

SCIENTIFIC REPORTS



OPEN

Generation of four-photon polarization entangled decoherence-free states with cross-Kerr nonlinearity

Meiyu Wang¹, Fengli Yan¹ & Ting Gao²

Received: 13 July 2016

Accepted: 07 November 2016

Published: 30 November 2016

We propose a theoretical protocol for preparing four-photon polarization entangled decoherence-free states, which are immune to the collective noise. With the assistance of the cross-Kerr nonlinearities, a two-photon spatial entanglement gate, two controlled-NOT gates, a four-photon polarization entanglement gate are inserted into the circuit, where X homodyne measurements are aptly applied. Combined with some swap gates and simple linear optical elements, four-photon polarization entangled decoherence-free states which can be utilized to represent two logical qubits, $|0\rangle_L$ and $|1\rangle_L$ are achieved at the output ports of the circuit. This generation scheme may be implemented with current experimental techniques.

Entanglement^{1–3} plays an important role in quantum information processing, mainly including quantum computation⁴ and quantum communication. It is the information carrier in some interesting branches of quantum communication, such as quantum key distribution⁵, quantum secret sharing^{6–8}, quantum secure direct communication^{9–11}, teleportation¹², quantum dense coding^{13,14}, and so on. Most of the above applications require maximally entangled states or noiseless quantum channels. However, in a realistic situation, decoherence, induced by uncontrolled coupling between a quantum system and the environment, is inevitable. When qubits are coupled to the environment, the quantum superposition and coherence are easily destructed, and as a result the maximally entangled state collapses into a non-maximally entangled one or even a mixed state. This will degrade the fidelity and security of quantum communication. To overcome this flaw, some specific entangled states, which are called decoherence-free states^{15–17}, are proposed. Decoherence-free states, no matter how strong the qubit-environment interaction, exhibit some symmetry, so the quantum states are invariant under this interaction. Therefore, the decoherence-free states are very useful for long-distance quantum information transmission and storage.

Due to the fact that photons have the merits of higher speed, lower decoherence, easier manipulation, and lower energy cost compared with more massive qubits, polarization photons are destined to have a central role in long-distance communication. Recently, the encoding in decoherence-free states of polarization photons to overcome collective decoherence attracts the extensively attention. An optical experiment has been reported to overcome collective noise by encoding quantum information into the decoherence-free state¹⁸. For two qubits, there is only one decoherence-free singlet state, i.e., $\frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$, where H and V denote horizontal and vertical linear polarizations respectively. Therefore, it is not sufficient to fully protect the quantum information of an arbitrary logical qubit against collective noise. Another nontrivial example is the four-photon polarization entangled decoherence-free state

$$|\Phi_0\rangle = \alpha|0\rangle_L + \beta|1\rangle_L, \quad (1)$$

with

$$|0\rangle_L = \frac{1}{2}(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2)(|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4), \quad (2)$$

¹College of Physics Science and Information Engineering, Hebei Normal University, Shijiazhuang 050024, China. ²College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang 050024, China. Correspondence and requests for materials should be addressed to F.Y. (email: flyan@mail.hebtu.edu.cn) or T.G. (email: gaoting@mail.hebtu.edu.cn)

$$\begin{aligned}
|1\rangle_L = & \frac{1}{2\sqrt{3}}(2|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4 + 2|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4 \\
& - |V\rangle_1|H\rangle_2|V\rangle_3|H\rangle_4 - |V\rangle_1|H\rangle_2|H\rangle_3|V\rangle_4 \\
& - |H\rangle_1|V\rangle_2|H\rangle_3|V\rangle_4 - |H\rangle_1|V\rangle_2|V\rangle_3|H\rangle_4). \tag{3}
\end{aligned}$$

The dimension of the above four-qubit decoherence-free state in Eq. (1) is 2, and thus it is sufficient to fully protect an arbitrary logical qubit against collective noise in contrast to the two-qubit state. With its interesting applications, Bourennane *et al.*¹⁹ have generated four-photon polarization entangled decoherence-free states via a spontaneous parametric down-conversion source. Recently, Zou *et al.*²⁰ and Gong *et al.*²¹ proposed two different schemes to generate four-photon polarization entangled decoherence-free states based on linear optical elements and postselection strategy. Subsequently, Xia *et al.*²² presented a protocol for the controlled generation of the four-photon polarization entangled decoherence-free state with conventional photon detectors. In 2010, Wang *et al.*²³ proposed a probabilistic linear-optics-based scheme for local conversion of four Einstein-Podolsky-Rosen photon pairs into four-photon polarization entangled decoherence-free states. In 2013, Xia *et al.*²⁴ also put forward a probabilistic protocol for preparation of four-photon polarization entangled decoherence-free states with the help of the cross-Kerr nonlinearity medium.

In this paper, we present an alternative scheme to generate the four-photon polarization entangled decoherence-free states with the assistance of the cross-Kerr nonlinearities. The states representing the logical qubits $|0\rangle_L$ and $|1\rangle_L$ can be achieved at different output ports of two beam splitters, combined with the output ports of the photon 3 and the photon 4. The rest of the paper is organized as follows. In Sec. II, we show how to generate these two logical qubits in the four-photon polarization entangled decoherence-free states based on the weak cross-Kerr nonlinearities. The discussion and conclusion are presented in Sec. III.

Generations of four-qubit entangled decoherence-free states

For the sake of the clearness, let us first introduce the cross-Kerr nonlinearity, which was first used by Chuang and Yamamoto to realize the simple optical quantum computation²⁵. The interaction Hamiltonian has the form $\hat{H}_k = -\hbar\kappa\hat{n}_s\hat{n}_p$, where \hat{n}_s (\hat{n}_p) is the photon-number operators of the signal (probe) mode, and κ is the strength of the nonlinearity. If the signal field contains n photons and the probe field is in an initial coherent state with amplitude α , the cross-Kerr nonlinearity interaction causes the combined signal-probe system to evolve as follows:

$$e^{-i\hat{H}_k t/\hbar}|n\rangle_s|\alpha\rangle_p = e^{i\kappa t\hat{n}_s\hat{n}_p}|n\rangle_s|\alpha e^{in\theta}\rangle_p, \tag{4}$$

where $\theta = \kappa t$ with t being the interaction time. It is easy to observe that the Fock state is unaffected by the interaction but the coherent state picks up a phase shift $n\theta$ directly proportional to the number of photons n in the signal mode. One can exactly obtain the information of photons in the Fock state but not destroy them by detecting the probe mode with a general homodyne-heterodyne measurement. The cross-Kerr nonlinearity between photons offers an ideal playground for quantum state engineering, and a number of applications have been studied, such as constructing nondestructive quantum nondemolition detectors (QND)^{26,27}, deterministic entanglement distillation²⁸, logic-qubit entanglement^{29,30}, generation of multi-photon entangled states and decoherence-free states^{24,31–36}.

In what follows, we explain the detailed procedures for generating the four-photon polarization entangled decoherence-free states abided by the following processes, which is also illustrated in Fig. 1.

Assume the four single photons are initially prepared in the state $|H\rangle_1 \otimes |H\rangle_2 \otimes \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_3 \otimes \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_4$, and let them enter into the circuit shown in Fig. 1 from the input ports. The first step is to create the spatial entanglement of the photons 1 and 2. Passing through beam splitters, BS₁ and BS₂, which have the following function between two input modes (a,b) and two output modes (c,d): $a^\dagger \rightarrow (c^\dagger + d^\dagger)/\sqrt{2}$, $b^\dagger \rightarrow (c^\dagger - d^\dagger)/\sqrt{2}$, the photons (1, 2) enter into the paths (S_{11} , S_{12}) and the paths (S_{21} , S_{22}) respectively. Accompanying with the coherent state, the photons (1, 2) enter into Kerr media. Then, the state of photons (1, 2) with the coherent state $|\alpha\rangle$ evolves as

$$\begin{aligned}
|\psi\rangle_{12\alpha} = & \frac{1}{2} \left[(|H\rangle_{S_{11}}|H\rangle_{S_{21}} + |H\rangle_{S_{12}}|H\rangle_{S_{22}}) |\alpha\rangle + |H\rangle_{S_{11}}|H\rangle_{S_{22}} |\alpha e^{i\theta}\rangle \right. \\
& \left. + |H\rangle_{S_{12}}|H\rangle_{S_{21}} |\alpha e^{-i\theta}\rangle \right]. \tag{5}
\end{aligned}$$

Performing an X homodyne measurement on the coherent state with α real, there are two measurement outcomes corresponding to scenarios of phase shift (0, $\pm\theta$). For the convenience of analysis, we expand the state in terms of the eigenstates of the X operator:

$$\begin{aligned}
|\psi\rangle_{12\alpha}^x = & f(x, \alpha)(|H\rangle_{S_{11}}|H\rangle_{S_{21}} + |H\rangle_{S_{12}}|H\rangle_{S_{22}}) \\
& + f(x, \alpha \cos \theta)(|H\rangle_{S_{11}}|H\rangle_{S_{22}} e^{i\phi_1(x)} \\
& + |H\rangle_{S_{12}}|H\rangle_{S_{21}} e^{-i\phi_1(x)}), \tag{6}
\end{aligned}$$

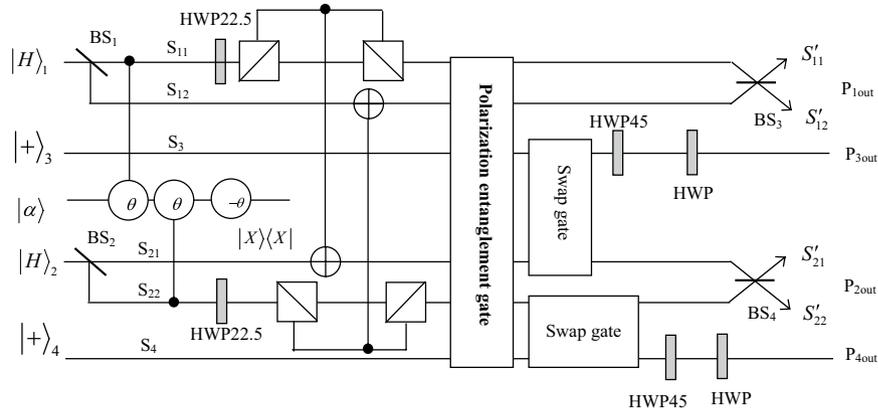


Figure 1. Illustration plot for generating four-photon polarization entangled decoherence-free state with the help of the cross-Kerr nonlinearities. The symbol BS denotes the beam splitter which has equal probability (50:50) of transmission and reflection. HWP22.5, HWP45, and HWP denote half-wave plates which realize the Hadamard transformation operation, single photon σ_x operation, and single photon σ_z operation respectively. In the construction of the circuit, after two Controlled-Not gates, a polarization entanglement gate and two swap gates need to be performed, which can be seen in Figs 2 and 3 respectively. Before four photons leave the circuit, the four potential paths of the photon (1, 2) are coherently combined by BS_3 and BS_4 for obtaining the different four-photon polarization entanglement decoherence-free states.

where the coefficients³⁷ are

$$\begin{cases} \langle x|\alpha\rangle = (2\pi)^{-1/4} \exp\left[-\frac{1}{4}(x - 2\alpha)^2\right] \equiv f(x, \alpha), \\ \langle x|\alpha e^{\pm i\theta}\rangle = (2\pi)^{-1/4} \exp\left[-\frac{1}{4}(x - 2\alpha \cos\theta)^2\right] e^{\pm i(-\alpha x \sin\theta - 2\alpha^2 \sin\theta \cos\theta)} \\ \equiv f(x, \alpha \cos\theta) e^{\pm i\phi_1(x)}. \end{cases} \quad (7)$$

Explicitly, if zero phase shift occurs, the spatial entangled state of the photons (1, 2) is created and can be written as

$$|\psi\rangle_{12, \text{zero}} = \frac{1}{\sqrt{2}}(|H\rangle_{S_{11}}|H\rangle_{S_{21}} + |H\rangle_{S_{12}}|H\rangle_{S_{22}}). \quad (8)$$

Otherwise, another measurement outcome (nonzero phase shift) is obtained, a phase shift operation $2\phi_1(x)$ should be performed on the photon 2 passing through the path S_{21} to erase the phase difference between two terms of $|H\rangle_{S_{11}}|H\rangle_{S_{22}}$ and $|H\rangle_{S_{12}}|H\rangle_{S_{21}}$. By omitting a global phase $\phi_1(x)$, the photons (1, 2) are in the following spatial entangled state

$$|\psi\rangle_{12, \text{nonzero}} = \frac{1}{\sqrt{2}}(|H\rangle_{S_{11}}|H\rangle_{S_{22}} + |H\rangle_{S_{12}}|H\rangle_{S_{21}}). \quad (9)$$

Without considering other conditions, there is a small probability of error to distinguish the state in Eq. (8) and the state in Eq. (9) from each other due to the overlap of the measurement functions $f(x, \alpha)$ and $f(x, \alpha \cos\theta)$, which is given by $P_{\text{error}} = \frac{1}{2} \left(1 - \text{erf}\left[\frac{x_d}{2\sqrt{2}}\right]\right)$, $x_d = 2\alpha(1 - \cos\theta)$. It is less than 10^{-5} when the distance $x_d \sim \alpha\theta^2 > 9^{26}$.

For simplifying description in the later processes, we take an example as the representative of two different scenarios of phase shift. If zero phase shift is witnessed by the X homodyne measurement, half wave plates, HWP22.5's, are inserted into the paths S_{11} , S_{22} at first, which function as Hadamard transformation operations to transform the state of the photons (1, 2) from $|H\rangle_{S_{11}}$ and $|H\rangle_{S_{22}}$ to $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_{S_{11}}$ and $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_{S_{22}}$. Then two controlled-NOT (CNOT) gates are performed on two paths (S_{11} , S_{21}) (photon 1 as control photon and photon 2 as target photon), and paths (S_{22} , S_{12}) (photon 2 as control photon and photon 1 as target photon) respectively. The CNOT gate is important in the experimental realization. Knill *et al.*³⁸ firstly proposed a probabilistic CNOT gate on two photonic qubits by using linear optical elements and postselection. The cross-Kerr nonlinearity has also been used to implement the CNOT gate^{26,39,40}. After two CNOT gates, the four-photon system will evolve into

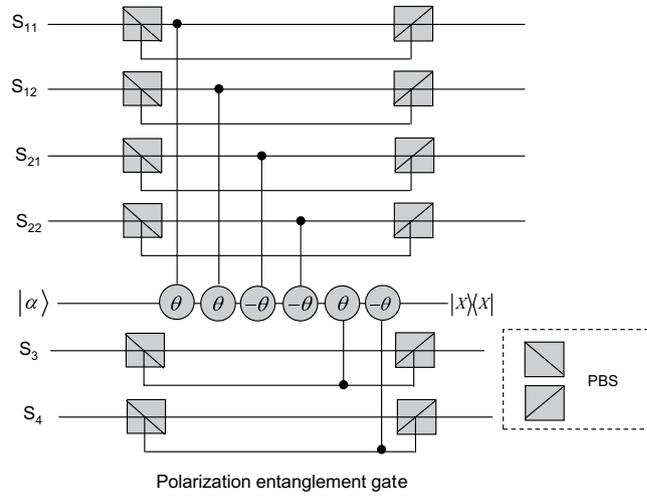


Figure 2. Illustration plot for depicting a polarization entanglement gate. The polarization beam splitters (PBS) reflect the vertical polarization $|V\rangle$ mode and transmit the horizontal polarization $|H\rangle$ mode. Influenced by cross-Kerr nonlinearities, the photon 1 in the horizontal polarization state and the photon 3 in the vertical polarization state enable the coherent state $|\alpha\rangle$ to pick up the phase shift θ , while the photon 2 in the horizontal polarization state and the photon 4 in the vertical polarization state enable the coherent state $|\alpha\rangle$ to pick up the phase shift $-\theta$.

$$\begin{aligned}
 |\psi\rangle_{1234} &= \frac{1}{\sqrt{2}}(|H\rangle_{S_{11}}|H\rangle_{S_{21}} + |H\rangle_{S_{12}}|H\rangle_{S_{22}})\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_{S_3} \\
 &\otimes \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_{S_4} \xrightarrow{\text{HWP}22.5^\circ, \text{CNOT gates}} \frac{1}{2}[(|H\rangle_{S_{11}}|H\rangle_{S_{21}} \\
 &+ |V\rangle_{S_{11}}|V\rangle_{S_{21}}) + (|H\rangle_{S_{12}}|H\rangle_{S_{22}} + |V\rangle_{S_{12}}|V\rangle_{S_{22}})] \\
 &\otimes \frac{1}{2}(|H\rangle_{S_3}|H\rangle_{S_4} + |H\rangle_{S_3}|V\rangle_{S_4} + |V\rangle_{S_3}|H\rangle_{S_4} + |V\rangle_{S_3}|V\rangle_{S_4}). \tag{10}
 \end{aligned}$$

A polarization entanglement gate illustrated in Fig. 2 is put into the paths ($S_{11}, S_{12}, S_{21}, S_{22}, S_3, S_4$) of the photons (1, 2, 3, 4) to entangle them with the polarization degree of freedom. Affected by cross-Kerr nonlinearities, the horizontal polarization mode of photons (1, 2) via the paths $S_{11}, S_{12}, S_{21}, S_{22}$ will accumulate the phase shift $\theta, -\theta$ respectively while the vertical polarization mode of photons (3, 4) will accumulate the phase shift $\theta, -\theta$ respectively on the coherent state $|\alpha\rangle$. As the consequence of the nonlinear interaction between photons and the coherent state, the state of the whole system can be expressed as

$$\begin{aligned}
 |\psi\rangle_{1234, \text{Kerr}} &= \frac{1}{4} \left\{ (|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle)_{S_{11}S_{21}S_3S_4} |\alpha\rangle \right. \\
 &+ (|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle)_{S_{12}S_{22}S_3S_4} |\alpha\rangle \left. \right\} \\
 &+ [(|HHVH\rangle + |VVVH\rangle)_{S_{11}S_{21}S_3S_4} |\alpha e^{i\theta}\rangle \\
 &+ (|HHHV\rangle + |VVHV\rangle)_{S_{11}S_{21}S_3S_4} |\alpha e^{-i\theta}\rangle] \\
 &+ [(|HHVH\rangle + |VVVH\rangle)_{S_{12}S_{22}S_3S_4} |\alpha e^{i\theta}\rangle \\
 &+ (|HHHV\rangle + |VVHV\rangle)_{S_{12}S_{22}S_3S_4} |\alpha e^{-i\theta}\rangle] \left. \right\}. \tag{11}
 \end{aligned}$$

After the photons leave Kerr media, the X homodyne measurement is performed on the coherent state. If zero phase shift occurs, no phase modulation is necessary. Otherwise, if nonzero phase shift of the coherent state presents on the measurement setup, a phase shift $2\phi(x)$ operation should be performed on the photon 3 in the path S_{31} . Moreover, a HWP45° should be inserted into the path S_3 to perform σ_x operation on the photon 3. So the four-photon state can be denoted as

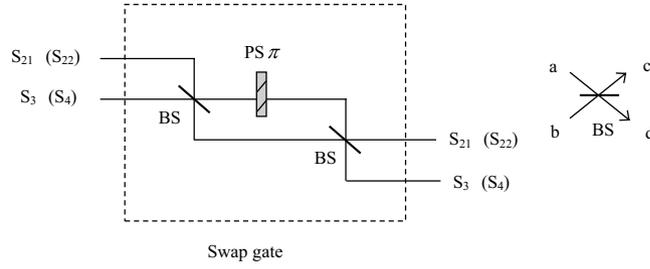


Figure 3. Illustration plot for depicting the swap gate. The symbol PS π denotes the phase shift π executed on the photon passing through the line it is inserted. A beam splitter has the following function between two input modes (a,b) and two output modes (c,d): $a^\dagger \rightarrow \frac{1}{\sqrt{2}}(c^\dagger + d^\dagger)$, $b^\dagger \rightarrow \frac{1}{\sqrt{2}}(d^\dagger - c^\dagger)$.

$$|\psi'\rangle_{1234} = \frac{1}{2\sqrt{2}} \left[(|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle)_{S_{11}S_{21}S_3S_4} + (|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle)_{S_{12}S_{22}S_3S_4} \right]. \quad (12)$$

Here, two swap gates need to be performed on two photons in the paths (S_{21} , S_3) and (S_{22} , S_4) respectively to swap them. A swap gate is an important two-qubit logic gate. In terms of the basis of $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the swap gate can be represented as the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (13)$$

In practice, the swap gate transformation can be yielded by the Hong-Ou-Mandel interference⁴¹ in the Mach-Zehnder interferometer^{39,42}, illustrated in Fig. 3. Two beam splitters constitute a Mach-Zehnder interferometer. Additionally, the phase shifter PS π denotes the phase shift π executed on the photon passing through the line it is inserted.

After two swap gate operations, the state denoted in Eq. (12) is changed to

$$|\psi'\rangle_{1234, \text{swap}} = \frac{1}{2\sqrt{2}} \left[(|HHHH\rangle + |HVHV\rangle + |VHVH\rangle + |VVVV\rangle)_{S_{11}S_{21}S_3S_4} + (|HHHH\rangle + |HVVH\rangle + |VHHV\rangle + |VVVV\rangle)_{S_{12}S_{22}S_3S_4} \right]. \quad (14)$$

Then, a local unitary operation σ_y should be performed on photon 3 and 4, respectively, which can be realized by the combination of a HWP45° and a HWP. So the above state can be denoted as

$$|\psi''\rangle_{1234} = \frac{1}{2\sqrt{2}} \left[(|VVHH\rangle + |HHVV\rangle - |VHHV\rangle - |HVVH\rangle)_{S_{11}S_{21}S_3S_4} + (|VVHH\rangle + |HHVV\rangle - |HVHV\rangle - |VHVH\rangle)_{S_{12}S_{22}S_3S_4} \right]. \quad (15)$$

Due to the presence of BS₃ and BS₄, the photons (1, 2) leave the paths (S_{11} , S_{12}) and the paths (S_{21} , S_{22}) to the paths (S'_{11} , S'_{12}) and the paths (S'_{21} , S'_{22}) according to the following rules, $a_{S_{11}}^\dagger \rightarrow \frac{1}{\sqrt{2}}(a_{S'_{11}}^\dagger + a_{S'_{12}}^\dagger)$, $a_{S_{12}}^\dagger \rightarrow \frac{1}{\sqrt{2}}(a_{S'_{12}}^\dagger - a_{S'_{11}}^\dagger)$, $a_{S_{21}}^\dagger \rightarrow \frac{1}{\sqrt{2}}(a_{S'_{21}}^\dagger + a_{S'_{22}}^\dagger)$, $a_{S_{22}}^\dagger \rightarrow \frac{1}{\sqrt{2}}(a_{S'_{22}}^\dagger - a_{S'_{21}}^\dagger)$. Correspondingly, at the output ports, the state of four photons expressed as Eq. (15) is transformed to

$$\begin{aligned} |\psi''\rangle_{1234, \text{out}} &= \frac{1}{4\sqrt{2}} \left[(2|VVHH\rangle + 2|HHVV\rangle - |VHHV\rangle - |HVVH\rangle - |HVHV\rangle - |VHVH\rangle)_{S'_{11}S'_{21}S_3S_4} \right. \\ &\quad + (|HVHV\rangle + |VHVH\rangle - |VHHV\rangle - |HVVH\rangle)_{S'_{11}S'_{22}S_3S_4} \\ &\quad + (|HVHV\rangle + |VHVH\rangle - |VHHV\rangle - |HVVH\rangle)_{S'_{12}S'_{21}S_3S_4} \\ &\quad + (2|VVHH\rangle + 2|HHVV\rangle - |VHHV\rangle - |HVVH\rangle - |HVHV\rangle - |VHVH\rangle)_{S'_{12}S'_{22}S_3S_4} \left. \right] \\ &= \frac{\sqrt{3}}{2\sqrt{2}} (|1\rangle_L)_{S'_{11}S'_{21}S_3S_4} + \frac{1}{2\sqrt{2}} (|0\rangle_L)_{S'_{11}S'_{22}S_3S_4} \\ &\quad + \frac{1}{2\sqrt{2}} (|0\rangle_L)_{S'_{12}S'_{21}S_3S_4} + \frac{\sqrt{3}}{2\sqrt{2}} (|1\rangle_L)_{S'_{12}S'_{22}S_3S_4}. \end{aligned} \quad (16)$$

From the above equation, we can see that by detecting the outputs of the four photons, the logical qubit $|0\rangle_L$ can be obtained with the total probability of 25% at the output ports of $(S'_{11}S'_{22}S_3S_4)$ or $(S'_{12}S'_{21}S_3S_4)$. As for the logical qubit $|1\rangle_L$, it can be obtained with the total probability of 75% at the output ports of $(S'_{11}S'_{21}S_3S_4)$ or $(S'_{12}S'_{22}S_3S_4)$.

As for another scenario, if we obtain the spatial entangled state denoted as Eq. (9), with a similar process, we can also obtain the four-photon polarization entangled decoherence-free states. It is worth noting that the state denoted as Eq. (9) is the same as Eq. (8) when a swap gate is inserted into the path S_{21} and S_{22} , so far, the preparation of four-photon polarization entangled decoherence-free states is fulfilled.

Discussion and Conclusion

We now give a brief discussion about the experimental feasibility of protocol with the current experimental technology. First of all, in the input ports, single-photon resources are used. The complete technology of these single photons is yet to be established^{43–46}. Currently single-photon sources in signal modes can be achieved from the collinear type II spontaneous parametric down conversion⁴⁷. As down conversion experiments are intrinsically probabilistic due to the statistical creation property of the photon pairs, the scheme will be in a sense probabilistic too in view of the usage of single-photon sources. Thus, more efficient ones are demanded for our setup. Second, in the present scheme, two CNOT gates are performed, which can be realized following the refs^{26,38–40}. However, these methods are at the best, nearly deterministic, so our scheme could be nearly deterministic. Third, in our protocol, we exploit the cross-Kerr nonlinearities medium in the spatial entanglement process and performing the polarization entanglement gate. It should be noted that in actual experiments, many factors will affect the perfect performance of cross-Kerr nonlinearities, such as dispersion, self-phase modulation, molecular vibrations in Kerr media, etc. Shapiro *et al.*⁴⁸ analyzed the cross-Kerr nonlinear interaction and showed that single-mode cross-Kerr nonlinearities is not available for quantum information processing. Recently, Gea-Banacloche⁴⁹ pointed out that the large phase shifts via the giant Kerr effect with single-photon wave packets is impossible at present. A proper physics systems providing larger strength of cross-Kerr nonlinearity should be atomic ensemble, and the fundamental problem with the cross-Kerr nonlinearity in atomic ensemble was discussed by Gea-Banacloche⁴⁹, and He and Scherer⁵⁰. Finally, the experiment feasibility of the present protocols also depends on the veracity of the X homodyne measurement. For the X homodyne measurement, we only consider the error chiefly coming from the overlap adjacent curves because of the fact that the coherent states of the probe beam with different phase shifts are not completely orthogonal. In fact, it is only one type of detection error in homodyne, other errors, such as the noises in detection, the reduced fidelity to the process in Eq. (4) due to multi-mode effect and decoherence, etc., also exist in a realistic implementation. Exploiting the appropriate measurement methods, the disadvantageous influence can be overcome or alleviated and the error probability will be decreased. In 2010, Wittmann *et al.*⁵¹ investigated quantum measurement strategies capable of discriminating two coherent states using a homodyne detector and a photon number resolving (PNR) detector. In order to lower the error probability, the postselection strategy is applied to the measurement data of homodyne detector as well as a PNR detector. They indicated that the performance of the new displacement controlled PNR is better than homodyne receiver.

To summarize, we have proposed a theoretical protocol for preparing four-photon polarization entangled decoherence-free states with the assistance of the cross-Kerr nonlinearity. In our protocol, combined with some swap gates and simple linear optical elements, a two-photon spatial entanglement gate, two CNOT gates, a four-photon polarization entanglement gate are applied. We hope our work will afford facilities for other practical implementations of quantum information processing based on optics.

References

- Horodecki, R., Horodecki, P., Horodecki, M. & Horodecki, K. Quantum entanglement. *Rev. Mod. Phys.* **81**, 865–942 (2009).
- Yan, F. L., Gao, T. & Chitambar, E. Two local observables are sufficient to characterize maximally entangled states of N qubits. *Phys. Rev. A* **83**, 022319 (2011).
- Gao, T., Yan, F. L. & van Enk, S. J. Permutationally invariant part of a density matrix and nonseparability of N -qubit states. *Phys. Rev. Lett.* **112**, 180501 (2014).
- DiVincenzo, D. P. Quantum gates and circuits. *Proc. R. Soc. London A* **454**, 261–276 (1998).
- Ekert, A. K. Quantum cryptography based on Bell's theorem. *Phys. Rev. Lett.* **67**, 661–663 (1991).
- Hillery, M., Bužek, V. & Berthiaume, A. Quantum secret sharing. *Phys. Rev. A* **59**, 1829–1834 (1999).
- Deng, F. G. *et al.* Multiparty quantum-state sharing of an arbitrary two-particle state with Einstein-Podolsky-Rosen pairs. *Phys. Rev. A* **72**, 044301 (2005).
- Yan, F. L. & Gao, T. Quantum secret sharing between multiparty and multiparty without entanglement. *Phys. Rev. A* **72**, 012304 (2005).
- Deng, F. G., Long, G. L. & Liu, X. S. Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block. *Phys. Rev. A* **68**, 042317 (2003).
- Gao, T., Yan, F. L. & Wang, Z. X. Deterministic secure direct communication using GHZ states and swapping quantum entanglement. *J. Phys. A* **38**, 5761–5770 (2005).
- Zhu, A. D., Xia, Y., Fan, Q. B. & Zhang, S. Secure direct communication based on secret transmitting order of particles. *Phys. Rev. A* **73**, 022338 (2006).
- Bennett, C. H. *et al.* Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **70**, 1895–1899 (1993).
- Bennett, C. H. & Wiesner, S. J. Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. *Phys. Rev. Lett.* **69**, 2881–2884 (1992).
- Liu, X. S., Long, G. L., Tong, D. M. & Feng, L. General scheme for superdense coding between multiparties. *Phys. Rev. A* **65**, 022304 (2002).
- Duan, L. M. & Guo, G. C. Preserving coherence in quantum computation by pairing quantum bits. *Phys. Rev. Lett.* **79**, 1953–1956 (1997).
- Zanardi, P. & Rasetti, M. Noiseless quantum codes. *Phys. Rev. Lett.* **79**, 3306–3309 (1997).
- Kempe, J., Bacon, D., Lidar, D. A. & Whaley, K. B. Theory of decoherence-free fault-tolerant universal quantum computation. *Phys. Rev. A* **63**, 042307 (2001).
- Altepeter, J. B. *et al.* Experimental investigation of a two-qubit decoherence-free subspace. *Phys. Rev. Lett.* **92**, 147901 (2004).
- Bourennane, M. *et al.* Decoherence-free quantum information processing with four-photon entangled states. *Phys. Rev. Lett.* **92**, 107901 (2004).

20. Zou, X. B., Shu, J. & Guo, G. C. Simple scheme for generating four-photon polarization-entangled decoherence-free states using spontaneous parametric down-conversions. *Phys. Rev. A* **73**, 054301 (2006).
21. Gong, Y. X. *et al.* Generation of arbitrary four-photon polarization-entangled decoherence-free states. *Phys. Rev. A* **77**, 042317 (2008).
22. Xia, Y., Song, J., Song, H. S. & Zhang, S. Controlled generation of four-photon polarization-entangled decoherence-free states with conventional photon detectors. *J. Opt. Soc. Am. B* **26**, 129–132 (2009).
23. Wang, H. F. *et al.* Local conversion of four Einstein-Podolsky-Rosen photon pairs into four-photon polarization-entangled decoherence-free states with non-photon-number-resolving detectors. *Opt. Exp.* **19**, 25433–25440 (2011).
24. Xia, Y. *et al.* Effective protocol for preparation of four-photon polarization-entangled decoherence-free states with cross-Kerr nonlinearity. *J. Opt. Soc. Am. B* **30**, 421–428 (2013).
25. Chuang, I. L. & Yamamoto, Y. Simple quantum computer. *Phys. Rev. A* **52**, 3489–3496 (1995).
26. Nemoto, K. & Munro, W. J. Nearly deterministic linear optical controlled-NOT gate. *Phys. Rev. Lett.* **93**, 250502 (2004).
27. Barrett, S. D. *et al.* Symmetry analyzer for nondestructive Bell-state detection using weak nonlinearities. *Phys. Rev. A* **71**, 060302 (2005).
28. Sheng, Y. B. & Zhou, L. Deterministic entanglement distillation for secure double-server blind quantum computation. *Sci. Rep.* **5**, 7815 (2015).
29. Ding, D., Yan, F. L. & Gao, T. Preparation of km -photon concatenated Greenberger-Horne-Zeilinger states for observing distinctive quantum effects at macroscopic scales. *J. Opt. Soc. Am. B* **30**, 3075–3078 (2013).
30. Sheng, Y. B. & Zhou, L. Two-step complete polarization logic Bell-state analysis. *Sci. Rep.* **5**, 13453 (2015).
31. He, Y. Q., Ding, D., Yan, F. L. & Gao, T. Exploration of multiphoton entangled states by using weak nonlinearities. *Sci. Rep.* **6**, 19116 (2016).
32. Ding, D., Yan, F. L. & Gao, T. Entangler and analyzer for multiphoton Greenberger-Horne-Zeilinger states using weak nonlinearities. *Sci. Chin. Phys. Mech. Astron.* **57**, 2098–2103 (2014).
33. He, Y. Q., Ding, D., Yan, F. L. & Gao, T. Exploration of photon-number entangled states using weak nonlinearities. *Opt. Exp.* **23**, 21671 (2015).
34. Dong, L. *et al.* Nearly deterministic preparation of the perfect W state with weak cross-Kerr nonlinearities. *Phys. Rev. A* **93**, 012308 (2016).
35. Lin, Q. & He, B. Highly efficient processing of multi-photon states. *Sci. Rep.* **5**, 12792 (2015).
36. Dong, Li. *et al.* Generation of three-photon polarization-entangled decoherence-free states. *Ann. Phys.* **371**, 287–295 (2016).
37. Gardiner, C. W. & Zoller, P. *Quantum Noise*, p. 113 (Springer, Berlin, 2000).
38. Knill, E., Laflamme, R. & Milburn, G. J. A scheme for efficient quantum computation with linear optics. *Nature* **409**, 46–52 (2001).
39. Lin, Q. & Li, J. Quantum control gates with weak cross-Kerr nonlinearity. *Phys. Rev. A* **79**, 022301 (2009).
40. Beenakker, C. W. J., DiVincenzo, D. P., Emary, C. & Kindermann, M. Charge detection enables free-electron quantum computation. *Phys. Rev. Lett.* **93**, 020501 (2004).
41. Hong, C. K., Ou, Z. Y. & Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. *Phys. Rev. Lett.* **59**, 2044–2046 (1987).
42. Milburn, G. J. Quantum optical fredkin gate. *Phys. Rev. Lett.* **62**, 2124–2127 (1989).
43. Brunel, C., Lounis, B., Tamarat, P. & Orrit, M. Triggered source of single photons based on controlled single molecule fluorescence. *Phys. Rev. Lett.* **83**, 2722–2725 (1999).
44. Michler, P. *et al.* A quantum dot single-photon turnstile device. *Science* **290**, 2282–2285 (2000).
45. Santori, C. *et al.* Triggered single photons from a quantum dot. *Phys. Rev. Lett.* **86**, 1502–1505 (2001).
46. Wang, S. M., Ma, H. Q. & Wu, L. A. A single photon source based on entangled photon pairs. *Acta Phys. Sin.* **58**, 717–721 (2009).
47. Su, S. L., Cheng, L. Y., Wang, H. F. & Zhang, S. An economic and feasible scheme to generate the four-photon entangled state via weak cross-Kerr nonlinearity. *Opt. Commun.* **293**, 172–176 (2013).
48. Shapiro, J. H. & Razavi, M. Continuous-time cross-phase modulation and quantum computation. *New J. Phys.* **9**, 16 (2007).
49. Gea-Banacloche, J. Impossibility of large phase shifts via the giant Kerr effect with single-photon wave packets. *Phys. Rev. A* **81**, 043823 (2010).
50. He, B. & Scherer, A. Continuous-mode effects and photon-photon phase gate performance. *Phys. Rev. A* **85**, 033814 (2012).
51. Wittmann, C., Andersen, U. L., Takeoka, M. & Leuchs, G. Discrimination of binary coherent states using a homodyne detector and a photon number resolving detector. *Phys. Rev. A* **81**, 062338 (2010).

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant Nos: 11475054, 11371005, and Hebei Natural Science Foundation of China under Grant Nos: A2014205060, A2014205064 and A2016205145.

Author Contributions

M.Y.W., F.L.Y. and T.G. contributed equally to this work. All authors wrote the main manuscript text and reviewed the manuscript.

Additional Information

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Wang, M. *et al.* Generation of four-photon polarization entangled decoherence-free states with cross-Kerr nonlinearity. *Sci. Rep.* **6**, 38233; doi: 10.1038/srep38233 (2016).

Publisher's note: Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.



This work is licensed under a Creative Commons Attribution 4.0 International License. The images or other third party material in this article are included in the article's Creative Commons license, unless indicated otherwise in the credit line; if the material is not included under the Creative Commons license, users will need to obtain permission from the license holder to reproduce the material. To view a copy of this license, visit <http://creativecommons.org/licenses/by/4.0/>

© The Author(s) 2016