Superconductor-metal transition in odd-frequency–paired superconductor in a magnetic field

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Edited by Piers Coleman, Rutgers University, Piscataway, NJ, and accepted by Editorial Board Member Zachary Fisk May 17, 2019 (received for review February 20, 2019)

It is shown that the application of a sufficiently strong magnetic field to the odd-frequency–paired pair-density wave state described in A. M. Tsvelik [*Phys. Rev. B* 94, 165114 (2016)] leads to formation of a low-temperature metallic state with zero Hall response. Applications of these ideas to the recent experiments on stripe-ordered $La_{1.875}Ba_{0.125}CuO_4$ (LBCO) are discussed.

superconductivity | Kondo lattice | strongly correlated systems

Recent magnetotransport measurements in $x = 1/8$ $La_{1.875}Ba_{0.125}CuO_4$ (LBCO) (1) have revealed yet additional extraordinary features of this otherwise highly usual system. It has turned out that when the applied magnetic field destroys the superconductivity in this layered material, the system becomes metallic with zero Hall response. This behavior is robust down to the lowest temperatures; the sheet resistance gradually increases, with the magnetic field leveling off at around $B \sim 30$ T at $G = 2e^2/h$.

At $x = 1/8$, doping the holes in the copper oxide layers of LBCO are arranged in static stripes at temperatures below ~ 40 K. The material undergoes Berezinskii–Kosterlitz–Thouless (BKT) transition at around $T_{BKT} = 16$ K into a 2D superconducting phase with a finite resistivity in the c direction (2). The Meissner effect is established at a much lower temperature, ~ 3 K. The theoretical explanation put forward in ref. 3 assigns these unusual properties to the formation in each CuO layer of pair density wave (PDW)—a superconducting state where the pairs have nonzero momentum $\mathbf{Q}$. If the direction of $\mathbf{Q}$ is different in neighboring copper oxide layers, then the pairs would not able to tunnel, and the layers would remain decoupled. The theory (3, 4) models the PDW state as an array of doped chains separated by undoped regions; the chains contain Luther–Emery liquids with gapped spin sector and enhanced superconducting fluctuations. An isolated chain has a quasi-long-range superconducting order with a spin gap; the chains interact through Josephson coupling (pair tunneling) and the long-range Coulomb interaction. Quasiparticles play no active role in this scenario. I argue that this standard picture of the stripe phase cannot explain the transport data of ref. 1—namely, the combination of metallic longitudinal resistivity and zero Hall conductivity. It will be shown that once the strong magnetic field makes the Josephson coupling irrelevant, the superconducting correlations in the transverse direction become short range, suppressing the transport in the direction perpendicular to the chains. To explain the metallic transport, one has to assume the presence of quasiparticles, as was done in ref. 5. However, then one has to explain zero Hall conductivity.

The present paper suggests a different scenario in which the above difficulties are resolved. It is based on the results obtained in ref. 6. This paper describes a version of a striped model where the spin gap and superconducting coupling on the hole doped stripes come as a result of exchange interactions between the holes and the surrounding spins and the Heisenberg exchange between the spins. This leads to the formation of PDW with the wave vector along the stripes, together with formation of hole- and electron-like Fermi pockets of gapless quasiparticles. The restrictions related to the Luttinger theorem guarantee the equality of the number of electrons and holes and, as a consequence, zero Hall response. The existence of ungapped quasiparticles is due to the fact that the PDW order parameter (OP) had a finite wave vector incommensurate with the Fermi surface which eliminates coupling between the OP and the quasiparticles. The superconductivity is essentially 2D, as in the standard layered model, but since nothing prevents quasiparticles from tunneling between the layers, their transport is 3D.

The present paper begins with a pedagogical description of this model adopted to the case of a layered 3D material with stripes in neighboring layers running perpendicular to each other, as is the case with LBCO. The model displays staggered odd-frequency PDW with a wave vector directed along the stripes. This staggering makes the interlayer coupling of the OPs difficult. Below, I will recall the main results of ref. 6, generalizing them for finite magnetic fields and putting them in the context of ref. 1.

Model

The adopted description of the striped state is one of a Kondo lattice. This is, however, a lattice of a special kind, where the conduction electrons and local moments are segregated into stripes. In the first approximation, we can consider a 2D arrangement of parallel stripes. The salient feature of the model is incommensurability between the Fermi wave vector of the holes occupying the conducting chains (stripes) and the lattice. The standard thinking about Kondo lattices considers its physics as a product of

Significance

It is generally expected that when a magnetic field destroys superconductivity in two dimensions, the system becomes an insulator. It is shown that there is a type of superconductivity—namely, the one where the wave function of pairs is odd in time—where the result is not an insulator, but a metal with a zero Hall response. It is suggested that the transition recently observed in the striped-ordered high-T_c superconductor $La_{1.875}Ba_{0.125}CuO_4$ may belong to this category.

Author contributions: A.M.T. performed research and wrote the paper.

The author declares no conflict of interest.

This article is a PNAS Direct Submission. P.C. is a guest editor invited by the Editorial Board.

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This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1902928116/-/DCSupplemental.

Published online June 10, 2019.

competition between the interspin and the Kondo interactions. If the former one wins, the spins decouple from the electrons, and when the latter wins, the spins fractionalize, hybridize with the itinerant electrons, and become a part of the conduction band, giving rise to a heavy fermion Fermi liquid. It has been frequently suggested (see, for example, refs. 7 and 8) that there are circumstances when the spin system left to its own devices will not magnetically order, but form a liquid—a strongly correlated state with short-range spin correlations. However, the experience of many years of research in this direction indicates that such disordered states are very difficult to realize. If interacting spins do not order magnetically, they tend to form so-called valence bond solids where the magnetic excitations are gapped, but the translational symmetry is still broken.

In ref. 6, I have considered a mechanism of spin-liquid formation based on cooperation between the Kondo and the Heisenberg exchange interactions. Somewhat paradoxically, such cooperation works better when the Heisenberg exchange is stronger than the Kondo one, provided the spin liquid has a Fermi surface, as is the case for solitary spin $S = 1/2$ Heisenberg chains (HCs). Such a situation takes place already for a single spin chain, and, indeed, a single HC coupled to 1D electron gas (1DEG) already provides a mechanism for PDW formation, as has been noticed in refs. 9 and 10. Hence, the simplest way to realize such situation is to consider an array of spin $S = 1/2$ HCs decoupled from each other. In LBCO, the doped and undoped stripes alternate. To simplify matters, I consider a somewhat different situation when doped chains lie on top of the Heisenberg one, as was done in ref. 6. In this case, each spin chain is coupled to only one conducting chain. This arrangement allows me not to consider additional details, which would only muddle the discussion.

A single Kondo–Heisenberg (KH) ladder consists of an antiferromagnetic spin $S = 1/2$ HC coupled to 1DEG via an anferromagnetic exchange interaction:

$$H = \sum_k \epsilon(k)\psi^+_{k\sigma}\psi_{k\sigma} + \frac{J_K}{2}\sum_{k,q}\psi^+_{k+q,\alpha}\sigma^a_{\alpha\beta}\psi_{k,\beta}S^a_q + J_H\sum_n \mathbf{S}_n\mathbf{S}_{n+1}, \quad [1]$$

where ψ^+, ψ are creation and annihilation operators of the 1DEG, σ^a are the Pauli matrices, $\mathbf{S}_n$ is the spin $S = 1/2$ operator on site n, and $\mathbf{S}_q$ is its Fourier transform. It is assumed that $J_K << J_H$ and the 1DEG is far from half filling, $|2k_F a_0 - \pi| \sim 1$ (k_F is the Fermi momentum of the electrons). Under these assumptions, one can formulate the low-energy description of Eq. **1**, taking into account that the backscattering processes between excitations in the HC and the 1DEG are suppressed by the incommensurability of the 1DEG. The effective theory is valid for energies much smaller than both the Fermi energy ϵ_F and the Heisenberg exchange interaction J_H of the model (Eq. **1**). It is integrable (10), and the exact solution was used as a springboard for a controllable approach to the model of a D-dimensional array of KH ladders developed in ref. 6.

At $J_K = 0$, both 1DEG and the HC are critical systems. The excitations of the HC are gapless spinons whose spectrum is linear at small momenta: $\omega = v_H|k|$ with $v_H = \pi J_H/2$. Spinons are fractionalized particles: They carry zero electric charge, and spin $S = 1/2$. In the absence of umklapp, the only smooth parts of the magnetizations of spin and electron chains couple. It is remarkable that in the spin $S = 1/2$ HC, the smooth part can be represented as a sum of the spin currents $\mathbf{j}_{R,L}$ (equation 2 in *SI Appendix*) which allow a fermionic representation: $j^a_R = \frac{1}{2}r^+\sigma^a r,\ \ j^a_L = \frac{1}{2}l^+\sigma^a l$, where r, l are noninteracting 1D fermions with dispersion $\pm v_H k_x$. These fermions carry a fictitious U(1) charge. However, the charge degrees of freedom do not affect the current–current commutation relations and hence do not partake in the interaction with the conduction electrons. When the Kondo coupling is much smaller than the electron bandwidth, we can linearize the electron spectrum close to the Fermi points, introducing right- and left-moving fermions $R(\mathbf{k}) = \psi(k_x + k_F, k_y),\ \ L(\mathbf{k}) = \psi(k_x - k_F, k_y)$. The resulting low-energy description is

$$H = H_+ + H_-, \quad [2]$$

$$H_+ = \sum_{\mathbf{k}}\left\{\epsilon_R(\mathbf{k})R^+_\alpha(\mathbf{k})R_\alpha(\mathbf{k}) + v_H k_x l^+_\alpha(\mathbf{k})l_\alpha(\mathbf{k})\right\} + J_K\int \mathrm{d}V R^+\sigma^a R(r)l^+\sigma^a l(r), \quad [3]$$

$$H_- = H_+(R \to L, L \to R, k_x \to -k_x). \quad [4]$$

We can choose $\epsilon_{R,L} = \pm v_F(k_x - k_F) + 2t(\cos k_y + \cos k_z)$.

Representation Eqs. **3** and **4** is similar to the one frequently adopted for the Kondo lattices; see, for example, ref. 11. However, there is one difference—namely, that in our approach, the spinon right- and left-moving fermionic operators r, l do not interact, and in the standard treatment where the spins are arranged on a 3D lattice with isotropic interactions, they do.

The Spectrum and the OPs

The following simple mathematical description gives the gist of what is going on. Although it is possible to carry on the calculations rigorously, as was done in ref. 6, I will resort to a simplified approach. Namely, I decouple the interaction with the Hubbard–Stratonovich transformation and look for the saddle point:

$$J_K R^+\sigma^a R l^+\sigma^a l \to |\Delta_+|^2/2J_K + (\Delta_+ R^+_\alpha l_\alpha + H.c.), \quad [5]$$

and the same for L and r. Then, the quasiparticle spectrum at the saddle point is

$$E_\pm(k) = \pm(k_x v_H - \epsilon_R)/2 + \sqrt{(k_x v_H + \epsilon_R)^2/4 + |\Delta_+|^2}, \quad [6]$$

where $\epsilon_R = v_F k_x + t_\perp(k)$, where $t_\perp$ is the Fourier transform of the interchain tunneling. Strictly speaking, this procedure is justified when the SU(2) symmetry is replaced by the SU(N) one with $N >> 1$. However, as was demonstrated in ref. 6, the results remain robust, even for $N = 2$. Some details can be found in *SI Appendix*. However, even on this level, we see that $\Delta_\pm$ are complex fields, and their phases must remain gapless.

If electrons are also 1D, their Fermi surface is flat. The processes which lead to creation of the spectral gaps in the spectrum of the spin excitations are depicted in Fig. 1, *Left*. It

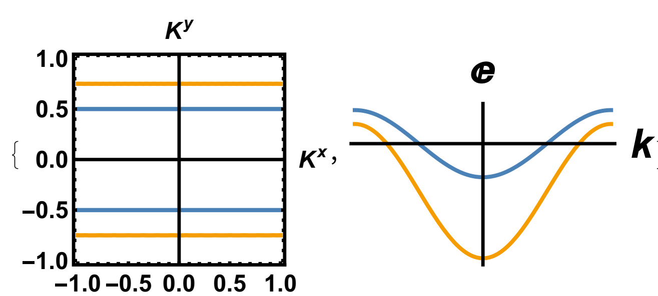

Fig. 1. (*Left*) Spinon (thick blue) and electron (yellow) Fermi surfaces in the array of 1D Kondo–Heisenberg ladders. In the limit when ladders are decoupled, the Fermi surfaces are flat and exhibit a perfect nesting. Then spinons and electrons with opposite chirality hybridize and create spin gaps. The spin subsystem decouples into two independent spin liquids. (*Right*) The bare spinon (blue) and electron (yellow) spectra. The Fermi momenta of electrons and spinons do not coincide.

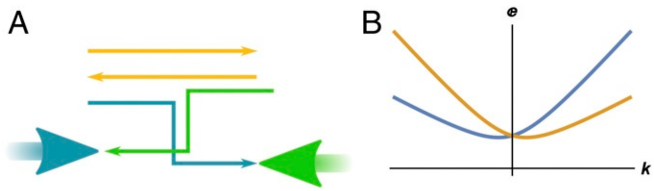

Fig. 2. Holons of 1DEG (orange arrows) do not pair. (*A*) Spinons of 1DEG (thin arrows) pair with the spinons of opposite chirality from the HC (thick arrows). This forms the gapped spinon dispersion (Eq. **6**) shown in *B*. $e = E/\Delta$, $q = k_x(v_H v_F)^{1/2}/\Delta$, and $v_F/v_H = 1/2$. The spinons of the Heisenberg model located at wave vectors $-\pi/2a_0$ ($+\pi/2a_0$) pair with the spinons of the 1DEG located at k_F ($-k_F$). The product of the corresponding pairing amplitudes forms the amplitude A of the composite OP (Eq. **8**).

is assumed that the Fermi momenta of electrons and spinons are different, so that there are no umklapp processes, and the hybridization takes place only between spinons and electrons of opposite chirality (Fig. 2). This opens a gap on the entire electron Fermi surface. If one allows an interchain tunneling, the nesting becomes imperfect, and pockets of electron- and hole-like quasiparticles will appear, as in Fig. 3.

The Hubbard–Stratonovich approach gives a somewhat simplified picture of the spectrum. It turns out that the gapped parts of the spectrum (Eq. **6**) correspond to neutral spinons–spin-1/2 incoherent excitations which remain confined to the chains. The gapless parts correspond to coherent quasiparticles, whose Fermi surfaces in the form of particle and hole pockets are shown in Fig. 3.

There are also gapless collective modes corresponding to fluctuations of the OP fields. The Hubbard–Stratonovich fields $\Delta_\pm$ contain a fictitious U(1) phase of the r, l fermions and hence are not gauge-invariant. The real OP fields in the KH ladder are their products $\Delta_+\Delta_-$ and $\Delta_+\Delta_-^*$.[†] These OPs can be expressed in terms of the electron and spin operators:

$$\mathcal{O}_{cdw} = \psi^+(x)\left[(\mathbf{S}_x\mathbf{S}_{x+a_0})\hat{I} + \mathrm{i}(\sigma^a S_x^a)\right]\psi(x)\mathrm{e}^{\mathrm{i}(\pi/a_0+2k_F)x}$$
$$\mathcal{O}_{sc} = \mathrm{i}(-1)^{x/a_0}\psi(x)\sigma^y\left[(\mathbf{S}_x\mathbf{S}_{x+a_0})\hat{I} + \mathrm{i}(\sigma^a S_x^a)\right]\psi(x), \qquad [7]$$

where $\hat{I}$ is a unit matrix. For a single KH chain, correlation functions of these composite OPs have a power-law decay at $T = 0$. These OPs can be conveniently written in the matrix form:

$$\hat{\mathcal{O}} = \begin{pmatrix} \mathcal{O}_{cdw} & \mathcal{O}_{sc}^+ \\ -\mathcal{O}_{sc} & \mathcal{O}_{cdw}^+ \end{pmatrix} = A\hat{g}, \qquad [8]$$

where $A \sim \Delta$ is an amplitude and g is the matrix field of the SU_1 (2) Wess–Zumino–Witten–Novikov model governing the dynamics of the collective charge excitations (*SI Appendix*, Eq. **S6**).

Using the equations of motion $\dot{\psi} = [H, \psi]$, where the dot stands for derivative in Matsubara time, one can show that these OPs have finite overlap with the OPs of the odd-frequency PDW and odd-frequency CDW:

$$\mathcal{O}_{osc} = \dot{\psi}(\tau, x)\sigma^y\psi(\tau, x)(-1)^{x/a_0},$$
$$\mathcal{O}_{ocdw} = \dot{\psi}^+(\tau, x)\psi(\tau, x)\mathrm{e}^{\mathrm{i}(2k_F+\pi/a_0)x}. \qquad [9]$$

One can find more detailed discussion of odd-frequency superconductivity in the review article (12). The idea that Kondo lattices may support odd-frequency superconductivity was put forward in the 1990s (13, 14), and its relation to the composite orders (Eq. **7**) was discussed in refs. 9, 10, and 15. However, the mean field theory presented in refs. 13 and 14 was too simple to account for interesting properties of the KH ladder encoded in its correlation functions.

Let us now turn to the quasiparticles. The best way to detect them is to calculate the single-particle Green's function. For the standard model, such calculations were done in ref. 5, with the result that the PDW leaves certain parts of the Fermi surface ungapped. However, this approach does not produce zero Hall response—one of the striking features on the metallic phase observed in ref. 1. In the PDW state discussed here, this feature comes as a consequence of the strong interactions and the spin-gap formation. The simplest way to see this is to consider the random phase approximation (RPA) form of the Green's function:

$$G_{RPA} = \left[G_{1D}^{-1}(\omega, k_x) - t_\perp(\mathbf{k})\right]^{-1}, \qquad [10]$$

where G_{1D} is the Green's function of a single KH ladder. This approximation allows one to take into account the strongest correlations on a single chain encoded in G_{1D} which can be calculated nonperturbatively. Its precise form is given in ref. 6 and *SI Appendix*. In the present context, we need to know that $G_{1D}(\omega = 0, k_x = \pm k_F) = 0$ which allows the purely 1D KH ladder to satisfy the Luttinger theorem, despite the absence of Fermi surface. This property translates to Eq. **10**, which guaranties that even for sufficiently large $t_\perp$ when Eq. **10** acquires quasiparticle poles at zero frequency, they will not contribute to the Luttinger volume already fixed by the zeroes. In other words, the poles will cancel each other, resulting in a compensated metal with zero Hall response (*SI Appendix*).

A Failure of the Standard Model

Below, I consider the standard model of stripes in the strong magnetic field and will show that once the Josephson tunneling is frustrated, the low-temperature transport of pairs becomes impossible. The standard model describes the charge sector of the stripe phase as an array of 1D Luther–Emery liquids coupled by Josephson tunneling. The effective Lagrangian density describing superconducting fluctuations of such system is

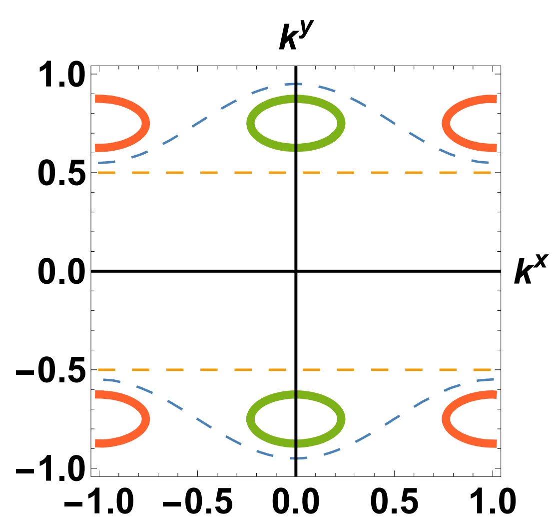

Fig. 3. Pockets of electron- and hole-like quasiparticles formed in the spin-liquid state with a sufficiently strong interstripe electron tunneling. The bare electron and spinon Fermi surfaces are gapped and marked by dashed lines.

[†]I remind the reader that in one dimension, there is only quasi-long-range order, meaning that the order parameter fields have power-law correlations at $T = 0$.

$$\mathcal{L} = \sum_y \left\{ \frac{1}{2}\left[v^{-1}(\partial_\tau\theta_y)^2 + v(\partial_x\theta_y)^2\right] + J\cos[\beta(\theta_y - \theta_{y+1}) - 2hx/v] \right\}, \quad [11]$$

where $h = eHa_0v/c$, with a_0 being the interchain distance and H the applied magnetic field; v is the velocity of the phase mode; and β is a parameter related to the interactions. In what follows, I set $v = 1$. I assume that the long-range Coulomb interaction is screened, for instance, by the gapless quasiparticles, as in ref. 5.

For our purposes, it will be sufficient to calculate the OP correlation function $\chi_P(\tau; x, y) = \langle\langle e^{i\beta\theta_y(\tau,x)} e^{-i\beta\theta_0(0,0)}\rangle\rangle$, for the large magnetic field when it can be done by using the perturbation theory. The expansion parameter of this theory is J/h^{2-2d}, where $d = \beta^2/4\pi$. In the leading order in this parameter, the correlation function for a given y must include only minimal number of Josephson interactions sufficient for the pair to tunnel for a given distance:

$$\chi_P(A, B; y) = J^y \int \prod_{i=1}^{y} d\tau_i dx_i \chi(A,1)\chi^*(1,2)\chi(2,3)\ldots\chi(y-1,B) \times e^{2ih(x_1 - x_2 + x_3 + \ldots)}, \quad [12]$$

where A, B are shorthand for $(\tau_A, x_A), (\tau_B, x_B)$, and $\chi(1,2)$ is just the correlation function $\chi_P(\tau_1 - \tau_2, x_1 - x_2; y = 0)$ at $J = 0$. Taking the Fourier transform and setting $k_x = 0$ to simplify the expressions, we get

$$\chi_P(\omega, k_x = 0, k_y) = \frac{G^{-1}(\omega, h) - J\cos k_y}{(G^{-1}(\omega, h) - J\cos k_y)^2 + J^2\sin^2 k_y},$$

where $G^{-1} = A(\omega^2 + h^2)^{1-d}$, with A being a nonuniversal dimensional parameter. At $J << h^{2-2d}$, the correlator is short-ranged, which means that the transverse tunneling of pairs is blocked. At these circumstances, one is left with conductivity along the chains, but since this is associated with charge density waves which are pinned by disorder, this will also vanish.

Summary and Discussion

Let us give a brief summary of physics of Kondo-Heisenberg array when the magnitude of the interstripe tunneling is of the order of the spin gap. At energies below the spin gap, the system effectively splits into two quasiindependent sectors. One sector is the collective modes—the superconducting and the CDW fluctuations. The other sector is the quasiparticles. The OPs are staggered with wave vectors incommensurate with the Fermi surface which has the most profound consequences for the low-temperature behavior. First of all, the incommensurability guarantees that the quasiparticle Fermi surface remains ungapped, even when there is a true long-range order. Then, in the layered system where the stripes in the neighboring planes are perpendicular to each other, the order becomes effectively 2D since the interlayer coupling is frustrated. The quasiparticle tunneling, however, is not frustrated, and the quasiparticles are free to propagate in all directions, which prevents their localization. At last, the total Fermi surface volume (the Fermi volume of the electron minus the volume of the hole-like parts) is zero. As is explained above, this is a property of the strongly correlated spin-liquid state from which the PDW originates. As a consequence, the Hall response is zero below the BKT transition.

Measurements of the specific heat produce a finite value of $\gamma(T \to 0) = 2.5$ mJ·K^{-2}mol^{-1} which increases to 2.8 mJ·K^{-2}mol^{-1} in H = 9 T (16) (about an order of magnitude smaller than in the normal state). This is consistent with the existence of a Fermi surface. The quasiparticle Fermi energy must be of the order of the spin gap which for a similar material $La_{2-x}Sr_xCuO_4$ is estimated as ∼9 meV (17). Such shallow Fermi sea could remain undetected by the angle-resolved photoemission spectroscopy (ARPES) measurements (18) in the stripe-ordered LBCO. In any case, the ARPES experiments represent problems for the standard model as well. Another potential problem is magnetic order in the stripe-ordered phase. Naturally, a strong order will destroy the spin gap which gives rise to the PDW. However, a weak order may coexist with PDW (6) (*SI Appendix*) and the measurements in a similar compound La1.48Nd0.4Sr0.12CuO4 with $x = 0.12$ yield a small Cu moment of 0.10 ± 0.03 μ_B (19). Besides zero Hall conductivity and finite γ, there is another feature which distinguishes the present theory from the standard one, it the direction of the wave vectors of the staggered OPs. They are directed along the stripes, and this presumably can be tested experimentally.

ACKNOWLEDGMENTS. I thank J. Tranquada for very useful discussions, as well as L. B. Ioffe and S. A. Kivelson for valuable remarks. The work was supported by Office of Basic Energy Sciences, Material Sciences and Engineering Division, US Department of Energy Contract DE-SC0012704.

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