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Research article

Comparison of fuzzy semi-Markov models for one unit with mixed standby units with and without preventive maintenance using regenerative point method

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ABSTRACT

This paper introduced a study for a new system that consists of one unit with mixed standby units. The mathematical model for the system is constructed using semi-Markov model with regenerative point technique in two cases: the first case when there is preventive maintenance provided to the main unit and the second case when there is no preventive maintenance in the system. Life and repair times of the units in the system are assumed to be generally distributed with fuzzy parameters defined by the bell-shaped membership function. A Numerical application is introduced to compare the performance of the system in the two cases.

1. Introduction

Markov models have many applications in the real life. Markov process is a stochastic process with the memoryless property in which the future state depends only on the current state and not on the past history (see [12]). Applications of Markov models were presented in literature such as [13, 14] and [15]. Semi-Markov models are the generalization of Markov models (see [16] and [17]). Semi-Markov models and their applications were introduced in [18, 19, 20, 21, 22]. Reliability measures such as mean time to failure is an effective measure when analyzing engineering models. The applications of reliability were introduced in many papers in literature (see for example [23] and [24]).

Fuzzy set theory had been introduced by Zadeh (1965). Fuzzy set theory is the generalization of the classical set theory. Fuzzy sets have many applications in the expert systems. Uncertainty and fuzziness are used in the models in which the parameters are vague and their exact values are unknown. Many applications of the fuzzy sets and systems were introduced in literature (see for example [25] and [26])

Goel et al. [1] analyzed profit of a cold standby system with two repair distributions. El-Said and El-Sherbeny [2] discussed the reliability of two units cold standby system with single repair. Kumar et al. [3] introduced the cost benefit analysis of a two-unit parallel system subject to degradation after repair. Malik and Barak [4] presented the reliability measures of a cold standby system with preventive maintenance and repair. Rathee and Chander [5] introduced a parallel system with priority to repair over preventive maintenance subject to maximum operation and repair times. Grabski [6] introduced reliability and maintainability characteristics in semi-Markov models. Singh et al. [7] analyzed the cost benefit of two identical warm standby system under heavy rain with partially operative after repair. Baweja and Kumar [8] obtained reliability measures for a two-unit cold standby repairable system with priority to operation over preventive maintenance. Manocha and Taneja [9] presented analysis of two unit cold standby system with life, repair and waiting times follow arbitrary distributions. Grabski [10] introduced semi-Markov reliability model of system composed of main subsystem and cold backup component. Kumar and Goel [11] introduced a two-unit cold standby system by considering the concepts of degradation, inspection, preventive maintenance and priority.

In this paper, analysis of a new system consists of one unit with mixed standby units is presented. The main unit works perfectly at the initial time t = 0. After a period of time, the main unit will undergo for preventive maintenance in order to increase the life time of the main unit and improve the performance of the system. When the main unit fails, it will be replaced immediately by a warm standby unit where the

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switch is assumed to work perfectly and hence the cold standby will operate as a warm standby unit. The mathematical model for the system is constructed using semi-Markov model with regenerative method technique. The life time of the system's units is assumed to follow general distribution with failure rate increases with time. The repair and preventive maintenance rates of the system's units are assumed to follow general distribution. An algorithm is introduced to obtain the measures of the system when the parameters of the general distribution are assumed to be fuzzy with bell-shaped membership function. An application is introduced to compare between the two introduced models and show the effect of preventive maintenance to improve the performance of the system.

2. Notations

S_i: state of the system ($i = 0, \ldots, 30$).

$q_{i,j}(t)$: probability density function of the transition from state S_i to state S_j during time interval $(0, t)$.

$A_i(t)$: probability that the system is up at time t given that the system entered regenerative state S_i at $t = 0$.

$Z_i(t)$: probability that the system is up initially in state S_i at time t without visiting any other regenerative state.

$R_i(t)$: probability that the system will be successfully operating without failure in time interval $(0, t)$ given that the system starts at regenerative state S_i.

$f(t)/F(t)$: probability density function/cumulative distribution function of the failure of the main unit.

$f_w(t)/F_w(t)$: probability density function/cumulative distribution function of the failure of the warm standby unit.

$h(t)/H(t)$: probability density function/cumulative distribution function of the repair of the main unit.

$h_w(t)/H_w(t)$: probability density function/cumulative distribution function of the repair of the warm standby unit.

$g(t)/G(t)$: probability density function/cumulative distribution function of the main unit to go under PM.

$m(t)/M(t)$: probability density function/cumulative distribution function of the main unit to complete the PM action.

λ/λ_w: parameter of the distribution of the lifetime of the main/warm standby unit.

μ/μ_w: parameter of the distribution of the repair of the main/warm standby unit.

g/m: parameter of the distribution of the main unit to go under PM/complete PM.

©: symbol used for Laplace convolution.

$*$: symbol used for Laplace transform.

$\sim$: symbol used for fuzziness.

3. States of the system

N_0: the unit is in operative mode.

N_c: the unit is in cold standby mode.

N_w: the unit is in warm standby mode.

F_0: the main unit is failed.

F_w: the warm standby unit is failed.

N_{pm}: the unit is under preventive maintenance.

The up states of the system will be given as follows:

$$S_0 \equiv (N_0, N_c, N_w), \quad S_1 \equiv (F_0, N_w, N_0), \quad S_2 \equiv (N_0, N_c, F_w),$$
$$S_3 \equiv (N_{pm}, N_w, N_0), \quad S_4 \equiv (F_0, N_0, F_0), \quad S_5 \equiv (F_0, F_w, N_0),$$
$$S_6 \equiv (F_0, N_0, N_{pm}), \quad S_7 \equiv (F_0, N_0, F_w), \quad S_8 \equiv (N_0, F_w, F_w),$$
$$S_9 \equiv (N_{pm}, N_0, F_w), \quad S_{10} \equiv (N_{pm}, N_0, F_0), \quad S_{11} \equiv (N_{pm}, F_w, N_0),$$
$$S_{12} \equiv (N_{pm}, N_0, N_{pm})$$

The failed states of the system will be given as follows:

$$S_{13} \equiv (F_0, F_0, F_0), \quad S_{14} \equiv (F_0, N_{pm}, F_0), \quad S_{15} \equiv (F_0, F_w, F_0),$$
$$S_{16} \equiv (F_0, F_w, N_{pm}), \quad S_{17} \equiv (F_0, F_0, N_{pm}), \quad S_{18} \equiv (F_0, N_{pm}, N_{pm}),$$
$$S_{19} \equiv (F_0, F_0, F_w), \quad S_{20} \equiv (F_0, N_{pm}, F_w), \quad S_{21} \equiv (F_0, F_w, F_w)$$
$$S_{22} \equiv (N_{pm}, F_w, F_w), \quad S_{23} \equiv (N_{pm}, F_0, F_w), \quad S_{24} \equiv (N_{pm}, N_{pm}, F_w),$$
$$S_{25} \equiv (N_{pm}, F_0, F_0), \quad S_{26} \equiv (N_{pm}, N_{pm}, F_0), \quad S_{27} \equiv (N_{pm}, F_w, F_0),$$
$$S_{28} \equiv (N_{pm}, F_w, N_{pm}), \quad S_{29} \equiv (N_{pm}, F_0, N_{pm}), \quad S_{30} \equiv (N_{pm}, N_{pm}, N_{pm})$$

where the regenerative states are $\{S_0, \ldots, S_{12}\}$ and the non-regenerative states are $\{S_{13}, \ldots, S_{30}\}$. All states of the system and transitions between them are illustrated in Fig. 1.

4. Model description

1. At initial time t = 0, the main unit is in operative mode, one unit is in cold standby mode and one unit is in warm standby mode.
2. After a period of time, the main unit will undergo for preventive maintenance.
3. When the main unit undergoes for preventive maintenance, it will be replaced by a standby unit if it is available.
4. If the main unit fails, it will be replaced by a warm standby unit and the cold standby unit will work as a warm standby unit.
5. When a unit is in operative mode, the warm standby unit can fail with failure rate less than the failure rate of the main unit.
6. All failed units are repairable.
7. The lifetimes of the units of the system follow general distribution with failure rate increases with time.
8. The repair and PM times of the units of the system follow any general distribution.

4.1. Mathematical model of the system under preventive maintenance

The equations for the system are constructed by using semi-Markov models with the regenerative point technique and the results are given as follows:

$$A_0(t) = Z_0(t) + q_{0,1}(t) © A_1(t) + q_{0,2}(t) © A_2(t) + q_{0,3}(t) © A_3(t) \quad (1.1)$$

$$A_1(t) = Z_1(t) + q_{1,0}(t) © A_0(t) + q_{1,4}(t) © A_4(t) + q_{1,5}(t) © A_5(t) + q_{1,6}(t) © A_6(t) \quad (1.2)$$

$$A_2(t) = Z_2(t) + q_{2,0}(t) © A_0(t) + q_{2,7}(t) © A_7(t) + q_{2,8}(t) © A_8(t) + q_{2,9}(t) © A_9(t) \quad (1.3)$$

$$A_3(t) = Z_3(t) + q_{3,0}(t) © A_0(t) + q_{3,10}(t) © A_{10}(t) + q_{3,11}(t) © A_{11}(t) + q_{3,12}(t) © A_{12}(t) \quad (1.4)$$

$$A_4(t) = Z_4(t) + q_{4,1}(t) © A_1(t) + q_{4,13}(t) © A_{13}(t) + q_{4,14}(t) © A_{14}(t) \quad (1.5)$$

$$A_5(t) = Z_5(t) + q_{5,1}(t) © A_1(t) + q_{5,15}(t) © A_{15}(t) + q_{5,16}(t) © A_{16}(t) \quad (1.6)$$

$$A_6(t) = Z_6(t) + q_{6,1}(t) © A_1(t) + q_{6,17}(t) © A_{17}(t) + q_{6,18}(t) © A_{18}(t) \quad (1.7)$$

$$A_7(t) = Z_7(t) + q_{7,2}(t) © A_2(t) + q_{7,19}(t) © A_{19}(t) + q_{7,20}(t) © A_{20}(t) \quad (1.8)$$

$$A_8(t) = Z_8(t) + q_{8,2}(t) © A_2(t) + q_{8,21}(t) © A_{21}(t) + q_{8,22}(t) © A_{22}(t) \quad (1.9)$$

$$A_9(t) = Z_9(t) + q_{9,2}(t) © A_2(t) + q_{9,23}(t) © A_{23}(t) + q_{9,24}(t) © A_{24}(t) \quad (1.10)$$

$$A_{10}(t) = Z_{10}(t) + q_{10,3}(t) © A_3(t) + q_{10,25}(t) © A_{25}(t) + q_{10,26}(t) © A_{26}(t) \quad (1.11)$$

$$A_{11}(t) = Z_{11}(t) + q_{11,3}(t) © A_3(t) + q_{11,27}(t) © A_{27}(t) + q_{11,28}(t) © A_{28}(t) \quad (1.12)$$

$$A_{12}(t) = Z_{12}(t) + q_{12,3}(t) © A_3(t) + q_{12,29}(t) © A_{29}(t) + q_{12,30}(t) © A_{30}(t) \quad (1.13)$$

$$A_{13}(t) = q_{13,4}(t) © A_4(t) \quad (1.14)$$

$$A_{14}(t) = q_{14,4}(t) © A_4(t) \quad (1.15)$$

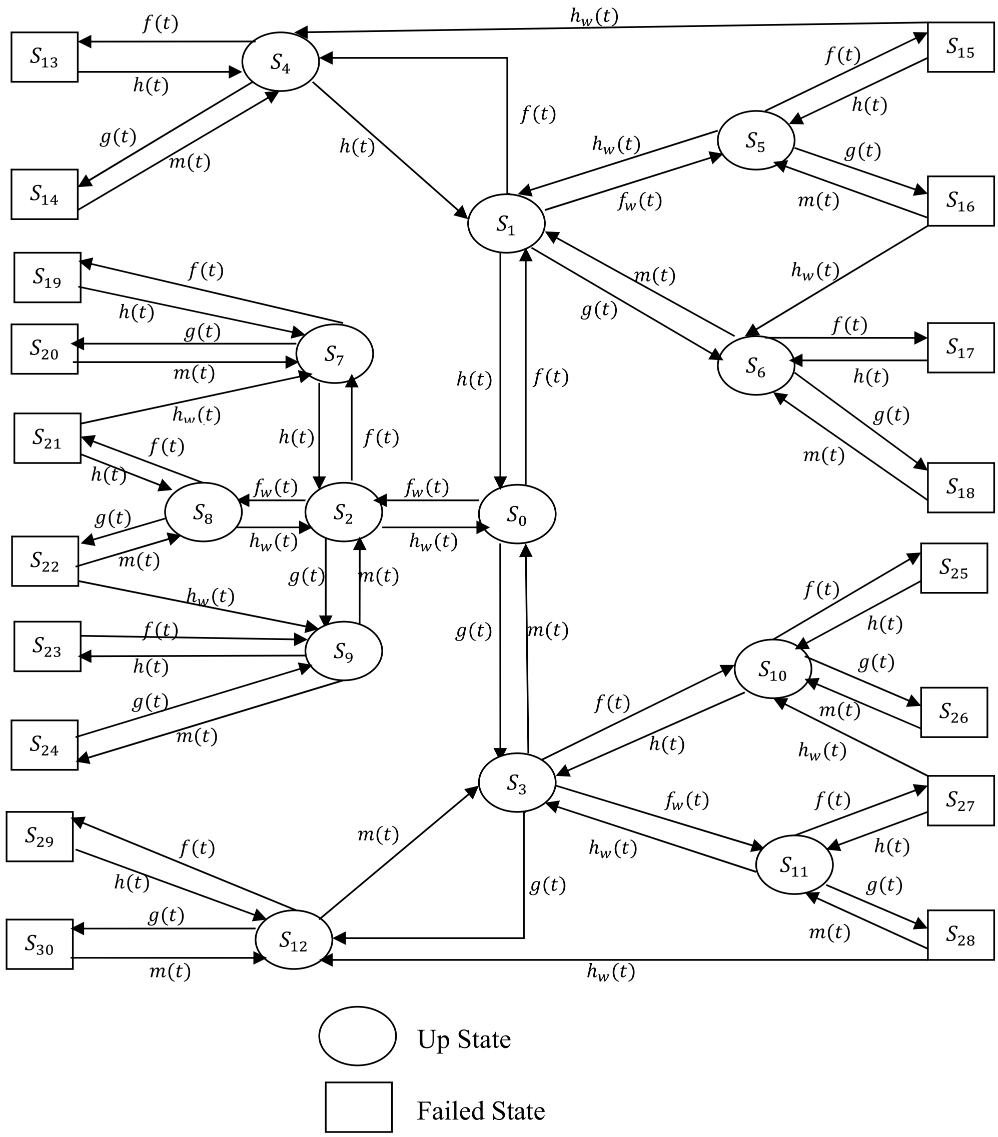

Fig. 1. State transition diagram for one unit with mixed standby units and PM.

$$A_{15}(t) = Z_{15}(t) + q_{15,4}(t) \copyright A_4(t) + q_{15,5}(t) \copyright A_5(t) \quad (1.16)$$

$$A_{16}(t) = Z_{16}(t) + q_{16,5}(t) \copyright A_5(t) + q_{16,6}(t) \copyright A_6(t) \quad (1.17)$$

$$A_{17}(t) = q_{17,6}(t) \copyright A_6(t) \quad (1.18)$$

$$A_{18}(t) = q_{18,6}(t) \copyright A_6(t) \quad (1.19)$$

$$A_{19}(t) = q_{19,7}(t) \copyright A_7(t) \quad (1.20)$$

$$A_{20}(t) = q_{20,7}(t) \copyright A_7(t) \quad (1.21)$$

$$A_{21}(t) = Z_{21}(t) + q_{21,7}(t) \copyright A_7(t) + q_{21,8}(t) \copyright A_8(t) \quad (1.22)$$

$$A_{22}(t) = Z_{22}(t) + q_{22,9}(t) \copyright A_9(t) + q_{22,8}(t) \copyright A_8(t) \quad (1.23)$$

$$A_{23}(t) = q_{23,9}(t) \copyright A_9(t) \quad (1.24)$$

$$A_{24}(t) = q_{24,9}(t) \copyright A_9(t) \quad (1.25)$$

$$A_{25}(t) = q_{25,10}(t) \copyright A_{10}(t) \quad (1.26)$$

$$A_{26}(t) = q_{26,9}(t) \copyright A_{10}(t) \quad (1.27)$$

$$A_{27}(t) = Z_{27}(t) + q_{27,10}(t) \copyright A_{10}(t) + q_{27,11}(t) \copyright A_{11}(t) \quad (1.28)$$

$$A_{28}(t) = Z_{28}(t) + q_{28,11}(t) \copyright A_{11}(t) + q_{25,12}(t) \copyright A_{12}(t) \quad (1.29)$$

$$A_{29}(t) = q_{29,12}(t) \copyright A_{12}(t) \quad (1.30)$$

$$A_{30}(t) = q_{30,12}(t) \copyright A_{12}(t) \quad (1.31)$$

where

$Z_0(t) = \overline{F}(t)\overline{F}_w(t)\overline{G}(t)$, $\quad Z_1(t) = \overline{F}(t)\overline{F}_w(t)\overline{G}(t)\overline{H}(t)$,

$Z_2(t) = \overline{H}_w(t)\overline{F}(t)\overline{G}(t)\overline{F}_w(t)$,

$Z_3(t) = \overline{F}(t)\overline{F}_w(t)\overline{G}(t)\overline{M}(t)$, $\quad Z_4(t) = \overline{F}(t)\overline{G}(t)\overline{H}(t)$,

$Z_5(t) = \overline{F}(t)\overline{G}(t)\overline{H}_w(t)$,

$Z_6(t) = \overline{M}(t)\overline{F}(t)\overline{G}(t)$, $\quad Z_7(t) = \overline{H}(t)\overline{F}(t)\overline{G}(t)$,

$Z_8(t) = \overline{H}_w(t)\overline{F}(t)\overline{G}(t)$,

$Z_9(t) = \overline{M}(t)\overline{F}(t)\overline{G}(t)$, $\quad Z_{10}(t) = \overline{F}(t)\overline{G}(t)\overline{H}(t)$,

$Z_{11}(t) = \overline{F}(t)\overline{G}(t)\overline{H}_w(t)$,

$Z_{12}(t) = \overline{M}(t)\overline{F}(t)\overline{G}(t)$, $\quad Z_{13}(t) = \overline{H}(t)$,

$Z_{14}(t) = \overline{M}(t)$, $\quad Z_{15}(t) = \overline{H}(t)\overline{H}_w(t)$,

$Z_{16}(t) = \overline{M}(t)\overline{H}_w(t)$, $\quad Z_{17}(t) = \overline{H}(t)$,

$Z_{18}(t) = \overline{M}(t)$,

$Z_{19}(t) = \overline{H}(t), \qquad Z_{20}(t) = \overline{M}(t),$

$Z_{21}(t) = \overline{H}(t)\overline{H}_w(t),$

$Z_{22}(t) = \overline{M}(t)\overline{H}_w(t), \qquad Z_{23}(t) = \overline{H}(t),$

$Z_{24}(t) = \overline{M}(t),$

$Z_{25}(t) = \overline{H}(t), \qquad Z_{26}(t) = \overline{M}(t),$

$Z_{27}(t) = \overline{H}(t)\overline{H}_w(t),$

$Z_{28}(t) = \overline{M}(t)\overline{H}_w(t), \qquad Z_{29}(t) = \overline{H}(t),$

$Z_{30}(t) = \overline{M}(t)$

and

$q_{0,1}(t) = f(t)\overline{F}_w(t)\overline{G}(t), \qquad q_{0,2}(t) = f_w(t)\overline{F}(t)\overline{G}(t),$

$q_{0,3}(t) = g(t)\overline{F}(t)\overline{F}_w(t),$

$q_{1,0}(t) = h(t)\overline{F}(t)\overline{F}_w(t)\overline{G}(t), \qquad q_{1,4}(t) = f(t)\overline{H}(t)\overline{F}_w(t)\overline{G}(t),$

$q_{1,5}(t) = f_w(t)\overline{H}(t)\overline{F}(t)\overline{G}(t),$

$q_{1,6}(t) = g(t)\overline{H}(t)\overline{F}_w(t)\overline{F}(t), \qquad q_{2,0}(t) = h_w(t)\overline{F}(t)\overline{G}(t)\overline{F}_w(t),$

$q_{2,7}(t) = f(t)\overline{H}_w(t)\overline{G}(t)\overline{F}_w(t),$

$q_{2,8}(t) = f_w(t)\overline{H}_w(t)\overline{G}(t)\overline{F}(t), \qquad q_{2,9}(t) = g(t)\overline{H}_w(t)\overline{F}(t)\overline{F}_w(t),$

$q_{3,0}(t) = m(t)\overline{F}(t)\overline{F}_w(t)\overline{G}(t),$

$q_{3,10}(t) = f(t)\overline{F}_w(t)\overline{G}(t)\overline{M}(t),$

$q_{3,11}(t) = f_w(t)\overline{F}(t)\overline{G}(t)\overline{M}(t), \qquad q_{3,12}(t) = g(t)\overline{F}(t)\overline{F}_w(t)\overline{M}(t),$

$q_{4,1}(t) = h(t)\overline{F}(t)\overline{G}(t),$

$q_{4,13}(t) = f(t)\overline{H}(t)\overline{G}(t), \qquad q_{4,14}(t) = g(t)\overline{H}(t)\overline{F}(t),$

$q_{5,1}(t) = h_w(t)\overline{F}(t)\overline{G}(t),$

$q_{5,15}(t) = f(t)\overline{H}_w(t)\overline{G}(t), \qquad q_{5,16}(t) = g(t)\overline{H}_w(t)\overline{F}(t),$

$q_{6,1}(t) = m(t)\overline{F}(t)\overline{G}(t),$

$q_{6,17}(t) = f(t)\overline{M}(t)\overline{G}(t), \qquad q_{6,18}(t) = g(t)\overline{M}(t)\overline{F}(t),$

$q_{7,2}(t) = h(t)\overline{F}(t)\overline{G}(t),$

$q_{7,19}(t) = f(t)\overline{H}(t)\overline{G}(t), \qquad q_{7,20}(t) = g(t)\overline{H}(t)\overline{F}(t),$

$q_{8,2}(t) = h_w(t)\overline{F}(t)\overline{G}(t),$

$q_{8,21}(t) = f(t)\overline{H}_w(t)\overline{G}(t), \qquad q_{8,22}(t) = g(t)\overline{H}_w(t)\overline{F}(t),$

$q_{9,2}(t) = m(t)\overline{F}(t)\overline{G}(t),$

$q_{9,23}(t) = f(t)\overline{M}(t)\overline{G}(t), \qquad q_{9,24}(t) = g(t)\overline{M}(t)\overline{F}(t),$

$q_{10,3}(t) = h(t)\overline{F}(t)\overline{G}(t),$

$q_{10,25}(t) = f(t)\overline{M}(t)\overline{G}(t), \qquad q_{10,26}(t) = g(t)\overline{M}(t)\overline{F}(t),$

$q_{11,3}(t) = h_w(t)\overline{F}(t)\overline{G}(t),$

$q_{11,27}(t) = f(t)\overline{H}_w(t)\overline{G}(t), \qquad q_{11,28}(t) = g(t)\overline{H}_w(t)\overline{F}(t),$

$q_{12,3}(t) = m(t)\overline{F}(t)\overline{G}(t),$

$q_{12,29}(t) = f(t)\overline{M}(t)\overline{G}(t), \qquad q_{12,30}(t) = g(t)\overline{M}(t)\overline{F}(t),$

$q_{13,4}(t) = h(t),$

$q_{14,4}(t) = m(t), \qquad q_{15,4}(t) = h_w(t)\overline{H}(t),$

$q_{15,5}(t) = h(t)\overline{H}_w(t),$

$q_{16,5}(t) = m(t)\overline{H}_w(t), \qquad q_{16,6}(t) = h_w(t)\overline{M}(t),$

$q_{17,6}(t) = h(t),$

$q_{18,6}(t) = m(t), \qquad q_{19,7}(t) = h(t),$

$q_{20,7}(t) = m(t) \qquad q_{21,7}(t) = h_w(t)\overline{H}(t),$

$q_{21,8}(t) = h(t)\overline{H}_w(t), \qquad q_{22,8}(t) = m(t)\overline{H}_w(t),$

$q_{22,9}(t) = h_w(t)\overline{M}(t),$

$q_{23,9}(t) = h(t), \qquad q_{24,9}(t) = m(t),$

$q_{25,10}(t) = h(t),$

$q_{26,10}(t) = m(t), \qquad q_{27,10}(t) = h_w(t)\overline{H}(t),$

$q_{27,11}(t) = h(t)\overline{H}_w(t),$

$q_{28,11}(t) = m(t)\overline{H}_w(t), \qquad q_{28,12}(t) = h_w(t)\overline{M}(t),$

$q_{29,12}(t) = h(t), \qquad q_{30,12}(t) = m(t)$

4.2. Mean time to system failure analysis under preventive maintenance

The relations for $R_i(t)$ can be constructed assuming that the failed states are absorbing states and hence set all transitions from them equal zero. Substituting in model (1.1)–(1.31) yields the following system of equations.

$$R_0(t) = Z_0(t) + q_{0,1}(t)©R_1(t) + q_{0,2}(t)©R_2(t) + q_{0,3}(t)©R_3(t) \tag{2.1}$$

$$R_1(t) = Z_1(t) + q_{1,0}(t)©R_0(t) + q_{1,4}(t)©R_4(t) + q_{1,5}(t)©R_5(t) + q_{1,6}(t)©R_6(t) \tag{2.2}$$

$$R_2(t) = Z_2(t) + q_{2,0}(t)©R_0(t) + q_{2,7}(t)©R_7(t) + q_{2,8}(t)©R_8(t) + q_{2,9}(t)©R_9(t) \tag{2.3}$$

$$R_3(t) = Z_3(t) + q_{3,0}(t)©R_0(t) + q_{3,10}(t)©R_{10}(t) + q_{3,11}(t)©R_{11}(t) + q_{3,12}(t)©R_{12}(t) \tag{2.4}$$

$$R_4(t) = Z_4(t) + q_{4,1}(t)©R_1(t) \tag{2.5}$$

$$R_5(t) = Z_5(t) + q_{5,1}(t)©R_1(t) \tag{2.6}$$

$$R_6(t) = Z_6(t) + q_{6,1}(t)©R_1(t) \tag{2.7}$$

$$R_7(t) = Z_7(t) + q_{7,2}(t)©R_2(t) \tag{2.8}$$

$$R_8(t) = Z_8(t) + q_{8,2}(t)©R_2(t) \tag{2.9}$$

$$R_9(t) = Z_9(t) + q_{9,2}(t)©R_2(t) \tag{2.10}$$

$$R_{10}(t) = Z_{10}(t) + q_{10,3}(t)©R_3(t) \tag{2.11}$$

$$R_{11}(t) = Z_{11}(t) + q_{11,3}(t)©R_3(t) \tag{2.12}$$

$$R_{12}(t) = Z_{12}(t) + q_{12,3}(t)©R_3(t) \tag{2.13}$$

The mean time to system failure can be obtained by using the following relation

$$MTTF = \lim_{s\to 0} R_0^*(s) \tag{3}$$

4.3. Mean time to system failure analysis without preventive maintenance

To obtain the mean time to system failure in case of no preventive maintenance is provided in the system, the following relations for $R_i(t)$ are constructed

$$R_0(t) = Z_0(t) + q_{0,1}(t)©R_1(t) + q_{0,2}(t)©R_2(t) \tag{4.1}$$

$$R_1(t) = Z_1(t) + q_{1,0}(t)©R_0(t) + q_{1,4}(t)©R_4(t) + q_{1,5}(t)©R_5(t) \tag{4.2}$$

$$R_2(t) = Z_2(t) + q_{2,0}(t)©R_0(t) + q_{2,7}(t)©R_7(t) + q_{2,8}(t)©R_8(t) \tag{4.3}$$

$$R_4(t) = Z_4(t) + q_{4,1}(t)©R_1(t) \tag{4.4}$$

$$R_5(t) = Z_5(t) + q_{5,1}(t)©R_1(t) \tag{4.5}$$

$$R_7(t) = Z_7(t) + q_{7,2}(t)©R_2(t) \tag{4.6}$$

$$R_8(t) = Z_8(t) + q_{8,2}(t)©R_2(t) \tag{4.7}$$

Table 1. Random samples generated from Erlang distribution.

Initial Parameters	Generated Random Samples	Values of point estimators
$n_1 = 10,\ k_1 = 3,\ \vartheta_1 = \lambda = 1.4$	$S_1 = (2.741,\ 2.719,\ 3.573,\ 5.199,\ 7.026,\ 3.712,\ 3.532, 2.979,\ 3.384,\ 2.677),$	$\hat{\lambda} = 0.799$
$n_2 = 7,\ k_2 = 2,\ \vartheta_2 = \mu = 1.1$	$S_2 = (2.749,\ 0.697,\ 2.851,\ 2.015,\ 1.794,\ 0.437,\ 0.566)$	$\hat{\mu} = 1.260$
$n_3 = 8,\ k_3 = 3,\ \vartheta_3 = \lambda_w = 1.6$	$S_3 = (8.197,\ 4.708,\ 5.361,\ 7.253,\ 4.082,\ 6.874,\ 4.706,\ 2.900),$	$\hat{\lambda}_w = 0.544$
$n_4 = 6,\ k_4 = 3,\ \vartheta_4 = \mu_w = 1.5$	$S_4 = (4.212,\ 4.210,\ 2.380,\ 4.370,\ 5.467,\ 2.515)$	$\hat{\mu}_w = 0.777$
$n_5 = 9,\ k_5 = 4,\ \vartheta_5 = g = 1.8$	$S_5 = (9.161,\ 7.828,\ 4.812,\ 2.517,\ 7.011,\ 8.483,\ 4.140,\ 3.447,\ 11.210),$	$\hat{g} = 0.614$
$n_6 = 7,\ k_6 = 2,\ \vartheta_6 = m = 1.3$	$S_6 = (4.471,\ 2.065,\ 1.739,\ 0.473,\ 2.641,\ 2.395,\ 1.088)$	$\hat{m} = 0.941$

Table 2. Intervals for fuzzy parameters $\tilde{\lambda}, \tilde{\mu}$ and $\tilde{\lambda}_w$.

$\alpha - cut$	$[\tilde{\lambda}_L, \tilde{\lambda}_U]$	$[\tilde{\mu}_L, \tilde{\mu}_U]$	$[\tilde{\lambda}_{wL}, \tilde{\lambda}_{wU}]$
0.1	[0.648, 0.950]	[1.109, 1.411]	[0.393, 0.695]
0.2	[0.672, 0.926]	[1.133, 1.387]	[0.417, 0.671]
0.3	[0.690, 0.908]	[1.151, 1.369]	[0.435, 0.653]
0.4	[0.699, 0.899]	[1.160, 1.360]	[0.444, 0.644]
0.5	[0.716, 0.882]	[1.177, 1.343]	[0.461, 0.627]
0.6	[0.722, 0.876]	[1.183, 1.337]	[0.467, 0.621]
0.7	[0.736, 0.862]	[1.197, 1.323]	[0.481, 0.607]
0.8	[0.745, 0.853]	[1.206, 1.314]	[0.490, 0.598]
0.9	[0.755, 0.843]	[1.216, 1.304]	[0.500, 0.588]

Table 3. Intervals for fuzzy parameters $\tilde{\mu}_w, \tilde{g}$ and $\tilde{m}$.

$\alpha - cut$	$[\tilde{\mu}_{wL}, \tilde{\mu}_{wU}]$	$[\tilde{g}_L, \tilde{g}_U]$	$[\tilde{m}_L, \tilde{m}_U]$
0.1	[0.626, 0.928]	[0.463, 0.765]	[0.790, 1.092]
0.2	[0.650, 0.904]	[0.487, 0.741]	[0.814, 1.068]
0.3	[0.668, 0.886]	[0.505, 0.723]	[0.832, 1.050]
0.4	[0.677, 0.877]	[0.514, 0.714]	[0.841, 1.041]
0.5	[0.694, 0.860]	[0.531, 0.697]	[0.858, 1.024]
0.6	[0.700, 0.854]	[0.537, 0.691]	[0.864, 1.018]
0.7	[0.714, 0.840]	[0.551, 0.677]	[0.878, 1.004]
0.8	[0.723, 0.831]	[0.560, 0.668]	[0.887, 0.995]
0.9	[0.733, 0.821]	[0.570, 0.658]	[0.897, 0.985]

where

$$Z_0(t) = \overline{F}(t)\overline{F}_w(t), \qquad Z_1(t) = \overline{F}(t)\overline{F}_w(t)\overline{H}(t),$$
$$Z_2(t) = \overline{H}_w(t)\overline{F}(t)\overline{F}_w(t),$$
$$Z_4(t) = \overline{F}(t)\overline{H}(t), \qquad Z_5(t) = \overline{F}(t)\overline{H}_w(t),$$
$$Z_7(t) = \overline{H}(t)\overline{F}(t), Z_8(t) = \overline{F}(t)\overline{H}_w(t)$$

and

$$q_{0,1}(t) = f(t)\overline{F}_w(t), q_{0,2}(t) = f_w(t)\overline{F}(t), \qquad q_{1,0}(t) = h(t)\overline{F}(t)\overline{F}_w(t),$$
$$q_{1,4}(t) = f(t)\overline{H}(t)\overline{F}_w(t), \qquad q_{1,5}(t) = f_w(t)\overline{H}(t)\overline{F}(t),$$
$$q_{2,0}(t) = h_w(t)\overline{F}(t)\overline{F}_w(t),$$
$$q_{2,7}(t) = f(t)\overline{H}_w(t)\overline{F}_w(t), \qquad q_{2,8}(t) = f_w(t)\overline{H}_w(t)\overline{F}(t)$$
$$q_{4,1}(t) = h(t)\overline{F}(t), \qquad q_{5,1}(t) = h_w(t)\overline{F}(t),$$
$$q_{7,2}(t) = h(t)\overline{F}(t), \qquad q_{8,2}(t) = h_w(t)\overline{F}(t)$$

Then the mean time to system failure can be obtained after taking Laplace transformation of model (4.1)–(4.7) and then using the relation (3)

4.4. The parameters of the model as fuzzy numbers

Now, let us consider that the parameters of the distribution of the life, PM and repair times are fuzzy numbers with bell shaped membership function (see [27]) which is defined as follows

$$\beta(\vartheta) = e^{-\left(\frac{\vartheta - u}{\varepsilon}\right)^2}, \quad u - \delta \le \vartheta \le u + \delta$$

For arbitrary values for δ and ε, the intervals for the fuzzy parameters ϑ (assuming that $u = \hat{\vartheta}$) are given as

$$[\tilde{\vartheta}_L, \tilde{\vartheta}_U] = \begin{cases} \left[\hat{\vartheta} - \sqrt{\ln\left(\frac{1}{\alpha^{\varepsilon^2}}\right)}, \hat{\vartheta} + \sqrt{\ln\left(\frac{1}{\alpha^{\varepsilon^2}}\right)}\right], & \alpha \ge e^{-\left(\frac{\delta}{\varepsilon}\right)^2} \\ [\hat{\vartheta} - \delta, \hat{\vartheta} + \delta], & \alpha < e^{-\left(\frac{\delta}{\varepsilon}\right)^2} \end{cases} \tag{5}$$

where $(0 < \alpha < 1)$ and $\hat{\vartheta}$ is the value of the point estimator of ϑ.

4.5. Algorithm

Steps of applying our considerations to find the measures of the mixed standby system are given in the following algorithm.

Step 1: Generate random samples of sizes n_i for $i = (1, 2, 3, 4, 5, 6)$ from a distribution with initial parameters ϑ_i, $i = 1, 2, 3, 4, 5, 6$.

Step 2: Obtaining the values of the point estimators for the population parameters ϑ_i, $i = 1, 2, 3, 4, 5, 6$.

Step 3: For $\alpha = 0.1, 0.2, \dots, 0.9$, the intervals for the fuzzy numbers of the parameters ϑ_i, $i = 1, 2, 3, 4, 5, 6$, can be obtained using formula (5).

Step 4: Substituting in model (2.1)–(2.13), the intervals for the fuzzy mean time to system failure in case of preventive maintenance can be obtained by using formula (3).

Step 5: By the same manner, substituting in model (4.1)–(4.7), the intervals for the fuzzy mean time to system failure in case of no preventive maintenance can be obtained by using formula (3).

5. Numerical application

Suppose that the life, preventive maintenance and repair times of the units of the system follow Erlang distribution with probability density function given by

$$\varphi(t) = \frac{\vartheta^k}{\Gamma(\mathrm{k})} t^{k-1} e^{-\vartheta t}, \; t, \vartheta > 0, \; k \in N$$

and hence the reliability function is given by

$$\overline{\Phi}(t) = \sum_{i=0}^{k-1} e^{-\vartheta t} \frac{(\vartheta t)^i}{i!}$$

The point estimator for the parameter ϑ is given by

$$\hat{\vartheta} = \frac{nk}{\sum_{i=1}^{n} t_i}$$

For $\delta = 1$ and $\varepsilon = 0.1$, the following samples are generated from Erlang distribution by the aid of MAPLE PACKAGE and the results are illustrated in Table 1.

The results for the intervals for the fuzzy parameters $\tilde{\lambda}, \tilde{\mu}$ and $\tilde{\lambda}_w$ are given in Table 2 and the results for the intervals for the fuzzy parameters $\tilde{\mu}_w, \tilde{g}$ and $\tilde{m}$ are given in Table 3.

The results for system fuzzy mean time to failure in the two cases when there a preventive maintenance action provided to the system and

Table 4. Comparison of the system fuzzy MTTF with and without PM.

$\alpha - cut$	$[M\tilde{T}TF_L, M\tilde{T}TF_U]$ with PM	$[M\tilde{T}TF_L, M\tilde{T}TF_U]$ without PM
0.1	[12.191, 23.994]	[11.487, 20.200]
0.2	[12.692, 22.305]	[11.878, 18.861]
0.3	[13.095, 21.183]	[12.191, 18.085]
0.4	[13.307, 20.662]	[12.353, 17.722]
0.5	[13.725, 19.743]	[12.674, 17.077]
0.6	[13.878, 19.437]	[12.791, 16.861]
0.7	[14.251, 18.759]	[13.074, 16.380]
0.8	[14.501, 18.347]	[13.263, 16.085]
0.9	[14.789, 17.909]	[13.480, 15.771]

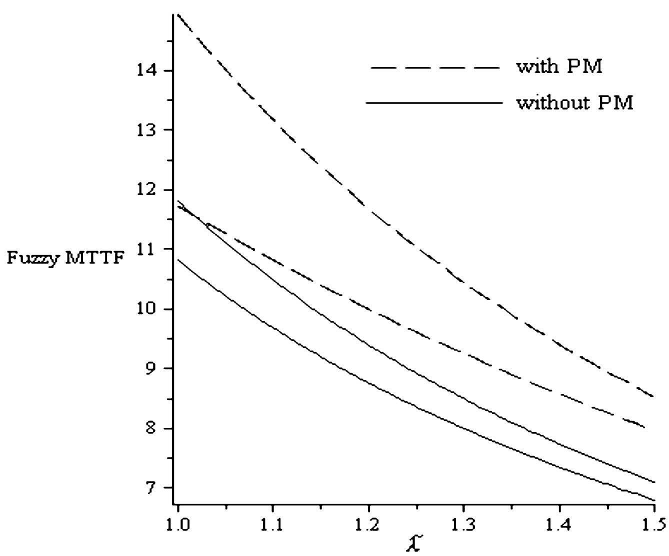

Fig. 2. Comparison of fuzzy mean time to system failure with and without PM versus $\tilde{\lambda}$ for $\alpha - cut = 0.1$.

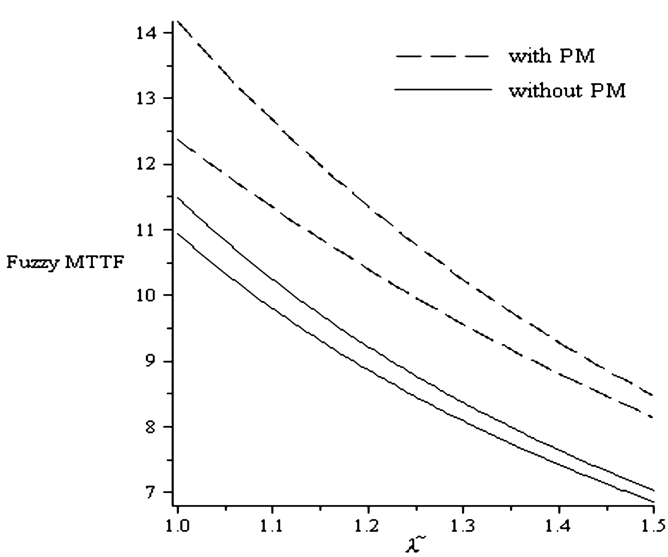

Fig. 3. Comparison of fuzzy mean time to system failure with and without PM versus $\tilde{\lambda}$ for $\alpha - cut = 0.5$.

when there is no preventive maintenance action provided to the system are illustrated in Table 4.

Comparison of the results for the fuzzy mean time to system failure versus the parameter $\tilde{\lambda}$, in the two cases of the introduced models with and without preventive maintenance, were illustrated in Figs. 2, 3 and 4 when $\alpha - cut = 0.1$, 0.5 and 0.9, respectively. It is obvious from the comparisons that the fuzzy mean time to system failure in case of the model in which preventive maintenance provided to the main unit is greater than the fuzzy mean time to system failure in case of the model without preventive maintenance provided to the main unit. And hence, the model with preventive maintenance is better than the model without preventive maintenance.

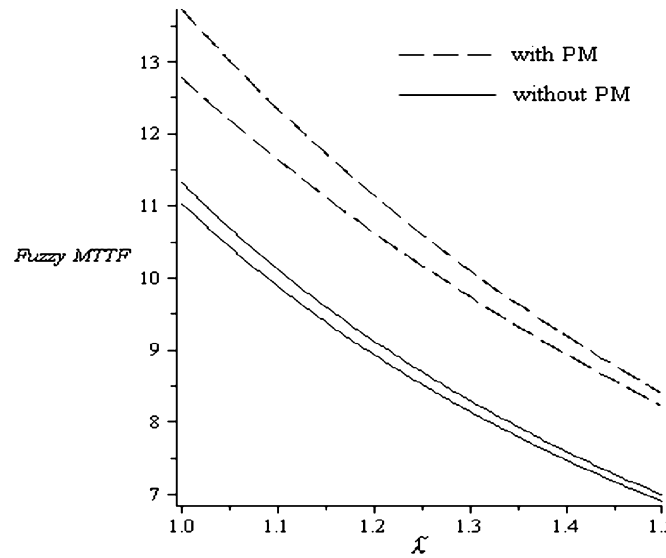

Fig. 4. Comparison of fuzzy mean time to system failure with and without PM versus $\tilde{\lambda}$ for $\alpha - cut = 0.9$.

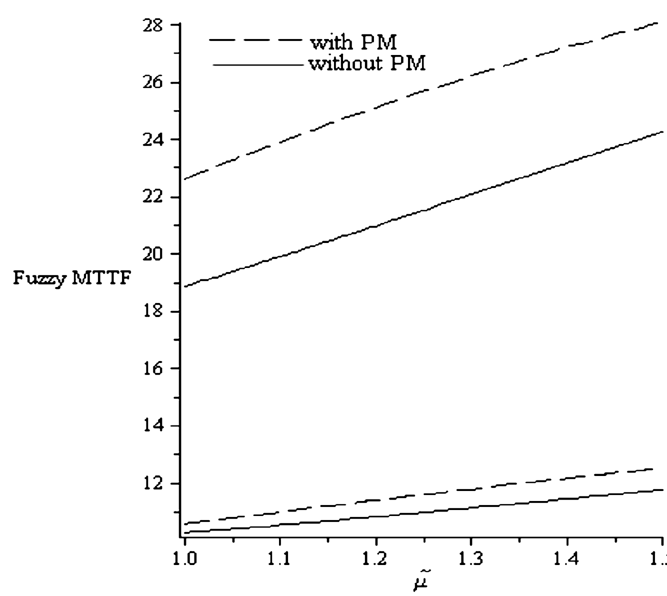

Fig. 5. Comparison of fuzzy mean time to system failure with and without PM versus $\tilde{\mu}$ for $\alpha - cut = 0.1$.

Comparison of the results for the fuzzy mean time to system failure versus the parameter $\tilde{\mu}$, in the two cases of the introduced models with and without preventive maintenance, were illustrated in Figs. 5, 6 and 7 when $\alpha - cut = 0.1$, 0.5 and 0.9, respectively. It is obvious from the comparisons that the fuzzy mean time to system failure in case of the model in which preventive maintenance provided to the main unit is greater than the fuzzy mean time to system failure in case of the model without preventive maintenance provided to the main unit. And hence, the model with preventive maintenance is better than the model without preventive maintenance.

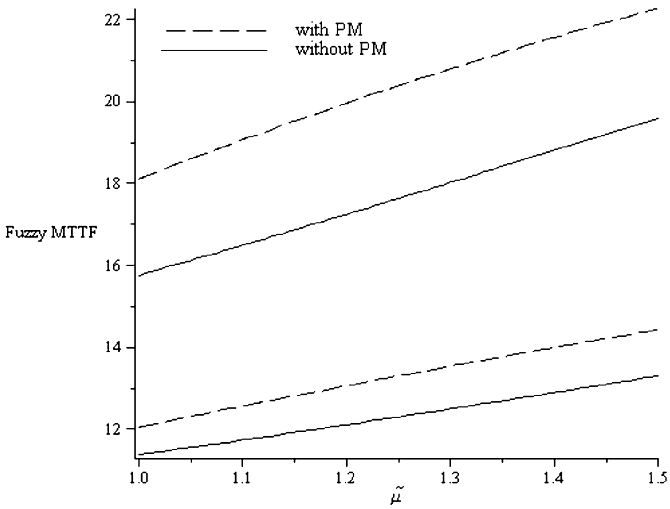

Fig. 6. Comparison of fuzzy mean time to system failure with and without PM versus $\tilde{\mu}$ for $\alpha - cut = 0.5$.

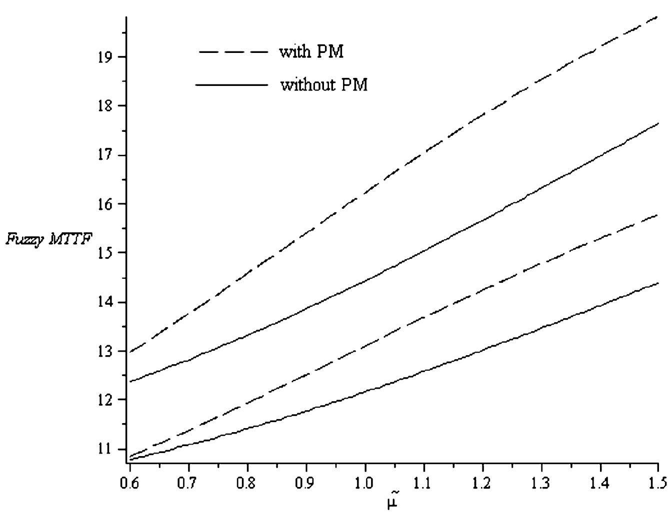

Fig. 7. Comparison of fuzzy mean time to system failure with and without PM versus $\tilde{\mu}$ for $\alpha - cut = 0.9$.

6. Conclusion

In this paper, analysis of a new mixed standby system was introduced in two cases. The first case considered that there is a preventive maintenance action is provided to the main unit after operating a period of time. The second case assumed that there is no preventive maintenance can be provided to the units of the system. The mathematical models for the two cases were constructed by using the semi-Markov models and regenerative point technique. All life, repair and preventive maintenance times of the units of the system were assumed to be generally distributed with fuzzy parameter defined by the bell-shaped membership function. An example was introduced assuming that the life, preventive maintenance and repair times of the units of the system follow Erlang distribution. Comparison of the fuzzy mean time to system failure of the two models was introduced. The results obtained in the numerical example showed that the performance of the model in which preventive maintenance provided to the main unit is better than the performance of model without preventive maintenance. Many papers in literature supposed that the failure, preventive maintenance and repair rates are constant values however in many cases the fuzziness of these rates must be considered.

Declarations

Author contribution statement

N. S. Y. Temraz: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Data availability statement

No data was used for the research described in the article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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