



## Review article

## Perceived ‘optimal efficiency’: theorization and conceptualization for development and implementation

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## ABSTRACT

We recently advanced the study of positive psychology by introducing *the theory of optimization*, which explains the underlying process of optimal best. Our continuing research interest has led us to a newly developed concept, termed as ‘optimal efficiency’. *Optimal efficiency*, we contend, focuses on the utilization of resources as well as the amount of time and effort that a person would have to expend during the course of his/her learning. How much time and effort, for example, should a student expend before it is perceived as being ‘inefficient’? Optimal efficiency, in this analysis, is concerned with an important relationship – namely: the minimization of expenditure of time, effort, resources, etc. *versus* the maximization in productivity.

Perceived efficiency is related to the teaching and training of judgment, decision making, autonomy, and self-determination – for example, in terms of successful schooling, a student has to decide whether it is worthwhile to expend so much time and effort on a given task when he/she may not necessarily pass. In our conceptual analysis and proposition of optimal efficiency, we consider the impact of *cognitive load theory*, which places emphasis on calculated investment and subsequent use of cognitive resources to process information for the purpose of achieving effective learning in a subject matter. Using cognitive load theory as a basis, we attempt to validate the concept of optimal efficiency by taking into account three main types of cognitive load imposition: extraneous, intrinsic, and germane. For example, we consider the possibility that a reduction in extraneous cognitive load imposition could instill a perception of efficiency, resulting in a person’s achievement of optimal best. Emphasis on encouragement of germane cognitive load, in contrast, could be perceived as being more efficient, likewise yielding exceptional outcomes in a subject matter.

## 1. Introduction: optimal best

Achieving *personal best* in terms of mastery and/or performance-based outcomes is an interesting topic of development (Martin, 2011; Martin and Liem, 2010; Phan et al., 2016). Optimal best, in brief, is related to personal fulfilment or accomplishment of an outcome (e.g., cognitive outcome: a half-yearly exam in mathematics) that is optimal – or, alternatively, optimal best reflects the maximization of a person’s capability (Phan et al., 2016). In the context of schooling, existing theorizations (e.g., motivation: Franken, 2007) and continuing pedagogical practices and subject contents play a notable role in helping students achieve their optimal academic and/or non-academic experiences. In their recent research, for example, Phan, Ngu, and Yeung (2019b) explored in detail the *process of optimization* and proposed an interesting theoretical concept known as the *index of optimization*. The index of optimization, according to the authors, is postulated to reflect the totality of the process of

optimization, which in turn would serve to facilitate the achievement of optimal best.

Optimization, we concur with existing theorizations (e.g., Fraillon, 2004; Phan et al., 2017; Phan et al., 2019b), is an important process for development. Achieving a state of optimal best, whether it is cognitive, physical, emotional, etc., requires some form of optimization (Phan et al., 2019b). In a practical sense, for example, a teacher may provide encouraging feedback to scaffold a student to achieve an optimal cognitive state of learning of Geometry. In a similar vein, a teacher could offer additional tutorial support before and/or after school hours to assist in the optimization of students’ academic learning experiences. An important question that we could ask, however, is this: how much time, effort, and/or resources should we use in order to optimize a person’s state of cognitive functioning in a subject matter? This question for us is related to a theoretical concept, which we term as *optimal efficiency* and, correspondingly, the ‘index of efficiency’, denoted as ‘IE’. Optimal

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efficiency is closely related to the nature of expediency, the appropriate use of resources, and self and collective regulation for optimal outcome. In this analysis, optimal efficiency places emphasis on a perceived ratio in judgment of two interrelated indexes: optimal outcome (i.e., maximum) versus expenditure of time, effort, and/or resources (i.e., minimum). This perceived ratio, situated within *the theory of cognitive load* (Sweller, 1994; Sweller et al., 2011), has considerable relevance for individuals, especially in terms of daily application (e.g., how can a student maximize his/her 'return' in terms of academic performance?) and research development (e.g., how would we advance empirical research development of optimal efficiency, especially given that this theoretical concept is seminal?).

**2. The process of optimization: an overview**

Recently, our quest to advance theoretical understanding into the topical theme of optimal best (Phan et al., 2020; Phan et al., 2016) led us to consider a related facet – namely, the utilization of time, effort, and resources (e.g., technologies, instructional designs, etc.) that would result in the *optimization of a state of functioning*. In other words, how 'much' optimization would be needed to ensure that optimal best is achieved? Non-academically, consider a football player who aspires to improve his scoring capability (e.g., from 50 to 85 goals). What does the football player have to do in order for him to fulfill this aspiration? The training of mental strength and personal resilience, and/or the use of encouraging feedback could act as precursors of the optimization of the football player's physical performance. In a similar vein, as an educational example, a Year 9 student wishes to experience an optimal state of mathematics learning in Geometry. What would 'operate' in this context to facilitate the achievement of optimal best in Geometry? The provision and exposure of different instructional designs, as research has shown (e.g., Ngu and Yeung, 2012; Ngu et al., 2014; Rittle-Johnson et al., 2017), may assist in the optimization of the child's mathematics learning.

In their research development into the concept of optimal best practice (Fraillon, 2004; Martin and Liem, 2010; Phan et al., 2016), Phan et al. (2017) introduced the *Framework of Achievement Bests*, which conceptualizes and depicts the process of optimization. The process of optimization, in particular, may help to explain a person's progression and achievement of optimal best (or notional best), L<sub>2</sub>, from his/her realistic best (or actual functioning), L<sub>1</sub>. There are different types of *psychological* (e.g., personal self-efficacy: Bandura, 1997), *educational* (e.g., an appropriate instructional design: Ngu et al., 2014), and *psycho-social* (e.g., the impact of the home environment: McCartney et al., 2007) agents that form the 'totality' of optimization. The activation and enactment of a psychological agent such as personal self-efficacy for learning (Bandura, 1977, 1997), for example, may initiate the sub-processes of persistence, effective functioning, and/or effort expenditure, resulting in the achievement of L<sub>2</sub>.

In their recent writing, likewise, Phan et al. (2019b) provided a comprehensive overview of optimal best and, more importantly, their

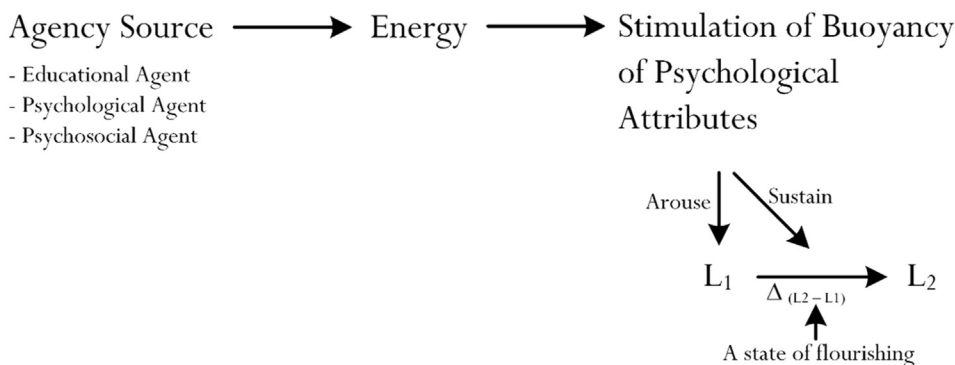
updated theorization of optimization. Specifically, drawing from previous understanding (Phan and Ngu, 2017; Phan et al., 2017), the authors proposed a core component of optimization known as *energy*, which in this case is defined as "feeling and experience of liveliness, vitality, and adrenaline". According to this reconceptualization, as shown in Figure 1, energy is postulated to serve as an 'optimizing force' that would stimulate the buoyancy of different types of psychological attributes (e.g., mental strength, personal resolve, effort expenditure, intrinsic motivation, effective functioning), resulting in the arousal and sustaining of a state of functioning from L<sub>1</sub> to L<sub>2</sub>. Indeed, from this analysis, a high level of energy is closely aligned with the 'operation' of optimization. A low level of energy, in contrast, would indicate an inaction of optimization – in other words, optimization is not taking place and instead there is sub-optimal experience of learning, etc.

The concept of energy (Phan et al., 2019b; Phan, Wang, Shih, Shi, Lin and Ngu, 2019c) is interesting and may provide a complete theoretical account of how a person achieves a state of optimal best in life. Optimization is positive in its nature and characteristics, coinciding with the *paradigm of positive psychology* (Seligman and Csikszentmihályi, 2000; Seligman et al., 2009). Existing theorizations (e.g., Fraillon, 2004; Phan et al., 2020; Phan et al., 2019b) have provided grounding for further development into the study of optimal best. Optimal best is not simply concerned with personal improvement; optimal best, in this case, also incorporates distinctive characteristics such as a person's experience of 'flow' (e.g., emotional flow of happiness) or a state of flourishing. From this description, we argue that in a person's quest to achieve optimal best, he/she would seek opportunities to attain the full gamut of positive and proactive life experiences.

Arising from the study of optimization (Fraillon, 2004; Phan et al., 2017, 2019b), we propose a comparable concept, which we term as 'efficiency'. As we discuss in the next section of this article, an important focus for examination is related to what is known as perceived 'optimal efficiency' – that is, the extent to which optimal best could be achieved via means of minimum expenditure of cost, time, effort, and/or the utilization of resource. This theorization is significant as it would address a person's planning and organization, and places emphasis on self-awareness, reflection, and personal judgment and decision making. Additionally, it includes the use of optimal cognitive resources arising from appropriate instructional designs. Perceived 'inefficiency' or 'sub-optimal efficiency', in contrast, is potentially related to a person's unstructured thought and behavior, extensive use of time, effort, resources, etc., and disorganization, all of which may combine to produce sub-optimal learning experiences.

**3. Introducing the concept of 'optimal efficiency'**

*Optimal efficiency*, as we briefly described, is concerned with the notion of judgment and assessment of 'cost-saving' in terms of a person's expenditure of time and effort, as well as his/her effective utilization of different types of resources. For example, in terms of academic learning, a



**Figure 1. A Summary of Optimization.** Note: This is a summary of the process of optimization, as detailed in Phan, Ngu et al.'s (2019b) article. In brief, the enactment of different types of agencies (e.g., educational agent) results in the creation of energy, denoted as 'E', which would then stimulate the buoyancy of different psychological attributes (e.g., effort expenditure), resulting in the arousal and sustaining of an internal state of functioning – that is, L<sub>1</sub> at T<sub>1</sub> to L<sub>2</sub> at T<sub>2</sub>. According to Phan et al. (2019b), the achievement of L<sub>2</sub> from L<sub>1</sub> is indicative of a state of flourishing, denoted as Δ(L<sub>2</sub> - L<sub>1</sub>) – in other words, Δ(L<sub>2</sub> - L<sub>1</sub>) is always positive (i.e., +ve).

student may excessively engage in cognitive resources, resulting in confusion, which would negate his/her thinking, planning, and organization (e.g., excessive and convoluted cognitive thoughts about a subject matter without any form of resolution). Our proposition of perceived optimal efficiency is drawn from the premise that, similar to human capitals, there is a need for a person, society, institution, organization, family, etc. to consider the availability of time, resources, etc. This premise, as we detail in the subsequent sections, is similar to *relevance theory* (Sperber & Wilson, 1986, 1995), which emphasizes the importance of cognitive relevance. How much is enough before a person recognizes that wasteful expenditure of time and/or effort has been made? In terms of mathematics learning, for example, how much effort would a student need to expend in order to achieve a level of optimal best in mastery and understanding of Algebra? How much time does a student spend before he/she perceives it as being 'ineffective'? Does the 'amount' of effort and/or time that one expends yield an adequate return (if any)? These sample questions make attempts to highlight the tenet of *cost-benefit analysis* or, alternatively, the balance in cost and productivity.

In a similar vein, Hoffman and Schraw's (2010) published work in *Educational Psychologist* is relevant, highlighting the extensive and diverse empirical history of the topic of efficiency. According to the two authors, within the fields of Education and Psychology, scholars have explored and emphasized the importance of efficient learning outcomes (Eysenck and Calvo, 1992; Hoffman and Schraw, 2009; Kulhavy et al., 1985; Mory, 1994; Walczyk and Griffith-Ross, 2006), "even though there is little consensus, at the present time, as to what constitutes the best conceptual definition of the most appropriate method to measure efficiency (Hoffman and Schraw, 2009; Hoffman and Spataru, 2008; Paas and Van Merriënboer, 1993; Tuovinen and Paas, 2004)..." (p. 1). We concur with this viewpoint, contending that at present, theoretical understanding and empirical and methodological contributions of efficiency are diverse in scope. Our proposition of efficiency is quite unique as we situate this concept within the context of optimal best practice (Liem et al., 2012; Martin, 2011; Phan, Ngu, Wang, Shih, Shi and Lin, 2019a; Phan et al., 2016). In other words, we consider the achievement of efficiency with reference to optimal productivity.

From the mentioned description, we consider optimal efficiency (denoted as OE) as a benchmark or a point of reference between two interrelated variables: (i) personal expenditure of time and effort, and the utilization of resources, etc., (denote as 'E') and (ii) subsequent performance outcome (denote as 'O'). Importantly, differing from existing research development (Hoffman & Schraw, 2009, 2010; Paas and Van Merriënboer, 1993; Tuovinen and Paas, 2004), we focus on the *maximization* of a person's perceived sense of efficiency. From our rationale, there are two possibilities from the perspective of an individual: expenditure outweighs the outcome accomplished (i.e.,  $E > O$ ) versus the accomplished outcome outweighs expenditure (i.e.,  $O > E$ ). Within the context of academic learning, in this case, the poignant point is concerned with a student's judgment of E and O and, more importantly, his/her understanding of both the  $E > O$  and  $O > E$  possibilities. It is non-desirable, in this case, to have a high level of E for a low level of O. It is more desirable, in contrast, to have a low level of E for a high level of O. Optimal efficiency then, from this analysis, is conceptualized as being:

a ratio by which a maximum outcome (e.g., optimal performance in mathematics learning) is achieved with the use of minimum expenditure (i.e., the least amount of expenditure that would be needed). This definition of optimal efficiency is depicted as shown:

$$\text{Optimal Efficiency (OE)} = \frac{\text{Maximum Outcome (Max - O)}}{\text{Minimum Expenditure (Min - E)}}$$

The uniqueness of optimal efficiency, as conceptualized, is related to the possibility we could 'quantify' the O-E ratio. The quantification of the O-E ratio (e.g., calculation of numerical values) is insightful, allowing

researchers to calculate, establish, and/or determine its magnitude, or strength. The term 'magnitude', recently introduced in the optimization literature (Phan and Ngu, 2017; Phan et al., 2019b), in this context, is concerned with the variation in strength of a person's perceived optimal efficiency. A high +ve magnitude value (e.g., say .70 out of 1.00) would indicate evidence of perceived optimal efficiency, whereas a -ve magnitude value (e.g., say -.15) would indicate perceived inefficiency, or a person's ineffective expenditure and usage of resources, etc. Inefficiency or, alternatively, 'sub-optimal efficiency', is conceptualized as being:

a ratio by which a minimum outcome (e.g., sub-optimal performance in mathematics learning) is achieved with the use of maximum expenditure (i.e., the most amount of expenditure that would be needed). This definition of inefficiency is depicted as shown:

$$\text{Inefficiency (IE)} = \frac{\text{Minimum Outcome (Min - O)}}{\text{Maximum Expenditure (Max - E)}}$$

Inefficiency is wasteful, indicating a person's wasteful expenditure of time and/or effort and/or usage of resources, etc. Inefficiency, reflecting a low level of optimal efficiency, is deficit for its ineffectiveness – for example, let us consider the case of a Year 11 student, Melissa, who has a low level of  $L_1$  in Algebra (e.g., the student knows how to solve linear equations with one unknown  $x$ : solve for  $x$  for  $x + 4 = -10$ ), and now wishes to achieve a level of  $L_2$  that is relatively complex (e.g., the student knows that she has the maximum capability to solve simultaneous equations with two unknowns: solve for  $x$  and  $y$  for  $2x + 5y = 10$  and  $-x + 6y = 20$ ). The question then for consideration is this: how much expenditure of time, effort, resources, etc., would be adequate before it is perceived as being inefficient? This question emphasizes what we previously termed as cost benefit analysis.

Delving into the cost benefit analysis in detail, Figure 2 provides two main variables: a student's expenditure of time, effort, and/or utilization of resources  $\times$  the achievement of  $L_2$ . The cost benefit analysis, in this case, is associated with a person's judgment and assessment of whether the benefit of his/her action (i.e., to achieve a level of optimal best) outweighs the involved cost of expenditure (e.g., expenditure of time). Referring to our previous example, the student has to decide whether the overall benefit of achieving mastery in problem solving of arithmetic with two unknowns is of value and outweighs her use of time, effort, and different types of resources. It is possible that regardless of advice from her parents, teachers, and friends, Melissa still perceives that her course of action is of significant value. Hence, this description contends that cost benefit analysis to ensure optimal efficiency may associate with personal acknowledgment, recognition, and exploration of other attributes – for example, perceived utility value (e.g., does achieving mastery of simultaneous equations with two unknowns worth excessive expenditure of time and effort?), comparison of other outcomes and their associated costs (e.g., are there other comparable topical themes in mathematics, other than simultaneous equations with two unknowns, that are more 'valuable' for learning?), and personal philosophical belief (e.g., does it matter if optimal best in solving simultaneous equations with unknowns is not achieved?).

#### 4. In-depth consideration of optimal efficiency

Optimal efficiency, from the preceding section, is concerned with a person's achievement of optimal best, assisted in this case by minimum expenditure of time, effort, and utilization of different types of resources. Understanding the significance of optimal efficiency requires examination of a personal cost benefit analysis – the weighing of productivity against associated cost(s). We contend that the concept of efficiency has a number of practical relevance, especially in relation to achievement of optimal best (Fraillon, 2004; Martin, 2011; Phan et al., 2016, 2020). Our rationalization is that achievement of optimal best is only *meaningful* and

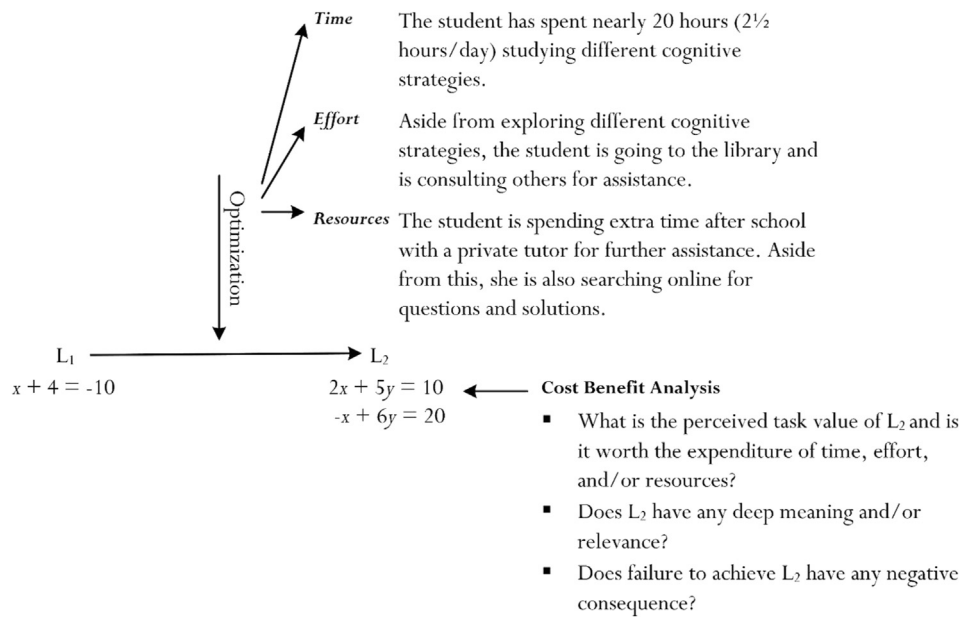


Figure 2. Cost benefit analysis.

of notable value when a person perceives its related cost (e.g., expenditure of time) as being minimal.

Our interest in perceived optimal efficiency is closely associated with the study of relevance theory (Sperber & Wilson, 1986, 1995; Wilson and Sperber, 2004), which posits that a person seeking meaning in any given communication situation will stop processing once he/she has found meaning that fits in with his/her expectation of relevance. Relevance theory, moreover, considers two major principles: (i) the cognitive principle of relevance and (ii) the communicative principle of relevance. We contend that the cognitive principle of relevance is similar to that of optimal efficiency. According to Wilson and Sperber (2004), the cognitive principle of relevance suggests that our internal cognitive processes (e.g., memory span) are guided by our consideration of efficiency – in other words, a person is more likely to attempt to allocate cognitive resources (e.g., memory, attention) that would yield maximum cognitive effect for the least processing investment. The main crux of similarity between the cognitive principle of relevance and our proposition, in this analysis, is related to the tenet of perceived efficiency in terms of investment or expenditure for maximum outcome – that one recognizes the importance of and the need for minimal investment (e.g., time, effort, resources, etc.) for the best course of action. Within the context of academic learning, relevance theory may entail a student's consideration of a learning strategy that would, in effect, minimize his/her working memory.

Perceive optimal efficiency, in accordance with our proposition, is closely aligned with the study of optimal best (Liem et al., 2012; Martin and Liem, 2010; Phan et al., 2020) and optimization (Phan and Ngu, 2020; Phan et al., 2017, 2019b). For example, our existing theory of optimization has not taken into account the perceived meaning and value of efficiency, and how this concept could account for a person's decision to strive for optimal best. We rationalize that perceived efficiency is related to personal analysis and judgment of a favorable adaptive outcome (e.g., improvement in academic performance), which importantly aligns with minimum expenditure of time, effort, resources, etc. In a similar vein, our theory of optimization has not addressed a related personal position – namely: when a person engages in high expenditure of time, effort, etc. in order to achieve optimal best in a subject matter. This consideration (i.e., high expenditure → optimal best) would, in this analysis, misalign with our proposition of optimal efficiency (i.e., optimal best/low expenditure ratio).

Over the past few years, we have used a term known as 'effective functioning' (Phan, Ngu and Alrashidi, 2018a; Phan, Ngu, Wang, Shih, Shi and Lin, 2018b; Phan et al., 2017). *Effective functioning*, from Phan et al.'s (2018a) definition, is concerned with an "individual's purposive state of organization, structured thoughts and behavioral patterns and his/her deliberate intent to succeed in life" (p. 414). Moreover, in accordance with Phan et al.'s (2017) *Framework of Achievement Bests* theory, effective functioning is a central sub-psychological component of the process of optimization. For example, proactive enactment of effective functioning would motivate a person to remain steadfast and, more importantly, to adhere to an effective study plan, regardless of the fact that it may be boring. By the same token, in their recent research, Phan et al. (2019b) introduced an interesting concept – namely, the 'quantification' of the process of optimization. According to the authors' conceptualization, the quantification of optimization involving the use of numerical values may produce concrete evidence that would attest to its magnitude – that is, for example, how 'much' optimization is needed to ensure that a person achieves optimal best in...? This theorization is interesting as it purports that optimization is not constant, but rather varies in terms of duration and strength, consequently as a result of the difference between L<sub>1</sub> and L<sub>2</sub> – that is, how much optimization is needed in order for a person to achieve L<sub>2</sub> from his/her current level of L<sub>1</sub>? According to Phan et al. (2019b), a 'large' difference between L<sub>1</sub> and L<sub>2</sub> (i.e., Δ(L<sub>1</sub>-L<sub>2</sub>)) would require an excessive amount of optimization. Thus, referring to our rationalization, the question then is whether an excessive amount of optimization to achieve L<sub>2</sub> from a low-modest level of L<sub>1</sub> would constitute evidence of efficiency?

From the preceding section, we contend that optimal efficiency is intricately linked to the process of optimization. Phan et al. (2019b) recently proposed what is known as the *index of optimization*. The quantification of the index of optimization is connoted as being the combined effects of three major pathways multiply by the difference between L<sub>1</sub> and L<sub>2</sub>: (i) *Pathway A* emphasizes the activation and enactment of psychological, educational, and psychosocial agents, which then transform and form sources of a person's positive 'energy', (ii) *Pathway B* entails the operational functioning of positive energy, which then stimulated the buoyancy of different psychological attributes (e.g., intrinsic motivation, personal resolve, mental strength, effective functioning, effort expenditure), and (iii) *Pathway C* highlights the buoyant effects of different psychological attributes (e.g., intrinsic motivation), which then arouse, improve, and sustain a person's state of functioning from T<sub>1</sub> to T<sub>2</sub>.



The index of optimization (Phan et al., 2019b) is an interesting tenet as it connotes the scientific measurement and validation of the totality of the process of optimization. A high numerical value, for example, could logically indicate the ‘excessive’ use of optimization, consequently because of the large quantitative difference between  $L_1$  and  $L_2$ . A small numerical value, in contrast, would simply imply a low level of optimization. In general, according to Phan et al. (2019b), the index of optimization reflects the degree of ‘work’ in optimization that would be required in order to facilitate an optimal state of functioning. This indication of variations in optimization suggests personal commitment in time, effort expenditure, and the utilization of different resources, pathways, means, etc. Consider the example shown in Figure 2 whereby a student's  $L_1$  consists of his cognitive capability to solve linear equations with one unknown  $x$  (e.g.,  $x + 4 = -10$ ), and he believes that he is able to solve simultaneous equations with two unknowns (i.e.,  $L_2$ ). How much optimization would he need in order to progress from  $L_1$  to  $L_2$ ? A low level of optimization may involve the activation and enactment of a psychological agent only (e.g., the use of personal self-efficacy). A high level of optimization would differ and could, in this case, involve the use of psychological (e.g., the use of personal self-efficacy), educational (e.g., exposing the student to two different instructional techniques), and psychosocial (e.g., additional peer tutorial support) agents for support.

Indeed, from the aforementioned example, it is evident that optimization is a complex process that largely relates to the cognitive complexity of  $L_2$  and, more importantly, the quantitative and qualitative difference between  $L_1$  and  $L_2$ . What is the magnitude between  $L_1$  and  $L_2$ ? A large magnitude and hence, perhaps, an extremely difficult level of  $L_2$  would require an excessive amount of optimization. This theoretical contention (i.e., the amount of optimization for usage) is interesting as it coincides with our proposition of perceived optimal efficiency. The main premise, in this analysis, is related to the amount of optimization for usage, which a person would judge and assess – for example, how much ‘optimization’ do I need before I believe that it is too much and/or too wasteful? One notable point to consider, which we discuss in the next section, is an association between optimal efficiency and the index of optimization.

#### 4.1. Relationship between optimal efficiency and index of optimization

The study of optimal best and, hence, optimization is of considerable interest, as detailed in Phan, Ngu, and Yeung's (2019b) recent conceptual article. For example, as we previously detailed, the proposed index of optimization is innovative for its depiction of the totality of the process of optimization (Phan et al., 2019b), which then serves to facilitate the achievement of optimal best (Martin, 2011; Phan et al., 2016, 2017). Conceptually, it is noteworthy to consider the relationship between optimal efficiency and the index of optimization. This proposition advances the study of optimal best by considering the extent to which we could integrate optimal efficiency and the index of optimization into one holistic theoretical model. We contend that successful enactment of the process of optimization would, in part, depend on a person's perception of efficiency versus inefficiency – in this case, perceived optimal efficiency would consist of an approved level of optimization. From this testament, let us consider side-by-side the potential relationship between perceived optimal efficiency and the index of optimization:

$$\text{Optimal Efficiency (OE)} = \frac{\text{Maximum Outcome (Max - O)}}{\text{Minimum Expenditure (Min - E)}} \Bigg| \text{Index of Optimization (IO)} = \gamma \times \Delta_{(L_2-L_1)}$$

From the above, we note that  $\gamma$ , defined as the combined effects of Pathway A, Pathway B, and Pathway C (i.e.,  $\gamma = \text{Pathway A} + \text{Pathway B}$

+ Pathway C), varies in accordance with the complexity of  $L_2$  and a person's current level of  $L_1$ . Importantly, the magnitude of  $\gamma$ , reflecting the operational nature of optimization (i.e., how much optimization is needed to optimize...?), indicates changes in a person's expenditure of time and effort (e.g., a student having to spend time seeking peer tutorial support and exploring different instructional techniques). This emphasis of  $\gamma$ , which changes, may associate with the mentioning of minimum expenditure (Min-E) – in other words, in combining both (i.e.,  $\gamma$  and Min-E), we have a case of minimum  $\gamma$  – that is, we want to minimize  $\gamma$  for the purpose of efficiency.

In addition to  $\gamma$ , we recognize that  $\Delta_{(L_2-L_1)}$  may also associate with the concept of perceived optimal efficiency. A difference between  $L_1$  and  $L_2$ , which indicates a person's internal state of flourishing (Phan et al., 2019a, 2019b), varies for various reasons/factors – for example: a person's current state of functioning,  $L_1$ , his/her motivation to achieve  $L_2$ , and the cognitive complexity of  $L_2$  (i.e., how difficult is  $L_2$ ?). What does a ‘large’ difference between  $L_1$  and  $L_2$  actually connote aside from indicating a person's experience of flourishing? Likewise, what does a ‘small’ difference between  $L_1$  and  $L_2$  connote other than to indicate a small level of flourishing? Consider the following:

- $L_1$ : knowing how to solve linear equations with one unknown:  
e.g.,  $x + 4 = 5$
- $L_{2A}$ : knowing how to solve simultaneous equations with two unknowns:  
e.g.,  $x + 2y = 10$  and  $3x - y = 5$
- $L_{2B}$ : knowing how to solve quadratic equations with two unknowns:  
e.g.,  $(x + y)^2 = 10$  and  $(x - 7y)^2 = 5$

From the above, the achievement of  $L_1$  to  $L_{2A}$  is easier to accomplish than the achievement of  $L_1$  to  $L_{2B}$  given the cognitive complexity of  $L_{2B}$  (compared to  $L_{2A}$ ). On this basis, when compared with  $L_1$  to  $L_{2A}$ , a student would need ‘more’ optimization (e.g., excessive use of time and effort to explore different instructional techniques) to accomplish  $L_1$  to  $L_{2B}$ . Having said this, however, we contend that of the two examples, accomplishment of  $L_{2B}$  from  $L_1$  would affirm evidence of a student's ‘more enriched’ experience of flourishing. Indeed, this testament espouses a student's personal decision making: does the achievement of  $L_{2B}$ , which is more difficult than that of  $L_{2A}$ , warrant the excessive expenditure of optimization that would be needed? In essence, this question emphasizes the importance of whether the end product justifies the means (i.e., the excessive use of optimization?). A large difference between  $L_1$  and  $L_2$  (e.g.,  $L_1$  and  $L_{2B}$ ) would, indeed, highlight a student's positive cognitive growth in a subject matter. In this analysis, because of increasing complex subject contents and the sequencing of learning objectives, achieving  $L_{2B}$  from  $L_1$  would require a student to first gain mastery and deep, meaningful understanding of other related knowledge (e.g., knowing how to solve quadratic equations with one unknown).

A large difference between  $L_1$  and  $L_2$  is a desirable outcome as this would reflect a person's enriching state of flourishing. A small difference, in contrast, would indicate minimal experience of flourishing. From the perspective of optimal efficiency, we could replace Maximum Outcome (Max-O) with a large difference between  $L_1$  and  $L_2$  so that, overall, we have  $\text{Max-O} \approx \text{Large } \Delta_{(L_2-L_1)}$ . In other words, from this equivalency, we contend that maximum outcome in a subject matter for accomplishment

is comparable to that of flourishing. Overall, we have the following ‘equivalency’ for optimal efficiency:

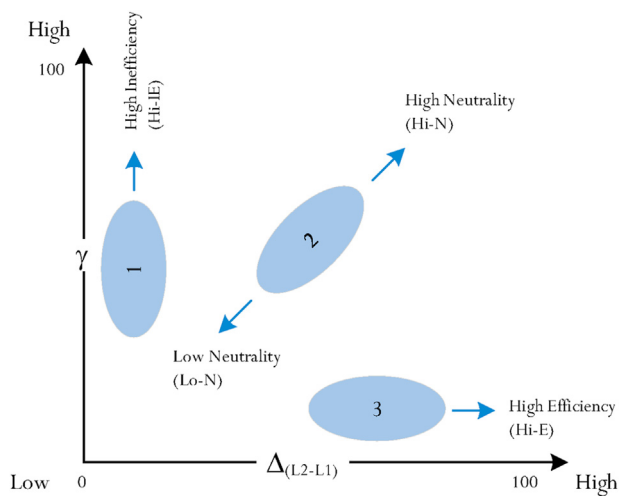


Figure 3. An example of optimal efficiency equivalency.

$$\text{Optimal Efficiency Equivalency} = \frac{\text{Large } \Delta_{(L2-L1)}}{\text{Minimum } \gamma}$$

In general, the notion of ‘Optimal Efficiency Equivalency’ may equate to the accomplishment of a large difference between  $L_1$  and  $L_2$  with the use of minimum expenditure – that is, minimum optimization. Minimum optimization, in this case, is reflected by a low numerical value of  $\gamma$ , where  $\gamma = \text{Pathway A} + \text{Pathway B} + \text{Pathway C}$ . To clarify this theorization, Figure 3 illustrates an example of optimal efficiency equivalency. The graph shows two axes with their denotations: the y-axis depicts  $\gamma$  and ranges from Low (0) to High (100), and the x-axis depicts  $\Delta_{(L2-L1)}$  and also ranges from Low (0) to High (100). There are three different regions (labelled as ‘1’, ‘2’, and ‘3’) that may shift: (i) Region 1 may shift along the y-axis from Low (0) to High (100), (ii) Region 2 may shift along the diagonal, and (iii) Region 3 may shift along the x-axis from Low (0) to High (100). From this example, Region 3 is most desirable when it moves towards the right of the x-axis (i.e., as it moves from 0 to 100), as this would indicate high efficiency (denoted as Hi-E). In contrast, Region 1 is undesirable as it moves up the y-axis (i.e., as it moves from 0 to 100), indicating a state of high inefficiency (denoted as Hi-IE). Region 2 is interesting, and we have termed this as ‘High Neutrality’ (denoted as Hi-N) and ‘Low Neutrality’ (denoted as Lo-N) as it moves up and down along the diagonal, respectively. What does the term ‘neutrality’ mean in this context? We propose that neutrality is a ‘reference point’ where a person judges and perceives that the amount of optimization needed (i.e.,  $\gamma$ ) actually equates to the amount of outcome that is accomplished (i.e.,  $\Delta_{(L2-L1)}$ ). Low Neutrality connotes a low level of  $\gamma$  as well as a small difference between  $L_1$  and  $L_2$ , whereas High Neutrality is the opposite.

Overall, then, the proposition regarding the relationship between perceived optimal efficiency and the process of optimization (Phan et al., 2017, 2019b) is logical and has daily relevance for educators, students, institutions, etc. to consider. In this analysis, the main premise for consideration entails one’s quest to achieve optimal best or, alternatively, an internal state of flourishing, using the process of optimization for assistance. “What is the appropriate amount of optimization?”, or “How much optimization?”, is a question that is related to a person’s cost-benefit analysis; for efficiency, in this case, it is poignant for a person or those involved to minimize, or not overdo, the amount of optimization that would be needed to achieve optimal best. This theoretical contention, we acknowledge, is not straightforward and, in fact, requires contemplation, judgment, decision making, and determination. Figure 3, depicting our rationalization, may be used as a guide to judge, assess, and determine three possibilities: inefficiency, neutrality, and efficiency. It is personal but, of course, it may also depend on society at large. What is

available (e.g., resource)? What is the expectation? Is there the availability of time?

Optimization, as detailed (Phan et al., 2017, 2019b), consists of the activation and enactment of different types of psychological, educational, and/or psychosocial agencies. From the perspective of schooling, for example, researchers, educators, institutions, etc. may engage in and use appropriate pedagogical practices to facilitate and optimize students’ academic learning experiences. An appropriate instructional design (e.g., concise instructions) that serves to facilitate mastery and deep learning, according to Phan et al. (2017), is perceived as being more efficient. Long, convoluted instructions with unstructured subject contents, in contrast, are more likely to facilitate the perception of inefficiency. In the next section of the article, we explore a related theory, which we believe could substantiate the topic of efficiency *versus* inefficiency: the importance of *cognitive load imposition*.

## 5. Cognitive load imposition: theoretical overview

*Cognitive load theory* (Sweller, 2010, 2012; Sweller et al., 2011), noted by scholars in the fields of Education and Psychology (Jalani and Sern, 2015; Kirschner et al., 2008; Ngu, Phan, Yeung and Chung, 2018a; Ngu, Yeung, Phan, Hong and Usop, 2018b; Seufert, 2018; Van Gog, Kester and Paas, 2011), is integral towards the facilitation and promotion of effective learning. Notably, closely aligned to the importance of information processing, cognitive load imposition plays a pivotal role in the design and development of comparable instructional designs for learners to use. In this analysis, according to the originator of cognitive load theory, John Sweller (2010, 2012), understanding of cognitive load imposition may offer insights and guide instructors to design and use appropriate pedagogies in-class to facilitate in-depth and meaningful understanding of subject matters. The potency of cognitive load theory, which we discuss in this section, is related specifically to its emphasis on the utilization of instructional designs that would, in turn, dampen a person’s cognitive load imposition. Cognitive load imposition (e.g., does a person expend too much mental effort?), in this analysis, may intricately link to ineffective instructional designs, cognitive complexities of subject matter, and student motivation.

There are *three types of cognitive load* (i.e., intrinsic, extraneous, germane), which are theorized to influence how a learner processes information from the contextual environment:

- i. *Extraneous cognitive load* occurs as a result of a suboptimal instructional design or designs. Investing cognitive resources to process element interactivity, which is impeding learning constitutes extraneous cognitive load.
- ii. *Intrinsic cognitive load* emphasizes the investment of cognitive resources to process element interactivity, which arises from the inherent complexity of the unit material. Intrinsic cognitive load depends on both the level of element interactivity of the material and a learner’s prior knowledge level. Intrinsic cognitive load decreases when the learner gains expertise in the domain; this occurs because the learner can ‘chunk’ multiple interactive elements that have been learned into schemas (Kalyuga et al., 2003).
- iii. *Germane cognitive load* is concerned with the use of working memory resources to process element interactivity, which is intrinsic to the nature of the learning material. Germane cognitive load, in this sense, depends on intrinsic cognitive load of the learning material.

The development of cognitive load theory has also consisted of the theoretical concept of *element interactivity* to explain the operational nature of the three mentioned types of cognitive load. Element interactivity, in its simplistic term, is defined as the interaction between elements within a unit learning material. An element, likewise, refers to anything that requires learning (e.g., number, symbol, concept) (Chen et al., 2017; Sweller, 2010). Element interactivity is regarded as a common factor

across intrinsic, extraneous and germane cognitive loads. The level of element interactivity imposed by the intrinsic nature of the material is regarded as intrinsic cognitive load. Intrinsic cognitive load decreases when a learner's knowledge base in the domain increases. Accordingly, intrinsic cognitive load of the material is fixed with a given level of the learner's expertise in the domain. The level of element interactivity imposed by extraneous cognitive load, consequently as a result of inappropriate instructional design is regarded as extraneous cognitive load. Extraneous cognitive load, in this sense, can be reduced via an appropriate instructional design. Germane cognitive load does not represent an independent source of cognitive load; rather, it refers to the investment of cognitive resources to process element interactivity that is essential for learning (i.e., intrinsic cognitive load).

There is research, spanning the course of three or so decades to affirm the potency of cognitive load theory within the context of academic learning (e.g., Leppink et al., 2013; Ngu et al., 2015; Saiphoo and Want, 2018; Sweller, 1994; van Merriënboer and Sluijsmans, 2008). One notable aspect of cognitive load research developments relates to the design and structuring of appropriate instructional designs for mathematics learning – for example, does the unitary approach for learning percentage impose higher cognitive load than the pictorial and/or the equation approach (Ngu et al., 2014)? In a similar vein, from a theoretical-conceptual point of view, Phan et al. (2017) recently provided an analysis to explain how different instructional designs, shaped by cognitive load imposition (Sweller, 2010, 2012; Sweller et al., 2011) could help explain the achievement of optimal best. In this analysis, according to the authors, an appropriate instructional design, shaped by perceived cognitive load imposition, could act as an educational optimizing agent, which would then facilitate a student's optimal best. Their updated work (Phan et al., 2019b), likewise, considered the importance of the potential relationship between cognitive load imposition and positive emotions and their combined effect to optimize a student's learning experience.

### 5.1. Indication of optimal efficiency: reducing cognitive load imposition

Within the context of the present article, we consider the extent to which cognitive load imposition could indeed associate with our rationalized theoretical concept of optimal efficiency. As a recap, we propose that efficiency is related to the capitalization of different types of resources (e.g., time) for the maximization of personal experiences (e.g., optimal academic best). Efficiency, in this analysis, is concerned with minimal expenditure of time, effort, and other types of resources, which then result in the achievement of optimal adaptive outcomes. By the same token then, from cognitive load theory (Sweller, 2010, 2012; Sweller et al., 2011), we emphasize a need to minimize a person's cognitive load imposition – this mentioning, in particular, aligns to the reduction and minimization of both intrinsic and extraneous cognitive load imposition. The case of germane cognitive load imposition, in contrast, is somewhat different as this type of cognitive load does produce positive yields for a learner.

The quest to minimize both intrinsic and extraneous cognitive load imposition is of interest in terms of achieving efficiency. From this understanding, we rationalize that optimal efficiency may inversely associate with a low level of intrinsic and/or extraneous cognitive load imposition. From the perspectives of motivation and positive learning experiences, a low level of intrinsic and extraneous cognitive load imposition is more effective as it imposes less cognitive constraint, time, and/or effort onto a learner. A high level of intrinsic and/or extraneous cognitive load imposition, in this case, would indicate and reflect a corresponding high level of mental effort, time, and cognitive processing to decipher, comprehend, and interpret the information for learning.

#### 5.1.1. Extraneous cognitive load and perceived efficiency

In terms of extraneous cognitive load, for example, let us consider how a sub-optimal instructional design for mathematics learning could relate to the notion of perceived efficiency/inefficiency. What is a sub-optimal instruction? In mathematics learning, educators and researchers have noted that there are some instructional designs that involve a high level of elementary interactivity, which would then impose high extraneous cognitive load (Sweller et al., 2011). For example, there are two contrasting instructional methods to help students learn coordinate geometry problems: the *split-attention* method and the *integrated* method. Any student for that matter can choose to use one or both instructional methods to learn this type of coordinate geometry problems, depending on his/her personal preference, experience, and/or perceived relevance. However, from the perspective of cognitive load imposition (Sweller, 2010, 2012; Sweller et al., 2011) and considering our proposed concept of optimal efficiency, we need to analyze, compare, and understand the two instructional methods.

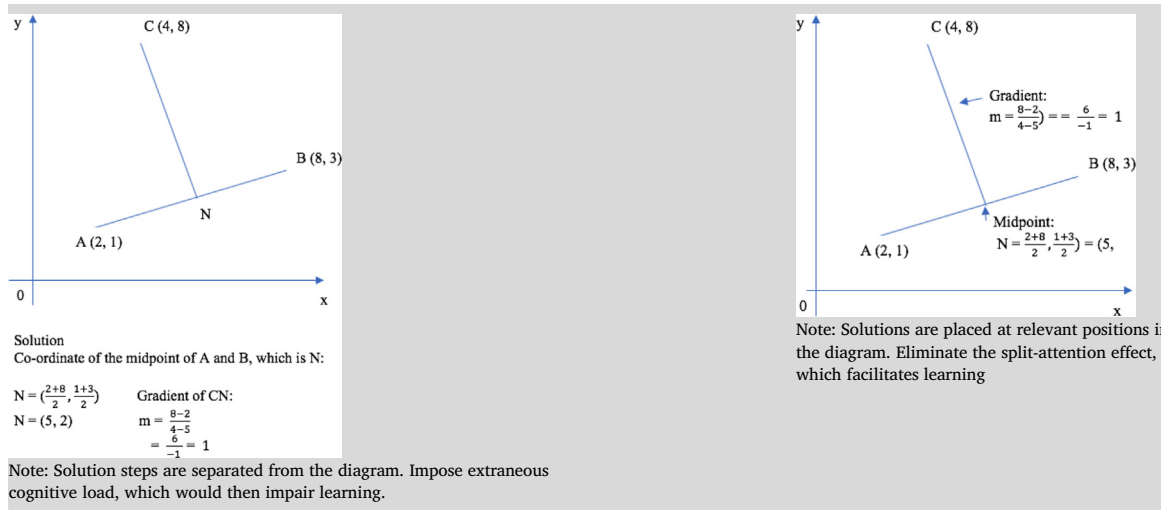
Cognitive load theory (Sweller, 2010, 2012) provides theoretical grounding to discern and recognize that the split-attention method and the integrated method differ in terms of element interactivity. The amount of element interactivity, in this case, imposes a certain level of extraneous cognitive load imposition onto a learner. In a prior study, Sweller et al. (1990) provided an in-depth analysis to explain the difference between the two instructional approaches. Table 1 provides a summation of Sweller et al.'s (1990) reasoning and research findings as to why the integrated method is more effective than that of the split-attention method.

For the integrated method, the solution steps are placed at relevant positions in the diagram to facilitate comprehension and processing. For the split-attention method, in contrast, the solutions steps are separated from the diagram. Switching from the solution steps in order to attend to the diagram would require the need for a student to hold the solution steps in his/her working memory, and at the same time to search and process element interactivity that is related to relevant referents of the diagram. Consequently, this split-attention effect unnecessarily increases the level of interactivity, which is unrelated to learning and thus would increase extraneous cognitive load. In other words, of the two methods, the split-attention method has additional elements associated with the integration of essential information from two sources for comprehension and processing.

From Table 1, in terms of comparison, there is a lower level of element interactivity for the integrated method, which then would associate with low extraneous cognitive load imposition. This brief example affirms the importance of element interactivity, especially in terms of its explanatory account of how it could reduce or increase extraneous cognitive load. From this understanding, it is preferable to have instructional designs with fewer elements for comprehension and processing. Moreover, when we compare with our proposed theorization of optimal efficiency, it would be sufficed to have instructional designs that consist of low level of element interactivity (e.g., the integrated method). In essence, we can equate effective/ineffective instructional designs with both Minimum Expenditure (Min-E) and the notation of  $\gamma$  – in this case, we want  $\gamma$  to be minimum.

In sum, referring to our proposed concept of optimal efficiency, we propose the use of appropriate instructional methods. One example of this consideration may entail the incorporation of solution steps in a diagram of the instructional method. Separating solution steps from the diagram may result in a split-attention effect, giving rise to high extraneous cognitive load (i.e., because of high level of element interactivity involved) for a learner. As such, as shown in Table 1, the integrated method has a lower level of element interactivity (Sweller et al., 2011). In this analysis, from our rationalization, a lower level of element interactivity does not require excessive expenditure of effort and/or time for processing. A higher level of element interactivity, as in the case of the

**Table 1.** The split-attention method and the integrated method.



split-attention method, in contrast, would ‘overburden’ a student’s processing of information as there are more elements for processing.

**5.1.2. Intrinsic cognitive load and perceived efficiency**

Intrinsic cognitive load differs somewhat from extraneous cognitive load for its emphasis on the inherent complexity of the subject content of the unit material (Sweller, 2010). The imposition of intrinsic cognitive load on a learner, in this case, is dependent on the level of element interactivity of the subject content. Moreover, as we described, intrinsic cognitive load is related to a person’s prior knowledge level – how much schemas in the area of Algebra, for example, does a Year 11 student have? From this understanding, how could we reduce a student’s intrinsic cognitive load? There are two considerations in this matter: (i) reducing the complexity of the unit material, and (ii) improve a learner’s prior knowledge by helping him/her gain expertise through scaffolding, peer tutorial support, etc.

It is interesting to note that intrinsic cognitive load is related to the importance of subject contents and the acquiring of knowledge. An instructor’s professional practice may involve the structuring of learning objectives that reflect increasing complexity. A learning objective that places emphasis on the ‘memorization of subject content’ (e.g., “... be able to describe the importance of Sigmund Freud”) is relatively simple when compared with a learning objective that emphasizes on ‘analytical reasoning’ (e.g., “... be able to critically analyze Sigmund Freud’s theory and its relevance to...”). What this means then, of course, is that simple learning objectives and subject contents are easier to comprehend and understand. However, from our point of view, this assertion contradicts with our emphasis on the striving of optimal best (Martin, 2011; Phan et al., 2016). That is, from the perspective of optimal best, we would want students to seek and strive for the maximization of their academic

capabilities. In other words, we want to encourage students to learn and acquire complex knowledge of subject matters in school. It is somewhat contradictory, in this analysis, to focus on and to foster the learning of ‘easier’ subject contents.

In Mathematics Education, sequencing linear equations hierarchically is a pedagogical strategy that may assist students to learn complex linear equations by building on their prior knowledge of simpler linear equation (Ngu and Phan, 2016b). As indicated in Table 2, the complex linear equation (i.e., the multi-step equation) has more solution steps than the simpler linear equation (i.e., the two-step equation) and thus, the former has a higher level of element interactivity. Moreover, the multi-step linear equation shares a subset of the solution steps of the two-step equation. Specifically, from Table 2, Lines 1, 2, 3, 4, and 5 of the two-step equation share similar structural elements as Lines 3, 4, 5, 6, and 7 of the multi-step equation. Therefore, if a learner has prior knowledge of the two-step equation then, by this account, the learning of the multi-step equation would simply entail the learning of Lines 1 and 2 only. Accordingly, capitalizing on prior knowledge of the two-step equation would help reduce the level of element interactivity involved (and hence, the intrinsic cognitive load of the multi-step equation) and thus, alleviating the strain on working memory load. From this rationalization, it is important for teachers to strengthen students’ prior knowledge of simpler linear equations before introducing complex linear equations.

It is important for us to consider the potential association between intrinsic cognitive load and the notion of optimal efficiency for learning. It is warranted that investment of cognitive and different types of resources (e.g., appropriate instructional methods) is used to process element interactivity, especially in light of one’s limited knowledge and/or the complexity of the subject material. Having said this, however, it is acknowledged that for optimal efficiency purposes, we want to minimize a person’s investment and/or use of resources with the aid of appropriate instructional methods designed from the cognitive load perspective (Sweller, 1994, 2012; Sweller et al., 2011). A minimum level of  $\gamma$ , in this context, would limit a student’s academic learning experience, consequently because of his/her modest level of  $L_1$  and the complexity of  $L_2$ . Minimization of  $\gamma$  is not desirable as this would not produce the means to assist a student in his/her quest to acquire in-depth knowledge and understanding of the subject content. With this in mind, using appropriate instructional methods would reduce intrinsic cognitive load, which then could motivate and facilitate students to master and achieve  $L_2$ . Hence from this rationalization, for optimal efficiency equivalency, we acknowledge that the level of  $\gamma$  would correspond with  $\Delta(L_2-L_1)$ . In other

**Table 2.** Two-step and multi-step equations.

Two-Step Equation				Multi-Step Equation			
Line 1	$7x - 3$	$=$	$11$	Line 1	$5x - 9$	$=$	$3x + 7$
Line 2	$+3$	$=$	$+3$	Line 2	$-3x$	$=$	$-3x$
Line 3	$7x$	$=$	$14$	Line 3	$2x - 9$	$=$	$7$
Line 4	$\div 7$	$=$	$\div 7$	Line 4	$+9$	$=$	$+9$
Line 5	$x$	$=$	$2$	Line 5	$2x$	$=$	$16$
Line 6				Line 6	$\div 2$	$=$	$\div 2$
Line 7				Line 7	$x$	$=$	$8$



words, for example, a low level of  $\gamma$  would indicate minimum achievement of  $L_2$ .

From the preceding paragraph, how then do we ensure optimal efficiency with reference to high intrinsic cognitive load? In this context, we contend there is a need to consider minimizing the complexity of  $L_2$  and, likewise, to emphasize the saliency of a student's prior learning experience. The emphasis here is that expenditure of time, effort, and other types of resources may rest with the instructor himself/herself. Overall, the reduction of intrinsic cognitive load imposition would depend on an instructor's expectation of complexity of knowledge for students to acquire. Moreover, a concerted effort is needed to ensure that capitalization of a student's prior knowledge is made, which would then connect his/her existing understanding (e.g.,  $L_1$ ) with the subject content for learning (i.e.,  $L_2$ ).

5.1.3. Germane cognitive load and perceived efficiency

Of the three types of cognitive load (Sweller, 2010; Sweller et al., 2011), germane cognitive load is the most interesting type for its potential positivity. Recall that germane cognitive load refers to the use of working memory resources to deal with intrinsic cognitive load of learning materials. Germane cognitive load investment, in this sense, is positive as this would improve a person's learning experience in a subject matter. In other words, investment in cognitive resources to process element interactivity is beneficial as this would assist a student to acquire knowledge and in-depth understanding of a subject content. For example, a student who wishes to master and achieve a state of  $L_2$  in Algebra (e.g., learning quadratic equations with two unknowns) would need to expend some time and effort, as well as to invest in the utilization of different resources in order to comprehend the unit material. With this in mind, we argue that it is somewhat premature and detrimental for a student to not invest in time, effort, etc.

More specifically, in relation to learning of percentage problems, for example, we can increase germane cognitive load by providing practice problems with varied problem contexts. Table 3 shows a worked example, a practice problem with low variability, and a practice problem with high variability. With reference to the worked example, a practice problem with low variability only differs by changing the values of the problem (e.g., 20% versus 30%). In contrast, the practice problem with high variability differs by changing not only the values (e.g., 20% versus 8%) but also the problem contexts (e.g., money versus injuries). Therefore, the number of interacting elements of the practice problem with high variability is more than that of the practice problem with low variability. In other words, the practice problem with high variability imposes a higher level of element interactivity than the practice problem with low variability. Accordingly, in the context of classroom learning, we would expect students to increase germane cognitive load in order to address the practice problems with high variability. Exposure to practice problems with high variability, however, would lead to the acquisition of

more flexible and sophisticated schemas. This would assist and enable students to solve transfer problems (i.e., non-standard problems) that differ from the standard problems in terms of problem contexts.

Germane cognitive load, indeed, is positive and may serve to produce achievement-related yields (e.g., improved comprehension and interest in the subject content). Nevertheless, within the context of academic learning, students need to have sufficient working memory resources to learn additional interacting elements, consequently as a result of an increase in the variability of the practice problems. Otherwise, an increase in variability practice, which would increase the investment of germane cognitive load would have a negative rather than positive learning effect.

Importantly of course, from our point of view, germane cognitive load is closely related to a person's objective, determination, motivation, and inner desire – for example, what is it that a student wishes to accomplish in a particular theme, module, etc.? From the perspective of optimal best (Martin, 2011; Martin and Liem, 2010; Phan et al., 2016), we postulate a close alignment between intrinsic cognitive load (e.g., the complexity of a subject content) and germane cognitive load (e.g., a student's intention to achieve mastery). We contend that it is possible to align germane cognitive load with the proposition of optimal efficiency equivalency. Minimizing  $\gamma$ , in this case, would reflect a reduction in germane cognitive load, whereas an increase in  $\gamma$  would indicate the opposite.

Germane cognitive load is not necessarily a deterrence for learning, as this investment in cognitive resources is viewed as a means towards a student achieving his/her optimal best. Having said this, from our proposition, optimal efficiency emphasizes a minimization in  $\gamma$  – in this case, the reduction of germane cognitive load. At the same time, of course, with a reduction in germane cognitive load, we also seek to maximize a student's learning experience in a subject matter (i.e.,  $L_2$ ). In other words, situating cognitive load theory (Sweller, 2010; Sweller et al., 2011) within the context of our proposition, we have an interesting but conflicting positioning: the maximization of  $L_2$  with an attempt to reduce germane cognitive load. This conceptualization, from our point of view, differs from the theoretical tenets of germane cognitive load – that is, an increase in investment of cognitive resources would benefit and indeed help improve student understanding.

Like intrinsic cognitive load, a focus on germane cognitive load is of considerable interest given its nature and purpose, as well as its related outcome(s). Ultimately, in conjunction with our proposition, we want to ensure that the level of  $\gamma$  is minimized. What is significant though, when we compare both intrinsic and germane cognitive load with extraneous cognitive load is that the latter is more 'negative' – for example, an inappropriate instruction has too many irrelevant interacting elements for processing, which would result in high cognitive expenditure.

Table 3. Difference between low variability and high variability practice problems.

Percentage problems	
<b>Worked example</b>	
I spent 20% of my money on a notebook which cost \$1,200. How much money did I have before the purchase?	
<i>Solution:</i>	
Let $x$ be my money	
$20\% \times x = \$1,200$	
$x = \$1,200 \div 20\%$	
$x = \$6,000$	
<b>A practice problem with low variability</b>	<b>A practice problem with high variability</b>
I spent 30% of my money on a smartphone which cost \$9.00. How much money did I have before the purchase?	The 120 injuries occurring on sports fields in December represented 8% of the year's total injuries on sports fields. How many injuries occurred in the year?
<i>Solution:</i>	<i>Solution:</i>
Let $x$ be my money	Let $x$ be my money
$30\% \times x = \$9,00$	$8\% \times x = 120$
$x = \$900 \div 30\%$	$x = 120 \div 8\%$
$x = \$3,000$	$x = 1,500$ injuries

## 6. Implications and caveats

The effective utilization of human and/or intellectual capital in educational settings is a significant feat for consideration. Over the past four decades, as Hoffman and Schraw (2010) noted, scholars in different discipline areas have studied and have provided diverse interpretations of the concept of efficiency. Studying efficiency is important for various reasons – for example, “to gain a better understanding of the time and effort needed to master core academic competencies such as literacy skills, mathematics, science, and writing” (p. 1). We concur with Hoffman and Schraw's (2010) viewpoint, especially with reference to the importance of the expenditure of time and effort in the conceptualization and measurement of efficiency. Our theoretical contribution is similar to existing research development but, in this case, we focus on the importance of optimal best (Liem et al., 2012; Martin, 2011; Phan et al., 2016, 2019a) as a point of reference. In this analysis, our interest seeks to understand the *ultimate cost* that would entail in the achievement of an optimal state of functioning (e.g., cognitive functioning).

It is poignant to acknowledge and mention that our proposition of the concept of optimal efficiency, as detailed in the preceding sections, is seminal and that, to date, empirical research development with reference to optimal best is still in its stage of infancy. Conceptual analysis of an educational or a psychological concept is largely derived from a researcher's use of theoretical psychology, philosophical reasoning, and/or personal intuition. It is sufficed to argue then, that acceptance and/or continuing interest in optimal efficiency would require some form of empirical validation using robust methodological designs. At present, however, we acknowledge that the theoretical tenets of optimal efficiency, situated within the framework of cognitive load imposition (Sweller, 1994, 2012; Sweller et al., 2011) are simply philosophical and may be subject to analytical critique and scrutiny. In the latter section, upon our discussion of educational implications, we explore a number of caveats and future directions for advancement.

### 6.1. Educational implication for consideration

In summary, our use of the term optimal efficiency is closely aligned to a person's maximization of his/her state of functioning. For example, in the context of academic learning, the maximization of a student's cognitive state of cognitive functioning may involve his/her in-depth understanding and mastery of the Solar System. This mastery and meaningful learning of the Solar System, reflecting the student's optimal best may consist of his/her ability to create a 3D-computer simulation for an in-class presentation. By this account, such optimal best achievement would require some form of expenditure of time and/or effort, as well as the student's utilization of different types of resources (e.g., the purchasing of an expensive software package to assist with the creation of a 3D model). The importance of efficiency, which is a desirable outcome, would call for a focus on the balance and cost ratio. One possibility to address this matter, as we previously explored, is to consider the impact of cognitive load imposition (Sweller, 2010, 2012; Sweller et al., 2011). From the perspective of cognitive load, which of the three types of cognitive load would we take into account? In terms of perceived efficiency, would an educator: (i) make a concerted attempt to reduce extraneous and/or intrinsic cognitive load? and/or (ii) make a concerted attempt to facilitate and strengthen germane cognitive load? In this analysis, a question for consideration may entail an educator's comparison of the following: investment in effort and time, say, to ensure that an instructional design is perceived as being effective *versus* investment in effort and time to reduce a person's investment in cognitive resources.

Perceived efficiency entails self-regulation, determination, and a minimization in personal and/or collective investment of time, effort,

and different types of resources. It is, in this sense, up to a person in terms of his/her perception and judgment in the ratio between outcome and expenditure. Is the intended outcome worth my time and effort? Am I investing and, possibly, ‘wasting’ too much time with this learning task? Is this intended outcome worth ignoring? These sample questions, from our point of view, are individualized, relying on the involved person to consider. Having said this, of course, it is plausible to argue that judgment of perceived efficiency may also involve others. For example, within the context of schooling and academic learning, a teacher may play a prominent role in helping to determine and/or establish an index of efficiency. A teacher may use his/her personal experiences, professional understanding, life wisdom, etc. to make a judgment regarding the ratio between outcome and expenditure. This testament may involve, say, a teacher's judgment and assessment as to whether it is logical to purchase a software package for learning – in this case, the teacher may consider weighing and judging the cost of the software package against the intended learning outcome (e.g. is it worth it?).

Indeed, aside from theoretical interest, the concept of perceived optimal efficiency has a number of potential daily relevance, which may apply to different areas of education, teaching, and learning. Our interest, in this case, is related to the applicability of perceived optimal efficiency with reference to a person's achievement of optimal best (Liem et al., 2012; Phan et al., 2016, 2019b, 2020), using cognitive load imposition (Sweller, 1994, 2010; Sweller et al., 2011) as a basis. Our proposition, as detailed, considers the importance of cognitive load imposition as a point of reference to assist an educator's cost-benefit analysis.

In terms of facilitating accurate and sound perceived efficiency for students, what would be some viable and/or feasible options for consideration? In other words, in terms of academic learning, what are some possible means, pathways, strategies, etc. that would ensure and/or instill and facilitate a perception of optimal efficiency? Given the limitation of space, our focus of examination considers the following:

- **Appropriate instructional design.** It is insightful to consider the use of appropriate instructional designs that would, in this case, facilitate effective (rather than ineffective) learning. Our previous research development (e.g., Ngu and Phan, 2016a; Ngu et al., 2018b; Ngu et al., 2014), as well as other cognitive load researchers' studies (Sweller et al., 2011), as cited in the preceding sections, place emphasis on designs of instructions and pedagogical practices to reduce cognitive load imposition. Interacting elements arising from inappropriate instructional designs, as we have noted, would impose extraneous cognitive load. For example, it would be ideal to place solution steps at relevant locations in a diagram within a particular instruction in order to minimize potential extraneous cognitive load imposition (Sweller et al., 1990). In a similar vein, where appropriate, it would be meaningful to capitalize on prior knowledge of simpler linear equations (Note: as an example) so as to reduce the intrinsic cognitive load of learning complex linear equations (Note: as an example) (Ngu and Phan, 2016a). The provision of variability practice, likewise, would encourage the investment of germane cognitive load, leading to in-depth processing of learning percentage problems (Note: as an example).
- **Subject contents and Learning outcomes.** *Self-efficacy theory* (Bandura, 1977, 1997) acknowledges the potency of prior experience, or enactive learning experience, as a source of information (e.g., a student's prior success may be used to formulate his/her state of self-efficacy) (Pajares et al., 2007; Phan, 2012; Usher and Pajares, 2006). It is therefore a beneficial feat to structure subject contents and learning outcomes (LOs) that capitalize on students' prior knowledge and experiences. This consideration, we contend, may involve teaching sequencing that seeks to emphasize on a student's current

level of knowledge (i.e.,  $L_1$ ) in a subject matter for the purpose of scaffolding – hence, a focus on intrinsic cognitive load imposition. A student's well-versed current level of best practice (i.e.,  $L_1$ ), or knowledge, which he/she could use for subsequent learning experience would, in this sense, help reduce cognitive load investment. In other words, it is important for a teacher to highlight the constructive alignment between a student's existing knowledge,  $L_1$  (e.g., simpler linear equations), and the intended learning outcome for accomplishment,  $L_2$  (e.g., complex linear equations).

By the same token, it is poignant for educators to pay attention to the complexity of subject content (e.g., how difficult is the subject content (e.g., the theme of Algebra) for learning?) as this would closely associate with expenditure of time, effort, resources, etc. In the absence of a constructive alignment between  $L_1$  and  $L_2$ , extreme difficulties in content of a subject would require excessive cost of time, effort, and/or use of resources for comprehension, which in turn could demotivate a student. This testament, from our point of view, suggests that capitalization of prior knowledge, understanding, and skills is prevalent in terms of countering the cognitive complexity of a subject matter, as well as helping to maximize efficiency in learning.

- **Authenticity, Interest, and Intellectual curiosity.** Authenticity, or real-life application, is an important focus as this element would complement potential cognitive load investment, scaffolding a student to learn new unit materials. Subject contents that have real-life relevance may, in this analysis, initiate and stimulate personal interest, intrinsic motivation, and/or intellectual curiosity, all of which may reduce a student's expenditure of time and effort. Personal interest in *Buddhist mindfulness* (Hanh, 1976; Master Sheng Yen, 2010; Yeshe and Rinpoche, 1976), for example, could instill intrinsic motivation for learning, which would then help alleviate a student's need to invest time, effort, etc. In other words, from our rationalization, personal interest, motivation, and intellectual curiosity may operate as 'expenditures' in place of, or in tandem with, the investment of time, effort, resources, etc. This point is significant in terms of recognizing the potentiality for intellectual curiosity, personal interest, and motivation (Hidi, 1990; Puamau, 1999; Zhu et al., 2009), via means of promotion of authenticity and daily relevance to facilitate optimal efficiency by helping to negate excessive expenditure of time, effort, etc.
- **The use of self-regulation.** Unlike the use of cognitive load imposition (Sweller, 1994, 2010; Sweller et al., 2011), we contend that personal self-regulation (Pintrich, 2000; Seufert, 2018; Zimmerman, 2002, 2008) could also feature in the judgment and assessment of optimal efficiency. Self-regulation, in this case, may accentuate and direct students to specific study habits and/or personal study-related strategies that could maximize academic learning experiences. Planning, organization, monitoring, and/or personal evaluation (e.g., ... what am I doing wrong?) are effective strategies that could operate to minimize a student's wasteful expenditure of time, effort, resources, etc (Effeney et al., 2013; Phan and Ngu, 2019; Rotgans and Schmidt, 2012). A student who is cognizant and well-organized, for example, is more likely to have additional time to engage in related endeavors, which could in turn help maximize his/her learning experiences. An 'unregulated' student (e.g., absent-minded with no sense of direction), in contrast, is more inclined to engage in wasteful expenditure of time, effort, etc. in order to comprehend and understand the subject content. Self-regulation indeed is an achievement-related approach that could operate in tandem with personal cost (e.g., expenditure of time) in order to facilitate the maximization in efficiency.

The preceding sections suggest there are numerous educational considerations (e.g., the use of appropriate instructional designs) that could coincide with the topic of perceived optimal efficiency. In terms of

academic learning, for example, there are pathways and/or means, which we could use to maximize efficiency in performance outcomes and learning experiences. The above examples (e.g., the use of self-regulation), indeed, showcase some interesting proposals for educators to pursue and develop. An important premise from our examination is that pathways and/or means to help facilitate efficiency are themselves comparative (e.g., consider the development and use of an appropriate instructional design *versus* the use of self-regulation), requiring personal judgment, assessment, and logical reasoning (e.g., which pathway and/or mean should I use?). In other words, from this account, a person would still have to expend a certain amount of time, effort, resources, etc. in order to determine which pathway and/or mean is most adequate for the purpose of efficiency.

## 6.2. Caveats and further development

Despite our recommendations and, more importantly, our conceptualization of optimal efficiency, there are a number of caveats, which we have identified that could advance further development. Foremost from this consideration is the fact that our proposition, to date, has been both theoretical and methodological, requiring in this case for robust scientific validation. Is it possible, in this analysis, for us to appropriately quantify a numerical value for the concept of optimal efficiency? This consideration, we contend, is similar to the quantification of optimization (e.g., the index of optimization) (Phan et al., 2019b), which so far is theoretical and does not have an empirical solution. From this emphasis, we propose for the design of an appropriate methodological design that could enable the measurement and assessment of the concept of optimal efficiency.

The question of whether we could, indeed, quantify and depict a numerical value for optimal efficiency is largely contestable. One questionable tenet, which is worth exploring is related to the consistency in applicability of the Max-O and Min-E relationship for different situations and contexts. Researchers need to consider a methodological strategy or strategies that could measure optimal efficiency. Does a quantifiable derivative of optimal efficiency sound logical, given that there are different contexts with distinctive interpretations? At this stage of development, we contend, our derivative of optimal efficiency is philosophical and, as such, there are pervasive questions remaining that require further analyses. For example, in terms of academic learning, we need to acknowledge and recognize that the *contextual nature* of a subject matter in itself could yield differing outcomes and interpretations (Becher, 1987, 1994). *Intellectual categorizations* of different academic subjects, according to Becher's (1987, 1994) theorization, indicate four distinctive categories: 'pure theoretical' (e.g., Calculus), 'soft pure theoretical' (e.g., Economics), 'hard applied' (e.g., Carpentry), and 'soft applied' (e.g., IT). This subject distinction in this sense emphasizes differing cognitive difficulties, resulting in different motivational beliefs, students' engagement of learning approaches, epistemologies, etc (Becher, 1994; Phan et al., 2017; Smith and Miller, 2005). Hence, from this differentiation, we argue that expenditure of effort, time spent, and the utilization of resources, etc. would differ between pure theoretical, soft pure theoretical, hard applied, and soft applied subjects. With this in mind, a student may exhibit different levels of efficiency depending on perceived interest, value, and/or cognitive difficulty of the learning task at hand.

Hence, as noted from the preceding discussion, establishing consistency of perception and interpretation of efficiency may not be easily achieved. A numerical value of optimal efficiency of .70 for Calculus, a pure theoretical subject, may not have the same connotation for Carpentry, a hard applied subject. One major difference between Calculus and Carpentry, in this case, is related to a learner's perceived cognitive complexity – in this sense, it is much more difficult to learn and comprehensively understand Calculus than Carpentry, which in this case

	Calculus	Carpentry
<i>Time spent</i>	10 hours	10 hours
<i>Expenditure of effort</i>	Revise the content learned everyday	Revise the content learned everyday
<i>Resources used</i>	Two 30-minute YouTube videos	Two 30-minute YouTube videos
<i>Accomplishment</i>	Able to achieve moderate understanding of <i>continuous function</i> . Still has a long way to go in terms of understanding of other topics, such as <i>derivative, fundamental theorem of calculus, integral, limit, non-standard analysis, and partial derivative</i> .	Able to achieve in-depth understanding of how to make a rocking chair with red-gum timber. At the same time, achieve in-depth understanding of different types of timber – for example: <i>bamboo, birch, cedar, cherry, and glulam</i> .
<i>Perceived Efficiency</i>	.20*	.50*

Note: \* For the purpose of discussion, consider the index of optimal efficiency of .20 for Calculus and .50 for Carpentry. This calculation is made on the basis that accomplishment, O, is judged as being more or higher than that of Calculus.

is practical and has life-related relevance. With this in mind, it is plausible to suggest that comparable levels of time and effort expenditure, as well as utilization of resources could result in *different accomplishments* for the two subjects – that is, as shown here, as an example:

The above example, for the purpose of discussion, shows that E (i.e., expenditure of effort, time spent, utilization of resources) is identical for both subjects (e.g., the learner spends 10 hours studying Calculus and 10 hours studying Carpentry). Having said this, however, perception of accomplishment, O, is perceived as being different for the two subjects – in this case, from the learner's point of view, Carpentry is judged as being 'more accomplished' than that of Calculus. With this judgment, the ratio of outcome *versus* expenditure is more efficient for Carpentry (i.e.,  $O > E$ ), and less efficient for Calculus (i.e.,  $E > O$ ). Having said this, some learners may view Calculus with a strong sense of subjective value (e.g., importance for future career plan) (Eccles, 2005; Eccles et al., 1983), resulting in their personal resolve, self-determination, and motivation to expend time and effort, as well as the utilization of resources regardless of their cost. From this emphasis, a learner's judgment of subjective value for Calculus, etc., may serve to warrant and justify the cost involved.

A possible line of inquiry for consideration may consist of the validation of a non-experimental approach or an experimental approach, which could measure the incremental differences of optimal efficiency for different subject disciplines (e.g., Calculus *versus* Carpentry). An in-class intervention, for example, could discern and identify distinctive cost patterns for different learning situations. In a similar vein, by means of a questionnaire, we could seek feedback from university students regarding the amount of time that they spend, as well as the different types of resources that they use for learning different subjects in a semester. By referencing this information against their end-of-semester academic grades, we could perhaps derive some form of consistency or inconsistency regarding the relationship between cost and benefits. This methodological approach could provide a basis for us to address the aforementioned issue of consistent interpretation and meaning of optimal efficiency. For example, it is plausible that comparing students' personal reflections of their study habits (e.g., amount of time spent), we could clarify the relationship between perceived cognitive complexity of a subject matter for accomplishment (e.g., Calculus) and  $\gamma$ , which is defined as the combined effects of Pathway A, Pathway B, and Pathway C (i.e.,  $\gamma = \text{Pathway A} + \text{Pathway B} + \text{Pathway C}$ ) in the process of optimization.

Finally, our attempt to incorporate cognitive load theory (Sweller, 2010, 2012) within the discussion of the proposed concept of optimal efficiency may provide grounding for further conceptual and/or empirical development. It would be of interest for researchers to construct a methodological design that could measure, assess, and gauge into the relationship between cognitive load imposition (i.e.,  $\gamma$ ) and the maximization of optimal best (Phan et al., 2017). The focus of inquiry then, in this analysis, is to potentially explore how reduction in cognitive load imposition (e.g., extraneous cognitive load) could maximize a student's

experience of optimal best. Discussions in the preceding section have provided a few points for consideration. For example, researchers may inquire into the positive effect of a reduction in extraneous cognitive load, via means of improving a specific instructional design for learning on the achievement of optimal best in a subject matter. It would be of theoretical significance in this case for researchers to devise, if possible, a methodological procedure that could enable some form of 'consistency' in terms of calculation between optimal best (i.e., Max-O) and, say, extraneous cognitive load imposition (i.e., Min-E).

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