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Research article

Multi-criteria decision support models under fuzzy credibility rough numbers and their application in green supply selection

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ABSTRACT

As the increasing environmental issues, various companies have take initiatives to produce green products or to select green suppliers which maximize the business performance and minimize the environmental pollution. The real numbers data have imbiguity and uncertainty due to described by classical tools. Therefore, we consider a new type of fuzzy set, fuzzy credibility rough sets. In fuzzy credibility rough set has credibility membership of positive membership and they reduced the imbiguity in data information. In this paper we have defined a new set called fuzzy credibility rough set (FCRS), after that we defined Frank operational laws for FCRS information. Using these operational laws, we defined a series of aggregation operators that is fuzzy credibility Frank rough weigthed averaging aggregation operators, fuzzy credibility Frank rough ordered weigthed averaging aggregation operators, fuzzy credibility Frank rough hybrid weigthed averaging aggregation operators and its basic properties like boundedness, monotonicity and idempotency.

As there is no work which is based on Frank norms aggregation operators under FCRS information. So, we defined a series of aggregation operators that can help us to collect the data for various green suppliers management.

We developed a new set called fuzzy credibility rough set (FCRS). We developed a new Frank norms operational laws under FCRS information. We developed a series of aggregation operators. We developed and extend various steps of GRA, VIKOR and TOPSIS method under FCRS information. We explained the application of our proposed work to a real life decision making problem (green supplier management).

All the proposed work is to applied to real life decision making problems (green supplier management) to find the best optimal result. Firstly we can collect the data from the decision makers using the proposed aggregation operators and then we applied all the steps of developed method to find the solution in case of green supplier management.

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1. Introduction

Multi criteria decision making is that type of decision making problems in which there is set of alternatives and criteria having a lack of information. So in this type of problems we can not decide that which alternative is best due to the lack of information. This type of problems is solved by Zadeh and define a new set called fuzzy set (FS) which is the generalization of crisp set and he define a FS which is a grading system and having one grade that grades must belong to the close interval zero and one. Afterthat Atanossov develop the concept of intuitionistic fuzzy set (IFS) and he define IFS as having two grades one is called membership grade (MG) and the other is called non-membership grade (NMG) with the condition that sum of all these two grades must be equal to close interval zero and one. Another concept called fuzzy credibility set (FCS) is develop by Ye which is also a grading system but here he define the degree of credibility (DoC) (accuracy/correctness) and MG having condition that MG and DoC belong to the close interval zero and one.

As there are some problems in which we have to define an equivalence relation, upper and lower approximation space which is not solved by FS, IFS and FCS. So, Palwak defined the concept of fuzzy rough set (FRS) having upper and lower approximation space and one grade system. Then, another defined the concept of intuitionistic fuzzy rough set (IFRS) which is basically two grades system and having upper and lower approximation space for these grades.

Now in the above studies we have conclude that there is no idea present about the fuzzy credibility set and fuzzy rough set. So we develop a new concept called fuzzy credibility rough set (FCRS) and after that we develop an operational laws of Frank norms under FCRS information. Using these develop operational laws a series of aggregation operators is defined under FCRS information and also a new score function is define to rank all the alternatives which is in the best position. Furthermore we have define the steps of some extended methods like TOPSIS, GRA and VIKOR under FCRS information. All these develop work can be used to find a best solution in any decision making problems (green suppliers management).

Related work The aggregation operators play an important rule in the collection of expert information and after that we applied various score function to rank all the alternatives or objects in any decision making problems. Furthermore some are discussed as follows which is base on the various aggregation operators on the basis of operational laws of various concepts. Gohain et al. [6] discussed the distance measure and its various types of distance measure under IFS information. Wang et al. [7] explained Jenson-Shannon divergence under interval-valued IFS and its application to real life decision making problems. Garg et al. [8] explained the distance measures for IFS and its uses to decision making (DM) problems. Senapati et al. [9] explained the series of aggregation operators (AoPs) under IFS information. Ohlan et al. [10] discussed the entropy measure, distance measures under interval-valued IFS information and its application to DM problems. Garge et al. [11] discussed an algorithm based on correlation coefficient under cubic IFS information and their application to DM problems. Rahman et al. [12] explained the series of AoPs and the distance measures under intuitionistic fuzzy hypersoft set. Kumar et al. [13] discussed the series of AoPs under linguistic IFS and their application to DM problems. Ye et al. [2] explained the series of AoPs under fuzzy credibility set information and their uses to real life DM problems. Ye et al. [14] discussed the AoPs under fuzzy cubic credibility set information and their application to DM problems. Talha et al. [15] discussed a series AoPs under fuzzy credibility set information and their application to DM problems. Qiyas et al. [16] discussed a series of AoPs under fuzzy credibility set and their application to DM problems. Yahya et al. [17] explained a series of AoPs under fuzzy credibility and their application to DM problems. Yahya et al. [18] discussed a series AoPs based on Frank norms under fuzzy credibility set information and their application to decision making problems. Wang et al. [19] explained the distance measure under fuzzy rough set and its application to decision making problems. Zhan et al. [20] discussed the properties of intuitionistic fuzzy rough sets (IFRS) and their uses to MCDM problems. Zhang et al. [21] explained the covering based on the IFRS information and their application to DM problems. Zhang et al. [22] discussed the covering-based methods and its various types of covering-based. Liu et al. [23] explained the distance measures and types of distance measure under IFRS information and their uses to real life problems. Senapati et al. [44] discussed a series of geometric aggregation operators and their application to real life problems. Hussain et al. [45] discussed a series of AoPs under Frank norms and their application to real life decision making problems. Mishra et al. [46] discussed a series of AoPs under IFS information. Mehmood et al. [47] discussed a series of AoPs under complex IFS information and their application to real life decision making problems. Seikh et al. [53] discussed a series of aggregation operators using the operational laws of Dombi norms under IFS information. Jana et al. [56] discussed steps of MABAC method and their application to real life decision making problems. Jana et al. [57] discussed IFS decision models and their application to real life decision making problems. Jana et al. [58] discussed a series of power aggregation operators under Dombi norms and steps of MABAC method and their application to real life decision making problems. Jana et al. [59] discussed steps of GRA method and their application to real life decision making problems. Seikh et al. [54] discussed a series of aggregation operators using the operational laws of Frank norms under picture fuzzy set setting. Seikh et al. [55] discussed a series of aggregation operators using the operational laws of Frank norms under Q-rung orthopair fuzzy set setting.

As we studied about aggregation operator that can help us in the aggregation process of expert information and also to help in find a best optimal results in the MCGDM problems. Now to solve the MCDM problems there are various methods to solve such type of MCDM problems and also to find a best optimal result. Bilgili et al. [24] discussed the various steps of TOPSIS method under IFS information. Deng et al. [25] explained the various steps of TOPSIS method and some types of distance measures under IFS information and their application to DM problems. Zoghi et al. [26] explained the steps of TOPSIS method and their uses to real life DM problems. Sadabadi et al. [27] explained the steps of TOPSIS method and their application to DM problems. Mahmood et al. [28] discussed the steps of TOPSIS method and the series of AoPs using bonferroni mean operator under cubic complex IFS information. Han et al. [29] explained the steps of TOPSIS method under linguistic Pythagorean hesitant fuzzy set information and their uses to real life problems. Huang et al. [30] discussed the steps of TOPSIS method and spherical rough AoPs and their application to DM problems. Chodha et al. [31] explained the steps TOPSIS method and their application to real life problems. Mahmood et al. [32] discussed the steps of TOPSIS method under T-bipolar soft set information. Khan et al. [33] discussed the steps of GRA method under linguistic pythagorean fuzzy set information and their application to DM problems. Azam et al. [34] explained the steps of GRA method under picture fuzzy set information and their application to DM problems. Chen et al. [35] discussed the steps of GRA and entropy-base TOPSIS method under IFS information and their uses to real life problems. Wu et al. [36] discussed the steps of GRA method and their application to MCDM problems. Wang et al. [37] discussed the step of GRA method and their application to MCDM problems. Delaram et al. [38] discussed the steps of VIKOR method under IFS information and their application to DM problem. Jabbir et al. [39] explained the steps of VIKOR method under pythagorean fuzzy set and their application to real life problems. Mishra et al. [40] discussed the steps of VIKOR method under Fermatean hesitant fuzzy set and their application to DM problems. Jishi et al. [41] explained the steps of VIKOR method under picture fuzzy set. Taghavifard et al. [42] discussed the steps of VIKOR method and their application to DM problems. Gupta et al. [43] discussed the steps of VIKOR method under linguistic IFS information and their application to DM problems. Erdebili et al. [48] discussed various steps of VIKOR and TOPSIS method under q-rung orthopair fuzzy set. Kirisci et al. [49] discussed various steps of TOPSIS method under Fermatean fuzzy set. Garg et al. [50] discussed various steps of GRA method and their application to real life decision making problems.

Motivation of proposed work As in literature review there are no work which is done in the field of fuzzy credibility set and rough set, as well as there is no work which is based on the Frank norms aggregation operators under fuzzy credibility rough set (FCRS) information. So here in this paper we defined a new set called fuzzy credibility rough set (FCRS) that is the generalization of two sets one is fuzzy rough set and the other is fuzzy credibility set. As fuzzy credibility set is defined as its a two degree system where one is called membership grade and other is called degree of credibility. The credibility degree show us the degree of correctness, degree of accuracy and talked more detailed about membership grades.

Firstly we define a new set called fuzzy credibility rough set (FCRS), then we define Frank norm operational laws under FCRS information. Using these proposed operational laws we define a series of aggregation operators like fuzzy credibility Frank rough weighted average aggregation operators, fuzzy credibility Frank rough ordered weighted average aggregation operators and fuzzy credibility Frank rough hybrid weighted average aggregation operators. Furthermore we define a new score function for ranking purposes in case of green supplier management. We can also define some steps of various methods like GRA, TOPSIS and VIKOR method under FCRS information.

Furthermore the main aim of our work is to find the best optimal results in green suppliers management. We applied our proposed aggregation operators to collect the data from green suppliers management. Afterthat we applied the developed steps of VIKOR, GRA and TOPSIS method to a green supplier management to find the best results.

Contribution of the proposed work There are some aggregation operators (AoPs) which is developed by many authors under various information. All these aggregation operators can help us in the collection of expert information for the multi criteria group decision making (MCGDM) problems and after that by using various developed score function we can find the best optimal result for any decision making problems. As in literature there are two types of aggregation operators one is called algebraic (simple) and the other is called norms based AoPs. But here in this paper we have defined a Frank norm based aggregation operators under FCRS information. Here some main objective and originality of our work is discussed as follows,

i) we have develop a new set called fuzzy credibility rough set (FCRS).
ii) we defined new operational laws of Frank norms under FCRS information.
iii) we defined a series of aggregation operators like fuzzy credibility Frank rough weighted average (FCRWA) AoPs, fuzzy credibility Frank rough ordered weighted average (FCROWA) AoPs and fuzzy credibility Frank rough hybrid weighted average (FCRHWA) AoPs.
iv) we defined a new score function for ranking purposes.
v) we defined some steps of various methods like GRA, TOPSIS, VIKOR under FCRS information.
vi) we discussed the application to a real life decision making (green supplier chain management) using our developed work.

Paper arrangement In the first section we defined introduction, in second section we defined basic concepts, in third section we defined fuzzy credibility rough set, in forth section we defined Frank operational laws for FCRS, in fifth section we defined fuzzy credibility Frank rough weighted average aggregation operators, in sixth section we defined fuzzy credibility Frank rough ordered weighted average aggregation operators, In sixth section we defined fuzzy credibility Frank rough hybrid weighted average aggregation operators, in seventh, eight, ninth, tenth, eleven and twelve section we defined decision support system, methodology-I, methodology-II, methodology-III and methodology-IV. In section 13, 14, 15, 16 and 17 we defined the real life example, computational results of methodology-I, II, III and IV. In section 18, 19, 20 and 21 we defined the sensitivity analysis, comparison, result and discussion and conclusion.

2. Basic concepts

Here some basic definitions are discussed like FS, IFS, FCS, FRS, IFRS.

Let $S \neq \phi$. Then, A FS E in S is define as [1],

$$E = \{\langle s, \nu(s)\rangle \,|\, s \in S\}$$

where $\nu(s)$ show the MG and where $\nu(s)$ belongs to [0,1].

Let $S \neq \phi$. Then, An IFS E in S is define as [3],

$$E = \{\langle s, \nu(s), \varkappa(s)\rangle \,|\, s \in S\}$$

where $\nu(s), \varkappa(s)$ show the MG and NMG and having condition that the sum of MG and NMG belongs to [0,1].

Let $S \neq \phi$. Then, a FCS S in E is defined as [2],

$$E = \{\langle x, \nu(s), \varkappa(s)\rangle \,|\, x \in X\}$$

where $\nu(s), \varkappa(s)$ show the MG and degree of credibility (DoC) respectively.

Let $S \neq \phi$. Then, a FRS is defined as [4],

$$\overline{E_{\nu(s)}} = \{s \in S \mid \nu(s) \neq \emptyset\}, \underline{E_{\nu(s)}} = \{s \in S \mid \nu(s) \subseteq \varphi\}$$

Therefore, $\left(\overline{E_{\nu(s)}}, \underline{E_{\nu(s)}}\right)$ is called FRS and $\overline{E_{\nu(s)}}$, $\underline{E_{\nu(s)}}$ is a lower and upper approximation space.

Let $S \neq \phi$. Then, an IFRS is defined as [5],

$$\overline{E_{\nu(s)}} = \left\{ s, \overline{\nu(s)}, \overline{\varkappa(s)} | s \in S \right\}$$

$$\underline{E_{\nu(s)}} = \left\{ s, \underline{\nu(s)}, \underline{\varkappa(s)} | s \in S \right\}$$

Where

$$\overline{\nu(s)} = \prod_{s \in S} [\nu(s) \vee \nu(s)],$$

$$\overline{\varkappa(s)} = \prod_{s \in S} [\varkappa(s) \wedge \varkappa(s)]$$

$$\underline{\nu(s)} = \prod_{s \in S} [\nu(s) \wedge \nu(s))],$$

$$\underline{\varkappa(s)} = \prod_{s \in S} [\varkappa(s) \vee \varkappa(s)]$$

Where $0 \leq \overline{\nu(s)} + \overline{\varkappa(s)} \leq 1$ and $0 \leq \underline{\nu(s)} + \underline{\varkappa(s)} \leq 1$ and $\overline{E_{\nu(s)}}$, $\underline{E_{\nu(s)}}$ is upper and lower approximation space. Then, the pair is called IFRS.

$$\left(\overline{E_{\nu(s)}}, \underline{E_{\nu(s)}}\right) = \left\{ s, \left(\overline{\nu(s)}, \overline{\varkappa(s)}\right), \left(\underline{\nu(s)}, \underline{\varkappa(s)}\right) \mid s \in S \right\}$$

3. Fuzzy credibility rough set

Consider $E \in (S \times S)$ is a FCS relation and defined on $S \neq \phi$. Then the pair (S, E) is called FCS approximation space. Consider for any subset $s \subseteq \overline{(S)}$. Then, the lower and upper approximations space of s w.r.t FCS approximation space (S, E) are two FCS, which is denoted by $\overline{E_{\nu(s)}}$ and $\underline{E_{\nu(s)}}$ and is defined as,

$$\overline{E_{\nu(s)}} = \left\{ s, \overline{\nu(s)}, \overline{\varkappa(s)} | s \in S \right\}$$

$$\underline{E_{\nu(s)}} = \left\{ s, \underline{\nu(s)}, \underline{\varkappa(s)} | s \in S \right\}$$

Where

$$\overline{\nu(s)} = \prod_{s \in S} \left[\nu_i(s) \vee \nu(s)\right],$$

$$\overline{\varkappa(s)} = \prod_{s \in S} \left[\varkappa_i(s) \wedge \varkappa(s)\right]$$

$$\underline{\nu(s)} = \prod_{s \in S} \left[\nu_i(s) \wedge \nu(s))\right],$$

$$\underline{\varkappa(s)} = \prod_{s \in S} \left[\varkappa_i(s) \vee \varkappa(s)\right]$$

Where $\overline{E_{\nu(s)}}$, $\underline{E_{\nu(s)}}$ is upper and lower approximation space. Then, the pair is called FCRS and that is denoted in (3.1).

$$\left(\overline{E_{\nu(s)}}, \underline{E_{\nu(s)}}\right) = \left\{ s, \left(\overline{\nu(s)}, \overline{\varkappa(s)}\right), \left(\underline{\nu(s)}, \underline{\varkappa(s)}\right) \mid s \in S \right\} \tag{3.1}$$

Score Function (s_f)

Let $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$ is a collection of fuzzy credibility rough numbers (FCRNs) then the score function is define as and further that can be denoted is (3.2),

$$s_f = \frac{1}{4}\left\{2 + \overline{\nu(s)} + \underline{\nu(s)} - \overline{\varkappa(s)} - \underline{\varkappa(s)}\right\} \tag{3.2}$$

4. Frank operational laws for fuzzy credibility rough set

Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$. Then, the basic Frank operational laws are define as follows,

$$(1)\quad AB_1 \oplus AB_2 = \left\{\begin{matrix} 1-\log_t\left(1+\frac{(\lambda^{1-\overline{\nu(s)}}-1)}{\lambda-1}\right) \\ 1-\log_t\left(1+\frac{(\lambda^{1-\underline{\nu(s)}}-1)}{\lambda-1}\right), \\ \log_t\left(1+\frac{(\lambda^{\overline{\varkappa(s)}}-1)}{\lambda-1}\right) \\ \log_t\left(1+\frac{(\lambda^{\underline{\varkappa(s)}}-1)}{\lambda-1}\right). \end{matrix}\right\}, (t>1).$$

$$(2)\quad AB_1 \otimes AB_2 = \left\{\begin{matrix} \log_t\left(1+\frac{(\lambda^{\overline{\nu(s)}}-1)}{\lambda-1}\right) \\ \log_t\left(1+\frac{(\lambda^{\underline{\nu(s)}}-1)}{\lambda-1}\right), \\ 1-\log_t\left(1+\frac{(\lambda^{1-\varkappa(s)}-1)}{\lambda-1}\right) \\ 1-\log_t\left(1+\frac{(\lambda^{1-\underline{\varkappa(s)}}-1)}{\lambda-1}\right). \end{matrix}\right\}, (t>1).$$

$$(3)\quad t(AB_1) = \left\{\begin{matrix} 1-\log_t\left(1+\frac{(\lambda^{1-\overline{\nu(s)}}-1)}{\lambda-1}\right) \\ 1-\log_t\left(1+\frac{(\lambda^{1-\underline{\nu(s)}}-1)}{\lambda-1}\right), \\ \log_t\left(1+\frac{(\lambda^{\overline{\varkappa(s)}}-1)}{\lambda-1}\right) \\ \log_t\left(1+\frac{(\lambda^{\underline{\varkappa(s)}}-1)}{\lambda-1}\right). \end{matrix}\right\}, (t>1).$$

$$(4)\quad (AB_1)^t = \left\{\begin{matrix} \log_t\left(1+\frac{(\lambda^{\overline{\nu(s)}}-1)}{\lambda-1}\right) \\ \log_t\left(1+\frac{(\lambda^{\underline{\nu(s)}}-1)}{\lambda-1}\right). \\ 1-\log_t\left(1+\frac{(\lambda^{1-\overline{\varkappa(s)}}-1)}{\lambda-1}\right) \\ 1-\log_t\left(1+\frac{(\lambda^{1-\underline{\varkappa(s)}}-1)}{\lambda-1}\right). \end{matrix}\right\}, (t>1).$$

5. Fuzzy credibility Frank rough weighted average (FCFRWA) AoPs

Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$. Then a mapping is define as $\Gamma^n \rightarrow \Gamma$. So the FCFRWA AoPs is define as follows,

$$FCFRWA(AB_1, AB_2, ..., AB_n) = \oplus_{i=1}^{n} AB_i w_k$$

$w_k = \left(w_1, w_2, ..., w_n\right)^T$, is a weight vectors with condition that $\Sigma_{i=1}^{n} w_k = 1$ and $w_k \in [0, 1]$.

Theorem 1. *Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$. Then, the aggregated value of the FCFRWA AoPs is also a FCRNs and using the proposed operational laws of Frank norms we can write FCFRWA AoPs as,*

$$FCFRWA(AB_1, AB_2, ..., AB_n)$$

$$= \oplus_{i=1}^{n} AB_i w_k$$

$$=\left\{\begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\overline{\nu(s)}}-1)^{w_k}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\underline{\nu(s)}}-1)^{w_k}\right), \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\overline{\varkappa(s)}}-1)^{w_k}\right) \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\underline{\varkappa(s)}}-1)^{w_k}\right) \end{array}\right\}$$

Proof. Using mathematical induction,

$$AB_1$$

$$=\left\{\begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\overline{\nu(s_1)}}-1)^{w_1}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\underline{\nu(s_1)}}-1)^{w_1}\right), \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\overline{\varkappa(s_1)}}-1)^{w_1}\right) \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\underline{\varkappa(s_1)}}-1)^{w_1}\right) \end{array}\right\}$$

$$AB_2$$

$$=\left\{\begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\overline{\nu(s_2)}}-1)^{w_2}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\underline{\nu(s_2)}}-1)^{w_2}\right), \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\overline{\varkappa(s_2)}}-1)^{w_2}\right) \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\underline{\varkappa(s_2)}}-1)^{w_2}\right) \end{array}\right\}$$

$$= AB_1 + AB_2$$

$$=\left\{\begin{array}{c} 1-\log_t\left\{\left(1+(\lambda^{1-\overline{\nu(s_1)}}-1)^{w_1}\right)+\left(1+(\lambda^{1-\overline{\nu(s_2)}}-1)^{w_2}\right)\right\} \\ 1-\log_t\left\{\left(1+\prod_{i=1}^{n}(\lambda^{1-\underline{\nu(s_1)}}-1)^{w_1}\right)+\left(1+\prod_{i=1}^{n}(\lambda^{1-\underline{\nu(s_2)}}-1)^{w_2}\right)\right\} \\ \log_t\left\{\left(1+\prod_{i=1}^{n}(\lambda^{\overline{\varkappa(s_1)}}-1)^{w_1}\right)+\left(1+\prod_{i=1}^{n}(\lambda^{\overline{\varkappa(s_2)}}-1)^{w_2}\right)\right\} \\ \log_t\left\{\left(1+\prod_{i=1}^{n}(\lambda^{\underline{\varkappa(s_1)}}-1)^{w_1}\right)+\left(1+\prod_{i=1}^{n}(\lambda^{\underline{\varkappa(s_2)}}-1)^{w_2}\right)\right\} \end{array}\right\}$$

$$=\left\{\begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{2}(\lambda^{1-\overline{\nu(s)}}-1)^{w_k}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{2}(\lambda^{1-\underline{\nu(s)}}-1)^{w_k}\right), \\ \log_t\left(1+\prod_{i=1}^{2}(\lambda^{\overline{\varkappa(s)}}-1)^{w_k}\right) \\ \log_t\left(1+\prod_{i=1}^{2}(\lambda^{\underline{\varkappa(s)}}-1)^{w_k}\right) \end{array}\right\}$$

it show that true for n=2. Now for n = k we have to check,

$$FCFRWA(AB_1, AB_2, ..., AB_k)$$

$$=\left\{\begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{k}(\lambda^{1-\overline{\nu(s)}}-1)^{w_k}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{k}(\lambda^{1-\underline{\nu(s)}}-1)^{w_k}\right), \\ \log_t\left(1+\prod_{i=1}^{k}(\lambda^{\overline{\varkappa(s)}}-1)^{w_k}\right) \\ \log_t\left(1+\prod_{i=1}^{k}(\lambda^{\underline{\varkappa(s)}}-1)^{w_k}\right) \end{array}\right\}$$

Further, we check for $n = k + 1$, we have,

$$FCFRWA(AB_1, AB_2, ..., AB_{k+1})$$

$$= \left\{ \begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{k+1}(\lambda^{1-\overline{\nu(s)}}-1)^{w_k}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{k+1}(\lambda^{1-\underline{\nu(s)}}-1)^{w_k}\right), \\ \log_t\left(1+\prod_{i=1}^{k+1}(\lambda^{\overline{\varkappa(s)}}-1)^{w_k}\right) \\ \log_t\left(1+\prod_{i=1}^{k+1}(\lambda^{\underline{\varkappa(s)}}-1)^{w_k}\right) \end{array} \right\}$$

The result is true for $n = k + 1$ and hence true for $n \geq 1$. □

The idempotency, boundedness and monotonicity properties for the developed FCFRWA AoPs is discussed as follows.

Idempotency

Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$ having weight vector $w_k = (w_1, w_2, ..., w_n)^T$. Then we can write idempotency property as follows and that is denoted in (5.1),

$$FCFRWA(AB_1, AB_2, ..., AB_n) = AB \tag{5.1}$$

Boundedness

Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$ having weight vector $w_k = (w_1, w_2, ..., w_n)^T$. Let $AB_{\min} = \left(\min_i \overline{\nu(s)}, \min_i \underline{\nu(s)}, \max_i \overline{\varkappa(s)}, \max_i \underline{\varkappa(s)}\right)$ and $AB_{\max} = \left(\max_i \overline{\nu(s)}, \max_i \underline{\nu(s)}, \min_i \overline{\varkappa(s)}, \min_i \underline{\varkappa(s)}\right)$ are the maximum and minimum collection of FCRNs. Then, we can write the boundedness property as follows and that is denoted in (5.2),

$$AB_{\min} \leq FCFRWA(AB_1, AB_2, ..., AB_n) \leq AB_{\max} \tag{5.2}$$

Monotonicity

Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$ having weight vector $w_k = (w_1, w_2, ..., w_n)^T$. Let $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$, $AB_i^* = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$ be the collection of FCRNs such that $AB_i \leq AB_i^*$. Then, we can write the monotonicity property as follows and that is denoted in (5.3),

$$\begin{aligned} & FCFRWA(AB_1, AB_2, ..., AB_n) \\ & \leq FCFRWA(AB_1^*, AB_2^*, ..., AB_n^*) \end{aligned} \tag{5.3}$$

Proof. Idempotency

Let we have a collection of FCRNs $AB_i = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)})$. Then,

$$\begin{aligned} & FCFRWA(AB_1, AB_2, ..., AB_n) \\ & = \oplus_{i=1}^{n} AB_i w_k \\ & = \left\{ \begin{array}{c} 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\overline{\nu(s)}}-1)^{w_k}\right) \\ 1-\log_t\left(1+\prod_{i=1}^{n}(\lambda^{1-\underline{\nu(s)}}-1)^{w_k}\right), \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\overline{\varkappa(s)}}-1)^{w_k}\right) \\ \log_t\left(1+\prod_{i=1}^{n}(\lambda^{\underline{\varkappa(s)}}-1)^{w_k}\right) \end{array} \right\} \\ & = \left\{ \begin{array}{c} \left(1-(1-\overline{\nu(s)})\right), \left(1-(1-\underline{\nu(s)})\right), \\ \left(\left(1-(1-\overline{\varkappa(s)}\right)\right), \left(\left(1-(1-\underline{\varkappa(s)}\right)\right) \end{array} \right\} \\ & = \left\{ \begin{array}{c} 1-(1-\overline{\nu(s)}), 1-(1-\underline{\nu(s)}), \\ 1-(1-\overline{\varkappa(s)}), 1-(1-\underline{\varkappa(s)}) \end{array} \right\} \\ & = (\overline{\nu(s)}, \underline{\nu(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)}) = AB \end{aligned}$$

Hence prove.

$$FCFRWA(AB_1, AB_2, ..., AB_n) = AB$$

Boundedness

Let $AB_{\min}$, $AB_{\max}$ are the FCRNs maximum and minimum numbers, as $AB_{\min} \leq AB_i \leq AB_{\max}$. Then,

$$\oplus_{i=1}^{n} AB_{\min} w_k \leq \oplus_{i=1}^{n} AB_i w_k \leq \oplus_{i=1}^{n} AB_{\max} w_k$$

$$AB_{\min} \le \oplus_{i=1}^{n} AB_i w_k \le AB_{\max}$$

which implies that,

$$AB_{\min} \le FCFRWA(AB_1, AB_2, ..., AB_n) \le AB_{\max}$$

Monotonicity

Let we have $AB_i \le AB_i^*, \oplus_{i=1}^{n} AB_i w_k \le \oplus_{i=1}^{n} AB_i^* w_k$ is exists. Then,

$$FCFRWA(AB_1, AB_2, ..., AB_n) \le FCFRWA(AB_1^*, AB_2^*, ..., AB_n^*) \quad \square$$

6. Fuzzy credibility frank rough ordered weighted average (FCFROWA) AoPs

Let we have a collection of FCRNs $AB_{\sigma i} = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)})$. Then a mapping is define as $\Gamma^n \to \Gamma$. So the FCFROWA AoPs is define as follows,

$$FCFROWA(AB_{\sigma 1}, AB_{\sigma 2}, ..., AB_{\sigma n})$$
$$= \oplus_{i=1}^{n} AB_{i_\sigma} w_k$$

where the weight vector is denoted by $w_k = (w_1, w_2, ..., w_n)^T$, with $\Sigma_{i=1}^{n} w_k = 1$ and $w_k \in [0, 1]$.

Theorem 2. *Let we have a collection of FCRNs $AB_{\sigma i} = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)})$. Then, the aggregated values of the FCFROWA AoPs is also a FCRNs and using the proposed Frank operational laws we can written as,*

$$FCFROWA(AB_{\sigma 1}, AB_{\sigma 2}, ..., AB_{\sigma n})$$
$$= \oplus_{i=1}^{n} AB_{i_\sigma} w_k$$
$$= \left\{ \begin{array}{c} 1 - \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{1-\overline{\nu(s\sigma)}} - 1)^{w_k}\right) \\ 1 - \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{1-\underline{\nu(s\sigma)}} - 1)^{w_k}\right), \\ \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{\overline{\varkappa(s\sigma)}} - 1)^{w_k}\right) \\ \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{\underline{\varkappa(s\sigma)}} - 1)^{w_k}\right) \end{array} \right\}$$

Proof. Obvious. $\square$

The idempotency, boundedness and monotonicity properties for the developed FCFROWA AoPs is discussed as follows.

Idempotency

Let we have a collection of FCRNs $AB_{\sigma i} = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)})$ having weight vector $w_k = (w_1, w_2, ..., w_n)^T$. Then we can write the idempotency property as follows and that is denoted in (6.1),

$$FCFROWA(AB_{\sigma 1}, AB_{\sigma 2}, ..., AB_{\sigma n}) = AB \tag{6.1}$$

Boundedness

Let we have a collection of FCRNs $AB_{\sigma i} = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)})$ having weight vector $w_k = (w_1, w_2, ..., w_n)^T$. Let $AB_{\min} = \left(\min_i \overline{\nu(s\sigma i)}, \min_i \underline{\nu(s\sigma i)}, \max_i \overline{\varkappa(s\sigma i)}, \max_i \underline{\varkappa(s\sigma i)}\right)$ and $AB_{\max} = \left(\max_i \overline{\nu(s\sigma i)}, \max_i \underline{\nu(s\sigma i)}, \min_i \overline{\varkappa(s\sigma i)}, \min_i \underline{\varkappa(s\sigma i)}\right)$ are the maximum and minimum FCRNs. Then, we can write the boundedness property as follows and that is denoted in (6.2),

$$AB_{\min} \le FCFROWA(AB_{\sigma 1}, AB_{\sigma 2}, ..., AB_{\sigma n}) \le AB_{\max} \tag{6.2}$$

Monotonicity

Let we have a collection of FCRNs $AB_{\sigma i} = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)})$ having weight vector $w_k = (w_1, w_2, ..., w_n)^T$. Let $AB_{\sigma i} = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)}), AB_{\sigma i}^* = (\overline{\nu(s\sigma i)}, \underline{\nu(s\sigma i)}, \overline{\varkappa(s\sigma i)}, \underline{\varkappa(s\sigma i)})$ be the collection of FCRNs such that $AB_i \le AB_i^*$. Then, we can write the monotonicity property as follows and that is denoted in (6.3),

$$FCFROWA(AB_{\sigma 1}, AB_{\sigma 2}, ..., AB_{\sigma n}) \tag{6.3}$$
$$\le FCFROWA(AB_{\sigma 1}^*, AB_{\sigma 2}^*, ..., AB_{\sigma n}^*)$$

Proof. Obvious. $\square$

7. Fuzzy credibility Frank rough hybrid weighted average (FCFHWA) AoPs

Let we have a collection of FCRNs $AB_i^\circ = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$. Then a mapping is define as $\Gamma^n \to \Gamma$. So the FCFRHWA AoPs is define as follows,

$$FCFRHWA(AB_1, AB_2, ..., AB_n)$$
$$= \oplus_{i=1}^n AB_i^\circ w_k$$

where weight vector is denoted by $w_k = \left(w_1, w_2, ..., w_n\right)^T$, with $\Sigma_{i=1}^n w_k = 1$ and $w_k \in [0, 1]$.

Theorem 3. *Let we have a collection of FCRNs $AB_i^\circ = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$. Then, the aggregated values of the FCFRHWA AoPs is also a FCRNs and using the proposed Frank operational laws we can written as,*

$$FCFHWA(AB_1, AB_2, ..., AB_n)$$
$$= \oplus_{i=1}^n l_i^\circ w_k$$
$$= \left\{ \begin{array}{c} 1 - \log_t \left(1 + \prod_{i=1}^n (\lambda^{1-\overline{\nu(s^\circ)}} - 1)^{w_k}\right) \\ 1 - \log_t \left(1 + \prod_{i=1}^n (\lambda^{1-\underline{\nu(s^\circ)}} - 1)^{w_k}\right), \\ \log_t \left(1 + \prod_{i=1}^n (\lambda^{\overline{\varkappa(s^\circ)}} - 1)^{w_k}\right) \\ \log_t \left(1 + \prod_{i=1}^n (\lambda^{\underline{\varkappa(s^\circ)}} - 1)^{w_k}\right) \end{array} \right\}$$

Proof. Obvious. □

The idempotency, boundedness and monotonicity properties for the developed FCFRHWA AoPs is discussed as follows.

Idempotency

Let we have a collection of FCRNs $AB_i^\circ = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$ having weight vector $w_k = \left(w_1, w_2, ..., w_n\right)^T$. Then we can write the idempotency property as follows and that is denoted in (7.1),

$$FCFRHWA(AB_1^\bullet, AB_2^\bullet, ..., AB_n^\bullet) = AB \tag{7.1}$$

Boundedness

Let we have a collection of FCRNs $AB_i^\circ = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$ having weight vector $w_k = \left(w_1, w_2, ..., w_n\right)^T$. Let a set $AB_{\min} = \left(\min_i \overline{\nu(si^\circ)}, \min_i \underline{\nu(si^\circ)}, \max_i \overline{\varkappa(si^\circ)}, \max_i \underline{\varkappa(si^\circ)}\right)$ and $AB_{\max} = \left(\max_i \overline{\nu(si^\circ)}, \max_i \underline{\nu(si^\circ)}, \min_i \overline{\varkappa(si^\circ)}, \min_i \underline{\varkappa(si^\circ)}\right)$ are the maximum and minimum FCRNs. Then, we can write the Boundedness property as follows and that is denoted in (7.2),

$$AB_{\min} \leq FCFRHWA(AB_1^\bullet, AB_2^\bullet, ..., AB_n^\bullet) \leq AB_{\max} \tag{7.2}$$

Monotonicity

Let we have a collection of FCRNs $AB_i^\circ = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$ having weight vector $w_k = \left(w_1, w_2, ..., w_n\right)^T$. Let $AB_i^\circ = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$, $AB_i^{\circ *} = (\overline{\nu(si^\circ)}, \underline{\nu(si^\circ)}, \overline{\varkappa(si^\circ)}, \underline{\varkappa(si^\circ)})$ be the collection of FCRNs such that $AB_i^\circ \leq AB_i^{\circ *}$. Then, we can write the monotonicity property as follows and that is denoted in (7.3),

$$FCFRHWA(AB_1^\circ, AB_2^\circ, ..., AB_n^\circ) \tag{7.3}$$
$$\leq FCFRHWA(AB_1^{\circ *}, AB_2^{\circ *}, ..., AB_n^{\circ *})$$

Proof. Obvious. □

8. Decision support system methodology under FCRS information

Here in this section we have discuss the detail about the set of alternatives, criteria and weight vectors which is denoted by $y_i = \{y_1, y_2, y_3, ..., y_n\}$, $x_i = \{x_1, x_2, x_3, ..., x_n\}$ and $w_k = (w_1, w_2, w_3, ..., w_n)^T$. But here in this paper we have taken five alternatives, five criteria and two types of weight vector which is basically used for two purposes, one is to collect the expert information and the other is for criteria wise aggregation process. Also we have used our proposed algorithms for the best optimal solution in any real-life decision-making problems.

Fig. 1. Show the proposed algorithm-I.

9. Methodology-I based on the develop AoPs

In the methodology-I we can explained some steps which is apply for further selection of best alternatives for any MCGDM problems. The description about the algorithm-I are as follows,

Step-1: Represents the Expert Information.

$$\text{Expert Information}\quad \begin{array}{c} Alternatives \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{array} \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_m \\ AB_{11} & AB_{12} & AB_{13} & \cdots & AB_{1m} \\ AB_{21} & AB_{22} & AB_{23} & \cdots & AB_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ AB_{n1} & AB_{n2} & AB_{n3} & \cdots & AB_{nm} \end{bmatrix}$$

Step-2: Represents the aggregated results of Expert Information.

$$FCFRWA(AB_1, AB_2, ..., AB_n)$$
$$= \oplus_{i=1}^{n} AB_i w_k$$
$$= \left\{ \begin{array}{c} 1 - \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{1-\overline{\nu(s)}} - 1)^{w_k} \right) \\ 1 - \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{1-\underline{\nu(s)}} - 1)^{w_k} \right), \\ \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{\overline{\varkappa(s)}} - 1)^{w_k} \right) \\ \log_t \left(1 + \prod_{i=1}^{n} (\lambda^{\underline{\varkappa(s)}} - 1)^{w_k} \right) \end{array} \right\}$$

Step-3: Evaluate the score functions and rank all the alternatives.

$$s_f = \frac{1}{4} \left\{ 2 + \overline{\nu(s)} + \underline{\nu(s)} - \overline{\varkappa(s)} - \underline{\varkappa(s)} \right\}$$

Flow Chart of Algorithm-I

The flow chart of algorithm-I is discussed as in the Fig. 1. In the Fig. 1 all the steps of proposed method like algorithm based on developed AoPs method is discussed in detailed.

10. Extended GRA methodology-II

The detail description of this methodology-II were explained as follows using the FCRS information.

Step-1: same as methodology-I.
Step-2: same as methodology-I.

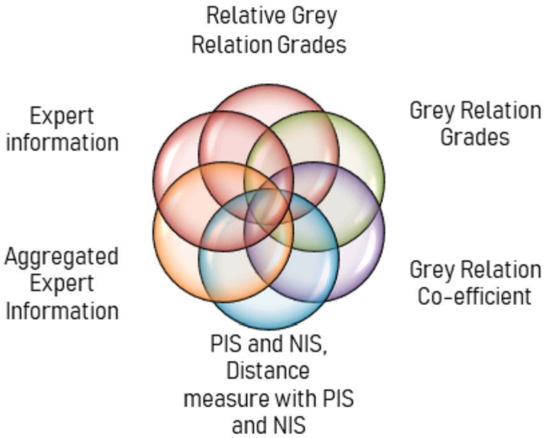

Fig. 2. Show the proposed algorithm-II.

Step-3: Find the positive ideal solution (PIS) and negative ideal solution (NIS). Find the distance measures of the aggregated values with positive ideal solution (PIS) and negative ideal solution (NIS) respectively and that is denoted in (10.1), (10.2), (10.3) and (10.4).

$$PIS(AB_1, AB_2, AB_3, ..., AB_n) = \max_i(\overline{v(s)}, \underline{v(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)}) \tag{10.1}$$

$$NIS(AB_1, AB_2, AB_3, ..., AB_n) = \min_i(\overline{v(s)}, \underline{v(s)}, \overline{\varkappa(s)}, \underline{\varkappa(s)}) \tag{10.2}$$

$$d\left((ab_{ij}, ab_{ij}), ab_{ij}^{+}\right) = \frac{1}{n}\sum_{i=1}^{n}\left(|ab_{ij} - ab_{ij}^{+}| + |ab_{ij} - ab_{ij}^{+}|\right). \tag{10.3}$$

$$d\left((ab_{ij}, ab_{ij}), ab_{ij}^{-}\right) = \frac{1}{n}\sum_{i=1}^{n}\left(|ab_{ij} - ab_{ij}^{-}| + |ab_{ij} - ab_{ij}^{-}|\right). \tag{10.4}$$

Step-4: Find the Grey relational coefficient of alternatives for PIS and NIS respectively, where t is scalar parameter and that is denoted in (10.5) and (10.6).

$$r_{ij}^{+} = r(ab_j^{+}, ab_{ij}) = \frac{\min_i \min_j d(ab_j^{+}, ab_{ij}) + t \max_i \max_j d(ab_j^{+}, Aab_{ij})}{d(ab_j^{+}, ab_{ij}) + t \max_i \max_j d(ab_j^{+}, ab_{ij})} \tag{10.5}$$

$$r_{ij}^{-} = r(ab_j^{-}, ab_{ij}) = \frac{\min_i \min_j d(ab_j^{-}, ab_{ij}) + t \max_i \max_j d(ab_j^{-}, ab_{ij})}{d(ab_j^{-}, ab_{ij}) + t \max_i \max_j d(ab_j^{-}, ab_{ij})} \tag{10.6}$$

Step-5: Find the Grey relational grades of alternatives and that is denoted in (10.7) and (10.8).

$$\upsilon_i^{+} = \sum_{j=1}^{n} r_{ij}^{+} \times w_k \tag{10.7}$$

$$\upsilon_i^{-} = \sum_{j=1}^{n} r_{ij}^{-} \times w_k \tag{10.8}$$

Step-6: Find the relative Grey relational grades and that is denoted in (10.9).

$$\alpha_i = \frac{\upsilon_i^{+}}{\upsilon_i^{+} + \upsilon_i^{-}} \tag{10.9}$$

Step-7: Rank all the alternatives on the basis of α_i and that is denoted in (10.9).

Flow Chart of Algorithm-II

The flow chart of algorithm-II is discussed as in the Fig. 2. In the Fig. 2 all the steps of proposed method like extended GRA method is discussed in detailed.

Fig. 3. Show the proposed algorithm-III.

11. Extended TOPSIS methodology-III

The detail description about methodology-III were explained as follows using the FCRS information.

Step-1: Same as methodology-I.
Step-2: Same as methodology-I.
Step-3: Same as methodology-II.
Step-4: Find the weighted values of alternatives and that is denoted in (11.1) and (11.2).

$$v_i^+ = \sum_{j=1}^{n} d_{ij}^+ \times w_k \tag{11.1}$$

$$v_i^- = \sum_{j=1}^{n} d_{ij}^- \times w_k \tag{11.2}$$

Step-6: Find the closeness indices that can help us in the ranking process of any decision making problems.

$$\alpha_i = \frac{v_i^+}{v_i^+ + v_i^-}$$

Step-7: Rank all the alternatives on the basis of α_i that is the last step of methodology-III.

Flow Chart of Algorithm-III

The flow chart of algorithm-III is discussed as in the Fig. 3. In the Fig. 3 all the steps of proposed method like extended TOPSIS method is discussed in detailed.

12. Extended VIKOR methodology-IV

There are some steps which is discussed in this section and will be used for the finding a best optimal solution for any decision making problems under FCRS information.

Step-1: Same as methodology-I.
Step-2: Same as methodology-I.
Step-3: Same as methodology-II.
Step-4: Find the distance measures between the aggregated matrix values with PIS and NIS respectively and the distance measures with PIS and NIS and that is denoted in (12.1) respectively.

$$d\left((ab_{ij}, ab_{ij}), ab_{ij}^+\right) = \frac{1}{n}\sum_{i=1}^{n}\left(|ab_{ij} - ab_{ij}^+| + |ab_{ij} - ab_{ij}^+|\right).$$

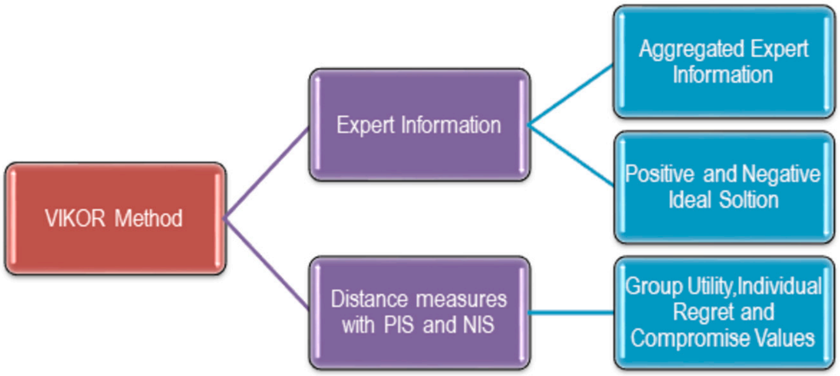

Fig. 4. Show the proposed algorithm-IV.

$$d\left((ab_{ij}, ab_{ij}), ab_{ij}^{-}\right) = \frac{1}{n}\sum_{i=1}^{n}\left(|ab_{ij} - ab_{ij}^{-}| + |ab_{ij} - ab_{ij}^{-}|\right).$$

$$d\left((ab_{ij}^{+}, ab_{ij}^{-}\right) = \frac{1}{n}\sum_{i=1}^{n}\left(|ab_{ij}^{+} - ab_{ij}^{-}| + |ab_{ij}^{+} - ab_{ij}^{-}|\right). \tag{12.1}$$

Step-5: Find the group utility values A_i, individual regret values B_i and compromise values D_i and that is denoted in (12.2), (12.3) and (12.4).

$$A_i = \sum_{i=1}^{n} w_j \left(\frac{d(a_{ij}^{+}, a_{ij})}{d(a_{ij}^{+}, a_{ij}^{-})}\right) \tag{12.2}$$

$$B_i = \max_j w_j \left(\frac{d(a_{ij}^{+}, a_{ij})}{d(a_{ij}^{+}, a_{ij}^{-})}\right) \tag{12.3}$$

$$D_i = \lambda\left(\frac{A_i - A_i^{*}}{A_i^{-} - A_i^{*}}\right) + (1-\lambda)\left(\frac{B_i - B_i^{*}}{B_i^{-} - B_i^{*}}\right) \tag{12.4}$$

where $A_i^{-} = \max_i A_i$, $A_i^{*} = \min_i A_i$, $B_i^{-} = \max_i B_i$, $B_i^{*} = \min_i B_i$.

Step-6: In this step we have rank all the alternatives on the basis of compromise values D_i.

Flow Chart of Algorithm-IV

The flow chart of algorithm-IV is discussed as in the Fig. 4. In the Fig. 4 all the steps of proposed method like extended VIKOR method is discussed in detailed.

13. A real life decision making problem

Recently several governmental and non-governmental companies have focused their attention in the promotion of eco-friendly resources. As the increasing environmental issues, various companies have take initiatives to produce green products or to select green suppliers which maximize the business performance and minimize the environmental pollution, emission and energy consumption. In the process of selecting the best green supplier, the decision makers have many alternative suppliers affected by several criteria. In this section, a case study of green supplier selection problem is evaluated by proposed various method.

In this problem, a manufacturer company wants to select the best supplier from a set of five alternatives $\{y_1, y_2, y_3, y_4, y_5\}$. After that a senior person of this company has made a set of three experts to handle the best selection problem of green supplier. As the alternatives are evaluated on the basis of five criteria. The detailed description is as follows.

x_1 represents the management system,

x_2 represents the commitment of manager to green supplier chain management (GSCM),

x_3 represents the use of green technology,

x_4 represents the use of green materials,

x_5 represents the quality management.

Now according to above information we have to find the best optimal solution in the green supplier chain management. So here we have taken three expert information under FCRS information having five alternatives and criteria, now to aggregate the expert information we have used the proposed aggregation operators like FCFRWA, FCFROWA and FCFRHWA using the weight vectors like $w = (0.21, 0.24, 0.22, 0.15, 0.18)$ and $w = (0.2, 0.3, 0.5)$. Furthermore we have also used all the steps of our proposed algorithms

Fig. 5. Representation of Green Supplier Chain Management (GSCM).

Table 1
Expert information.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	$\left\{\begin{matrix}(.8,.1),\\(.2,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.5,.4),\\(.7,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.3,.5),\\(.7,.5)\end{matrix}\right\}$	$\left\{\begin{matrix}(.5,.6),\\(.5,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.2,.7),\\(.7,.8)\end{matrix}\right\}$
y_2	$\left\{\begin{matrix}(.3,.8),\\(.4,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.2),\\(.4,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.2,.9),\\(.1,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.4,.2),\\(.4,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.4,.6),\\(.8,.6)\end{matrix}\right\}$
y_3	$\left\{\begin{matrix}(.2,.1),\\(.6,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.9,.6),\\(.1,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.9),\\(.6,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.9),\\(.7,.1)\end{matrix}\right\}$	$\left\{\begin{matrix}(.8,.3),\\(.7,.3)\end{matrix}\right\}$
y_4	$\left\{\begin{matrix}(.7,.8),\\(.6,.2)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.8),\\(.6,.3)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.2),\\(.6,.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(.1,.8),\\(.9,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.9,.2),\\(.5,.4)\end{matrix}\right\}$
y_5	$\left\{\begin{matrix}(.7,.4),\\(.3,.2)\end{matrix}\right\}$	$\left\{\begin{matrix}(.9,.8),\\(.6,.3)\end{matrix}\right\}$	$\left\{\begin{matrix}(.2,.2),\\(.9,.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(.4,.8),\\(.2,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.2),\\(.5,.3)\end{matrix}\right\}$

Table 2
Expert information.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	$\left\{\begin{matrix}(.2,.7),\\(.1,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.3),\\(.8,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.5),\\(.7,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.4,.6),\\(.5,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.7),\\(.8,.5)\end{matrix}\right\}$
y_2	$\left\{\begin{matrix}(.9,.8),\\(.7,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.6,.5),\\(.2,.6)\end{matrix}\right\}$	$\left\{\begin{matrix}(.4,.9),\\(.1,.7)\end{matrix}\right\}$	$\left\{\begin{matrix}(.3,.2),\\(.4,.7)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.3),\\(.9,.5)\end{matrix}\right\}$
y_3	$\left\{\begin{matrix}(.3,.5),\\(.9,.8)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.6),\\(.8,.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.5),\\(.6,.3)\end{matrix}\right\}$	$\left\{\begin{matrix}(.5,.9),\\(.7,.1)\end{matrix}\right\}$	$\left\{\begin{matrix}(.2,.3),\\(.7,.9)\end{matrix}\right\}$
y_4	$\left\{\begin{matrix}(.4,.8),\\(.6,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.8,.8),\\(.6,.7)\end{matrix}\right\}$	$\left\{\begin{matrix}(.8,.2),\\(.6,.7)\end{matrix}\right\}$	$\left\{\begin{matrix}(.2,.8),\\(.9,.9)\end{matrix}\right\}$	$\left\{\begin{matrix}(.5,.2),\\(.9,.7)\end{matrix}\right\}$
y_5	$\left\{\begin{matrix}(.3,.8),\\(.6,.2)\end{matrix}\right\}$	$\left\{\begin{matrix}(.7,.8),\\(.9,.7)\end{matrix}\right\}$	$\left\{\begin{matrix}(.5,.2),\\(.6,.4)\end{matrix}\right\}$	$\left\{\begin{matrix}(.1,.8),\\(.9,.3)\end{matrix}\right\}$	$\left\{\begin{matrix}(.5,.9),\\(.9,.7)\end{matrix}\right\}$

in green supplier chain management to find the best optimal results. And also the computational results of all steps of our develop work is as follows. Fig. 5 show the graphically representation of the green suppliers selection.

14. Computational results of methodology-I

In this section we can show the computational results of develop methodology-I under FCRS information.

Step-1: In Table 1, Table 2 and Table 3 the expert information under FCRS information were presented.
Step-2: The Table 4, Table 5 and Table 6 show the aggregated result using our develop AoPs.
Step-3: The score values of each alternatives is as follows that is represented in the form of Table 7.

In the Fig. 6 the ranking results is shown in which y_3 is ranked as first having high score function.

15. Computational results methodology-II

In this section we can show the computational results of develop methodology-II under FCRS information.

Table 3
Expert information.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	$\{(.1,.2),(.1,.8)\}$	$\{(.3,.3),(.8,.6)\}$	$\{(.3,.5),(.7,.2)\}$	$\{(.2,.5),(.5,.6)\}$	$\{(.6,.9),(.9,.5)\}$
y_2	$\{(.4,.8),(.6,.9)\}$	$\{(.3,.5),(.2,.9)\}$	$\{(.5,.9),(.1,.6)\}$	$\{(.3,.2),(.4,.9)\}$	$\{(.8,.3),(.3,.3)\}$
y_3	$\{(.4,.5),(.9,.8)\}$	$\{(.6,.6),(.9,.4)\}$	$\{(.3,.5),(.9,.3)\}$	$\{(.3,.9),(.5,.1)\}$	$\{(.2,.3),(.1,.9)\}$
y_4	$\{(.4,.6),(.6,.1)\}$	$\{(.3,.8),(.6,.1)\}$	$\{(.3,.2),(.6,.9)\}$	$\{(.2,.6),(.9,.3)\}$	$\{(.3,.5),(.7,.7)\}$
y_5	$\{(.3,.6),(.7,.4)\}$	$\{(.3,.6),(.8,.1)\}$	$\{(.3,.3),(.4,.2)\}$	$\{(.4,.6),(.8,.3)\}$	$\{(.9,.2),(.5,.4)\}$

Table 4
The aggregated expert information by develop FCFRWA AoPs.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	$\{(.45,.25),(.13,.80)\}$	$\{(.41,.42),(.70,.66)\}$	$\{(.44,.29),(.66,.54)\}$	$\{(.47,.62),(.53,.42)\}$	$\{(.49,.76),(.81,.58)\}$
y_2	$\{(.68,.64),(.67,.78)\}$	$\{(.78,.51),(.37,.27)\}$	$\{(.67,.78),(.10,.63)\}$	$\{(.41,.32),(.47,.58)\}$	$\{(.68,.38),(.75,.45)\}$
y_3	$\{(.33,.50),(.85,.68)\}$	$\{(.68,.53),(.72,.53)\}$	$\{(.60,.61),(.74,.44)\}$	$\{(.47,.56),(.60,.16)\}$	$\{(.48,.30),(.56,.63)\}$
y_4	$\{(.43,.66),(.56,.47)\}$	$\{(.55,.51),(.54,.36)\}$	$\{(.64,.34),(.63,.41)\}$	$\{(.41,.20),(.73,.90)\}$	$\{(.66,.27),(.75,.58)\}$
y_5	$\{(.60,.36),(.57,.48)\}$	$\{(.55,.63),(.65,.82)\}$	$\{(.82,.42),(.73,.62)\}$	$\{(.70,.56),(.64,.68)\}$	$\{(.76,.35),(.73,.55)\}$

Table 5
The aggregated expert information by develop FCFROWA AoPs.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	$\{(.45,.50),(.56,.80)\}$	$\{(.41,.42),(.10,.66)\}$	$\{(.44,.90),(.26,.54)\}$	$\{(.72,.62),(.53,.20)\}$	$\{(.90,.76),(.81,.85)\}$
y_2	$\{(.34,.64),(.67,.80)\}$	$\{(.78,.16),(.73,.27)\}$	$\{(.67,.87),(.90,.63)\}$	$\{(.10,.32),(.47,.85)\}$	$\{(.84,.38),(.75,.50)\}$
y_3	$\{(.25,.40),(.58,.76)\}$	$\{(.80,.53),(.72,.30)\}$	$\{(.40,.61),(.74,.48)\}$	$\{(.74,.56),(.90,.16)\}$	$\{(.18,.30),(.56,.30)\}$
y_4	$\{(.34,.56),(.66,.70)\}$	$\{(.45,.51),(.54,.63)\}$	$\{(.49,.34),(.63,.81)\}$	$\{(.21,.20),(.73,.60)\}$	$\{(.56,.27),(.75,.85)\}$
y_5	$\{(.30,.63),(.77,.80)\}$	$\{(.50,.63),(.65,.28)\}$	$\{(.52,.42),(.73,.20)\}$	$\{(.30,.56),(.64,.80)\}$	$\{(.30,.35),(.73,.78)\}$

Table 6
The aggregated expert information by develop FCFRHWA AoPs.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	$\{(.25,.25),(.13,.80)\}$	$\{(.41,.33),(.80,.66)\}$	$\{(.46,.29),(.66,.40)\}$	$\{(.72,.62),(.53,.27)\}$	$\{(.98,.76),(.81,.84)\}$
y_2	$\{(.80,.64),(.67,.18)\}$	$\{(.78,.41),(.75,.27)\}$	$\{(.17,.78),(.10,.30)\}$	$\{(31,.32),(.47,.87)\}$	$\{(.28,.38),(.75,.50)\}$
y_3	$\{(.23,.50),(.85,.68)\}$	$\{(.68,.93),(.12,.53)\}$	$\{(.50,.61),(.74,.40)\}$	$\{(.46,.56),(.60,.67)\}$	$\{(.84,.30),(.56,.38)\}$
y_4	$\{(.38,.66),(.56,.75)\}$	$\{(.55,.81),(.69,.36)\}$	$\{(.46,.34),(.63,.17)\}$	$\{(.16,.20),(.73,.10)\}$	$\{(.86,.27),(.75,.87)\}$
y_5	$\{(.50,.36),(.57,.86)\}$	$\{(.55,.36),(.56,.82)\}$	$\{(.62,.42),(.73,.28)\}$	$\{(.40,.56),(.64,.80)\}$	$\{(.64,.35),(.73,.50)\}$

Step-1: Same as algorithm-I.
Step-2: Same as algorithm-II.
Step-3: The Table 8 show the result of PIS and NIS.
Step-4: The Table 9 and Table 10 show the distances measures with PIS and NIS respectively.
Step-5: The Table 11 show the result of Grey relational coefficient.
Step-6: The Table 12 show the result of relative Grey relational grades.
Step-7: In this step, we have find the relative Grey relational grades which is represented in the form of Table 13. (See Table 14.)

Table 7
The ranking results of develop AoPs methodology-I.

Proposed AoPs	score values					Ranking
	y_1	y_2	y_3	y_4	y_5	
FCFRWA	.425	.394	.435	.405	.360	$y_3 > y_1 > y_4 > y_2 > y_5$
FCFROWA	.489	.415	.571	.539	.459	$y_3 > y_4 > y_1 > y_5 > y_2$
FCFRHWA	.506	.488	.512	.479	.481	$y_3 > y_1 > y_2 > y_4 > y_5$

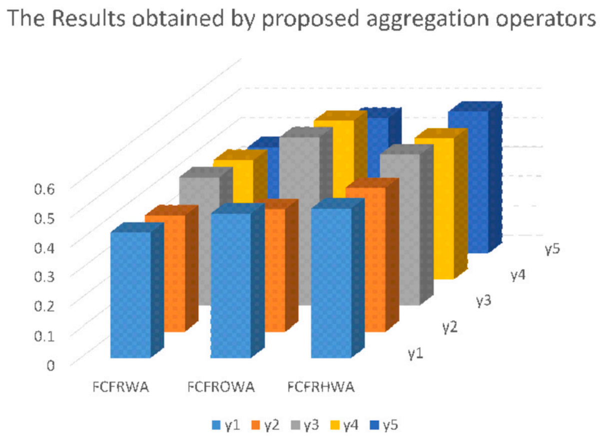

Fig. 6. Show the graphically representation of algorithm-I.

Table 8
The result of PIS and NIS.

	x_1	x_2	x_3	x_4	x_5
PIS	$\left\{ \begin{array}{c} (.49,.76), \\ (.81,.58) \end{array} \right\}$	$\left\{ \begin{array}{c} (.67,.78), \\ (.10,.63) \end{array} \right\}$	$\left\{ \begin{array}{c} (.47,.56), \\ (.60,.16) \end{array} \right\}$	$\left\{ \begin{array}{c} (.55,.51), \\ (.54,.36) \end{array} \right\}$	$\left\{ \begin{array}{c} (.60,.36), \\ (.57,.48) \end{array} \right\}$
NIS	$\left\{ \begin{array}{c} (.41,.42), \\ (.70,.68) \end{array} \right\}$	$\left\{ \begin{array}{c} (.41,.32), \\ (.47,.58) \end{array} \right\}$	$\left\{ \begin{array}{c} (.48,.30), \\ (.56,.63) \end{array} \right\}$	$\left\{ \begin{array}{c} (.20,.73), \\ (.90,.61) \end{array} \right\}$	$\left\{ \begin{array}{c} (.33,.55), \\ (.63,.65) \end{array} \right\}$

Table 9
The result of distance measures of aggregated values with PIS.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	.20	.03	.02	.01	.12
y_2	.02	.07	.07	.03	.04
y_3	.05	.05	.01	.03	.01
y_4	.10	.04	.04	.09	.04
y_5	.12	.03	.01	.01	.07

Table 10
The result of distance measures of aggregated values with NIS.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	.01	.06	.03	.07	.09
y_2	.01	.09	.04	.02	.01
y_3	.03	.01	.08	.01	.04
y_4	.01	.03	.01	.03	.01
y_5	.03	.07	.02	.02	.04

In the Fig. 7 the ranking results is shown in which y_3 is ranked as first having high score function.

16. Computational results of methodology-III

In this section we can show the computational results of develop algorithm-III under FCRS information.

Table 11
The result of Grey Relational Coefficient with PIS.

Alternatives	x_1	x_2	x_3	x_4	x_5
y_1	.05	.76	.35	.47	.09
y_2	.33	.09	.17	.24	.22
y_3	.18	.11	.12	.26	.51
y_4	.10	.14	.25	.11	.21
y_5	.09	.70	.09	.09	.14
The result of Grey Relational Coefficient with NIS.					
y_1	.01	.06	.03	.07	.09
y_2	.01	.09	.04	.02	.01
y_3	.03	.01	.08	.01	.04
y_4	.01	.03	.01	.03	.01
y_5	.03	.07	.02	.02	.04

Table 12
The result of relative Grey relational grades.

Criteria	υ_i^+	υ_i^-
x_1	.27	.17
x_2	.22	.18
x_3	.30	.09
x_4	.18	.21
x_5	.17	.11

Table 13
The result of Relative Grey relational grades.

Alternatives	α_i
y_1	.610
y_2	.547
y_3	.767
y_4	.460
y_5	.595

Table 14
The ranking results Of Extended GRA methodology-II.

Proposed Methods	score values					Ranking
	y_1	y_2	y_3	y_4	y_5	
Extended GRA Method	.610	.547	.767	.460	.595	$y_3 > y_1 > y_4 > y_5 > y_2$

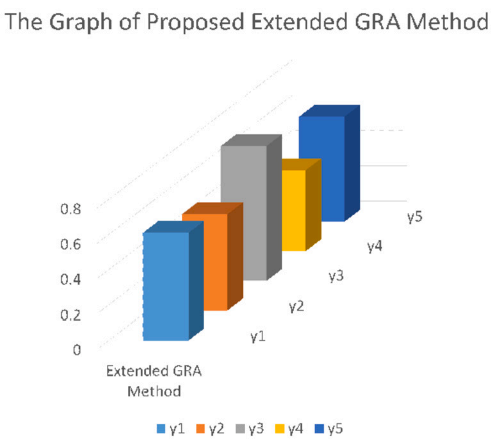

Fig. 7. Show the graphically representation of algorithm-II.

Table 15
The result of weighted values.

Criteria	υ_i^+	υ_i^-
x_1	.08	.07
x_2	.04	.05
x_3	.04	.07
x_4	.06	.01
x_5	.09	.05

Table 16
The result of closeness indices.

Alternatives	α_i
y_1	.470
y_2	.531
y_3	.605
y_4	.189
y_5	.373

Table 17
The ranking results of Extended TOPSIS methodology-III.

Proposed Methods	score values					Ranking
	y_1	y_2	y_3	y_4	y_5	
Extended TOPSIS Method	.470	.531	.605	.189	.373	$y_3 > y_2 > y_1 > y_5 > y_4$

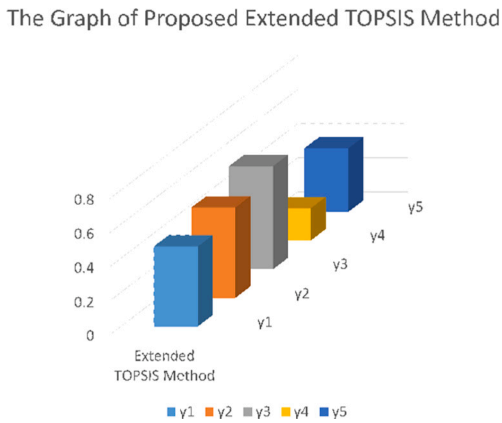

Fig. 8. Show the graphically representation of algorithm-III.

Step-1: Same as algorithm-I.
Step-2: Same as algorithm-I.
Step-3: Same as algorithm-II.
Step-4: Same as algorithm-II.
Step-5: The Table 15 show the result of weighted values.
Step-6: The Table 16 show the result of closeness indices. (See Table 17.)

In Fig. 8 the ranking results is shown in which y_3 is ranked as first having high score function.

17. Computational results of methodology-IV

In this section we can show the computational results of develop methodology-IV under FCRS information.

Step-1: Same as methodology-I.
Step-2: Same as methodology-I.
Step-3: Same as methodology-II.

Table 18
The result of group utility, individual regret and compromise values.

E_i	F_i	I_i	Ranking Position	Ranking
.006	.154	.016	2	$y_3 > y_1 > y_4 > y_2 > y_5$
.019	.086	.062	4	$y_3 > y_1 > y_4 > y_2 > y_5$
.012	.048	.010	1	$y_3 > y_1 > y_4 > y_2 > y_5$
.013	.019	.023	3	$y_3 > y_1 > y_4 > y_2 > y_5$
.003	.023	.084	5	$y_3 > y_1 > y_4 > y_2 > y_5$

Table 19
The ranking results of Extended VIKOR methodology-IV.

Proposed Methods	score values					Ranking
	y_1	y_2	y_3	y_4	y_5	
Extended VIKOR Method	.016	.023	.084	.062	.010	$y_3 > y_1 > y_4 > y_2 > y_5$

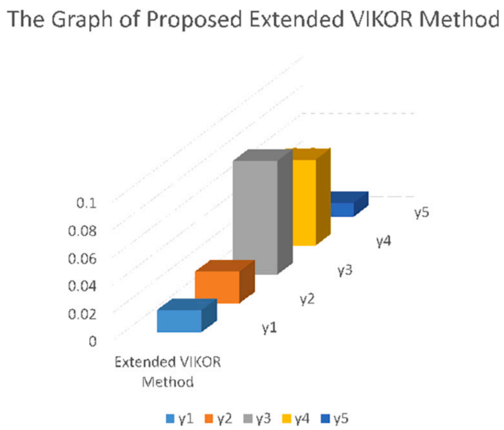

Fig. 9. Show the graphically representation of algorithm-IV.

Step-4: Same as methodology-IV.
Step-5: The Table 18 show the result of group utility values E_i, individual regret values F_i and compromise values I_i respectively.
Step-6: The Table 19 show the result of final ranking.

In the Fig. 9 the ranking results is shown in which y_3 is ranked as first having high score function.

18. Sensitivity analysis

In sensitivity analysis, we have assign various values to the parameter that is present in the VIKOR method and in the proposed aggregation operators. In this section there are two way of sensitivity analysis one is of the form that is named as VIKOR method and the other is name as aggregation operators. So in VIKOR method there is some scalar (parameter, (t)) and we have assign various values to that parameter to check the stability and whether we have to check that the result is same or varies with the passage of time due to the assigns of various values. The result is presented in the form of Table 20.

As we have provide various values to the compromise values like $\lambda = .2$. Then, the resulted values are .026, .043, .029, .032, .048 here the small value is the y_3 so we rank y_3 as best or first (high) because in VIKOR method smaller values of compromise solution will be rank first and up to so on for all alternatives, if we assign $\lambda = 3$. Then, the resulted values are .013, .034, .011, .023, .040 here the small value is the y_3 so we rank y_3 as best. And from the Table 20 we see that the ranking results is same in case of VIKOR method. In the above Fig. 10 the ranking results is shown in which y_3 is ranked as first having high score function.

Afterthat we have explained the sensitivity analysis by apply the various values to the proposed aggregation operators scalar values (parameter) t. To check the stability of our proposed AoPs we have to assign values to the t which is basically involve in the proposed AoPs and the result is presented in the form of Table 21. As we have provide various values to the aggregation operators like $t = 2$. Then, the resulted values are .425, .349, .435, .405, .360 and provide that y_3 is best. And if we assign $t = 3$, to the proposed AoPs. Then, the resulted values are .420, .560, .530, .400, .413 and provide that y_3 is best. The overall ranking result and score function for providing various values of parameters t that is involve in the proposed AoPs are given in Table 21 and that provide us the same best optimal results. In the above Fig. 11 the ranking results is shown in which y_3 is ranked as first having high score function.

Table 20
The Ranking results for different values of parameter t incase of VIKOR method.

t	D_1	D_2	D_3	D_4	D_5	Ranking
.2	.026	.043	.029	.032	.048	$y_3 > y_1 > y_4 > y_2 > y_5$
.3	.013	.034	.011	.023	.040	$y_3 > y_1 > y_4 > y_2 > y_5$
.5	.016	.023	.010	.026	.034	$y_3 > y_1 > y_4 > y_2 > y_5$
.7	.026	.034	.032	.029	.054	$y_3 > y_1 > y_4 > y_2 > y_5$
.8	.026	.032	.030	.029	.044	$y_3 > y_1 > y_4 > y_2 > y_5$
.9	.026	.038	.040	.032	.070	$y_3 > y_1 > y_4 > y_2 > y_5$
.11	.036	.053	.030	.042	.064	$y_3 > y_1 > y_4 > y_2 > y_5$
.12	.036	.030	.040	.042	.074	$y_3 > y_1 > y_4 > y_2 > y_5$
.14	.060	.079	.050	.072	.080	$y_3 > y_1 > y_4 > y_2 > y_5$

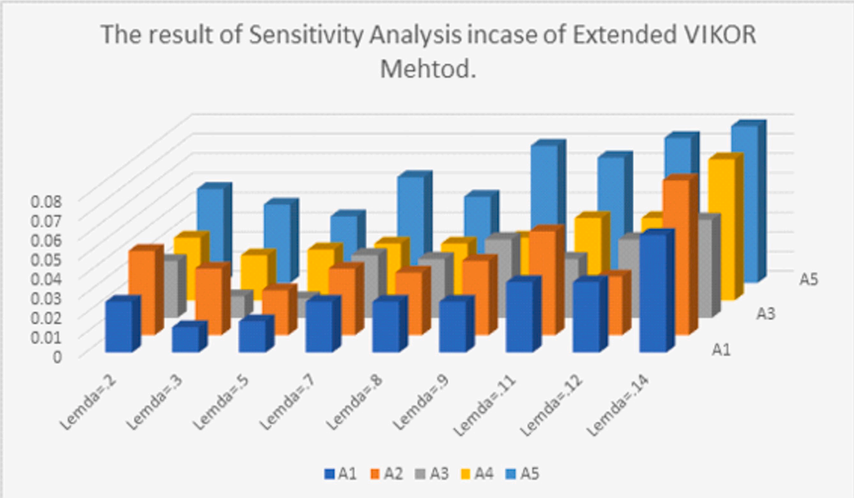

Fig. 10. Show the graphically representation of algorithm-V.

Table 21
The Ranking results for different values of t incase of proposed AoPs.

t	s_{f1}	s_{f2}	s_{f3}	s_{f4}	s_{f5}	Ranking
2	.425	.394	.435	.405	.360	$y_3 > y_2 > y_1 > y_5 > y_4$
3	.420	.530	.560	.400	.413	$y_3 > y_2 > y_1 > y_5 > y_4$
5	.523	.624	.630	.465	.500	$y_3 > y_2 > y_1 > y_5 > y_4$
7	.634	.646	.650	.587	.597	$y_3 > y_2 > y_1 > y_5 > y_4$
9	.621	.734	.750	.605	.609	$y_3 > y_2 > y_1 > y_5 > y_4$

19. Comparison and discussion

In the comparison section we have to check the stability, application and validation of our proposed work. Furthermore there are two way of comparison one is to compare with various AoPs and the other is to compare with various methods. The detailed is as follows.

Comparison with IFWA AoPs

Existing AoPs IFS information

In this section we can compare our develop algorithms with the existing AoPs. In this existing AoPs the basic idea is IFS information and some aggregation operators based on the operational laws of IFS information, which is named as IFWA [3], IFOWA [3] and IFHWA [3] AoPs.

Proposed Aggregation Operators (Algorithm-I)

But here in this paper we have take the data in the form of FCRNs and the basic operational laws of Frank t-norm and t-conorm to develop a series aggregation operators like FCFRWA, FCFROWA and FCFRHWA. Which can help us to aggregate the MCGDM information and after that we apply our proposed score function that can help in the final ranking of the object (alternatives) which is in the best position.

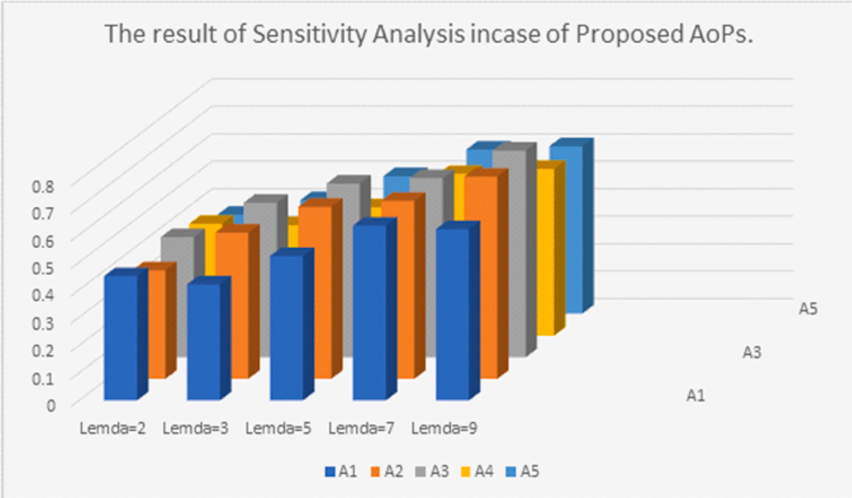

Fig. 11. Show the graphically representation of sensitivity analysis.

Result of comparison

The result of comparison is that we have compare our results with existing AoPs like IFS information. So the results obtained is same as existing AoPs which show that our proposed work is valid and applicable. The ranking result is show as below.

Approach	Score values					Ranking
IFWA [3]						$y_3 > y_1 > y_4 > y_2 > y_5$
FCFRWA	.425	.394	.435	.405	.360	$y_3 > y_1 > y_4 > y_2 > y_5$

As we see that the score function of alternative three (y_3) is high. So we rank as first and also in the existing method there is y_3 which have high score function and rank as first.

Comparison with IFRWA AoPs

Existing AoPs IFRS information

In this section we can compare our develop algorithms with the existing AoPs. In this existing AoPs the basic idea is IFRS information and some aggregation operators based on the operational laws of IFRS information, which is named as IFRWA [51], IFROWA [51] and IFRHWA [51] AoPs.

Proposed Aggregation Operators (Algorithm-I)

But here in this paper we have take the data in the form of FCRNs and the basic operational laws of Frank t-norm and t-conorm to develop a series aggregation operators like FCFRWA, FCFROWA and FCFRHWA. Which can help us to aggregate the MCGDM information and after that we apply our proposed score function that can help in the final ranking of the object (alternatives) which is in the best position.

Result of comparison

The result of comparison is that we have compare our results with existing AoPs like IFRS information. So the results obtained is same as existing AoPs which show that our proposed work is valid and applicable. The ranking result is show as below.

Approach	Score values					Ranking
IFRWA [51]						$y_3 > y_1 > y_4 > y_2 > y_5$
FCFRWA	.425	.394	.435	.405	.360	$y_3 > y_1 > y_4 > y_2 > y_5$

As we see that the score function of alternative three (y_3) is high. So we rank as first and also in the existing method there is y_3 which have high score function and rank as first.

Comparison with IFFRWA AoPs

Existing AoPs IFRS information

In this section we can compare our develop algorithms with the existing AoPs. In this existing AoPs the basic idea is IFRS information and some aggregation operators based on the Frank norms operational laws of IFRS information, which is named as IFFRWA [52], IFFROWA [52] and IFFRHWA [52] AoPs.

Proposed Aggregation Operators (Algorithm-I)

But here in this paper we have take the data in the form of FCRNs and the basic operational laws of Frank t-norm and t-conorm to develop a series aggregation operators like FCFRWA, FCFROWA and FCFRHWA. Which can help us to aggregate the MCGDM information and after that we apply our proposed score function that can help in the final ranking of the object (alternatives) which is in the best position.

Result of comparison

The result of comparison is that we have compare our results with existing AoPs like IFRS information. So the results obtained is same as existing AoPs which show that our proposed work is valid and applicable. The ranking result is show as below.

Approach	Score values					Ranking
IFFRWA [52]						$y_3 > y_1 > y_4 > y_2 > y_5$
FCFRWA	.425	.394	.435	.405	.360	$y_3 > y_1 > y_4 > y_2 > y_5$

As we see that the score function of alternative three (y_3) is high. So we rank as first and also in the existing method there is y_3 which have high score function and rank as first.

19.1. Comparison with EDAS method existing EDAS method under IF information

We can compare our develop methodology with existing methods in this section. In this existing method, The method of Evaluation based on Distance from Average Solution (EDAS) [30] and their application to decision making problems under IFS information.

Proposed Various Methods (Algorithm-II, III and IV)

We have also discuss some methods like TOPSIS, GRA and VIKOR under FCRNs information to find best optimal solution for any MCDM problems.

Result of comparison

The result of comparison is that we have compare our results with existing methods under IFS information. So the results obtained is same as existing methods which show that our proposed algorithm is valid and applicable. The ranking result is show as below.

Approaches	Score values					Ranking
EDAS [30]						$y_3 > y_1 > y_4 > y_2 > y_5$
Extended TOPSIS Method	.470	.531	.605	.189	.373	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended GRA Method	.610	.547	.767	.460	.595	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended VIKOR Method	.065	.025	.015	.250	.258	$y_3 > y_1 > y_4 > y_2 > y_5$

As we see that the score function of alternative-three (y_3) is high. So we rank as first and also in the existing method there is y_3 which have high score function and rank as first alternatives.

19.2. Comparison with TODIM method existing TODIM method under IF information

We can compare our develop methodology with existing methods in this section. In this existing method, the TODIM method and their application to decision making problems under IFS information were discussed by [57].

Proposed Various Methods (Algorithm-II, III and IV)

We have also discuss some methods like TOPSIS, GRA and VIKOR under FCRNs information to find best optimal solution for any MCDM problems.

Result of comparison

The result of comparison is that we have compare our results with existing methods under IFS information. So the results obtained is same as existing methods which show that our proposed algorithm is valid and applicable. The ranking result is show as below.

Approaches	Score values					Ranking
TODIM [57]						$y_3 > y_1 > y_4 > y_2 > y_5$
Extended TOPSIS Method	.470	.531	.605	.189	.373	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended GRA Method	.610	.547	.767	.460	.595	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended VIKOR Method	.065	.025	.015	.250	.258	$y_3 > y_1 > y_4 > y_2 > y_5$

As we see that the score function of alternative-three (y_3) is high. So we rank as first and also in the existing method there is y_3 which have high score function and rank as first alternatives.

The Table 22, show the overall comparison analysis of proposed work with existing methods. And from the Table 22, we can see that the best optimal result is y_3 due the high value of score function. Also the Table 22 show that there are two way of comparison in which we there is existing method as well as existing aggregation operators which provide that the score of alternative y_3 is high and rank as first.

Table 22
The Ranking Results With Various Existing Work.

Various AoPs	score values					Ranking
	y_1	y_2	y_3	y_4	y_5	
IFWA [3]						$y_3 > y_1 > y_4 > y_2 > y_5$
IFRWA [51]						$y_3 > y_1 > y_4 > y_2 > y_5$
IFFRWA [52]						$y_3 > y_1 > y_4 > y_2 > y_5$
TODIM method [57]						$y_3 > y_1 > y_4 > y_2 > y_5$
EDAS method [30]						$y_3 > y_1 > y_4 > y_2 > y_5$
FCFRWA AoPs	.425	.394	.435	.405	.360	$y_3 > y_1 > y_4 > y_2 > y_5$
FCFROWA AoPs	.489	.415	.571	.539	.459	$y_3 > y_1 > y_4 > y_2 > y_5$
FCFRHWA AoPs	.506	.488	.512	.479	.481	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended TOPSIS Method	.470	.531	.605	.189	.373	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended GRA Method	.610	.547	.767	.460	.595	$y_3 > y_1 > y_4 > y_2 > y_5$
Extended VIKOR Method	.065	.025	.015	.250	.258	$y_3 > y_1 > y_4 > y_2 > y_5$

20. Result and discussion

In this paper we have developed a new set called fuzzy credibility rough set (FCRS). We defined the Frank norms operational laws for FCRS. Afterthat we defined a series of aggregation operators using these developed Frank operational laws under FCRS information. These developed aggregation operators we can collect the expert information for green supplier management. Furthermore we defined a new score function for ranking of green suppliers management. Also we have discussed the various steps of various extended methods like VIKOR, TOPSIS and GRA under FCRS information.

As our work is more effective than existing because there are some parameters that is involved in the proposed aggregation operators and we can assign various values to these parameters to check the stability and correctness. Furthermore our work is applicable to solve any decision making problems and to find the best optimal results.

21. Conclusion and future directions

In this research we have develop a new set called Fuzzy credibility rough set which is the combination of fuzzy credibility set and fuzzy rough set, Afterthat we proposed a series of aggregation operators based on the Frank norms which is name as FCFRWA, FCFROWA and FCFRHWA AoPs. Under FCRS information we have defined some develop algorithms which is used for the selection of best optimal solution in any decision making problems (like green supplier management). Furthermore there are some main objectives which is discussed in detailed as follows.

a) First we defined a series of AoPs like FCFRWA, FCFROWA and FCFRHWA AoPs and its basic properties like boundedness, monotonicity and idempotency.

b) Second, the conversion of the FCFRWA, FCFROWA and FCFRHWA AoPs to the earlier AoPs for FCRS setting demonstrates the adaptability of the suggested AoPs.

c) Third, the results obtained by the FCFRWA, FCFROWA and FCFRHWA AoPs are accurate and dependable when compared to other existing AoPs for MCGDM problems under FCRS setting, demonstrating their applicability in real-world problems.

d) The methods for MCDM that are suggested in this paper are able to further acknowledge more association between attributes and alternatives, demonstrating that they have a higher accuracy and a larger reference value than the methods currently in use, which are unable to take into account the inter-relationships of attributes in practical applications. This indicates that the methods for MCGDM that are suggested in this paper can recognize even more associations between attributes.

Future Work

We will examine the conceptual framework of FCRS for Einstein, Dombi, Yager and Hamcher norms operations in future work using innovative decision-making approaches like as CODAS, TODIM and EDAS. We'll also talk about how these techniques are used in domains including analytical thinking, intelligent systems, finance, robotics, navigation, soft computing, S-box image Encryption, Neural Network, Machine learning, Deep learning, to EV charging station site selection problem, bio-medical waste management and many others.

Reliability of the Method

The proposed work is more reliable than other existing work because in the proposed work we have a parameter that is involved in the aggregation operators and we can assign various values to check the reliability of our work and that can provide same ranking results by assigning various values to the parameters.

Capabilities of the Study

The proposed work is capable to solve decision making problems and also can help us to find the best optimal results.

Research's limitations

As we developed a new set called FCRS, new Frank operational laws for FCRS and series of aggregation operators under FCRS information. This work has some limitations. The detail of limitations are as following; 1) This work only discuss membership with credibility number of the fuzzy membership. 2) This work describe only the membership of an object. 3) This work also have extenion in the two dimentional membership. 4) This work only discuss the fuzzy credibility real numbers. Due to these limitation we can extend this concept in anothe direction in future work. i.e Complex fuzzy credibility numbers, Linguistic fuzzy credibility numbers and also apply to other type of t-norms.

CRediT authorship contribution statement

Muhammad Yahya: Writing – original draft. **Saleem Abdullah:** Supervision. **Faisal Khan:** Validation. **Kashif Safeen:** Formal analysis. **Rafiaqat Ali:** Funding acquisition, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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