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Research article

Improved estimation of population distribution function using twofold auxiliary information under simple random sampling

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ABSTRACT

In this article, our main aim is to suggest enhanced families of estimators for estimating the population distribution function (DF) using twofold auxiliary evidence within the framework of simple random sampling. Numerical analysis is performed on four different actual data sets. The precision of the estimators is further investigated exhausting a simulation study. As equated with existing estimators, the suggested families of estimators have minimum mean square error (MSE) and higher percentage relative efficiency (PRE). The succeeding recommended family of estimators outperforms the first family of estimators across all data sets. These are positive indicators of its performance. The theoretical result shows that the recommended family of estimators performs better than the existing estimators. The extent of improvement in efficiency is noteworthy, indicating the superiority of the suggested estimators in terms of minimum MSE.

1. Introduction

The utilization of auxiliary information in survey sampling can significantly enhance the exactness of population mean or total estimates. By incorporating auxiliary information effectively, researchers can develop sampling strategies that outperform those that do not take advantage of such information. The estimation of population mean is of great importance across diverse fields of study. Researchers can create more efficient sample techniques than those based purely on the study data by including auxiliary information in the sampling design and estimation process. This approach is known as model-assisted or model-based estimation. When auxiliary information is available, it can be used in various ways to enhance the estimation process. One common approach is using regression models to establish a relationship among the study and the auxiliary variable. These models can then be utilized to predict the values of the study variable for non-sampled units based on their observed auxiliary information. The forecast values, and the study variable values for the sampled units are used to compute the population mean or total estimate. More reliable estimates can be obtained with

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the help of supplementary data by decreasing sampling error and maximizing productivity gains. It can also help reduce non-sampling, such as coverage error or measurement errors by incorporating relevant information into the estimation process. Overall, all proper utilization of auxiliary information in survey sampling can lead to more robust and accurate estimates, making it a valuable tool in various disciplines of study. Many investigators have recommended numerous estimators by satisfactorily adjusting the auxiliary variables.

For estimating various finite population parameters using different sampling designs, there are many estimators in the literature. To calculate the proportion of a set of numbers that are less than or equal to a given threshold, it is necessary to have a firm grasp of the finite DF. A physician may wonder, for instance, how many patients derive at least half of their daily energy from cholesterol in their diet. A soil scientist is curious to know how deep the water table is in a particular land. The percentage of individuals living in poverty is a topic of interest to the government. Distribution functions play a significant role in influencing the approaches employed in the design and investigation of survey, which are an ever-developing area. Several interesting topics of future research in DF within the framework of sampling have emerged as a consequence of technological development and the emergence of new obstacles. These research emphases have the potential to improve the precision, effectiveness, and stability of sampling procedures and their uses. The potential for further study into distribution functions in survey sampling is immense. Improvements in this area may have far-reaching implications for the design, administration, and analysis of surveys in a variety of fields, allowing for the generation of more reliable and informative findings. Survey statisticians rely on population DF estimation in these situations. Some significant work exists in literature including [1] creating a novel estimator for determining the mean of a population [2]. recommended estimating the DF of a population using supplementary information based on stratified sample design [3]. estimating the mean using outliers from a PPS [4]. discussed multivariate supplementary data is proposed for estimating the DF of a population [5]. estimating the population mean with two supplementary variables in a stratified random sample [6]. investigated use of supplementary data in simple random sampling [7]. investigated estimation DF using additional information [8]. discussed on an enhanced category of estimators obtained using stratified sampling [9]. discussed some new families of estimators. The [10] proposed simple random sampling with a predictive estimator of the logarithmic type [11]. estimating population mean with a Stratified Ranked Sample [12]. using survey data to estimate DF [13]. discussed characteristics of estimators for the probability density function of a finite population [14]. recommended estimators of the finite population DF from both design and model perspectives are compared and contrasted [15]. discussed distribution based estimators [16]. recommended estimating the DF of a finite population [17]. discussed a modified estimator under stratified random sampling, we develop estimators of the finite population distribution function [18]. suggested difference-type estimators based on auxiliary evidence [19]. recommended exponential estimator for the DF using auxiliary variables [20]. The authors in [21] discussed estimation of a symmetric distribution function. The proposed optimal sample design for ratio-based mean population estimation utilizing supplementary data [22]. selected strategies for enhanced ratio and regression estimators [23]. recommended the estimation of mean using auxiliary variables under SRS [24]. suggested effective strategy for estimating the mean of a finite population using two supplementary variables [25]. recommended the median ranked set sampling for population mean [26]. suggested estimators for missing data [27]. The authors in [28] suggested an improved estimator for estimating population mean under simple random sampling. recommended a class of estimators for the DF of a finite population [29]. recommended rank-based simultaneous sampling for estimating DF [30]. discussed ranked set sampling for the DF. The [31,32] discussed robust ratios to estimate the mean of a finite population.

Section 1, provides an introduction to the topic of the article. It presents the background information, states purposes, and frameworks the significance of the study. The introduction typically includes a literature review to provide context and identify gaps in existing knowledge. Section 2, describe the materials used in the study and the methodology employed. The materials may include equipment, instruments, software, or any other resources used in the research. The methods section outlines the procedures followed to collect data, conduct experiments, or analyze information. In Section 3, we have reviewed some of the latest estimators for the estimation of population DF. Section 4, presents the two improved proposed classes of estimators. It explains the theoretical basis or rationale behind the estimators and provides a detailed description of their implementation. The section may include equations, algorithms or statistical models used in the estimation process. Section 5, includes data description of four actual data to regulate the efficiency of the recommended estimator. Section 6, comprises data description of simulated data to determine the competence of the recommended estimator. Section 7, analyzes and interprets the consequences acquired from the proposed estimators. It compares the findings with previous studies, explores any limitations or assumptions made in the research, and discusses the result's implications. Section 8, summarizes the study main findings. It restates the research problem, highlights the key results and discusses the implications of the study findings. The conclusion may also suggest areas for further investigation or practical application of the research. It provides closure to the article by summarizing the main points and their significance.

2. Materials and methods

Let a population Q_i $(i = 1,2,\ldots,N)$ consists of N recognized and divergent units. A sample of size n is chosen from Q using *SRSWOR* to assess a population DF. Let Y_i and X_i be the values of the features of the study variable and the auxiliary variable. The indicator function of the study and the auxiliary variables are signified by $I(Y_i \leq \mathrm{Y})$ and $I(X_i \leq \mathrm{X})$, respectively. Let $\left[F(y) = \frac{\sum_{i=1}^{N} I(Y_i \leq \mathrm{Y})}{N}\right]$, $F(x) = \left[\frac{\sum_{i=1}^{N} I(X_i \leq \mathrm{X})}{N}\right]$, and $[\widehat{F}\,(\mathrm{y}) = \frac{\sum_{i=1}^{n} I(Y_i \leq \mathrm{Y})}{n},\ \widehat{F}\,(\mathrm{x}) = \frac{\sum_{i=1}^{n} I(X_i \leq \mathrm{X})}{n}]$ are the population DF conforming to the sample DF.

Let $\xi_0 = \frac{\widehat{F}_y - F_y}{F_y}$, $\xi_1 = \frac{\widehat{F}_x - F_x}{F_x}$, $\xi_2 = \frac{\widehat{\overline{X}} - \overline{X}}{\overline{X}}$, $\xi_3 = \frac{\widehat{\overline{R}}_x - \overline{R}_x}{\overline{R}_x}$ such that

$E(\xi_0^2) = \lambda\ C^2_{F(y)} = \theta_{2000}$, $E(\xi_1^2) = \lambda\ C^2_{F(x)} = \theta_{0200}$, $E(\xi_2^2) = \lambda\ C^2_{\overline{X}} = \theta_{0020}$, $E(\xi_3^2) = \lambda\ C^2_{\overline{R}_x} = \theta_{0002}$,

$E(\xi_0\xi_1) = \lambda\ \rho_{(F(y)F(x))} C_{F(y)} C_{F(x)} = \theta_{1100}$, $E(\xi_0\xi_2) = \lambda\ \rho_{(F(y)\overline{X})} C_{F(y)} C_{\overline{X}} = \theta_{1010}$, $E(\xi_0\xi_3) = \lambda\ \rho_{(F(y)\overline{R}_x)} C_{F(y)} C_{\overline{R}_x} = \theta_{1001}$, $E(\xi_1\xi_2) = \lambda\ \rho_{(F(x)\overline{X})} C_{F(y)} C_{\overline{X}} = \theta_{0110}$, $E(\xi_1\xi_3) = \lambda\ \rho_{(F(x)\overline{R}_x)} C_{F(x)} C_{\overline{R}_x}$.

$C^2_{F(y)} = \frac{S^2_{F(y)}}{F^2(Y)}$, $C^2_{F(x)} = \frac{S^2_{F(x)}}{F^2(X)}$, $C^2_{\overline{X}} = \frac{S^2_{\overline{R}_x}}{\overline{X}^2}$, $C^2_{\overline{X}} = \frac{S^2_{\overline{R}_x}}{\overline{R}_x{}^2}$, .where.

$S^2_{F(y)} = \frac{\sum_{i=1}^{N}\{I(Y_i \le Y) - F(y)\}^2}{N-1}$, $S^2_{F(x)} = \frac{\sum_{i=1}^{N}\{I(X_i \le X) - F(x)\}^2}{N-1}$, $S^2_{\overline{X}} = \frac{\sum_{i=1}^{N} I\ \{X_i - \overline{X}\}^2}{N-1}$, $S^2_{F(x)} = \frac{\sum_{i=1}^{N}\{I(R_{xi} \le \overline{R}_x) - (X_i - \overline{X})\}^2}{N-1}$, $\rho_{F(y)F(x)} = \sum_{i=1}^{n}[\{I(Y_i \le Y) - F(y)\}\{I(X_i \le X) - F(x)\}]$,

$\rho_{F(y)\overline{X}} = \sum_{i=1}^{n}[\{I(Y_i \le Y) - F(y)\}\{X_i - \overline{X}\}]$, $\rho_{F(x)\overline{X}} = \sum_{i=1}^{n}[\{I(X_i \le X) - F(x)\}\{X_i - \overline{X}\}]$.

$\rho_{F(y)\overline{R}_x} = \sum_{i=1}^{n}[\{I(Y_i \le Y) - F(y)\}\{R_{Xi} - \overline{R}_x\}]$, $\rho_{F(x)\overline{R}_x} = \sum_{i=1}^{n}[\{I(X_i \le X) - F(x)\}\{R_{Xi} - \overline{R}_x\}]$, $\lambda = (\frac{1}{n} - \frac{1}{N})$.

The main purpose of this study is given by.

1. In this paper, the prime goal is to estimate the population DF exhausting twofold auxiliary variable under simple random sampling.
2. The bias and MSE of the mentioned estimators, are ensuing up to the first degree approximation.
3. The efficiency of the mentioned estimator is emphasized over the use of real data and simulation study from numerous fields.

3. Literature review

In this section, we review various existing counterparts of DF.

1. The conventional estimator of (y), given in equation (1):

$$\widehat{F}_{usual} = \frac{1}{n}\sum_{i=1}^{n} I(Y_i \le Y). \tag{1}$$

The variance of $\widehat{F}_{usual}$, given in equation (2):

$$\mathrm{Var}(\widehat{F}_{usual}) = F^2(y)\theta_{2000}. \tag{2}$$

2 [33]. presented ratio estimator, given in equation (3):

$$\widehat{F}(\mathrm{R}) = \widehat{F}(y)\left(\frac{F(x)}{\widehat{F}(x)}\right). \tag{3}$$

The bias of $\widehat{F}(\mathrm{R})$, is given by:

$$\mathrm{Bias}(\widehat{F}(\mathrm{R})) \cong F(y)(\theta_{0200} - \theta_{1100}),$$

The MSE of $\widehat{F}(\mathrm{R})$, given in equation (4):

$$\mathrm{MSE}(\widehat{F}(\mathrm{R})) \cong F^2(y)(\theta_{2000} + \theta_{0200} - 2\theta_{1100}). \tag{4}$$

3 [34]. recommended product estimator, given in equation (5):

$$\widehat{F}(\mathrm{P}) = \widehat{F}(y)\left(\frac{\widehat{F}(x)}{F(x)}\right). \tag{5}$$

The properties of $\widehat{F}(\mathrm{P})$, are given by

$$\mathrm{Bias}(\widehat{F}(\mathrm{P})) = F(y)\theta_{1100},$$

The MSE of $\widehat{F}(\mathrm{P})$, given in equation (6):

$$\mathrm{MSE}(\widehat{F}(\mathrm{P})) \cong F^2(y)(\theta_{2000} + \theta_{0200} + 2\theta_{1100}) \tag{6}$$

4. The regression estimator, given in equation (7):

$$\widehat{F}_{Reg} = [\widehat{F}(y) + \psi_1(F(x) - \widehat{F}(x))], \tag{7}$$

The optimal value of ψ_1, given in equation (8):

$$\psi_1 = (F(y)\theta_{1100}) / (F(x)\theta_{0200}) = \psi_{1(\mathrm{opt})} \tag{8}$$

The variance of $\widehat{F}_{Reg}$, given in equation (9):

$$\mathrm{Var}_{\min}\left(\widehat{F}_{Reg}\right) = F^2(y)\theta_{2000}\left(1 - \rho^2_{F(y)F(x)}\right). \tag{9}$$

5 [22]. difference type etimator, given in equation (10):

$$\widehat{F}_{RD} = \Psi_2\,\widehat{F}(y) + \Psi_3(F(x) - \widehat{F}(x)) \tag{10}$$

The ideal values of Ψ_2 and Ψ_3 are:

$$\Psi_{2\,(\mathrm{opt})} = \frac{1}{1 + \lambda C^2_{F_y}\left(1 - \rho^2_{F(y)F(x)}\right)},\ \Psi_{3\,(\mathrm{opt})} = \frac{F(t_y)\rho_{F(y)F(x)}C_{F_y}}{F(t_x)C_{F_x}\{1 + \lambda C^2_{F_y}(1 - \rho^2_{F(y)F(x)})\}}.$$

The properties of $\widehat{F}_{RD}$, are given by:

$$\mathrm{Bias}(\widehat{F}_{RD}) = F(\mathrm{y})(\Psi_2 - 1),$$

The MSE of $\widehat{F}_{RD}$, given in euation (11):

$$\mathrm{MSE}_{\min}(\widehat{F}_{RD}) \cong \left[\frac{\lambda F^2(\mathrm{y})C^2_{F_y}\left(1 - \rho^2_{F(y)F(x)}\right)}{1 + \lambda C^2_{F_y}\left(1 - \rho^2_{F(y)F(x)}\right)}\right] \tag{11}$$

6 [35]. recommended exponential type estimators, given in euation (12) and (13):

$$\widehat{F}_{BTR} = \widehat{F}(y)\exp\left(\frac{F(x) - \widehat{F}(x)}{F(x) + \widehat{F}(x)}\right) \tag{12}$$

$$\widehat{F}_{BTP} = \widehat{F}(y)\exp\left(\frac{\widehat{F}(x - F(x))}{\widehat{F}(x + F(x)}\right) \tag{13}$$

The bias and MSE of $\widehat{F}_{BTR}$ and $\widehat{F}_{BTP}$, are given by:

$$\mathrm{Bias}(\widehat{F}_{BTR}) \cong F(y)\left(\frac{3}{8}\theta_{0200} - \frac{1}{2}\theta_{1100}\right),$$

The MSE of $\widehat{F}_{BTR}$, given in euation (14):

$$\mathrm{MSE}(\widehat{F}_{BTR}) \cong \frac{F^2(y)}{4}(4\theta_{2000} + \theta_{0200} - 4\theta_{1100}) \tag{14}$$

$$\mathrm{Bias}(\widehat{F}_{BTP}) \cong F(y)\left(\frac{1}{2}\theta_{1100} - \frac{1}{8}\theta_{0200}\right),$$

The MSE of $\widehat{F}_{BTR}$, given in euation (15):

$$\mathrm{MSE}(\widehat{F}_{BTP}) \cong \frac{F^2(y)}{4}(4\theta_{2000} + \Theta_{0200} + 4\theta_{1100}), \tag{15}$$

7 [29]. recommended enhanced exponential estimator, given in euation (16):

$$\widehat{F}_{Singh} = \widehat{F}(y)\exp\left(\frac{\alpha(F(x) - \widehat{F}(x))}{\alpha(F(x) + \widehat{F}(x)) + 2\beta}\right), \tag{16}$$

where α and β are known population parameters of the auxiliary variable.

$$\mathrm{Bias}\left(\widehat{F}_{Singh}\right) \cong F(y)\left(\frac{3}{8}\vartheta^2\theta_{0200} - \frac{1}{2}\vartheta\theta_{1100}\right),$$

The MSE of $\widehat{F}_{Singh}$, given in euation (17):

$$\mathrm{MSE}\left(\widehat{F}_{Singh}\right) \cong \frac{F^2(y)}{4}\left(4\theta_{2000} + \vartheta^2\theta_{0200} - 4\vartheta\theta_{1100}\right), \tag{17}$$

where $\vartheta = \left[\frac{\alpha F(x)}{\alpha F(x) + \beta}\right]$.

8 [36]. recommended improvede estimator, given in equation (18):

$$\widehat{F}_{Grover} = \{\Psi_4 \widehat{F}(y) + \Psi_5 (F(x) - \widehat{F}(y))\} \exp\left(\frac{\alpha(F(x) - \widehat{F}(x))}{\alpha(F(x) + \widehat{F}(x)) + 2\beta}\right), \tag{18}$$

The bias of $\widehat{F}_{Grover}$, given in equation (19):

$$\text{Bias}(\widehat{F}_{Grover}) \cong F(y)(\Psi_4 - 1) + \frac{3}{8}\Theta^2 \Psi_4 F(y) + \frac{1}{2}\Theta \Psi_5 F(x)\theta_{0200} - \frac{1}{2}\Theta F(y)\theta_{1100}, \tag{19}$$

The MSE of $\widehat{F}_{Grover}$, given in equation (20):

$$\begin{aligned}\text{MSE}(\widehat{F}_{Grover}) \cong\ & \Psi_5^2 F^2(x)\theta_{0200} + \Psi_4^2 F^2(y)\theta_{2000} + 2\Theta\Psi_4\Psi_5 F(y)F(x)\theta_{0200} \\ & -2\Psi_4\Psi_5 F(y)F(x)\theta_{1100} + F^2(y) - 2\Psi_4 F^2(y) + \Theta\Psi_4^2 F^2(y) \\ & +\Psi_4 F^2(y)\theta_{1100} - \Theta\Psi_5 F(y)F(x)\theta_{0200} - 2\Theta\Psi_4^2 F^2(y)\theta_{1100} \\ & -\frac{3}{4}\Theta^2\Psi_4 F^2(y)\theta_{0200} + \Theta^2\Psi_4^2 F^2(y)\theta_{0200}.\end{aligned} \tag{20}$$

The least values of Ψ_4 and Ψ_5 are given as:

$$\Psi_{4\,(\text{opt})} = \frac{\theta_{0200}(\Theta^2\theta_{0200} - 8)}{8(-\theta_{2000}\theta_{0200} + \theta_{1100}^2 - \theta_{0200})},$$

and

$$\Psi_{5\,(\text{opt})} = \frac{F(y)\left(\Theta^3\theta_{0200}^2 - \Theta^2\theta_{0200}\theta_{1100} + 4\Theta\theta_{2000}\theta_{0200} - 4\Theta\theta_{1100}^2 - 4\Theta\theta_{0200} + 8\theta_{1100}\right)}{8F(x)\left(\theta_{2000}\theta_{0200} - \theta_{1100}^2 + \theta_{0200}\right)}.$$

The least MSE of $\widehat{F}_{Grover}$, given in equation (21):

$$\text{MSE}_{\min}(\widehat{F}_{Grover}) \cong \text{Var}_{\min}(\widehat{F}_{Reg}) - \frac{F^2(y)\left(\Theta^2\theta_{0200}^2 - 8\theta_{1100}^2 + 8\theta_{0200}\theta_{2000}\right)^2}{64\theta_{0200}^2\left\{1 + \theta_{2000}\left(1 - \rho_{F(y)F(x)}^2\right)\right\}}, \tag{21}$$

4. Suggested efficient family of estimators

Auxiliary variables can be used to progress estimators accuracy throughout the design and estimation processes. The estimators use the correlation concerning the auxiliary variable and the study variable to produce more precise estimates by factoring in additional information from the auxiliary variable. When the auxiliary variable is connected with the research variable, the auxiliary variable's order becomes significant. The order reflects the ordering of the values in the auxiliary variable relative to each other. If the correlation exists, the order of the auxiliary variable can serve as a substitution for the rank of the study variable, providing a valuable indicator for estimation purposes. These estimators exhibit more excellent elasticity and efficiency compared to existing estimators. Elasticity states the capability of the estimators to adapt and adjust to different circumstances or conditions, allowing for more flexibility in estimation. Efficiency, on the other hand, pertains to the estimators ability to produce estimates with smaller variances, thereby reducing the sampling error. Delightful incentive from Ref. [16], we propose two improved families of estimators that are more effective as likened to existing estimators. These are given as.

4.1. *First family of estimator*

In the first class of estimators, we used the auxiliary variable and sample mean of the auxiliary variable. The first propose class of estimators, given in equation (22):

$$\widehat{F}_{(sohaib1)} = [\Psi_{12}\widehat{F}(y) + \Psi_{13}(F(x) - \widehat{F}(x)) + \Psi_{14}(\overline{X} - \widehat{\overline{X}})] \exp\left(\frac{\alpha(F(x) - \widehat{F}(x))}{\alpha(F(x) + \widehat{F}(x)) + (\alpha - 1)\alpha F(x) + (\alpha + 1)\beta}\right), \tag{22}$$

where Ψ_i ($i = 12,13,14$) are constant.

Family members of the recommended generalized estimators (**see Appendix C**).

Solving equation (22), we get equation (23):

$$\widehat{F}_{(sohaib1)} = [\Psi_{12}F(y)(1 + \xi_0) - \Psi_{13}F(x)\xi_1 - \Psi_{14}\overline{X}\xi_2]\left[1 - \Theta\xi_1 + \frac{3}{8}\Theta^2\xi_1^2 + \ldots\right], \tag{23}$$

where $\theta = \frac{\alpha F(x)}{[(\alpha+1)(\alpha F(x)+\beta)]}$, is a known quantity.

The expanded form given in equation (24):

$$\widehat{F}_{(sohaib1)} - F(y) = -F(y) + F(y)\psi_{12} + F(y)\psi_{12}\xi_0 - F(y)\psi_{12}\theta\xi_1 - F(y)\psi_{12}\theta\xi_0\xi_1 + \frac{3}{2}F(y)\psi_{12}\theta^2\xi_1^2 - \psi_{13}F(x)\xi_1 - \psi_{14}\overline{X}\xi_2 + \psi_{13}F(x)\xi_1^2 + \psi_{14}\overline{X}\xi_1\xi_2 \tag{24}$$

The bias of $\widehat{F}_{(sohaib1)}$, is given in equation (25):

$$\mathrm{Bias}\left(\widehat{F}_{(sohaib1)}\right) = F(y)\left[(\psi_{12}-1) + \lambda\psi_{12}\theta C_{F(x)}\left(\frac{3}{2}\theta C_{F(x)} - \rho_{F(y)F(x)}C_{F(y)}\right) + \lambda\theta C_{F(x)}\left(\psi_{13}R_1C_{\overline{X}} + \psi_{14}R_2C_{\overline{X}}\rho_{F(x)\overline{X}}\right)\right], \tag{25}$$

where $R_1 = \frac{F(x)}{F(y)}$, $R_2 = \frac{\overline{X}}{F(y)}$.

Squaring equation (25), we get equations (26) and (27):

$$\begin{aligned}\left(\widehat{F}_{(sohaib1)} - F(y)\right)^2 &= F^2(y) + F^2(y)\psi_{12}^2 + F^2(y)\psi_{12}^2\xi_0^2 + 4F^2(y)\psi_{12}^2\xi_1^2 + 4F^2(x)\psi_{13}^2\xi_1^2 + \overline{X}^2\psi_{14}^2\xi_2^2 \\ &- 4F^2(y)\psi_{12}\theta\xi_1\xi_2 + 4F(y)F(x)\psi_{12}\psi_{13}\theta\xi_1^2 + 4F(y)\overline{X}\psi_{12}\psi_{13}\theta\xi_1\xi_2 - 2F(y)F(x)\psi_{12}\psi_{13}\xi_0\xi_1 - 2F(y)\overline{X}\psi_{12}\psi_{14}\xi_0\xi_2 \\ &+ 2F(y)F(x)\overline{X}\psi_{13}\psi_{14}\xi_1\xi_2 - 2F^2(y)\psi_{12} - 3F^2(y)\psi_{12}\theta^2\xi_1^2 - 2F^2(y)\psi_{12}\theta\xi_0\xi_1 - \left\{2F(y)F(x)\psi_{13}\theta\xi_1^2\right\} - \left\{2F(y)F(x)\psi_{14}\theta\psi_{14}\xi_1\xi_2\right\}\end{aligned} \tag{26}$$

$$\begin{aligned}\mathrm{MSE}\left(\widehat{F}_{(sohaib1)}\right) &= F^2(y)\Big[1 + \psi_{12}^2\Big\{1 + \lambda\Big(C_{F(y)}^2 + 4\theta\Big(\theta C_{F(x)}^2 - \rho_{F(y)F(x)}C_{F(y)}\Big)\Big) - \lambda R_1\psi_{13}C_{F(x)}^2(2\theta - R_1\psi_{13})\Big\} + \lambda R_2^2\psi_{14}^2C_{\overline{X}}^2 \\ &- 2\lambda R_1\psi_{13}\psi_{14}\Big(\rho_{F(y)F(x)}C_{F(y)}C_{F(x)} - 2\theta C_{F(x)}^2\Big) - 2\lambda R_2\psi_{12}\psi_{14}C_{\overline{X}}\Big(\rho_{F(y)\overline{X}}C_{F(y)}C_{\overline{X}} + 2\rho_{F(y)\overline{X}}\theta C_{F(x)} + 2\lambda R_1R_2\psi_{13}\psi_{14}\rho_{F(x)\overline{X}}C_{F(x)}C_{\overline{X}} \\ &- 2\lambda R_1\psi_{14}\theta\rho_{F(x)\overline{X}}C_{F(x)}C_{\overline{X}} - 2\psi_{12}\Big\{1 + \lambda\theta C_{F(x)}\Big(\frac{3}{2}\theta C_{F(x)} - \rho_{F(y)F(x)}\theta C_{F(y)}\Big)\Big\}\Big)\Big]\end{aligned} \tag{27}$$

The optimum values of ψ_{12}, ψ_{13} and ψ_{14}, are given by:

$$\psi_{12} = \left[\frac{1-\frac{1}{2}\theta^2 C_{F(x)}^2}{1+\lambda C_{F(y)}^2(1-\mho_{F(y).F(x)\overline{X}}^2)}\right],$$

$$\psi_{13} = \left(\frac{F(y)\left[\lambda\theta^3\left(\left\{-1+\rho_{F(x)\overline{X}}^2 - C_{F(y)}\left(1-\frac{1}{2}\lambda\theta^2 C_{F(x)}^2\right)\right\}\{\rho_{F(y)F(x)} - \rho_{F(x)\overline{X}}\rho_{F(y)\overline{X}}\} + \theta C_{F(x)}\{-1+\rho_{F(x)\overline{X}}^2\}\right)\left(-1+\lambda C_{F(x)}^2(1-\mho_{F(y).F(x)\overline{X}}^2)\right)\right]}{F(y)C_{F(x)}\{-1+\rho_{F(x)\overline{X}}^2\}(1+\lambda C_{F(x)}^2(1-\mho_{F(y).F(x)\overline{X}}^2))}\right).$$

$$\psi_{14} = \left(\frac{F(y)\left(1-\frac{1}{2}\lambda\theta^2 C_{F(x)}^2\right)C_{F(y)}\{\rho_{F(y)\overline{X}}\rho_{F(y)F(x)} - \rho_{F(x)\overline{X}}\}}{\overline{R}_x C_{\overline{R}_x}\{-1+\rho_{F(x)\overline{X}}^2\}[1+\lambda C_{F(x)}^2\{1-\mho_{F(y).F(x)\overline{X}}^2\}]}\right).$$

Putting the values of ψ_{12}, ψ_{13} and ψ_{14} in (27), we get equation (28):

$$\mathrm{MSE}\left(\widehat{F}_{(sohaib1)}\right) = \lambda F^2(y)\left[C_{F(y)}^2\left\{\frac{1-\mho_{F(y).F(x)\overline{X}}^2\Big\} - \frac{1}{2}\lambda\theta^4 C_{F(x)}^4 - \lambda\theta^2 C_{F(x)}^2\left\{1-\mho_{F(y).F(x)\overline{X}}^2\right\}\Big]}{1+\lambda C_{F(y)}^2\left\{1-\mho_{F(y).F(x)\overline{X}}^2\right\}}\right.\right., \tag{28}$$

where $\mho_{F(y).F(x)\overline{R}_x}^2 = \frac{\rho_{F(y)F(x)}^2 + \rho_{F(y)\overline{X}}^2 - 2\rho_{F(y)F(x)}\rho_{F(y)\overline{X}}\rho_{F(x)\overline{X}}}{1-\rho_{F(x)\overline{X}}^2}$.

4.2. Second family of estimator

In the second class of estimators, we included the auxiliary variable's rank as an additional auxiliary variable. The second proposed estimator, which is given in equation (29):

$$\widehat{F}_{(sohaib2)} = [\psi_{15}\widehat{F}(y) + \psi_{16}(F(x) - \widehat{F}(x)) + \psi_{17}(\overline{R}_x - \widehat{\overline{R}}_x)]\exp\left(\frac{\alpha(F(x) - \widehat{F}(x))}{\alpha(F(x) + \widehat{F}(x)) + (\alpha-1)\alpha F(x) + (\alpha+1)\beta}\right) \tag{29}$$

where ψ_i $(i = 15,16,17)$ are constants.

Solving equation (29), we get equation (30):

$$\widehat{F}_{(sohaib2)} = [\psi_{15}F(y)(1+\xi_0) - \psi_{16}F(x)\xi_2 - \psi_{17}\overline{R}_x\xi_3]\left[1 - \theta\xi_1 + \frac{3}{2}\theta^2\xi_1^2 + \ldots\right], \tag{30}$$

where $\theta = \frac{\alpha F(x)}{[(\alpha+1)(\alpha F(x)+\beta)]}$, is a known quantity.

The expanded form of (30), is given in equation (31):

$$\widehat{F}_{(sohaib2)} - F(y) = \left[- F(y) + F(y)\Psi_{15} + F(y)\Psi_{15}\xi_0 + \frac{5}{8} F(y)\Psi_{15}\xi_1^2 - \Psi_{16}\overline{R}_x\xi_2 + F(x)\Psi_{17}\Theta^2\xi_1^2 \right] \quad (31)$$

The bias of $\widehat{F}_{(sohaib2)}$, which is given in equation (32):

$$\text{Bias}\left(\widehat{F}_{(sohaib2)}\right) = F(y)\left[\Psi_{15} - 1 + \frac{5}{8}\lambda\Psi_{15}\ C_{F(x)}^2 + \lambda\Psi_{17}\overline{R}_x\Theta\ C_{F(x)}^2\right] \quad (32)$$

Squaring equation (31), we get equations (33) and (34):

$$\left(\widehat{F}_{(sohaib2)} - F(y)\right)^2 = F^2(y) + F^2(y)\Psi_{15}^2 + F^2(y)\Psi_{15}^2\xi_0^2 + \frac{5}{4}F^2(y)\Psi_{15}^2\xi_1^2 + \overline{R}_x^2\Psi_{16}^2\xi_2^2 + F^2(x)\Psi_{17}^2\xi_1^2$$

$$-\{2\ F(y)\overline{R}_x\Psi_{15}\Psi_{16}\xi_0\xi_2\} + \{2\ F(y)F(x)\Psi_{15}\Psi_{17}\Theta\xi_1^2\} - \{2\ F(y)F(x)\Psi_{15}\Psi_{17}\xi_0\xi_1\} + 2\ F(x)\overline{R}_x\Psi_{16}\Psi_{17}\xi_1\xi_2 - 2\ F^2(y)\Psi_{15} - \frac{5}{4}F^2(y)\xi_1^2$$
$$- 2\ F(y)F(x)\Psi_{17}\Theta\xi_1^2 \quad (33)$$

$$\text{MSE}\left(\widehat{F}_{(sohaib2)}\right) = F^2(y)\left[1 + \Psi_{15}^2\left\{1 + \lambda\left(C_{F(y)}^2 + \frac{5}{4}C_{F(x)}^2 \right) + \lambda\overline{R}_x^2\Psi_{16}^2\ C_{\overline{R}_x}^2 {}_{-}\lambda R_1\Psi_{17}\ C_{F(x)}^2(2\Theta + R_1\Psi_{17})\right\} - 2R_2\Psi_{15}\Psi_{16}\rho_{F(y)\overline{R}_x}C_{F(y)}C_{\overline{R}_x}\right.$$
$$\left. - 2\lambda R_1\Psi_{15}\Psi_{17}\left(\rho_{F(y)F(x)}C_{F(y)}C_{F(x)} - \Theta\ C_{F(x)}^2\right) + 2\lambda R_1R_2\Psi_{16}\Psi_{17}\rho_{F(y)\overline{R}_x}C_{F(x)}C_{\overline{R}_x}\right], \quad (34)$$

where $R_3 = \frac{\overline{R}_x}{F(y)}$..

The optimum values of Ψ_{15}, Ψ_{16} and Ψ_{17}, are given by:

$$\Psi_{15} = \left[\frac{1-\frac{1}{2}\Theta^2\ C_{F(x)}^2}{1+\lambda\ C_{F(y)}^2(1-\mho_{F(y).F(x)\overline{R}_x}^2)}\right],$$

$$\Psi_{16} = \left[\frac{F(y)\left[\lambda\Theta^3\left(-1+\rho_{F(x)\overline{R}_x}^2 - C_{F(y)}\left(1-\frac{1}{2}\lambda\Theta^2\ C_{F(x)}^2\right)\left(\rho_{F(y)F(x)} - \rho_{F(x)\overline{X}}\rho_{F(y)\overline{R}_x}\right) + \Theta C_{F(x)}\{-1+\rho_{F(x)\overline{R}_x}^2\}\right)\left(-1+\lambda\ C_{F(x)}^2\{1-\mho_{F(y).F(x)\overline{R}_x}^2\}\right)\right]}{F(y)C_{F(x)}\{-1+\rho_{F(x)\overline{R}_x}^2\}(1+\lambda\ C_{F(x)}^2\{1-\mho_{F(y).F(x)\overline{R}_x}^2\})}\right].$$

$$\Psi_{17} = \left[\frac{F(y)\left(1-\frac{1}{2}\lambda\Theta^2\ C_{F(x)}^2\right)C_{F(y)}(\rho_{F(y)\overline{X}}\rho_{F(y)F(x)} - \rho_{F(x)\overline{X}})}{\overline{R}_xC_{\overline{R}_x}\{-1+\rho_{F(x)\overline{R}_x}^2\}[1+\lambda\ C_{F(x)}^2\{1-\mho_{F(y).F(x)\overline{R}_x}^2\}]}\right].$$

Using Ψ_{15}, Ψ_{16} and Ψ_{17}, in (34), we get equation (35):

$$\text{MSE}\left(\widehat{F}_{(sohaib2)}\right) = \left[\frac{\lambda F^2(y)\left[C_{F(y)}^2\left\{1 - \mho_{F(y).F(x)\overline{R}_x}^2\right\} - \frac{1}{2}\lambda\Theta^4\ C_{F(x)}^4 - \lambda\Theta^2\ C_{F(x)}^2\left\{1 - \mho_{F(y).F(x)\overline{R}_x}^2\right\}\right]}{1 + \lambda\ C_{F(y)}^2\left\{1 - \mho_{F(y).F(x)\overline{R}_x}^2\right\}}\right], \quad (35)$$

where

$$\mho_{F(y).F(x)\overline{R}_x}^2 = \left[\frac{\rho_{F(y)F(x)}^2 + \rho_{F(y)\overline{R}_x}^2 - 2\rho_{F(y)F(x)}\rho_{F(y)\overline{R}_x}\rho_{F(x)\overline{R}_x}}{1 - \rho_{F(x)\overline{R}_x}^2}\right].$$

5. Numerical results

We use four sets of real data to determine how well estimators work in PRE. The subsequent equation is used to calculate the PRE.

$$\text{PRE} = \frac{V(\widehat{F}_{usual})}{MSE(\widehat{F}_i)} \times 100,$$

where (i= $\widehat{F}$(R), $\widehat{F}$(P), …, $\widehat{F}_{(sohaib1)}$, $\widehat{F}_{(sohaib2)}$).

Population-I [Source: [37]]:

Y= Number of fish in 1995,

X = number of fish in 1994, R_x = rank of X variable

$N = 69$, $n = 10$, $F(y) = 0.4927536$, $S_{F(y)}^2 = 0.2536223$, $C_{F(y)} = 1.022032$, $S_{F(x)}^2 = 0.4927536$, $C_{F(x)} = 1.022032$, $\rho_{F(y)F(x)} = 0.9601401$, $\rho_{F(y)\overline{R}_x} = 0.7851271$, $\rho_{F(x)\overline{R}_x} = 0.7401045$.

Population-II: [Source: PBS (2021–2022)]

Y= Covid-19 test performed in 2021.

X = Covid-19 inveterate circumstances in 2021.

R_x = rank of X variable

$N = 228$, $n = 50$, $F(y) = 0.5$, $S_{F(y)}^2 = 0.2511013$, $C_{F(y)} = 1.022$, $S_{F(x)}^2 = 0.2511013$, $C_{F(x)} = 1.0022$, $\rho_{F(y)F(x)} = 0.8473361$, $\rho_{F(y)\overline{R}_x} =$

0.5123973, $\rho_{F(x)\overline{R}_x} = 0.2552666$.

Population-III [Source: [37]]:

Y= Interval of sleep, X = age of person, R_x = rank of X variable

$N = 30$, $n = 5$, $F(y) = 0.5$, $S^2_{F(y)} = 0.2586207$, $C_{F(y)} = 1.017095$, $S^2_{F(x)} = 0.5$, $C_{F(x)} = 1.077095$, $\rho_{F(y)F(x)} = 0.7816666$, $\rho_{F(y)\overline{R}_x} = 0.977474$, $\rho_{F(x)\overline{R}_x} = 0.9889946$.

Population-IV: [Source: Source: PBS (2021–2022)]

Y= Sales of fertilizers by district of Punjab during 2020, X = sales of fertilizers by the district of Punjab during 2018, R_x = rank of X variable

$N = 36$, $n = 8$, $F(y) = 0.5$, $S^2_{F(y)} = 0.2571429$, $C_{F(y)} = 1.014185$, $S^2_{F(x)} = 0.2571429$, $C_{F(x)} = 1.014185$, $\rho_{F(y)F(x)} = 0.7649152$, $\rho_{F(y)\overline{R}_x} = 0.942648$, $\rho_{F(x)\overline{R}_x} = 0.9538315$.

The consequences based on actual data sets are in Tables 1–8 (**see Appendix A**).

6. Simulation study

Tables 9–20 illustrate the outcomes of a generated data evaluating the effectiveness of the two recommended estimators using SRS, with and without the auxiliary variable and in varying orders of auxiliary variable use. We have created two populations of size 1000 from a multivariate normal distribution with diverse covariance conditions and diverse sample of sizes (100, 150, 200).

Population-**I**

$$\mu_1 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

and

$$\sum_1 = \begin{bmatrix} 4 & 9.6 \\ 9.6 & 64 \end{bmatrix}$$

$$\rho_{YX} = 0.613800$$

Population-II

$$\mu_2 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\sum_2 = \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$$

$$\rho_{XY} = 0.902645$$

The generated outcomes are given in Tables 9–20 (**see Appendix B**).

7. Results and discussion

The numerical results from the four real data sets, as presented in Tables 1–8 (Appendix A), support these findings. It's worth noting that the efficiency of the recommended estimators varied depending on the choice of a and b. The specific details of this choice and its impact on MSE and PRE were likely discussed in the Tables we mentioned. Tables 1–8 (Appendix A) show the numerical results of MSE and PRE based on four actual data sets. Tables 1–8 show numerical findings that illustrate how different values of α and β affect the MSE and PRE of our proposed class of estimators. The recommended estimators in these tables have executed enhanced than the existing estimators in expressions of minimum MSE and higher PRE. Simulation consequences given in Tables 9–20 (Appendix A), which demonstrated similar behavior. Using a multivariate normal distribution, we generated population of size 1000 with different covariance matrices and sample sizes (100, 150, and 200). The results varied between populations due to the use of varying sample sizes. Among the two proposed estimators, $\widehat{F}_{(sohaib2)}$ exhibited greater efficiency gain than $\widehat{F}_{(sohaib1)}$.

8. Conclusion

In this paper, we have recommended two comprehensive classes of estimators that can be used for assessing population distribution functions in the context of SRS. These estimators utilize a twofold auxiliary variables approach. The proposed estimators fall into two generalized classes, each offering a different approach to estimating the population DF. These classes might be based on different mathematical formulations, providing flexibility in the estimation procedure. The evaluation was conducted using four actual data sets. Regarding MSE and PRE, the numerical findings indicate that both classes of estimators perform admirably. Furthermore, the recommended estimators, the robustness and generalizability were assessed by a simulation study. This study likely involved generating artificial data sets with known properties and comparing the performance of the estimators under different scenarios. The generated data helps to understand how the estimators perform in situations where actual data may not be readily available. The contemporary work can be easily prolonged to produce comprehensive estimators for stratified random sampling, PPS, measurement error, and two-stage sampling by making use of the auxiliary variables.

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Data availability statement

Data will be made available on request.

CRediT authorship contribution statement

Sohaib Ahmad: Writing - original draft. **Sardar Hussain:** Formal analysis. **Aned Al Mutairi:** Funding acquisition, Data curation. **Mustafa Kamal:** Conceptualization. **Masood Ur Rehman:** Validation, Data curation. **Manahil SidAhmed Mustafa:** Investigation, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Table 1
Mean square error using Population-I

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.02168662	0.01850729	0.002416243	0.002380577	0.002275665
$\widehat{F}(\mathrm{R})$	0.002514919	0.01850557	0.002416241	0.00238057	0.002275658
$\widehat{F}(\mathrm{P})$	0.0847302	0.01850557	0.002416241	0.00238057	0.002275658
$\widehat{F}_{Reg}$	0.002442007	0.0184192	0.002416156	0.002380204	0.002275306
$\widehat{F}_{RD}$	0.002417692	0.01833842	0.002416074	0.002379849	0.002274965
$\widehat{F}_{BTR}$	0.006679115	0.01829268	0.002416026	0.002379643	0.002274767
$\widehat{F}_{BTP}$	0.04753744	0.01862857	0.002416357	0.002381068	0.002276136
		0.01833666	0.002416072	0.002379842	0.002274958
		0.01858627	0.002416318	0.0023809	0.002275975
		0.02154097	0.002417689	0.002386541	0.00228138

Table 2
PRE using Population-I.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	117.1788	897.5349	910.9816	952.9796
$\widehat{F}(\mathrm{R})$	862.3188	117.1897	897.5355	910.9843	952.9825
$\widehat{F}(\mathrm{P})$	25.59491	117.1897	897.5355	910.9843	952.9825
$\widehat{F}_{Reg}$	888.0653	117.7392	897.5671	911.1244	953.1298
$\widehat{F}_{RD}$	896.9969	118.2578	897.5976	911.2602	953.2726
$\widehat{F}_{BTR}$	324.693	118.5535	897.6153	911.3392	953.3557
$\widehat{F}_{BTP}$	45.62009	116.4159	897.4923	910.7937	952.7821

(*continued on next page*)

Table 2 (*continued*)

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
		118.2692	897.5983	911.2632	953.2758
		116.6809	897.5069	910.8581	952.8498
		100.6761	896.998	908.7052	950.592

Table 3
MSE using Population-II.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.003920705	0.003570093	0.002579386	0.002545898	0.002310368
$\widehat{F}(R)$	0.003301646	0.003569618	0.002579385	0.002545894	0.002310365
$\widehat{F}(P)$	0.01238984	0.003570093	0.002579386	0.002545898	0.002310368
$\widehat{F}_{Reg}$	0.002606563	0.003569144	0.002579384	0.002545891	0.002310362
$\widehat{F}_{RD}$	0.002579667	0.003447381	0.002579121	0.002544827	0.002309393
$\widehat{F}_{BTR}$	0.002630999	0.003446885	0.002579119	0.002544822	0.002309388
$\widehat{F}_{BTP}$	0.007170763	0.003678663	0.002579539	0.002546512	0.002310926
		0.003447381	0.002579121	0.002544827	0.002309393
		0.003678271	0.002579539	0.00254651	0.002310924
		0.003915753	0.002579666	0.002547015	0.002311383

Table 4
PRE using Population-II.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	109.8208	152.0015	154.0009	169.7005
$\widehat{F}(R)$	118.75	109.8354	152.0015	154.0011	169.7007
$\widehat{F}(P)$	31.64452	109.8208	152.0015	154.0009	169.7005
$\widehat{F}_{Reg}$	150.4167	109.85	152.0016	154.0013	169.7009
$\widehat{F}_{RD}$	151.9849	113.73	152.0171	154.0657	169.7721
$\widehat{F}_{BTR}$	149.0196	113.7463	152.0172	154.066	169.7724
$\widehat{F}_{BTP}$	54.67626	106.5796	151.9924	153.9637	169.6595
		113.73	152.0171	154.0657	169.7721
		106.591	151.9925	153.9639	169.6596
		100.1264	151.985	153.9333	169.6259

Table 5
MSE using Population-III.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.04310345	0.0486049	0.01843113	0.01698011	0.01686391
$\widehat{F}(R)$	0.1494253	0.04867098	0.01843063	0.01697822	0.01686204
$\widehat{F}(P)$	0.02375035	0.0486049	0.01843113	0.01698011	0.01686391
$\widehat{F}_{Reg}$	0.01992337	0.0487375	0.01843012	0.0169763	0.01686013
$\widehat{F}_{RD}$	0.0184528	0.0216067	0.01739801	0.01140316	0.01130837
$\widehat{F}_{BTR}$	0.08548851	0.02113619	0.01725897	0.0104629	0.01037049
$\widehat{F}_{BTP}$	0.02227011	0.03505003	0.01838893	0.01681664	0.01670149
		0.0216067	0.01739801	0.01140316	0.01130837
		0.03485638	0.0183854	0.01680262	0.01668756
		0.04361608	0.0184526	0.01706051	0.01694376

Table 6
PRE using Population-III.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	88.68127	233.8623	253.8466	255.5957
$\widehat{F}(R)$	28.84615	88.56089	233.8686	253.8749	255.6242
$\widehat{F}(P)$	181.4855	88.68127	233.8623	253.8466	255.5957
$\widehat{F}_{Reg}$	216.3462	88.44001	233.875	253.9037	255.6532
$\widehat{F}_{RD}$	233.5875	199.4911	247.7493	377.9958	381.164
$\widehat{F}_{BTR}$	50.42017	203.932	249.7453	411.9647	415.6357
$\widehat{F}_{BTP}$	193.5484	122.9769	234.3989	256.3143	258.0815
		199.4911	247.7493	377.9958	381.164
		123.6601	234.4439	256.5281	258.2969
		98.82468	233.5902	252.6505	254.3912

Table 7
MSE using Population-IV.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.02959184	0.02599346	0.0111593	0.009425252	0.008595108
$\widehat{F}(R)$	0.01315193	0.02596136	0.01115912	0.009424648	0.008594555
$\widehat{F}(P)$	0.105647	0.02599346	0.0111593	0.009425252	0.008595108
$\widehat{F}_{Reg}$	0.0116906	0.02592914	0.01115895	0.009424036	0.008593994
$\widehat{F}_{RD}$	0.01116834	0.02537192	0.01115562	0.009412309	0.008583249
$\widehat{F}_{BTR}$	0.01397392	0.02533814	0.01115539	0.009411527	0.008582532
$\widehat{F}_{BTP}$	0.06000567	0.02654427	0.01116198	0.009434583	0.008603646
		0.02537192	0.01115562	0.009412309	0.008583249
		0.02651461	0.01116185	0.009434128	0.00860323
		0.02928216	0.01116828	0.009456187	0.008623375

Table 8
PRE using Population-IV.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	113.8434	265.1766	313.9633	344.287
$\widehat{F}(R)$	225	113.9842	265.1807	313.9835	344.3091
$\widehat{F}(P)$	28.0101	113.8434	265.1766	313.9633	344.287
$\widehat{F}_{Reg}$	253.125	114.1258	265.1848	314.0039	344.3316
$\widehat{F}_{RD}$	264.9617	116.6322	265.264	314.3951	344.7627
$\widehat{F}_{BTR}$	211.7647	116.7877	265.2693	314.4212	344.7915
$\widehat{F}_{BTP}$	49.31507	111.4811	265.1129	313.6528	343.9453
		116.6322	265.264	314.3951	344.7627
		111.6058	265.116	313.6679	343.9619
		101.0576	264.9632	312.9363	343.1584

Appendix B

Table 9
MSE of Population-I, using sample of size 100.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.002252252	0.002110277	0.001841178	0.001820921	0.001669448
$\widehat{F}(R)$	0.002612613	0.002110235	0.001841178	0.001820921	0.001669447
$\widehat{F}(P)$	0.006397524	0.002110277	0.001841178	0.001820921	0.001669448
$\widehat{F}_{Reg}$	0.001854955	0.001999436	0.001840828	0.001837942	0.001668134
$\widehat{F}_{RD}$	0.001841293	0.002036780	0.001840985	0.001820141	0.001668731
$\widehat{F}_{BTR}$	0.001869369	0.001904421	0.001840040	0.001834510	0.001664984
$\widehat{F}_{BTP}$	0.003761261	0.002174437	0.001841262	0.001821257	0.001669756
		0.002036780	0.001840985	0.001820141	0.001668731
		0.002174406	0.001841262	0.001821256	0.001669756
		0.002251780	0.001841293	0.001821380	0.001669869

Table 10
MSE of Population-I, using sample of size 150.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.001418085	0.001328693	0.001162458	0.001149715	0.001053839
$\widehat{F}(R)$	0.001644978	0.001328667	0.001162458	0.001149715	0.001053839
$\widehat{F}(P)$	0.004028071	0.001328693	0.001162458	0.001149715	0.001053839
$\widehat{F}_{Reg}$	0.001167935	0.001258904	0.001162319	0.001160831	0.001053317
$\widehat{F}_{RD}$	0.001162504	0.001282417	0.001162381	0.001149405	0.001053554
$\widehat{F}_{BTR}$	0.001177010	0.001199080	0.001162006	0.001159467	0.001052065
$\widehat{F}_{BTP}$	0.002368202	0.001369090	0.001162491	0.001149849	0.001053962
		0.001282417	0.001162381	0.001149405	0.001053554
		0.001369071	0.001162491	0.001149849	0.001053961
		0.001417787	0.001162504	0.001149898	0.001054007

Table 11
MSE of Population-I, using sample of size 200.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.00100100	0.00093790	0.00082169	0.00081270	0.00074484
$\widehat{F}$(R)	0.00116116	0.00093788	0.00082169	0.00081270	0.00074484
$\widehat{F}$(P)	0.00284334	0.00093790	0.00082169	0.00081270	0.00074484
$\widehat{F}_{Reg}$	0.00082442	0.00088864	0.00082162	0.00082069	0.00074458
$\widehat{F}_{RD}$	0.00082171	0.00090524	0.00082165	0.00081255	0.00074470
$\widehat{F}_{BTR}$	0.00083083	0.00084641	0.00082147	0.00082001	0.00074396
$\widehat{F}_{BTP}$	0.00167167	0.00096642	0.00082171	0.00081277	0.00074491
		0.00090524	0.00082165	0.00081255	0.00074470
		0.00096640	0.00082171	0.00081277	0.00074491
		0.00100079	0.00082171	0.00081279	0.00074493

Table 12
MSE of Population-II, using sample of size 100.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.0022523	0.0019947	0.0010541	0.0010191	0.0008935
$\widehat{F}$(R)	0.0012252	0.0019946	0.0010541	0.0010191	0.0008935
$\widehat{F}$(P)	0.0077849	0.0019947	0.0010541	0.0010191	0.0008935
$\widehat{F}_{Reg}$	0.0010586	0.0017681	0.0010539	0.0010517	0.0008928
$\widehat{F}_{RD}$	0.0010541	0.0019418	0.0010540	0.0010190	0.0008934
$\widehat{F}_{BTR}$	0.0011757	0.0016779	0.0010537	0.0010511	0.0008923
$\widehat{F}_{BTP}$	0.0044550	0.0020436	0.0010541	0.0010192	0.0008936
		0.0019418	0.0010540	0.0010190	0.0008934
		0.0020435	0.0010541	0.0010192	0.0008936
		0.0022514	0.0010541	0.0010194	0.0008937

Table 13
MSE of Population-II using sample of size 150.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.0014181	0.0012559	0.0006647	0.0006427	0.0005634
$\widehat{F}$(R)	0.0007714	0.0012559	0.0006647	0.0006427	0.0005634
$\widehat{F}$(P)	0.0049016	0.0012559	0.0006647	0.0006427	0.0005634
$\widehat{F}_{Reg}$	0.0006665	0.0011133	0.0006646	0.0006635	0.0005631
$\widehat{F}_{RD}$	0.0006648	0.0012226	0.0006647	0.0006427	0.0005633
$\widehat{F}_{BTR}$	0.0007402	0.0010564	0.0006646	0.0006632	0.0005629
$\widehat{F}_{BTP}$	0.0028050	0.0012867	0.0006647	0.0006428	0.0005634
		0.0012226	0.0006647	0.0006427	0.0005633
		0.0012866	0.0006647	0.0006428	0.0005634
		0.0014176	0.0006648	0.0006428	0.0005635

Table 14
MSE of Population-II using sample of size 200.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	0.0010010	0.0008865	0.0004696	0.0004540	0.0003980
$\widehat{F}$(R)	0.0005445	0.0008865	0.0004696	0.0004540	0.0003980
$\widehat{F}$(P)	0.0034600	0.0008865	0.0004696	0.0004540	0.0003980
$\widehat{F}_{Reg}$	0.0004705	0.0007858	0.0004695	0.0004688	0.0003978
$\widehat{F}_{RD}$	0.0004696	0.0008630	0.0004696	0.0004540	0.0003979
$\widehat{F}_{BTR}$	0.0005225	0.0007457	0.0004695	0.0004687	0.0003977
$\widehat{F}_{BTP}$	0.0019800	0.0009083	0.0004696	0.0004541	0.0003980
		0.0008630	0.0004696	0.0004540	0.0003979
		0.0009082	0.0004696	0.0004541	0.0003980
		0.0010006	0.0004696	0.0004541	0.0003980

Table 15
PRE of Population-I using sample of size 100.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	106.72780	122.32670	123.68750	134.91000

(*continued on next page*)

Table 15 (*continued*)

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}(R)$	86.206900	106.72991	122.32670	123.68760	134.91000
$\widehat{F}(P)$	35.205060	106.72780	122.32670	123.68750	134.91000
$\widehat{F}_{Reg}$	121.41820	106.73205	122.32670	123.68760	134.91010
$\widehat{F}_{RD}$	122.31910	110.57910	122.33950	123.74050	134.96800
$\widehat{F}_{BTR}$	120.48193	110.58125	122.33950	123.74060	134.96800
$\widehat{F}_{BTP}$	59.880240	103.57870	122.32110	123.66470	134.88510
		110.57910	122.33950	123.74050	134.96800
		103.58010	122.32110	123.66480	134.88510
		100.02098	122.31910	123.65630	134.87600

Table 16
PRE of Population-I using sample of size 150.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	106.72780	121.99020	123.34230	134.56370
$\widehat{F}(R)$	86.206900	106.72991	121.99020	123.34230	134.56370
$\widehat{F}(P)$	35.205060	106.72780	121.82200	123.16970	134.39050
$\widehat{F}_{Reg}$	121.41820	106.73205	121.99020	123.34230	134.56370
$\widehat{F}_{RD}$	121.98540	110.57910	121.99820	123.37550	134.60010
$\widehat{F}_{BTR}$	120.48193	110.58125	121.99830	123.37550	134.60010
$\widehat{F}_{BTP}$	59.880240	103.57870	121.98670	123.32800	134.54810
		110.57910	121.99820	123.37550	134.60010
		103.58010	121.98670	123.32800	134.54810
		100.02098	121.98540	123.32270	134.54230

Table 17
PRE of Population-I using sample of size 200.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	106.72780	121.82200	123.16970	134.39050
$\widehat{F}(R)$	86.206900	106.72780	121.82200	123.16970	134.39050
$\widehat{F}(P)$	35.205060	106.72780	121.82200	123.16970	134.39050
$\widehat{F}_{Reg}$	121.41820	106.73205	121.82200	123.16970	134.39060
$\widehat{F}_{RD}$	121.81860	110.57910	121.82760	123.19310	134.41620
$\widehat{F}_{BTR}$	120.48193	110.58125	121.82760	123.19310	134.41620
$\widehat{F}_{BTP}$	59.880240	103.57870	121.81950	123.15960	134.37950
		110.57910	121.82760	123.19310	134.41620
		103.58010	121.81950	123.15960	134.37950
		100.02098	121.81860	123.15580	134.37550

Table 18
PRE results of Population II, using sample size 100.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	112.91180	213.67300	220.99340	252.07120
$\widehat{F}(R)$	183.82353	112.91180	213.67300	220.99340	252.07120
$\widehat{F}(P)$	28.930990	112.91180	213.67300	220.99340	252.07120
$\widehat{F}_{Reg}$	212.75870	112.92095	213.67300	220.99350	252.07120
$\widehat{F}_{RD}$	213.65960	115.98870	213.67970	221.02170	252.10360
$\widehat{F}_{BTR}$	191.57088	115.99377	213.67970	221.02170	252.10370
$\widehat{F}_{BTP}$	50.556120	110.21160	213.66820	220.97300	252.04790
		115.98870	213.67970	221.02170	252.10360
		110.21550	213.66820	220.97310	252.04790
		100.03637	213.65960	220.93730	252.00700

Table 19
PRE results of Population II, using sample size 150.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	112.91180	213.33440	220.63890	251.71370
$\widehat{F}(R)$	183.82353	112.91180	213.33440	220.63890	251.71370
$\widehat{F}(P)$	28.930990	112.91180	213.33440	220.63890	251.71370

(*continued on next page*)

Table 19 (*continued*)

Estimators	Value	$\widehat{F}_{Singh}$	F_{Grover}	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{Reg}$	212.75870	112.92095	213.33440	220.63890	251.71370
$\widehat{F}_{RD}$	213.32590	115.98870	213.33860	220.65670	251.73410
$\widehat{F}_{BTR}$	191.57088	115.99377	213.33860	220.65670	251.73410
$\widehat{F}_{BTP}$	50.556120	110.21160	213.33130	220.62610	251.69900
		115.98870	213.33860	220.65670	251.73410
		110.21550	213.33130	220.62610	251.69910
		100.03637	213.32590	220.60360	251.67340

Table 20
PRE results of Population II, using sample size 200.

Estimators	Value	$\widehat{F}_{Singh}$	$\widehat{F}_{Grover}$	$\widehat{F}_{(sohaib1)}$	$\widehat{F}_{(sohaib2)}$
$\widehat{F}_{usual}$	100	112.91180	213.16510	220.46160	251.53500
$\widehat{F}(\mathrm{R})$	183.82353	112.91180	213.16510	220.46160	251.53500
$\widehat{F}(\mathrm{P})$	28.930990	112.91180	213.16510	220.46160	251.53500
$\widehat{F}_{Reg}$	212.75870	112.92095	213.16510	220.46170	251.53500
$\widehat{F}_{RD}$	213.15910	115.98870	213.16800	220.47420	251.54940
$\widehat{F}_{BTR}$	191.57088	115.99377	213.16800	220.47420	251.54940
$\widehat{F}_{BTP}$	50.556120	110.21160	213.16290	220.45260	251.52460
		115.98870	213.16800	220.47420	251.54940
		110.21550	213.16290	220.45260	251.52470
		100.03637	213.15910	220.43680	251.50650

Appendix C

Table 21
Special cases of $\widehat{F}^{*(\alpha,\beta)}_{(Prop)}$ using changed values of α and β.

α	β	$\widehat{F}^{*(\alpha,\beta)}_{(sohaib)}$
1	$C_{F(x)}$	$\widehat{F}^{*(1)}_{(sohaib)}$
1	$\beta_{2(F(x))}$	$\widehat{F}^{*(2)}_{(sohaib)}$
$\beta_{2(F(x))}$	$C_{F(x)}$	$\widehat{F}^{*(3)}_{(sohaib)}$
$C_{F(x)}$	$\beta_{2(F(x))}$	$\widehat{F}^{*(4)}_{(sohaib)}$
1	$\rho_{F(y)F(x)}$	$\widehat{F}^{*(5)}_{(sohaib)}$
$C_{F(x)}$	$\rho_{F(y)F(x)}$	$\widehat{F}^{*(6)}_{(sohaib)}$
$\rho_{F(y)F(x)}$	$C_{F(x)}$	$\widehat{F}^{*(7)}_{(sohaib)}$
$\beta_{2(F(x))}$	$\rho_{F(y)F(x)}$	$\widehat{F}^{*(8)}_{(sohaib)}$
$\rho_{F(y)F(x)}$	$\beta_{2(F(x))}$	$\widehat{F}^{*(9)}_{(sohaib)}$
1	$N\,F(x)$	$\widehat{F}^{*(10)}_{(sohaib)}$

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