

# Experimental Measurement and Thermodynamic Modeling of the Wax Disappearance Temperature (WDT) for a Quaternary System of Normal Paraffins

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ABSTRACT: Normal paraffin (N-alkane)-based wax is well known as a severe problem in petroleum production, transportation, and processing. Implementing suitable solutions for wax-related problems requires vast technical knowledge and investigation of the wax disappearance temperature (WDT) of multicomponent systems in petroleum-dominated systems. In this study, the WDTs of a quaternary system comprising different mixtures of *n*-undecane + *n*-tetradecane + *n*-hexadecane + *n*-octadecane were measured using a visual-based diagnosis apparatus under atmospheric pressure. On the other hand, the WDTs of the studied systems are predicted by applying a solid solution model without any adjustable parameter. Two approaches namely  $\gamma-\varphi$  and  $\gamma-\gamma$  are assessed. In the  $(\gamma-\varphi)$  approach, perturbed-chain statistical associating fluid theory (PC-SAFT) is applied for liquid phase modeling, while the solid phase is described using different activity coefficient models. In the  $(\gamma-\gamma)$  approach, nonidealities of both the liquid and solid phases are investigated using different combinations of activity coefficient models such as ideal solution, regular solution theory, predictive Wilson, predictive UNIQUAC, and UNIFAC. Comparison of experimental data and thermodynamic modeling results indicates that applying the predictive UNIQUAC model for describing the nonideality of the solid phase and the regular solution model for the liquid phase is the best combination for the aforementioned system with the average absolute deviation (AAD) of 0.8 K.

# 1. INTRODUCTION

It is estimated that oil extraction growth will be continued until  $2040<sup>1</sup>$  $2040<sup>1</sup>$  $2040<sup>1</sup>$  For enhancing oil production over a short period of time, there should be a significant improvement in areas related to extraction/production science and technology. It should be particularly mentioned that there are a significant number of oil fields that contain waxy compounds with high molecular weights. Under certain thermodynamic circumstances, n-alkanes deposition can decrease oil production efficiency by crystallizing on equipment.<sup>[2](#page-9-0)−[4](#page-9-0)</sup> To vanquish the serious adversities in the way of oil production and processing and predominantly its transportation, detailed comprehension of the solid−liquid equilibria of waxy oils is required. Hereupon, studying the behaviors of  $n$ -alkanes with the measurement and prediction of the wax appearance temperature (WAT) is of interest. WAT describes a temperature at

which the initial crystal of wax starts to precipitate when cooling a liquid mixture.<sup>[5](#page-9-0)</sup> Based on previous research studies,<sup>[6](#page-9-0)−[8](#page-9-0)</sup> WAT is not a reliable parameter because of weak reproducibility, repeatability, and its dependency on measurement conditions. Moreover, WAT is more dependent on the kinetics of crystal formation like cooling rates or experimental measurement techniques, so the experimental conditions and the measurement techniques have great impacts on it; in other words, it is a path function property. On the other hand, wax

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disappearance temperature (WDT) is a temperature wherein the last tiny particles of wax crystals break down in the liquid phase on heating and represents an actual equilibrium state, a point function property.<sup> $\dot{b}$ ,9−[11](#page-9-0)</sup> Several studies indicate that the difference between the measured WAT and WDT by one method can be considerable. $12,13$  $12,13$  $12,13$ 

Researchers have conducted different experimental studies on WATs and WDTs of mixtures of alkanes and waxy crude oils using different methods such as light transmittance, $12$ cross-polar microscopy (CPM),<sup>8</sup> differential scanning calorimetry  $(DSC)$ ,<sup>[14](#page-9-0)</sup> and viscometry.<sup>15</sup> For example, Daridon et al.<sup>16</sup> utilized a high-pressure cell mounted on a polarizing microscope to measure WDT for several  $n$ -alkane mixtures from *n*-C<sub>13</sub> to *n*-C<sub>24</sub> under pressures up to 100 MPa. Due to the limitations of conventional laboratories, although this apparatus is highly accurate, it is expensive and difficult to access. Pauly et al.<sup>17</sup> investigated the quantity and composition of the precipitated wax in five mixtures of an  $n-C_{10} + (n-C_{24} +$  $n-C_{25} + n-C_{26}$  quaternary system at different temperatures under a WAT applying chromatographic technique and filtration. They also measured the WAT of each mixture at atmospheric pressure. It is evident that due to the quaternary system's complexity, WAT measuring for only five different mixtures of a quaternary system is not sufficient to estimate system behavior. Mansourpoor et  $al.^{18}$  $al.^{18}$  $al.^{18}$  experimentally measured the WATs of 12 samples of Iranian crude oils and condensates using DSC and viscometry methods. They used an artificial neural network (ANN) model to estimate the WDTs of crude oils. They also introduced a precise correlation that determines the WDT with good agreement with laboratory data. Due to the expensiveness, time consumption, and difficulty of the experimental approaches, the development of thermodynamic models can help operating engineers predict wax deposition conditions in production facilities, oil transportation pipelines, or process equipment. Currently, there are two viewpoints for thermodynamic modeling of wax deposition. First, the solid solution (SS) approach states that only one solid phase of wax containing miscellaneous hydrocarbon components can exist.<sup>19,[20](#page-9-0)</sup> Second, the multi solid (MS) approach states that the wax phase is composed of several pure solid phases.<sup>[21](#page-9-0)</sup> Won<sup>[19](#page-9-0)</sup> developed one of the initial SS models for wax deposition. In this model, for the description of the solid phase, a modified regular solution theory is employed, and the Soave−Redlich−Kwong (SRK) equation of state  $(Es)^{22}$  is applied for vapor-liquid equilibria calculations. Also, Won corrected fusion temperatures and fusion enthalpies of pure n-alkanes as a function of molecular weights. Pedersen et al.<sup>[23](#page-9-0)</sup> developed Won's model and, for WAT prediction, calculated the activity coefficients of both solid and liquid phases from the modified regular solution theory. Coutinho and Stenby<sup>[20](#page-9-0)</sup> surveyed the orthorhombic solid phase by Wilson's model. Afterward, Coutinho et al. $^{24}$ presented a modified UNIQUAC model<sup>[24](#page-9-0)</sup> to describe the orthorhombic solid-phase in solid−liquid equilibria of hydro-carbon mixtures. In comparison with Wilson's model,<sup>[25](#page-9-0)</sup> the UNIQUAC model<sup>[25](#page-9-0)</sup> was more practical. They tested a combination of UNIFAC and Flory free-volume approaches<sup>25</sup> to describe the nonideality behavior of the liquid phase, but the results are not satisfying enough. Ji et al.<sup>6</sup> presented a thermodynamic model according to the SS theory, $19$  which calculates the WDT and determines the quantity and composition of precipitated wax. According to their model, the UNIQUAC excess Gibbs energy model<sup> $25$ </sup> is used for solid

fugacity calculation in the solid−liquid equilibrium. Also, to obtain fugacity in the vapor−liquid equilibrium, the SRK and Peng-Robinson (PR) EoSs<sup>22,[26](#page-9-0)</sup> are employed. New correlations are provided for heat capacity and fusion properties, which consider the impact of even or odd  $n$ -paraffin carbon numbers to achieve more accurate results. They also measured the WDTs of some binary systems to validate their proposed correlations. Esmaeilzadeh et al. $^{27}$  $^{27}$  $^{27}$  used combinations of different activity coefficient models to predict the WATs of some binary, ternary, quaternary, and multicomponent systems. They realized that when the carbon numbers of system components are consecutive, the SS model<sup>[19](#page-9-0)</sup> is more accurate, and the MS model $^{21}$  $^{21}$  $^{21}$  predicts WAT lower than the actual value in general. Aftab et al. $11$  conducted some experiments to measure the WDTs of two different ternary systems containing  $n - C_{11} + n - C_{16} + n - C_{18}$  and  $n - C_{14} + n - C_{16} +$  $n-C_{18}$  by a visual method. They also used two different SS models for thermodynamic modeling. In the first method, PC-SAFT  $EoS<sup>28</sup>$  and the predictive version of the UNIQUAC  $(p. UNIQUAC)$  activity coefficient model<sup>[24](#page-9-0)</sup> are used to estimate the nonideality of the liquid and solid phases, respectively. In the second method, they assumed that both phases could be described incorporating different activity coefficient models. The results indicate that the combination of regular solution theory<sup>[19](#page-9-0)</sup> for the liquid phase with the predictive version of the Wilson  $(p.Wilson)^{20}$  $(p.Wilson)^{20}$  $(p.Wilson)^{20}$  for the solid phase leads to the best precision relative to other activity coefficient models (AADs of 0.48 and 0.76 K for  $n-C_{11} + n-C_{16}$ ) + n-C<sub>18</sub> and n-C<sub>14</sub> + n-C<sub>16</sub> + n-C<sub>18</sub> systems, respectively). Parsa et al.<sup>[29](#page-9-0)</sup> measured the WDTs of three different binary systems  $(n-C_{11} + n-C_{18}, n-C_{16} + n-C_{18}, n-C_{14} + n-C_{16})$  and compared experimental WDTs with the results of thermodynamic models based on  $SS<sup>19</sup>$  $SS<sup>19</sup>$  $SS<sup>19</sup>$ . The best combination of the activity coefficient models for the  $(n-C_{11} + n-C_{18})$  and  $(n-C_{16} + n-C_{18})$  systems was obtained by the ideal solution theory<sup>25</sup> for the liquid phase and p.UNIQUAC model<sup>[24](#page-9-0)</sup> for the solid phase (AADs of  $0.22$ ) and 0.32 K, respectively). The  $n-C_{14} + n-C_{16}$  system had the lowest AAD value of 0.43 K with the ideal solution<sup>25</sup> + p.Wilson package.<sup>[20](#page-9-0)</sup>

There are a few experimental studies on WATs and WDTs of *n*-alkanes in quaternary systems.<sup>[16,17](#page-9-0)</sup> However, no extensive and methodical studies have yet been performed on quaternary systems. This work aims to investigate a quaternary system in various compositions accurately and validate different thermodynamic models to predict the WDT. In this study, WDTs of a quaternary system containing  $n-C_{11} + n-C_{14} + n$ - $C_{16}$  + n- $C_{18}$  were experimentally measured using a visual-based diagnosis apparatus under the atmospheric pressure (about 0.9 bar). Then, a thermodynamic model based on the SS model<sup>19</sup> is proposed to determine the WDT of the studied quaternary system. For describing the liquid phase, PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  and different activity coefficient models such as ideal behavior,<sup>[25](#page-9-0)</sup> regular solution theory,  $19$  and UNIFAC<sup>[25](#page-9-0)</sup> are applied. The solid phase degree of nonideality behavior is characterized by ideal behavior,<sup>[25](#page-9-0)</sup> regular solution theory,<sup>[19](#page-9-0)</sup> p. Wilson,<sup>[20](#page-9-0)</sup> p.UNIQUAC, $^{24}$  $^{24}$  $^{24}$  and UNIFAC<sup>[25](#page-9-0)</sup> activity coefficient models. Eventually, the modeling results are compared with the experimental data, and then the best combination of thermodynamic sets for the liquid and solid phases, which can precisely determine the experimental results, are introduced.

# <span id="page-2-0"></span>2. RESULTS AND DISCUSSION

The reliability of the WDT measurement technique used in this study was confirmed previously[.29](#page-9-0) WDTs of 30 mixtures of the  $n - C_{11} + n - C_{14} + n - C_{16} + n - C_{18}$  quaternary system were experimentally measured. The composition of these mixtures is reported in Table 1.

Table 1. Experimental Data of WDTs at Atmospheric Pressure (about 0.9 bar) for the  $n - C_{11} + n - C_{14} + n - C_{16} + n$ - $C_{18}$  Quaternary System Depending on Weight Fractions<sup>a</sup>

	weight fraction				
mixture number	$n-$ $C_{11}H_{24}$	$n-$ $C_{14}H_{30}$	n- $C_{16}H_{34}$	$n-$ $\rm{C_{18}H_{38}}$	Exp. WDT (K)
1	0.3114	0.2224	0.4124	0.0538	275.8
$\mathbf{2}$	0.0752	0.5791	0.2404	0.1052	276.8
3	0.1347	0.4032	0.3161	0.1460	278.9
$\overline{4}$	0.2477	0.1851	0.4023	0.1649	280.0
5	0.2857	0.0476	0.5476	0.1191	281.8
6	0.1036	0.2518	0.5443	0.1003	282.4
7	0.1830	0.2340	0.3401	0.2429	282.4
8	0.2556	0.0612	0.4833	0.1999	282.9
9	0.1925	0.1048	0.5270	0.1757	283.0
10	0.2439	0.2927	0.2195	0.2439	283.1
11	0.1170	0.1504	0.5773	0.1553	284.4
12	0.1291	0.1211	0.5748	0.1750	284.5
13	0.1025	0.1115	0.6826	0.1034	285.1
14	0.1029	0.5379	0.0718	0.2874	286.0
15	0.3075	0.1963	0.1925	0.3037	286.2
16	0.4413	0.1019	0.1103	0.3465	286.3
17	0.1470	0.0625	0.4778	0.3127	287.0
18	0.0747	0.1969	0.3895	0.3389	287.2
19	0.1250	0.2001	0.2714	0.4035	287.3
20	0.3354	0.1573	0.1360	0.3713	287.5
21	0.0910	0.0957	0.5266	0.2866	287.6
22	0.1404	0.1788	0.2798	0.4010	287.6
23	0.2374	0.2290	0.1695	0.3641	288.4
24	0.1197	0.0852	0.3563	0.4388	289.6
25	0.2102	0.1748	0.1542	0.4608	291.1
26	0.0732	0.0488	0.4390	0.4390	291.5
27	0.0765	0.1133	0.2616	0.5486	292.1
28	0.2406	0.1036	0.0799	0.5760	293.3
29	0.0636	0.2920	0.0796	0.5648	293.5
30	0.1505	0.2159	0.0768	0.5568	293.6
<sup>a</sup> Expanded uncertainty $U_c$ is $U_c(T) = 0.1$ K, $U_c(w) = 0.0001$ , and $U_c$ $(P) = 0.1$ bar (0.95 level of confidence).					

Light n-alkanes play an important role in the oil industry. For example, they are used as the feed of the isomerization process.  $C_4-C_7$  and  $C_7-C_{15}$  *n*-alkanes are used to produce high octane gasoline and diesel fuel, respectively.  $C_{15+}$  nalkanes make up greater than 80 wt % wax and commonly, distillate cuts have large amounts of these components, which their probable presence in the feed of isomerization causes the freezing point of products to be higher than the standard level. According to the 8th and 17th mixtures or the 9th and 21st mixtures in Table 1, it can be seen that when the weight fractions of  $n-C_{14}$  and  $n-C_{16}$  are approximately fixed and the weight fraction of  $n$ -C<sub>18</sub> is increased, the WDTs in mixture numbers 17 and 21 are increased. This observation reflects the fact that increasing the weight fractions of the heavier components will cause an increase in the  $WDT<sup>11</sup>$  $WDT<sup>11</sup>$  $WDT<sup>11</sup>$ . This has been mentioned in [Section 4.3](#page-4-0).

Fifteen different sets of activity coefficient models and five various sets of the PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  + activity coefficient models were applied to calculate WDTs without any adjustable parameters. The results of using various activity coefficient models including ideal behavior,  $25$  regular solution theory,  $19$ UNIFAC,<sup>[25](#page-9-0)</sup> and PC-SAFT EoS<sup>[28](#page-9-0)</sup> for the liquid phase and combining them with the ideal behavior, $25$  regular solution theory,<sup>[19](#page-9-0)</sup> p.Wilson,<sup>[20](#page-9-0)</sup> p.UNIQUAC,<sup>[24](#page-9-0)</sup> and UNIFAC<sup>25</sup> models for the solid phase were investigated by calculating the average absolute deviation (AAD) with and without considering the solid−solid phase transition. The calculated AADs (K) have been classified from low to high values in Table 2.

Table 2. Average Absolute Deviations (AADs) for WDT



 ${}^a\text{AAD}(K) = \frac{1}{n} \sum_{i=1}^n \text{INDT}_i^{\text{Cal}} - \text{WDT}_i^{\text{Exp}}.$ 

Comparing rows 1−4 of Table 2 illustrates that the liquidphase behavior is close to the ideal solution because by changing the activity coefficient models or using PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  for describing the liquid phase while the p.UNIQUAC  $model<sup>24</sup>$  is applied to characterize the solid-phase behavior, outputs are not varied significantly. This result is in line with the formerly published literature.<sup>[11,29](#page-9-0)</sup> According to Table 2, it can be observed that the most suitable sets of activity coefficient models that lead to the lowest deviation are the regular solution theory<sup>[19](#page-9-0)</sup> and p.UNIQUAC model<sup>24</sup> for the liquid and solid phases, respectively. The reason is that the

p.UNIQUAC model $^{24}$  $^{24}$  $^{24}$  is a modified version of the UNIQUAC<sup>25</sup> to calculate solid–liquid equilibria of alkanes and can accurately describe the nonidealities of the paraffinic solid solutions and can predict the phase behavior of complex hydrocarbon mixtures at low and high pressures. The high accuracy of the p.UNIQUAC model $^{24}$  is due to several reasons: (1) this model takes into account the size, shape, and structure of the molecules to compute the entropic term. Therefore, this model is suitable for systems containing light to heavy hydrocarbons. (2) It considers the impact of temperature on the molecular interactions in the enthalpic term. (3) It can also determine the phase split, and (4) by assuming the pair interaction energies as a function of the heat of sublimation, it becomes a fully predictive model. Moreover, it can be interpreted from [Table 2](#page-2-0) that PC-SAFT  $EoS<sup>28</sup>$  with no binary interaction parameter can successfully calculate the nonidealities of the liquid phase. The advantage of PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  is its accuracy in computing the fugacities of the heavy components in the fluid phases.  $11,28,30,31$  $11,28,30,31$  $11,28,30,31$  $11,28,30,31$  Experimental and predicted (regular solution<sup>19</sup> + p.UNIQUAC<sup>[24](#page-9-0)</sup>) WDTs are depicted in Figure 1.



Figure 1. Comparison of experimental and predicted WDTs by regular solution theory<sup>[19](#page-9-0)</sup> for the liquid phase and p.UNIQUAC<sup>24</sup> for the solid phase for different mixtures of the  $n - C_{11} + n - C_{14} + n - C_{16} + n$ - $C_{18}$  quaternary system considering the solid–solid phase transition.

Figure 1 shows that the set of the regular solution model<sup>19</sup> for the liquid phase and the p. UNIQUAC model<sup>24</sup> for the solid phase can accurately predict the WDT of the quaternary system studied in this work at a variety of paraffin concentrations. The reason is the superiority of the p. UNIQUAC model $^{24}$  in calculating the nonidealities of the solid phase. It can be realized in Figure 2 that using some activity coefficient models such as ideal solution, $25$  regular solution theory,<sup>19</sup> and UNIFAC<sup>[25](#page-9-0)</sup> for the solid phase leads to the WDT overestimation.

Furthermore, it is concluded that an accurate calculation of the nonideality of the solid phase is a key parameter for the system studied in this work. It is also seen in Figures 1 and 2 that the p.UNIQUAC<sup>[24](#page-9-0)</sup> and p.Wilson<sup>20</sup> models are suitable for the representation of the solid-phase nonideality, while applying the UNIFAC model<sup>25</sup> for the solid phase has the least precision in predicting the WDT. One of the reasons for the low accuracy of the UNIFAC model<sup>25</sup> is that this model



Figure 2. Comparison of experimental and predicted WDTs by different sets of activity coefficient models vs mixture number for the  $n-C_{11} + n-C_{14} + n-C_{16} + n-C_{18}$  quaternary system considering the solid−solid phase transition.

was used in its original format and no modification was applied to it. Figure 3 gives a comparison between the experimental



Figure 3. Comparison of experimental and predicted WDTs by PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  for the liquid phase and different activity coefficient models for the solid phase considering the solid−solid phase transition.

data of the WDT measured in this work and various models that employ PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  for the liquid phase and different activity models for the solid phase.

Figure 3 depicts that the combination of PC-SAFT  $EoS^{28}$  $EoS^{28}$  $EoS^{28}$ and the p.UNIQUAC $^{24}$  $^{24}$  $^{24}$  activity coefficient models gives the best results. The results of the PC-SAFT<sup>28</sup> + p. Wilson<sup>2</sup> package are satisfactory, while using the ideal solution,<sup>25</sup> regular solution,<sup>19</sup> and the UNIFAC<sup>[25](#page-9-0)</sup> models for the solid phase leads to poor results. [Figure 4](#page-4-0) compares the fractional deviation of WDTs for multiple thermodynamic packages employed in this work.

<span id="page-4-0"></span>

Figure 4. Comparison between fractional deviations of three sets of activity coefficient models in prediction of WDTs.

Figure 4 shows that applying the ideal liquid−ideal solid approach leads to the overestimation of WDTs for all of the mixtures. This is consistent with the results obtained by Ji et al.<sup>[6](#page-9-0)</sup> Both regular solution-p.UNIQUAC<sup>[24](#page-9-0)</sup> and regular solution<sup>19</sup> -p.Wilson<sup>20</sup> combinations result in fractional -p.Wilson<sup>[20](#page-9-0)</sup> combinations result in fractional deviations lower than 1% for all of the mixtures, which indicates the high accuracy of these two models.

By evaluating a variety of activity coefficient models and PC-SAFT EoS, $^{28}$  it is concluded that the solid-phase nonideality is a key parameter in the accuracy of a thermodynamic model. Therefore, applying the p.UNIQUAC<sup>[24](#page-9-0)</sup> and p.Wilson<sup>[20](#page-9-0)</sup> activity coefficient models is recommended for the solid phase. Employing various models for the liquid phase leads to approximately the same results. The regular solution<sup>[19](#page-9-0)</sup> + p.UNIQUAC model<sup>[24](#page-9-0)</sup> is the best combination for calculating the WDTs studied in this work. The input parameters of this model are the physical properties, the pressure, and the compositions of the n-alkanes in the mixtures. By changing the pressure and composition of n-alkanes in the model, it is concluded that the pressure has a negligible impact on WDT, while the compositions of  $n$ -alkanes have a significant effect on it.

# 3. CONCLUSIONS

In this study, WDTs of the *n*-undecane + *n*-tetradecane + *n*hexadecane + n-octadecane quaternary system were measured using a visual-based diagnosis apparatus at atmospheric pressure of about 0.9 bar. The WDTs were predicted by combining the different activity coefficient models together and with PC-SAFT EoS<sup>[28](#page-9-0)</sup> for both solid and liquid phases without any adjustable parameter. The investigations indicate that using the different activity coefficient models or PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  for the liquid phase approximately gives the same results. The UNIFAC model<sup>25</sup> is not appropriate for the explanation of the nonideality of the solid phase, while the p.UNIQUAC<sup>24</sup> and the p.Wilson<sup>[20](#page-9-0)</sup> models show the best performances for the solid phase description in turn. The results indicate that among the investigated activity coefficient models, the use of the p.UNIQUAC model<sup>24</sup> for the solid phase and its combination with the regular solution model<sup>19</sup>

for the liquid phase is the most precise one and leads to the best agreement with the experimental data with the AAD of 0.8 K.

# 4. EXPERIMENTAL SECTION

4.1. Materials. The information and purities of materials used in the experiments are reported in Table 3.

#### Table 3. Materials Used in This Work



4.2. Apparatus. In the current work, the key part of the experimental setup is an equilibrium cell (SS-316) of stainless steel with a  $75 \text{ cm}^3$  internal volume that has a window for sample observation. Inside the equilibrium cell, there is a stirrer that makes adequate turbulence for reaching equilibrium. The cell is placed in an ethanol bath with a cooling/heating capability. The regulation of the ethanol bath temperature is accomplished using a manageable circulator (TCS) with scheduling capability (Julabo FP-50). To measure the temperature accurately, a Pt-100 thermometer with an uncertainty of less than 0.1 K is used. An overview of the experimental setup is illustrated in Figure 5.



Figure 5. Schematic plan of the experimental setup; T, thermometer and TCS, temperature controller system.

4.3. Experimental Procedure. It has been indicated that using the equilibrium step heating method, the measured WDT can be reliable and reproducible.<sup>[6](#page-9-0)</sup> The visual method has long been used as a technique by researchers.<sup>[9](#page-9-0),[11,29](#page-9-0)</sup> Some researchers have previously carried out studies on the precision and validity of this method. For example, Parsa et al. $^{29}$  $^{29}$  $^{29}$  showed that there is good agreement between the measured WDTs based on this method and others. They reported that the difference between the measured WDTs of the  $n-C_{14} + n-C_{16}$ binary system by CPM and visual methods would be just up to 0.2 K. Also, their results for experimental melting point

<span id="page-5-0"></span>

Figure 6. Distribution of experimental data in a tetrahedral plot diagram for the quaternary system.

measurements of pure  $n-C_{14}$ ,  $n-C_{16}$ , and  $n-C_{18}$  are in agreement with the NIST Data Bank. Therefore, it has been concluded that the visual method can be a valid technique for measuring the WDTs of multicomponent systems. Hence, in this study, a visual method was used to measure the WDTs of the quaternary system. For each experiment, the quaternary mixture was prepared by a gravimetric method using a digital A&D balance (HR-200) with an accuracy of 0.0001 gr. First, the well-mixed mixture was inserted into the equilibrium cell. After that, the mixture temperature was decreased by approximately 10 °C lower than the predictable WAT. Then, the system temperature was kept constant for an hour using a controllable circulator to achieve a thermodynamically stable state. When wax crystals were formed completely, the heating process was started by raising the temperature of the ethanol bath. A very low heating rate  $(0.1 \text{ K h}^{-1})$  was applied to make sure that the equilibrium was reached at each step. As the temperature increased, the wax crystals were melted very slowly. This process was carefully observed until the last crystal was redissolved in the mixture, and this exact temperature was determined as the WDT of the mixture. The procedure was repeated twice for each mixture.

Using this method, 30 experimental data were obtained over a wide range of compositions of studied compounds. Weight fractions of all components were changed in the range between 0.04 and 0.70. To enhance the understanding of the distribution of quaternary system compositions, a well-known three-dimensional (3D)-tetrahedral plot was applied, which is a suitable visualization method for quaternary systems in petroleum and mineral sciences.<sup>[32](#page-9-0)</sup> Figure 6 presents this plot for compounds used in this study. According to this figure, the proper distribution of compositions in the studied space is visible. In addition, there is a color scale that indicates the experimental WDTs related to each composition. Based on Figure 6, it is evident that the presence of a higher amount of the  $n-C_{18}$  compound, as the heaviest component of the mixtures, causes an increase in the value of the respective WDTs.

4.3.1. Thermodynamic Modeling: γ−γ Approach. In this work, the solid solution model<sup>[19](#page-9-0)</sup> was used because based on studies on the crystal structure, the multisolid approach<sup> $21$ </sup> with

the actual behavior of the wax crystal is not consistent.<sup>[6](#page-9-0)</sup> The accuracy of the solid solution  $\text{model}^{19}$  for describing the liquid−solid equilibrium significantly depends on the selection of the appropriate activity coefficient model.<sup>27</sup> The solid− liquid equilibrium of waxy systems can be defined by $^{11,29}$  $^{11,29}$  $^{11,29}$  $^{11,29}$  $^{11,29}$ 

$$
f_i^1(T, P, x_i^1) = f_i^s(T, P, x_i^s) \quad i = 1, 2, ... N
$$
 (1)

where  $f_i$  stands for the fugacity of component "i" in which the superscripts l and s indicate the liquid and solid phases, respectively.  $f_i$  is a function of the temperature  $(T)$ , pressure  $(P)$ , and mole fraction  $(x_i)$ . The fugacities of the liquid and solid phases in an *n*-alkane mixture can be obtained by the  $\gamma$ - $\gamma$ approach $11,29$ 

$$
f_i^1 = x_i^1 r_i^1 f_{\text{pure},i}^1 \exp\left(\int_0^P \frac{v_i^1}{RT} dP\right)
$$
 (2)

$$
f_i^s = x_i^s \gamma_i^s f_{pure,i}^s \exp\left(\int_0^P \frac{v_i^s}{RT} dP\right)
$$
 (3)

where  $\gamma_i$  and  $f_{pure,i}$  are the activity coefficient and fugacity of the pure component "i", respectively.  $x_i$  stands for the mole fraction, as mentioned earlier,  $v_i$  stands for the molar volume of component " $i$ ",  $T$  indicates the temperature, as mentioned earlier, and R is the universal gas constant. Prausnitz et al.<sup>25</sup> presented a relationship for the fugacity ratio of the pure liquid and solid states, but due to the effects of the solid phase transition at high temperatures and before fusion, this relationship requires to be slightly changed.<sup>[6,10,33](#page-9-0)</sup> The fugacity ratio of the pure liquid and solid states can be expressed as  $follows<sup>11,29</sup>$  $follows<sup>11,29</sup>$  $follows<sup>11,29</sup>$ 

$$
\ln \frac{f_{\text{pure},i}^{\text{l}}}{f_{\text{pure},i}^s} = \frac{\Delta H_i^{\text{tr}}}{RT} \left( 1 - \frac{T}{T_i^{\text{tr}}} \right) + \frac{\Delta H_i^{\text{f}}}{RT} \left( 1 - \frac{T}{T_i^{\text{f}}} \right) + \frac{1}{RT} \int_{T_i^{\text{f}}}^T \Delta C_{\text{p},i}^{\text{ls}} dT - \frac{1}{RT} \int_{T_i^{\text{f}}}^T \frac{\Delta C_{\text{p},i}^{\text{ls}}}{T} dT \tag{4}
$$

where  $T_i^{\rm f}$  and  $T_i^{\rm tr}$  are the solid–liquid (fusion) and solid–solid transition temperatures of the pure component "i", respec(5)

tively.  $\Delta H_i^{\rm f}$  and  $\Delta H_i^{\rm tr}$  are the enthalpies of fusion and solid– solid transition of the pure component " $i$ ", respectively.  $\Delta C^{\rm ls}_{\rm p,i}$ represents the difference between the heat capacities of component "i" in the liquid and solid phases. Correlations used for calculating the fusion temperature and solid−solid transition temperature are varied considering even or odd carbon numbers of *n*-alkanes $\delta$  and will be described using eqs 5−9.

• *n*-alkanes with an even carbon number.<sup>[6](#page-9-0)</sup> For  $C_{10} < C_n \le C_{42}$  $T^{f}(K) = 0.0031 C_n^3 - 0.3458 C_n^2 + 14.277 C_n$ 

+ 137.73

 $T<sup>f</sup>$  indicates the fusion temperature. For  $C_{22} \leq C_n \leq C_{42}$ 

$$
\text{For } \mathsf{C}_{22} \leq \mathsf{C}_n \leq \mathsf{C}_4
$$

$$
T^{\text{tr}}(K) = 0.0032C_n^3 - 0.3249C_n^2 + 12.78C_n
$$
  
+ 154.19 + ln(C\_n) (6)

 $T<sup>tr</sup>$  denotes the transition temperature.

Others

$$
T^{\text{tr}}(K) = T^{\text{f}}(K) \tag{7}
$$

• *n*-alkanes with an odd carbon number.<sup>[6](#page-9-0)</sup> For  $C_9 < C_n \leq C_{43}$ 

$$
T^{f}(K) = 0.0122C_n^2 - 2.0861C_n - \frac{755.598}{C_n}
$$
  
+ 76.2189 ln(C\_n) + 156.9 (8)

$$
T^{tr}(K) = 0.0039C_n^3 - 0.4239C_n^2 + 17.28C_n
$$
  
- ln(C<sub>n</sub>) + 95.4 (9)

Correlations used for calculating the fusion heat and enthalpy of solid−solid transitions, which are a function of fusion temperature and molecular weight, are given  $\mathrm{bv}^6$  $\mathrm{bv}^6$ 

 $n$ -alkanes with an even carbon number.<sup>[6](#page-9-0)</sup> For  $C_n \leq C_{20}$ 

$$
\Delta H^{\text{f}}(\text{cal mol}^{-1}) = 0.180 M_{\text{w}} \times T^{\text{f}} + 522.7 \tag{10}
$$

 $\Delta H^{\text{f}}$  describes the fusion heat<sup>[6](#page-9-0)</sup>

$$
\Delta H^{\text{tr}}(\text{cal mol}^{-1}) = 0 \tag{11}
$$

 $\Delta H^{\text{tr}}$  stands for the enthalpy of solid–solid transitions. • *n*-alkanes with an odd carbon number.<sup>[6](#page-9-0)</sup>

For 
$$
C_9 < C_n \leq C_{43}
$$
  
\n
$$
\Delta H^{\text{f}}(\text{cal mol}^{-1}) = 0.74(0.167M_w \times T^{\text{f}} + 432.47) \tag{12}
$$

$$
\Delta H^{\text{tr}}(\text{cal mol}^{-1}) = 0.26(0.167 M_{\text{w}} \times T^{\text{f}} + 432.47)
$$
\n(13)

 $\Delta C_{\text{p}i}$  the specific heat capacity difference for component "*i*" is calculated using the following correlation<sup>22</sup>

$$
\Delta C_{\text{P}_i}(\text{cal mol}^{-1}) = 0.3033 M_{w_i} - 4.635 \times 10^{-4} M_{w_i}
$$
  
 
$$
\times T \tag{14}
$$

In eqs 5, 6, 8, and 9,  $C_n$  represents the carbon number of each component. Molecular weights, critical temper-

# Table 4. Molecular Weights, Critical Temperatures, and Acentric Factors of the Components Used in This Study<sup>[34](#page-9-0)a,b,c</sup>



atures, and acentric factors of all components are reported in Table 4.

By incorporating [eqs 1](#page-5-0)−[4,](#page-5-0) the solid−liquid equilibrium ratio of component "i",  $K_i^{\text{sl}}$  can be obtained<sup>[11,29](#page-9-0)</sup>

$$
K_i^{\rm sl} = \frac{x_i^{\rm s}}{x_i^1} = \frac{\gamma_i^1 f_{\text{pure},i}^1}{\gamma_i^s f_{\text{pure},i}^s}
$$
(15)

4.3.2. Thermodynamic Modeling:  $\gamma-\varphi$  Approach. In this approach, the nonidealities of the solid phase and the liquid phase are calculated using the activity coefficient models and PC-SAFT EoS,<sup>[28](#page-9-0)</sup> respectively. The  $K_i^{sl}$  is computed as follows<sup>[11](#page-9-0)</sup>

$$
K_i^{sl} = \frac{x_i^s}{x_i^l} = \frac{\varphi_i^l P}{\gamma_i^s f_{\text{pure},i}^s}
$$
(16)

In eq 16,  $\varphi^{\text{l}}_i$  stands for the fugacity coefficient of component " $\it i$ " in the liquid phase that can be calculated using PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$ 

$$
\ln(\varphi_i^1) = \left[\frac{\partial(n a^{\text{res}})}{\partial n_i}\right]_{T, \rho, n_j \neq i} + z - 1 - \ln(z)
$$
\n(17)

In eq 17,  $a^{\text{res}}$  represents the reduced residual Helmholtz free energy, and  $z$  denotes the compressibility factor. Subscript  $n_i$ indicates that all mole numbers except  $n_i$  are held constant. In PC-SAFT EoS,<sup>28</sup> when no associating and polar forces exist,  $a^{res}$  and z can be written as follows<sup>2</sup>

$$
a^{\text{res}} = a^{\text{hc}} + a^{\text{disp}} \tag{18}
$$

$$
z = zhc + zdisp
$$
 (19)

In eqs 18 and 19, the superscript hc expresses the hard-chain molecules and superscript disp is the dispersive force contribution. The details of PC-SAFT  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  $EoS<sup>28</sup>$  are presented in previous studies.<sup>28,[30,31,35](#page-9-0)</sup> The input parameters of the models are reported in [Table 5.](#page-7-0) In this study, the binary interaction parameter,  $k_{ij}$ , is set to zero.<sup>[28,36](#page-9-0)</sup>

# 5. ACTIVITY COEFFICIENT MODELS

5.1. Regular Solution Theory. Some researchers have applied the regular solution theory<sup>19</sup> to characterize the nonideality behaviors of paraffin wax mixtures[.19,23](#page-9-0),[29,37](#page-9-0) In this approach, the activity coefficient is defined by the following  $correlation<sup>19,25</sup>$  $correlation<sup>19,25</sup>$  $correlation<sup>19,25</sup>$ 

$$
\ln \gamma_i = \frac{\nu_i (\bar{\delta} - \delta_i)^2}{RT}
$$
\n(20)

<span id="page-7-0"></span>

where  $\delta$  is the solubility parameter. Pedersen et al.<sup>[23](#page-9-0)</sup> proposed two separate correlations to calculate the liquid and solid solubility parameters.

$$
\delta_i^1 = 7.41 + 0.5149 \left( \ln C_{ni} - \ln 7 \right) \tag{21}
$$

$$
\delta_i^s = 8.5 + 5.763 \left( \ln C_{ni} - \ln 7 \right) \tag{22}
$$

 $\delta_{i}^{\rm l}$  and  $\delta_{i}^{\rm s}$  are input parameters for regular solution theory  $^{19}$  $^{19}$  $^{19}$  that were calculated for all components (Table 5).  $v_i$  and  $\overline{\delta}$  are the molar volume and average solubility parameters, respectively. In this work, the molar volumes of both phases are considered to be equal. $19,25$ 

$$
\overline{\delta}_i = \sum \varphi_i \delta_i \tag{23}
$$

$$
v_i^1 = v_i^s = \frac{M_{w_i}}{d_{i,2s}^1} \tag{24}
$$

$$
\varphi_i^1 = \frac{x_i^1 v_i^1}{\sum x_i^1 v_i^1} \tag{25}
$$

$$
\varphi_i^s = \frac{x_i^s v_i^s}{\sum x_i^s v_i^s} \tag{26}
$$

where  $\varphi_i^{\mathrm{l}}$  and  $\varphi_i^{\mathrm{s}}$  are the liquid- and solid-phase volume fractions of component "i", respectively. The following correlation, which is molecular-weight  $(M_w)$ -dependent, is presented to compute the density of each component at 25 °C  $(d_{i,25}^{\rm l})$ .<sup>[38](#page-10-0)</sup>

$$
d_{i,25}^1 = 0.8155 + 0.6272 \times 10^{-4} M_{w_i} - \frac{13.06}{M_{w_i}}
$$
 (27)

5.2. Predictive Wilson Model. Another model applied to calculate the activity coefficients of  $n$ -alkane mixture components is predictive Wilson (p.Wilson).<sup>2</sup>

$$
\ln \gamma_i = 1 - \ln \left( \sum_{j=1}^n x_j \Lambda_{ij} \right) - \sum_{k'=1}^n \frac{x_{k'} \Lambda_{k'i}}{\sum_{j=1}^n x_j \Lambda_{k'j}}
$$
(28)

where *n* represents the number of components and  $\Lambda_{ij}$  is the interaction parameter between "i" and  $\ddot{q}$ " molecules, which is calculated by the following relation $14$ 

$$
\Lambda_{ij} = \exp\left(-\frac{\lambda_{ij} - \lambda_{ii}}{RT}\right) \tag{29}
$$

The characteristic interaction energy that exists between two identical molecules,  $\lambda_{ii}$ , depends on the sublimation enthalpy  $(\Delta H_{sub,i})$  and temperature.<sup>[20](#page-9-0)</sup>

$$
\lambda_{ii} = -\frac{2}{Z} (\Delta H_{\text{sub},i} - RT) \tag{30}
$$

The alkyl chains in the wax lead to an orthorhombic structure creation.<sup>22,[39](#page-10-0),[40](#page-10-0)</sup> As a result, the coordination number,  $Z_2$  is set to 6 for *n*-alkanes in the orthorhombic phase.<sup>[20](#page-9-0)</sup> The sublimation enthalpy of pure n-alkanes is estimated by summing the component latent heats of the solid−solid transition  $(\Delta H^{\text{tr}})$ , melting  $(\Delta H^{\text{f}})$ , and vaporization  $(\Delta H^{\text{vap}})^{19}$  $(\Delta H^{\text{vap}})^{19}$  $(\Delta H^{\text{vap}})^{19}$ 

$$
\Delta H_{\text{sub},i} = \Delta H_i^{\text{vap}} + \Delta H_i^{\text{f}} + \Delta H_i^{\text{tr}} \tag{31}
$$

The correlation of Morgan and Kobayashi to calculate the enthalpy of vaporization is given as follows<sup>4</sup>

$$
\frac{\Delta H_i^{\text{vap}}}{RT_{\text{c}}} = \Delta H_v^{(0)} + \omega \Delta H_v^{(1)} + \omega^2 \Delta H_v^{(2)} \tag{32}
$$

$$
\Delta H_{\rm v}^{(0)} = 5.2804x^{0.3333} + 12.865x^{0.8333} + 1.171x^{1.2083}
$$

$$
- 13.116x' + 0.485x^{0.2} - 1.088x^{0.3}
$$
(33)

$$
\Delta H_{\rm v}^{(1)} = 0.800x'^{0.3333} + 273.23x'^{0.8333} + 465.08x'^{1.2083} - 638.51x' - 145.12x'^2 + 74.049x'^3
$$
 (34)

$$
\Delta H_{\rm v}^{(2)} = 7.2543x^{\prime 0.3333} - 346.35x^{\prime 0.8333} - 610.48x^{\prime 1.2083} + 839.89x^{\prime} + 160.05x^{\prime 2} - 50.71x^{\prime 3}
$$
 (35)

$$
x' = \left(1 - \frac{T}{T_c}\right) \tag{36}
$$

In eq 36,  $T_c$  represents the critical temperature. The pair interaction energies between nonidentical molecules ("i" and "j") are equal to those between two identical molecules with a shorter chain  $(j)$  of pair  $ij$ .<sup>[42](#page-10-0)</sup>

$$
\lambda_{ij} = \lambda_{ji} = \lambda_{jj} \tag{37}
$$

5.3. Predictive UNIQUAC Model. The predictive UNIQUAC (p.UNIQUAC) model<sup>24</sup> comprises two terms: combinatorial and residual. The first term is an entropic term that computes the molecules' differences in shape, structure, and size. The second term is an enthalpic term that explains the energetic interaction between the various molecules.<sup>2</sup>

$$
\ln \gamma_i = \ln \gamma_i^{\text{comb}} + \ln \gamma_i^{\text{res}} \tag{38}
$$

$$
\ln \gamma_i^{\text{comb}} = \ln \left( \frac{\Phi_i}{x_i} \right) + 1 - \left( \frac{\Phi_i}{x_i} \right) - \frac{Z}{2} q_i \left[ \ln \left( \frac{\Phi_i}{\theta_i} \right) + 1 - \frac{\Phi_i}{\theta_i} \right]
$$
\n(39)

<span id="page-8-0"></span>
$$
\ln \gamma_i^{\text{res}} = q_i \left[ 1 - \ln \left( \sum_{j=1}^n \theta_j \tau_{ji} \right) - \sum_{j=1}^n \frac{\theta_j \tau_{ij}}{\sum_{k'=1}^n \theta_k \tau_{k'j}} \right]
$$
(40)

$$
\tau_{ji} = \exp\left(-\frac{\lambda_{ji} - \lambda_{ii}}{q_i RT}\right) \tag{41}
$$

The pair interaction energies are calculated in the same way as for the p. Wilson model. $^{2}$ 

$$
\Phi_i = \frac{x_i r_i}{\sum_{j=1}^n x_j r_j} \tag{42}
$$

$$
\theta_i = \frac{x_i q_i}{\sum_{j=1}^n x_j q_j} \tag{43}
$$

In eqs 42 and 43,  $\Phi_i$  and  $\theta_i$  are segment and area fractions, respectively. The structural parameters of size,  $r_{\nu}$  and external surface,  $q_{\nu}$  can be obtained from the following relation<sup>24</sup>

$$
r_i = 0.1 C_{ni} + 0.0672 \tag{44}
$$

$$
q_i = 0.1 C_{ni} + 0.1141 \tag{45}
$$

 $r_i$  and  $q_i$  parameters are the calculated input data for the p.UNIQUAC model that are presented in [Table 5](#page-7-0).

**5.4. UNIFAC Model.** The UNIFAC model<sup>[25](#page-9-0)</sup> is another model to demonstrate the nonideality of wax mixtures containing n-alkanes. This model also consists of two parts: combinatorial and residual. In this study, the combinatorial term is only used and the residual term is eliminated.<sup>[43](#page-10-0)–[45](#page-10-0)</sup>

$$
\ln \gamma_i = \ln \left( \frac{\Phi_i}{x_i} \right) + 1 - \left( \frac{\Phi_i}{x_i} \right) - \frac{Z}{2} q_i \left( \ln \left( \frac{\Phi_i}{\theta_i} \right) + 1 - \frac{\Phi_i}{\theta_i} \right) \tag{46}
$$

The segment fraction,  $\Phi_{ij}$  and the area fraction,  $\theta_{ij}$  are determined by eqs 42 and 43, respectively. The structural parameters,  $r_i$  and  $q_i$ , are calculated as follows<sup>[43](#page-10-0)</sup>

$$
r_i = 0.6744C_{ni} + 0.4534\tag{47}
$$

$$
q_i = 0.54C_{ni} + 0.616
$$
\n(48)

The values of these parameters, which are the input data of the UNIFAC model, $^{25}$  $^{25}$  $^{25}$  are calculated for all components and are reported in [Table 5](#page-7-0).

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## ■ NOMENCLATURE

AAD average absolute deviation

- $C_n$  carbon number<br> $C_p$  specific heat ca
- $C_p$  specific heat capacity<br>  $d$  density
- density
- $f$  fugacity<br> $a^{\text{res}}$  reduced
- $a^{\text{res}}$  reduced residual Helmholtz free energy<br>H enthalpy
- $H$  enthalpy<br> $i$  compone
- component counter
- *j* component counter<br>*k* Boltzmann constant
- $k$  Boltzmann constant<br> $k'$  component counter
- component counter
- $k_{ij}$  binary interaction parameter<br>K equilibrium constant
- equilibrium constant
- m number of segments per chain
- $M_{\rm w}$  molecular weight
- $n$  number of components  $P$
- pressure
- q molecular external surface parameter
- *r* molecular size parameter<br>R Universal gas constant
- Universal gas constant
- $T_c$  temperature<br> $T_c$  critical temp
- $T_c$  critical temperature<br>  $v$  molar volume
- molar volume
- $x$  mole fraction<br> $z$  coordination
- coordination number
- z compressibility factor

## **■ GREEK CHARACTERS**

- $\epsilon$  depth of pair potential
- γ activity coefficient
- $\delta$  solubility parameter<br> $\overline{\delta}$  average solubility pa
- average solubility parameter
- $\Delta$  variation<br> $\theta$  area fract
- area fraction
- $\lambda$  interaction energy
- $\sigma$  segment diameter
- Λ interaction parameter
- $\tau$  characteristic energy parameter
- $\varphi$  volume fraction
- Φ segment fraction
- $\omega$  acentric factor
- $\varphi_i^{\iota}$ fugacity coefficient of component "i" in the liquid

## **SUPERSCRIPTS**

Cal calculated comb combinatorial

<span id="page-9-0"></span>

## ■ SUBSCRIPTS

component number

j component number<br>k' component number

component number

n component number

sub sublimation

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