0.1998	0.2095
3	0.2124	0.2172	0.1988	0.2049	0.2007	0.2034	0.2072	0.2062	0.1997
4	0.1922	0.1879	0.1927	0.2067	0.2121	0.2100	0.1948	0.1988	0.1968
5	0.1911	0.1892	0.1925	0.2107	0.2202	0.1996	0.1932	0.1986	0.1984

maintenance measures.

To verify the practicality and effectiveness of the proposed method. Based on the evaluation index system and weight data of this study, a mature fuzzy comprehensive evaluation method [21] has been adopted to evaluate the safety status of the tunnel structure, and the evaluation result is also at level 3, which is the same as the evaluation result in this article. As described earlier, the characteristic of this study is that the evaluation process well reflects the complex and ever-changing relationships between uncertain information.

5. Conclusion

(1) Due to the limited rationality of humans and the complex and variable mapping relationship between monitoring indicators and the structural safety of highway tunnels. Combing possibility theory and prospect theory, a new approach is proposed for evaluating the structural safety of operating highway tunnels. This method solves the comprehensive impact of multiple indicators on the evaluation results of highway tunnel structural safety. This method is applied to a case of operating highway tunnel, and the safety level of the highway tunnel is evaluated.

(2) The obtained information has fuzziness and uncertainty because of the differences in experts' understanding, the tunnel testing personnel's ability, and the equipment. Fuzzy information can be represented by the possibility distribution function of information, which can be constructed using the possibility theory. This change well reflects the fuzziness of the evaluation process and is beneficial to the accuracy of the evaluation results.

(3) Possibility set-valued mapping provides a new method for establishing the mapping relationship between sets and solves the problem that the deterministic function relationship cannot change as information varies when describing the complex and variable relationship between uncertain information.

(4) The research method of this article is difficult for ordinary engineers to understand. However, the evaluation process of this technique is programmable, which facilities future dynamic control of the safety status of operating highway tunnels.

Author contribution statement

Hailin Liu: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Data availability statement

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

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