

Modeling growth curve parameters in Peruvian llamas using a Bayesian approach

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ABSTRACT

The objective of this study was to fit four nonlinear models (Brody, von Bertalanffy, Gompertz and Logistic) to realizations of llama weight, using frequentist and Bayesian approaches. Animals from both sexes and types (K'ara and Ch'accu) were observed. Data consisted of 43,332 monthly body weight records, taken from birth to 12 months of age from 3611 llamas, collected from 1998 to 2017 in the Quimsachata Experimental Station of the Instituto Nacional de Innovación Agraria (INIA) in Peru. Parameters for Non-linear models for growth curves were estimated by frequentist and Bayesian procedures. The MCMC method using the Metropolis-Hastings algorithm with noninformative prior distributions was applied in the Bayesian approach. All non-linear functions closely fitted actual body weight measurements, while the Brody function provided the best fit in both frequentist and Bayesian approaches in describing the growth data of llamas. The analysis revealed that female llamas reached higher asymptotic weights than males, and K'ara-type llamas exhibited higher asymptotic weights compared to Ch'accu-type animals. The asymptotic body weight, estimated for all data using the Brody model, was 42 kg at 12 months of age in llamas from Peru. The results of this research highlight the potential of applying nonlinear functions to model the weight-age relationship in llamas using a Bayesian approach. However, limitations include the use of historical data, which may not fully represent current growth patterns, and the reliance on non-informative priors, which could be improved with prior knowledge. Future studies should refine these aspects.

1. Introduction

It is estimated that more than 5.5 million South American camelids, including only alpacas and llamas, are found in Peru, among which the llama (*Lama glama*) represents 19 % of the total. Most llamas are found in the regions of Puno (33.4 %), Cusco (13.5 %), and Huancavelica (12 %) (Midagri, 2021). The habitat of these animals is primarily high-altitude areas, which extend from northern Peru to northern Argentina, including the respective mountainous zones of Bolivia and Chile. Llama and alpaca farming is one of the main socio-economic

activities for a large sector of the Andean population in Peru (Huanca Mamani et al., 2012), as they provide a variety of products such as fibers, meat, and manure, and are used as pack animals. In Peru, llamas are traditionally classified into two different types: Ch'accu, characterized by greater fiber coverage, and K'ara, which lack fibers at the extremities, head, and ears and have reduced fiber growth on the neck. However, K'ara llamas are more hardy and are often used as a pack animals. Both types of llamas provide meat with high protein content, which is regarded as healthy and nutritious, making it an important food source for the inhabitants of the Andean region (Polidori et al., 2007; Quispe

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Table 1
Non-linear models used to describe the growth the growth curve of young llamas.

Model	Equation
Brody	$y_{ij} = A_i(1 - B_i e^{-k_i t_{ij}}) + \varepsilon_{ij}$
Gompertz	$y_{ij} = A_i e^{-B_i e^{-k_i t_{ij}}} + \varepsilon_{ij}$
Logistic	$y_{ij} = A_i(1 + B_i e^{-k_i t_{ij}})^{-1} + \varepsilon_{ij}$
von Bertalanffy	$y_{ij} = A_i(1 - B_i e^{-k_i t_{ij}})^3 + \varepsilon_{ij}$

et al., 2009).

Currently, llama meat and fiber are being progressively commercialized, reflecting a growing demand for these products. However, the lack of infrastructure and the quality and quantity standards of llama meat and fiber are the main factors hindering more market-oriented farming. Additionally, challenges such as disease management and climate change impacts further complicate sustainable production. This issue arises because llamas are handled and produced in small production systems by low-income farmers, facing the consequences of the marginalization of subsistence systems (Quispe et al., 2009). Furthermore, while small-scale farmers have clear production objectives (e.g., meat, fiber, or other purposes), these objectives are not typically aligned with the specific goals required for structured genetic improvement programs. Under these production conditions, where small-scale farmers prioritize specific objectives (e.g., meat or fiber production), an important phenotypic indicator of meat production capacity is the live weight of animals over time. This is because meat production is directly influenced by the growth rate and body size of the animals, which are key traits for improving productivity in genetic improvement programs.

Mathematical modeling of growth data allows for better explanation and interpretation of growth events, which in turn helps improve overall productivity. Growth curves, using nonlinear functions, describe the animal's growth over a period of time. There are many nonlinear mathematical functions, such as Gompertz (Winsor, 1932), Brody (Brody, 1945), von Bertalanffy (von Bertalanffy, 1957), and Logistic (Nelder, 1961) that have been widely used in different livestock species because they provide biologically meaningful parameters and describe the growth patterns underlying body weight development in most animals (e.g., Domínguez-Viveros et al., 2023; Araujo et al., 2023, in cattle; Ozturk et al., 2023, in sheep; Tavares et al., 2023, in pigs; and Yadav et al., 2023, in chickens). Riek and Gerken (2007) adjusted growth curves in llamas using simple linear regression and the Gompertz equation and concluded that linear regression could be adequate to describe body weight development from birth to 27 weeks post-partum. Recent studies have fitted growth curves in one-year-old young llamas using different nonlinear functions. Canaza-Cayo et al. (2015) evaluated four nonlinear growth models—Brody, von Bertalanffy, Gompertz, and Logistic—and identified the Gompertz function as the most suitable, with K'ara llamas reaching higher mature weights but lower precocity, while their peak growth occurred earlier than in Ch'accu llamas. In contrast, Maquera (2023) found that the Brody and von Bertalanffy models best described llama growth, with males maturing earlier than females.

Growth curve parameter estimates are often obtained using the maximum likelihood (ML) approach, also known as the frequentist method. In this framework, model parameters are treated as fixed constants, while their estimates are random variables obtained through iterative algorithms due to the nonlinear nature of the models. The ML approach is computationally efficient and provides asymptotically unbiased estimates; however, it is sensitive to small sample sizes and model misspecification (Casella and Berger, 2002). An alternative is the Bayesian approach, which estimates parameters by integrating the posterior probability distribution, combining prior knowledge (represented by a prior distribution) with a probability model for the observed

data (likelihood) (Iqbal et al., 2019a). The Bayesian method allows for the incorporation of prior information and provides full posterior distributions of parameters, offering a more comprehensive uncertainty quantification. However, it is computationally intensive and requires careful selection of priors, as these can significantly influence the results (Gelman et al., 2013).

The Bayesian approach has been widely applied in modeling growth curves across various livestock species, demonstrating its effectiveness in parameter estimation and uncertainty incorporation. For instance, Salles et al. (2020) employed Bayesian methods to model the growth curve of Santa Inês sheep, highlighting the approach's flexibility in integrating prior information and providing more precise parameter estimates. Similarly, Iqbal et al. (2019b) used Bayesian inference to fit nonlinear growth functions to Thalli sheep, identifying the Brody model as the most suitable for describing the weight-age relationship. These studies underscore the applicability of the Bayesian approach in livestock growth modeling, providing a relevant context for its application in llamas.

In general, this approach produces accurate point estimates along with respective credibility intervals. The present study is, as far as we know, the first to model weight-age relationship of llamas (*Lama glama*) using Bayesian methods, in contrast to previous studies that have relied on frequentist approaches (e.g., Riek and Gerken, 2007; Canaza-Cayo et al., 2015; Maquera, 2023). In addition, the growth characteristics of llamas from birth to their first year of age have not been adequately studied, likely due to data availability limitations and the fact that weight recording is often prioritized within this period for animal selection purposes. Therefore, the objective of this study was to fit four nonlinear models (Brody, von Bertalanffy, Gompertz and Logistic) to the realizations of weight during the first year of age of peruvian llamas from both sexes and types (K'ara and Ch'accu). Results were compared with frequentist and Bayesian methods based on goodness-of-fit measures.

2. Materials and methods

2.1. Data

The dataset used in this study was obtained from the Quimsachata Experimental Station, of the Instituto Nacional de Inovacion Agraria (INIA) located in Puno, Peru. The Quimsachata Station is located at 4025 meters above sea level, 15°45' 38.9" south latitude, 70° 34' 18.9" western longitude. The temperature at the station varies between −5 and 18°C, and rainfall reaches 700 mm/year. The animals are managed under extensive grazing systems, with access to natural pastures supplemented by cultivated forages and mineral supplements as needed. Health protocols include regular deworming and vaccination programs to prevent common diseases such as enterotoxemia. Additionally, reproductive management involves careful monitoring of females during gestation and parturition. The initial database contained 70,113 records from 5,924 llamas. Data were edited to ensure consistency in the number of records per animal, considering only those with 12 monthly weight records from birth to one year of age. Records with inconsistent dates or weights were removed, as well as those from animals outside the age range. After editing, the final dataset comprised 43,332 monthly body weight records from 3,611 young llamas (1702 males and 1909 females), belonging to two types: K'ara (2,133 animals: 974 males and 1,159 females) and Ch'accu (1,478 animals: 728 males and 750 females) from birth to 365 days of age, born from 1998 to 2017. Individual body weight of animals was obtained by using a digital weighing scale.

2.2. Statistical analysis

The growth trajectory of animals was described by applying four non-linear models: Brody, Von Bertalanffy, Logistic, and Gompertz (Table 1), where y_{ij} is the observed body weight of individual i ($i = 1, \dots$,

Table 2
Descriptive statistics of live weight (kg) of llamas.

Factor	n	Mean	SD	Minimum	Maximum	Lower quartile	Upper quartile
Sex							
Male	20424	28.44	10.29	5.00	61.5	21.5	36.0
Female	22908	28.75	10.59	5.00	63.5	21.5	36.5
Type							
K'ara	25596	29.27	10.67	5.00	63.5	22	37.0
Ch'accu	17736	27.64	10.06	5.00	57.0	20.5	35.0

n: total number of weight records, SD: Standard deviation.

Table 3
Estimates (mean ± standard error) of growth curve parameters under four models in llamas through the frequentist approach.

Factor	N	Parameters	Model Brody	Logistic	Gompertz	von Bertalanffy
All	43332	A	42.12±0.14	38.25±0.07	39.42±0.09	40.06±0.10
		B	0.77±0.01	57.47±0.41	25.29±0.36	0.36±0.00
		k	0.01±0.00	0.02±0.00	0.01±0.00	0.01±0.00
		R ² adj	0.731658	0.712344	0.7236901	0.7270336
		AIC	269357.4	270725.8	269929	269698
		BIC	269392.1	270760.5	269963.7	269732.7
Sex						
Male	20424	A	41.34±0.19	37.75±0.1	38.84±0.12	39.44±0.14
		B	0.76±0.00	55.55±0.58	23.88±0.51	0.36±0.00
		k	0.01±0.00	0.02±0.00	0.01±0.00	0.01±0.00
		R ² adj	0.7302318	0.7112328	0.7224713	0.7257576
		AIC	126428.9	127057.3	126688.8	126582.9
		BIC	126460.6	127088.9	126720.5	126614.6
Female	22908	A	42.84±0.21	38.71±0.1	39.95±0.13	40.62±0.14
		B	0.77±0.00	59.2±0.57	26.59±0.5	0.37±0.00
		k	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00
		R ² adj	0.7335713	0.7140041	0.7254319	0.7288212
		AIC	142850.6	143592.4	143163	143037.7
		BIC	142882.7	143624.6	143195.2	143069.8
Type						
K'ara	25596	A	43.25±0.18	39.22±0.09	40.43±0.11	41.09±0.13
		B	0.77±0.00	58.34±0.51	26.16±0.45	0.37±0.00
		k	0.01±0.00	0.02±0.00	0.01±0	0.01±0.00
		R ² adj	0.7466748	0.7275633	0.7388373	0.7421447
		AIC	158679	159545.6	159039	158892.6
		BIC	158711.6	159578.2	159071.6	158925.2
Ch'accu	17736	A	40.47±0.21	36.84±0.1	37.95±0.13	38.55±0.15
		B	0.76±0.00	56.09±0.65	23.93±0.57	0.36±0.00
		k	0.01±0.00	0.01±0.00	0.01±0.00	0.01±0.00
		R ² adj	0.7210768	0.7016264	0.7129922	0.7163556
		AIC	109570.4	110107.3	109796.6	109706
		BIC	109601.6	110138.4	109827.7	109737.1

n) at measurement time j ($j = 1, \dots, n_i$) for animal i ; A_i , the asymptotic body weight of animal i ; B_i , the proportion of the asymptotic mature weight to be gained after birth for animal i ; k_i , is the maturity index of growth rate relative to the mature weight of animal i ; t_{ij} is age of animal i in days at time j and ε_{ij} is the random residual term. Detailed biological interpretations of these nonlinear growth models can be found in Freitas (2005).

Initially, a frequentist or classical approach was performed to estimate the growth curve parameters from Brody, Von Bertalanffy, Logistic, and Gompertz models. The NLIN procedure from the SAS software package (SAS Institute Inc., 2013) was used to estimate the least-squares estimates and the standard errors of parameters A , B and k . Parameter estimation was performed using the Gauss-Newton algorithm. The goodness of fit to these growth models was evaluated using R^2_{Adj} , the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Lower values of AIC and BIC indicate a better model fit while balancing model complexity and goodness of fit.

Secondly, a Bayesian approach was used to fit the nonlinear models previously mentioned. Bayesian hierarchical model, was adjusted to obtain the joint posterior distribution for the model parameters of each growth model separately by sex and breed of llamas. A growth model for

a single animal was assumed as $y_{ij} = f(t_{ij}, \theta) + \varepsilon_{ij}$, where $f(\cdot)$ represents one of the four nonlinear growth curve models outlined in Table 1, and the subscripts follow the same notation. The distribution of the sample data for the growth model was $y|\theta, \sigma_y^2 \sim N(f(t_{ij}, \theta), \sigma_y^2)$, where $\theta = [A, B, k]$ and σ_y^2 is the sample variance. To estimate the probability posterior density of θ given the sample y , denoted by $p(\theta|y)$, obtained from the expression: $p(\theta|y) \propto p(\theta)L(\theta|y)$, where $p(\theta)$ is the prior density and $L(\theta|y)$ is likelihood function which was obtained as: $L(\theta, \sigma_y^2) \propto \prod_{i=1}^n \prod_{j=1}^{n_i} \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left\{-\frac{1}{2\sigma_y^2} \sum_{i=1}^n [y_{ij} - f(t_{ij}, \theta)]^2\right\}$. The non-informative prior distributions for the parameters of every growth model as well as for the precision parameter were: $A \text{ Normal}(0, \tau_a)$; $B \text{ Normal}(0, \tau_b)$; $k \text{ Normal}(0, \tau_k)$ and $\tau \sim \text{gamma}(0.001, 0.001)$, respectively. In all models, chain lengths of 40,000 cycles were considered to ensure convergence and adequate exploration of the parameter space, with a burn-in period of first 15,000 cycles. Every 4th sample is retained in the next 40,000 samples to reduce the auto-correlation and improve the efficiency of posterior distribution estimation, resulting in a total of 10,000 effective samples.

Table 4

Pairwise DIC differences comparing four growth curve models for llama body weight according to sex and animal type.

Model	DIC	Comparison	Difference	Significance
All data				
Brody	269361.6	-	-	
Logistic	270729.7	Logistic - Brody	1368.1	*
Gompertz	269932.9	Gompertz - Brody	571.3	*
von Bertalanffy	269702	von Bertalanffy - Brody	340.4	*
Male				
Brody	126436.4	-	-	
Logistic	129745.7	Logistic - Brody	3309.3	*
Gompertz	126696.6	Gompertz - Brody	260.2	*
von Bertalanffy	126590.7	von Bertalanffy - Brody	154.3	*
Female				
Brody	142858.6	-	-	
Logistic	146192	Logistic - Brody	3333.4	*
Gompertz	143170.8	Gompertz - Brody	312.2	*
von Bertalanffy	143045.6	von Bertalanffy - Brody	187.0	*
K'ara type				
Brody	158685.8	-	-	
Logistic	162461.6	Logistic - Brody	3775.8	*
Gompertz	159045.3	Gompertz - Brody	359.5	*
von Bertalanffy	160298.6	von Bertalanffy - Brody	1612.8	*
Ch'accu				
Brody	109579.1	-	-	
Logistic	112736.9	Logistic - Brody	3157.8	*
Gompertz	111420.6	Gompertz - Brody	1841.5	*
von Bertalanffy	110812.2	von Bertalanffy - Brody	1233.1	*

*, Significant; DIC, Deviance Information Criteria; Lowest values of DIC are in bold.

The deviance information criterion (DIC) was used to assess the performance of the Bayesian models as described by Spiegelhalter et al. (2002). It is calculated as: $DIC = D(\hat{\theta}) + 2p_D$, where $D(\hat{\theta})$ is the deviance at the posterior mean of the parameters, and p_D is the effective number of parameters. Lower DIC values indicate a better fit with fewer parameters. According to Iqbal et al. (2019a) small DIC values differences between models require additional analysis. Therefore, the next criterion was used to assess the significance of DIC differences: a difference of less than 3 is considered not significant, while a difference of 3 or more is considered significant (Spiegelhalter et al., 2002).

For the frequentist models, model fit was assessed using the adjusted coefficient of determination ($R^2_{adj} = 1 - \left(\frac{(1-R^2)(n-1)}{n-k-1} \right)$), the Akaike Information Criterion ($AIC = -2\log L + 2k$), and the Bayesian Information Criterion ($BIC = -2\log L + k\log(n)$), where R^2 is the coefficient of determination, L is the likelihood of the model, n is the sample size,

and k is the number of parameters. Higher R^2_{adj} values indicate a better model fit, while lower AIC and BIC values suggest a more parsimonious model with better predictive performance.

The convergence diagnostics from Geweke and Raftery-Lewis were used to assess the Markov Chain Monte Carlo (MCMC) algorithms (Metropolis-Hastings). Models with lower DICs values are preferred. Bayesian posterior distribution analysis was performed using PROC MCMC in SAS, version 9.4, software (SAS Institute Inc., 2013). The Metropolis-Hastings algorithm, a type of MCMC method, was used to sample from the posterior distributions of the model parameters. This algorithm generates a sequence of samples by proposing new parameter values based on a proposal distribution and accepting or rejecting these values according to an acceptance probability that ensures convergence to the target posterior distribution. Finally, the Shapiro-Wilk, Durbin-Watson and Breusch-Pagan tests were performed to assess the normality, independence and homoscedasticity of residuals. These tests were performed on the residuals obtained from the mean model to evaluate whether the assumptions of the nonlinear models were met.

3. Results and discussion

3.1. Residual analysis and descriptive statistics

The results of the Shapiro-Wilk ($p = 0.8285$), Durbin-Watson ($p = 0.3823$) and Breusch-Pagan ($p = 0.0671$) tests suggested that the residuals meet the assumptions of normality, independence, and heteroscedasticity, so the results obtained from the analysis of the studied models are valid. Descriptive statistics of llama weights according to sex and animal type are presented in Table 2. It is noted that llamas of both sexes exhibited similar average weights, ranging from 5 to 63.5 kg. It can also be observed that the average weight of K'ara llamas was higher than that of Ch'accu llamas, ranging from 5 to 63.5 kg and from 5 to 57 kg, respectively. It is also noted that the weight for 25 % of the data from llamas of both sexes did not exceed 21.5 kg, and 75 % of the data did not exceed 36.5 kg. The availability of such information is essential to achieving the best fitting weight-age relationship of one-year-old llamas.

3.2. Growth model estimates and comparison by using frequentist approach

The estimates of growth curve parameters, their standard errors and goodness of fit statistics obtained from the frequentist approach for Brody, Logistic, Gompertz and von Bertalanffy models in llamas from birth to one year of age for both sexes and type of animal are presented in Table 3. For all data, and both sex and llama types, Brody model shows the best fit to the data due to the highest R^2_{adj} as well as the lowest AIC and BIC values for estimated growth curve parameters in one-year-old llamas, followed by von Bertalanffy, Gompertz and Logistic models. The estimation of parameter A , representing the asymptotic

Table 5

Posterior estimates of growth curve parameters under four models in llamas through the Bayesian approach for all data.

Model	Parameter	Median	Mean	SD	95 % Credible Interval (BCI)	
					Lower (2.5 %)	Upper (97.5 %)
Brody	A	42.033	42.034	0.140	41.764	42.309
	B	0.769	0.769	0.002	0.766	0.773
	k	0.007	0.007	0.000	0.007	0.007
Logistic	A	38.224	38.225	0.074	38.084	38.374
	B	2.370	2.370	0.017	2.339	2.405
	k	0.015	0.015	0.000	0.015	0.015
Gompertz	A	39.381	39.383	0.092	39.209	39.570
	B	1.317	1.317	0.006	1.305	1.328
	k	0.011	0.011	0.000	0.011	0.011
von Bertalanffy	A	40.008	40.009	0.104	39.806	40.214
	B	0.365	0.365	0.001	0.362	0.367
	k	0.010	0.010	0.000	0.009	0.010

Table 6

Posterior estimates of growth curve parameters under four models in llamas through the Bayesian approach by sex.

Model	Parameter	Median	Mean	SD	95 % Credible Interval (BCI)	
					Lower (2.5 %)	Upper (97.5 %)
Brody						
Sex						
Male	A	41.170	41.176	0.189	40.812	41.556
	B	0.765	0.765	0.002	0.760	0.770
	k	0.007	0.007	0.000	0.007	0.007
Female	A	42.652	42.653	0.199	42.242	43.020
	B	0.773	0.773	0.002	0.769	0.777
	k	0.007	0.007	0.000	0.006	0.007
Logistic						
Sex						
Male	A	49.367	49.365	0.008	49.347	49.377
	B	2.379	2.377	0.006	2.365	2.385
	k	0.007	0.007	0.000	0.007	0.008
Female	A	49.848	49.847	0.012	49.826	49.869
	B	2.428	2.429	0.012	2.410	2.451
	k	0.008	0.008	0.000	0.008	0.008
Gompertz						
Sex						
Male	A	38.771	38.770	0.126	38.524	39.013
	B	1.304	1.304	0.008	1.289	1.320
	k	0.011	0.011	0.000	0.011	0.011
Female	A	39.867	39.869	0.130	39.611	40.116
	B	1.328	1.328	0.008	1.312	1.345
	k	0.011	0.011	0.000	0.011	0.011
von Bertalanffy						
Sex						
Male	A	39.348	39.349	0.137	39.095	39.630
	B	0.362	0.362	0.002	0.358	0.365
	k	0.010	0.010	0.000	0.010	0.010
Female	A	40.520	40.521	0.147	40.250	40.816
	B	0.367	0.367	0.002	0.364	0.371
	k	0.009	0.009	0.000	0.009	0.010

Table 7

Posterior estimates of growth curve parameters under four models in llamas through the Bayesian approach by type of llama.

Model	Parameter	Median	Mean	SD	95 % Credible Interval (BCI)	
					Lower (2.5 %)	Upper (97.5 %)
Brody Type						
K'ara	A	43.099	43.097	0.181	42.758	43.457
	B	0.773	0.773	0.002	0.769	0.777
	k	0.007	0.007	0.000	0.007	0.007
Ch'accu	A	40.276	40.274	0.211	39.873	40.687
	B	0.764	0.764	0.003	0.758	0.769
	k	0.007	0.007	0.000	0.007	0.007
Logistic Type						
K'ara	A	49.029	49.030	0.008	49.017	49.043
	B	2.469	2.468	0.015	2.442	2.490
	k	0.008	0.008	0.000	0.008	0.008
Ch'accu	A	51.901	51.902	0.005	51.891	51.910
	B	2.578	2.578	0.007	2.565	2.589
	k	0.007	0.007	0.000	0.007	0.007
Gompertz Type						
K'ara	A	40.373	40.375	0.116	40.155	40.611
	B	1.329	1.329	0.007	1.315	1.343
	k	0.011	0.011	0.000	0.011	0.011
Ch'accu	A	49.904	49.904	0.006	49.894	49.916
	B	1.323	1.323	0.005	1.312	1.333
	k	0.005	0.005	0.000	0.005	0.005
von Bertalanffy Type						
K'ara	A	49.336	49.336	0.004	49.329	49.347
	B	0.367	0.367	0.001	0.364	0.369
	k	0.006	0.006	0.000	0.006	0.006
Ch'accu	A	48.052	48.054	0.005	48.048	48.064
	B	0.363	0.363	0.002	0.360	0.366
	k	0.005	0.005	0.000	0.005	0.005

Table 8

Diagnostics by the Geweke and Raftery–Lewis test for the growth curve parameters A, B, and k based on all data.

Parameter	Geweke (Pr > z)	Raftery–Lewis
A	0.9593	1.9746
B	0.7451	1.9413
k	0.9106	2.0299

mature weight of the adult animal, was highest for the Brody (42.12 kg), followed by Von Bertalanffy (40.06 kg), but lowest in Gompertz (39.42 kg) and Logistic (38.25 kg) models for all data. Parameter estimate for B, referred to as the integrated constant, is associated with the proportion of the asymptotic mature weight to be gained after birth for an animal, but without any biological interpretation. The values were highest for Logistic and Gompertz models (57.47 and 25.29) and lowest for Brody and von Bertalanffy models (0.77 and 0.36). Parameter estimate for k, associated with the maturity index of growth rate relative to the mature weight of an animal. A high maturity rate indicates early maturation in animals, and conversely, a low maturity rate suggests slower maturation. These values were similar for Brody, von Bertalanffy and Gompertz models (0.01 kg) and slightly higher in the Logistic model (0.02 kg), indicting that llamas reach maturity later in life. These findings have practical implications for optimizing feeding strategies, health interventions, and breeding schedules, ultimately improving farm efficiency and animal welfare.

Table 3 also presents the estimated growth curve parameters for both sexes and animal types (K'ara and Ch'accu) from birth to one year of age. For the four models studied, female llamas showed the highest estimated mature weights (A) than those of males. These results were highest for the Brody model (41.34 kg for males and 42.84 kg for females), followed by the von Bertalanffy model (39.44 kg for males and 40.62 kg for females). The lowest estimated mature weights were obtained from the Gompertz model (38.84 kg for males and 39.95 kg for females), followed by the logistic model (37.75 kg for males and 38.71 kg for females). The maturity rate estimates (k) were similar for both sexes across all models, except in the logistic model, where males showed a slight increase over females.

The K'ara llamas reached higher estimated mature weights than the Ch'accu llamas in all four models studied. For both animal types, the Brody model showed the highest mature weights (43.25 kg for K'ara and 40.47 kg for Ch'accu), followed by the von Bertalanffy model (41.09 kg for K'ara and 38.55 kg for Ch'accu). However, the lowest estimated mature weights were shown by the Gompertz model (40.43 kg for K'ara and 37.95 kg for Ch'accu) and the logistic model (39.32 kg for K'ara and 36.84 kg for Ch'accu). The maturity rate estimates (k) were similar for both animal types across all models, except in the logistic model, where K'ara llamas showed a slight increase over Ch'accu llamas. Similar results were reported for both sexes and animal types in the llama data, where the Brody model outperformed the others. Maquera (2023) noted that K'ara llamas had higher estimated mature weights than Ch'accu llamas and attributed this difference to their genetic potential for growth, slightly faster growth rates. These findings align with Canaza-Cayo et al. (2015), who reported that K'ara llamas have a greater genetic potential for growth and are more responsive to nutritional improvements compared to Ch'accu llamas.

The Brody and von Bertalanffy models better described the growth of K'ara and Ch'accu llamas under natural feeding conditions. The overall evaluation of the growth models studied in this research suggests that the Brody model provides the best fit for the body weight data of one year old llamas due to its ability to accurately capture the growth patterns of llamas, and their flexibility in describing both early and late growth phases makes it particularly suitable for llamas, which exhibit a prolonged growth period compared to other livestock species.

3.3. Comparison models using the Bayesian approach

Table 4 displays the Deviance Information Criterion (DIC) values as well as the pairwise DIC differences comparing four growth curve models. The DIC is a measure used in Bayesian analysis to assess the quality of a model and a lower DIC value indicates a better model. Among the four models, the Brody growth model produced the lowest DIC value for all data, as well as for both sex and type of animal. The second-best fitting model was the Von Bertalanffy model, followed by the Gompertz and Logistic models for all four cases (all, male, female, K'ara and Ch'accu data). Large differences between the DIC values of the Brody model (Table 4) and those of other models across all data, sexes, and animal types indicate that the Brody model provides a significantly better fit than the remaining models. These findings further confirmed that, out of the four nonlinear growth models, the Brody growth model provided the best fit. Therefore, both the frequentist and Bayesian approaches identified the same model, namely the Brody model, as the best growth model to fit the weight-age data compared to the other candidate nonlinear models for one year old llamas of both sex and type of animal. However, the Gompertz and Logistic models do not appear to be suitable for describing the weight-age relationship of llamas.

3.4. Growth model estimates using the Bayesian approach

Table 5 displays the growth curve parameters, median, mean, standard deviation as well as the 95 % Bayesian credible intervals for each growth curve model fitted using a Bayesian approach to the weight of llamas for all data. In all four models, the estimated parameters fall within the 95 % Bayesian credible intervals, indicating precise estimates. For instance, in the Brody model, A ranged from 41.764 to 42.309 kg and k from 0.007 to 0.007 kg/day, while in the Gompertz model, A ranged from 39.209 to 39.570 kg and k from 0.011 to 0.011 kg/day. These narrow intervals suggest high confidence in the estimates, enhancing model reliability and predictive accuracy. The estimation of asymptotic mature weight of the adult animal (parameter A), was highest for the Brody (42.03 kg) and Von Bertalanffy (40.01 kg) models. In contrast, the Gompertz (39.38 kg) and Logistic (38.23 kg) models yielded lower estimates for all data. Similar results were obtained using the frequentist approach for all data (Table 3), where the Brody model also outperformed the other models. Under this approach, parameter A was estimated to be 42.12, 40.06, 39.42 and 38.25 kg for the Brody, Von Bertalanffy, Gompertz, and Logistic models, respectively. Therefore, both frequentist and Bayesian methods produced nearly identical parameter estimates, indicating that both approaches can provide consistent results when analyzing large datasets. Although the differences between the methods were minimal, the Bayesian approach offers the additional advantages of model flexibility, which allows us to capture complex relationships in our data; incorporation of prior knowledge, particularly useful given the limited data available in some aspects of our study; and robust uncertainty estimation, which provides a more comprehensive understanding of the variability in our results (Gelman et al., 2013). These advantages make the Bayesian approach particularly valuable in situations with more complex models.

From Table 6, females showed a higher estimated mature weight than males in all the models analyzed. Similar trends were reported by Canaza-Cayo et al. (2015), who found higher asymptotic weights in females (47.64 kg) than in males (45.47 kg), as well as by Maquera (2023), who also observed a slight superiority in male body weight compared to females. However, in Kivircik sheep populations, Ozturk et al. (2023) reported that male lambs had a higher asymptotic weight (44.19 kg) compared to female lambs (35.24 kg), as estimated by the Gompertz model. Similarly, Kheirabadi (2024) found that male Zandi lambs reached a higher asymptotic weight (35.49 kg) than females (31.87 kg) using the Logistic model. Both studies attributed these differences to sexual dimorphism, which results in heavier adult weights for male lambs, despite females often exhibiting a higher rate of maturity. These

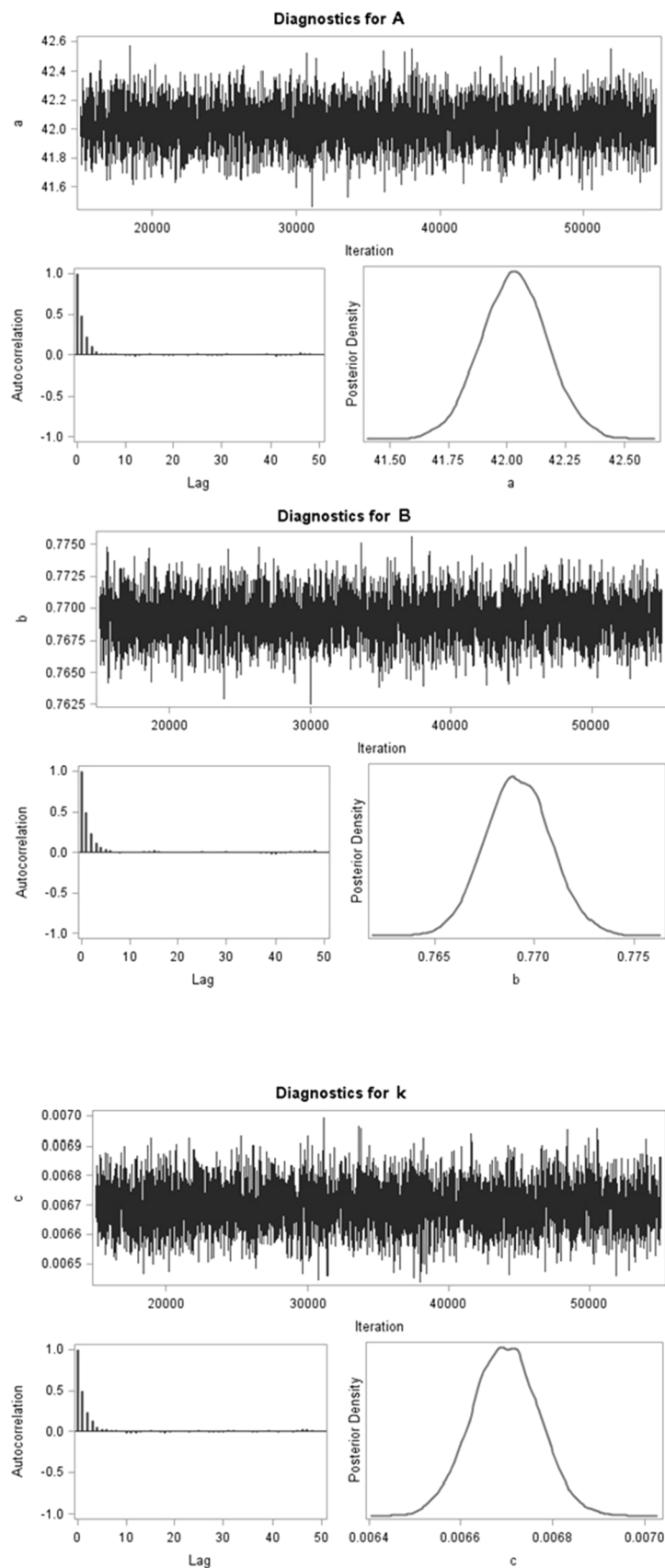


Fig. 1. Convergence diagnostics plots (trace plot, autocorrelation and posterior density plots) for the Brody growth model parameters.

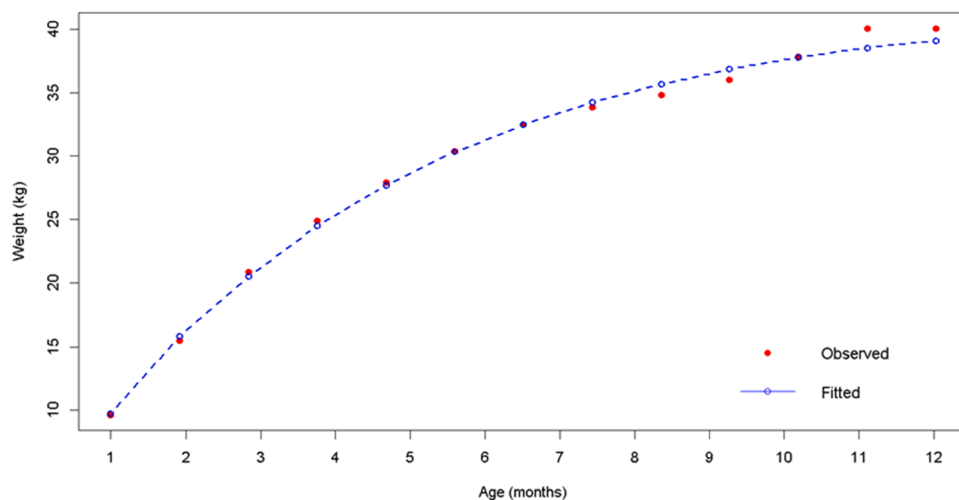


Fig. 2. Observed and fitted body weight (kg) using the Brody function for all data using the Bayesian approach in llamas.

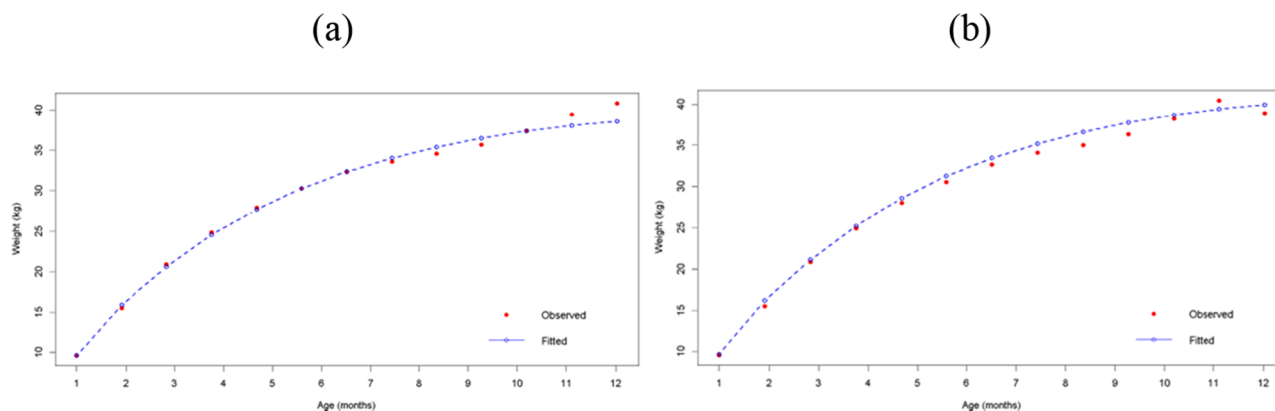


Fig. 3. Observed and fitted body weight (kg) using the Brody function for male (a) and female (b) llamas using the Bayesian approach.

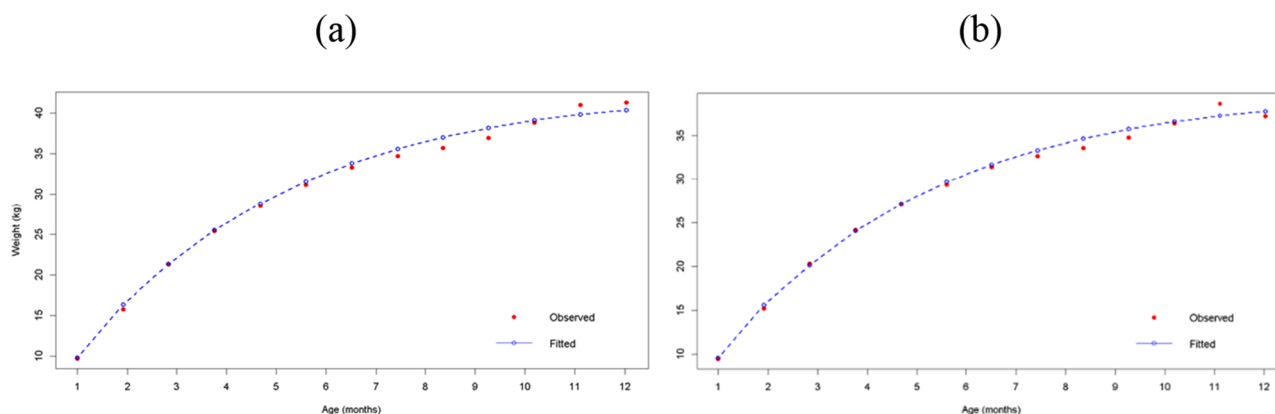


Fig. 4. Observed and fitted body weight (kg) using the Brody function for K'ara (a) and Ch'accu (b) llamas using the Bayesian approach.

differences reported by those authors compared to our results may be attributed to variations in feeding practices or genetic differences within the studied population. Furthermore, it is possible that females have been selectively bred for traits favoring higher body weight, such as increased reproductive capacity.

The highest estimated mature weights for llamas of both sexes were observed in the Logistic model (49.37 kg for males and 49.85 kg for females), followed by the Brody model (41.18 kg for males and 42.65 kg for females). Meanwhile, lower mature weight estimates were observed

in the von Bertalanffy (39.35 kg for males and 40.52 kg for females) and Gompertz models (38.77 kg for males and 39.87 kg for females). However, in both sexes, low DIC values were observed in the Brody model, followed by the von Bertalanffy model, while higher DIC values were observed in the Gompertz and Logistic models. These results suggest that the model providing the best fit for the growth data of llamas was the Brody model. The estimates of the maturity rate (k) were similar across both sexes in all models. However, for both sexes, lower values were observed in the Brody model (0.007), while higher values were reported

in the Gompertz model (0.011). Similarly, Ozturk et al. (2023), using the Gompertz model (selected as the best-fitting model), found that the k values were similar for both male and female Kivircik lambs (0.01), indicating no significant differences in the rate of maturity between sexes. In contrast, Kheirabadi (2024), using the Logistic model (selected as the best-fitting model), reported that female Zandi lambs exhibited a slightly higher rate of maturity ($k = 0.028$) compared to males ($k = 0.027$). These variations in k values across studies may reflect differences in breed characteristics, management practices, or environmental conditions.

According to Table 7, the Logistic model provided the highest estimated mature weights, with 49.03 kg for K'ara llamas and 51.90 kg for Ch'accu llamas. The lowest mature weights estimates were obtained from the Brody model, with 43.10 kg for K'ara llamas and 40.27 kg for Ch'accu llamas. However, in both animal types, low DIC values were observed in the Brody model while higher DIC values were observed in the Logistic model, indicating that the Brody model best fit the llama data for both animal types.

The convergence diagnostics for Brody growth model are displayed in Table 8. The Geweke diagnostic indicated adequate convergence of the MCMC chains for all parameters of the Brody growth model, with p -values ($\Pr > |z|$) above 0.05, suggesting no significant differences between the initial and final segments of the chains. Meanwhile, the Raftery-Lewis diagnostic, revealed a small correlation between samples (which is less than 5), suggesting that the chains were effectively sampling the posterior distribution with minimal autocorrelation. Together, these findings confirm the proper convergence of the MCMC chains and the reliability of the parameter estimates. The majority of the plots for all models demonstrated satisfactory convergence of the Markov chains. Since the Brody model provided the best fit compared to the other models analyzed, the convergence diagnostic plot based on the weight-age relationship of llamas for all data is shown in Fig. 1. This plot illustrates that the Markov chains meet the convergence diagnostics presented in Table 8 for global data of the Brody model.

The Bayesian Brody growth curve for both sexes and animal types are shown in Figs. 2, 3 and 4. The growth curves in all cases were accurate in predicting the observed body weights of the llamas, based on the results of the frequentist approach (Table 3) and the Bayesian approach (Tables 5 and 6), where the goodness-of-fit measures for the Brody model were superior to those of the other models studied. Additionally, slight differences in growth curves between sexes and animal types were observed in the middle and final stages. Overestimation occurred at the middle stage (8-9 months) and underestimation at the final stage (11-12 months) for all, male and K'ara data. For females, overestimation was observed at the middle stage (7-9, 12 months), and underestimation at the final stage (11 months). For Ch'accu data, overestimation occurred at the middle stage (8-10 months), and underestimation at the final stage (11 months). The higher body weights of female llamas may be attributed to hormonal and physiological differences compared to male llamas. Significant sex differences in Kh'ara and Th'ampulli llamas from Bolivia were reported by Wuzinger et al. (2005). However, no sex differences were reported by Maquera (2023), working with K'ara and Ch'accu llamas.

According to the authors' best understanding, this is the first time that results on the modeling of the growth curve in llamas using the Bayesian approach have been reported, which makes a detailed comparison difficult. However, studies on growth curves using the frequentist approach have been reported in the literature very few studies on llamas (Canaza-Cayo et al., 2015; Maquera, 2023). In most of these studies, the Brody model has been found to be the best nonlinear model for fitting the weight-age relationship in llamas.

This study has any limitations that should be acknowledged. The data used were collected between 1998 and 2017, which may not fully reflect current growth patterns or management practices. The Bayesian approach relied on non-informative priors, which, while robust, may not fully incorporate prior knowledge or expert information that could

enhance parameter estimation. Future research should address these limitations to refine the models further.

4. Conclusion

The Bayesian approach was compared to the usual frequentist analysis in models of the weight-age relationship in the first year of peruvian llamas from both sexes. Usual nonlinear growth models such as Brody, Von Bertalanffy, Gompertz, and Logistic, were fitted. The estimates of the median, mean, standard deviation, and credible intervals were obtained from the posterior distributions of the parameters using non-informative priors. This research highlights the Brody model as the best-fitting growth model for Peruvian llamas, providing a reliable tool for accurately describing the weight-age relationship. These findings can assist breeders and farmers in optimizing feeding strategies, selecting animals with desirable growth traits, and improving farm management. Although both Bayesian and frequentist methods produced similar parameter estimates, the Bayesian approach offers distinct advantages, such as its ability to incorporate uncertainty through credible intervals, flexibility in modeling complex relationships, and the potential to integrate prior knowledge in future studies. These features make Bayesian methods a robust framework for modeling complex growth patterns, supporting the development of more advanced models for sustainable livestock production.

CRediT authorship contribution statement

Ali William Canaza-Cayo: Conceptualization, Formal analysis, Writing – original draft. **Rubén Herberth Mamani-Cato:** Writing – review & editing, Methodology, Formal analysis. **Roxana Churata-Huacani:** Writing – review & editing, Methodology, Formal analysis. **Francisco Halley Rodríguez-Huanca:** Writing – review & editing, Resources. **Maribel Calsin-Cari:** Writing – review & editing, Resources. **Ferdinand Marcos Huacani-Pacori:** Validation, Writing – review & editing. **Oscar Efrain Cardenas Minaya:** Validation, Writing – review & editing. **Júlio Sílvio de Sousa Bueno Filho:** Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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