



## Research article

# A novel adaptive resampling for sequential Bayesian filtering to improve frequency estimation of time-varying signals



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## ABSTRACT

This paper presents a new algorithm for adaptive resampling, called percentile-based resampling (PBR) in a sequential Bayesian filtering, i.e., particle filter (PF) in particular, to improve tracking quality of the frequency trajectories under noisy environments. Since the conventional resampling scheme used in the PF suffers from computational burden, resulting in less efficiency in terms of computation time and complexity as well as the real time applications of the PF. The strategy to remedy this issue is proposed in this work. After state updating, important high particle weights are used to formulate the pre-set percentile in each sequential iteration to create a new set of high quality particles for the next filtering stage. The number of particles after PBR remains the same as the original. To verify the effectiveness of the proposed method, we first evaluated the performance of the method via numerical examples to a complex and highly nonlinear benchmark system. Then, the proposed method was implemented for frequency estimation for two time-varying signals. From the experimental results, via three measurement metrics, our approach delivered better performance than the others. Frequency estimates obtained by our method were excellent as compared to the conventional resampling method when number of particles were identical. In addition, the computation time of the proposed work was faster than those recent adaptive resampling schemes in literature, emphasizing the superior performance to the existing ones.

## 1. Introduction

Frequency estimation is one of the most important tasks in many areas that are related to signal and information processing, especially in electrical engineering, mechanical engineering, and environmental studies [1, 2, 3, 4]. For environmental study, frequency estimation is a crucial step that must be done with accurate estimating results, and then these results are used to obtain the environmental parameters for inversion problem. This can be seen in the ocean acoustic inversion where the signals received from the field were utilized to extract the frequency content from ocean acoustics time-series which is typically done by using particle filtering framework [5, 6, 7, 8].

To track the objects or targets that are moving, changing, or evolving over time, a sophisticated framework is required for such task. Recently, sequential Bayesian filtering has attracted researchers and engineers who are working on tracking applications. Since the establishment of

a well-known Kalman Filter (KF) that it can estimate the parameters of interest for tremendous number of problems in the cases of additive and Gaussian perturbations in the evolution of the unknown parameters, many challenging problems could be resolved [9]. Under the assumption that the system follows additive Gaussian noise in the measured data, and a linear relationship between state vector and measurements, KF can work efficiently. Unfortunately, under highly nonlinear/non-Gaussian systems, simple KF barely provides satisfactory estimating results and it is no longer efficient for highly nonlinear/non-Gaussian systems [10, 11, 12, 13]. Particle Filtering, a numerical technique to estimate or track the parameters of interest via a recursive computation, hence, comes into consideration. Recently, particle filter (PF) has been used in many science and engineering applications containing wireless networks, economic, multimedia, geography, and medicine [14, 15, 16, 17, 18, 19, 20], for examples. The need of PF stems from the fact that it can handle the highly nonlinear/non-Gaussian systems, the restriction

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of Gaussianity is now relaxed. PF is a class of sequential Monte Carlo approach that can sequentially approximate the probability distribution of the parameters of interest. The available measured or observed data is used to update the predicted state values.

Typical PF conducts important process called resampling in the PF iteration. Resampling has been implemented and used successfully in various fields of applications. In social sciences, the income of the population is calculated by using the resampling, the results show good accuracy [21]. Moreover, in engineering problems, the PF was employed in robot localization for position tracking [22] and hardware usage. In addition to that, resampling was also used to calculate battery endurance [23]. The main idea of resampling is that the particles with small weight will be replaced by new particles in the area around or close to the particles with large weight [24], resulting in the duplication of significant quantity (particle containing the parameters of interest, frequency in our work) to represent the posterior probability density functions (PDFs) of the investigating parameters. Nevertheless, a problem that resampling process can cause is a diversity problem. This is known as sample impoverishment and this problem occurs when all particles are identical. Some scenarios, the conventional resampling does not perform well under some noisy environments.

From the drawback mentioned above, the development to conquer the problem has continuously conducted [25, 26]. The technique called a diversity enhanced particle filter was introduced to improve the estimation accuracy. The technique is done by generating new sets of secondary particles from high weight primary particles, and then combining those sets to create the updated particles [27]. Residual-systematic resampling for fixed duration of the new randomness and partial resampling for reducing the burden of traffic through hardware networks has been proposed [28]. The development of the weight distribution of the abandoned particles for higher tracking efficiency [29], and adaptive fission particle filter (AF-PF) to increase the efficiency of parameter tracking by creating a better set of particles [30, 31] has been reported in literature for enhancing the resampling process. The above works performed resampling satisfactory by providing excellent tracking results, but those algorithms are still complicated and take substantial amount of processing time, preventing the realtime applications to be possible. To address the above mentioned issue, this paper therefore intends to present a new technique created for faster resampling while the tracking or estimating accuracy of the PF remains the same. Since the complexity of the proposed algorithm is reduced, lower processing time is required, allowing the applicability of the method for realtime state estimation.

The rest of this paper is organized as follows. In Section 2, we present a background of particle filter (PF), and the methodology of new adaptive resampling proposed in this paper is described in section 3. Time-frequency representation of the signal and particle filter implementation are described in Section 4. Tracking results at different noise levels will be illustrated, and the proposed algorithm is compared with the other two methods which include conventional resampling and adaptive fission resampling, there tracking results are presented in Section 5. Conclusions of the paper can be found in Section 6.

## 2. Particle filtering

To begin the particle filtering, the state-space model formulation for parameter estimation for this framework is required, it needs the following two state equations

$$\mathbf{X}_k = \mathbf{f}_{k-1}(\mathbf{X}_{k-1}, \mathbf{V}_{k-1}) \quad (1)$$

and

$$\mathbf{Y}_k = \mathbf{g}_k(\mathbf{X}_k, \mathbf{W}_k) \quad (2)$$

where  $k$  is the time index. Vector  $\mathbf{X}_k = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k\}$  contains tracking parameters and  $\mathbf{Y}_k = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_k\}$  contains the data observed in

the field (measured signals). The noise quantities in the state transition equation, Eq. (1), and the observation equation, Eq. (2), are given by  $\mathbf{V}_{k-1}$  and  $\mathbf{W}_k$ , respectively. Both components were assumed to follow Gaussian distributions. The nonlinear functions  $\mathbf{f}_{k-1}$  and  $\mathbf{g}_k$  portray the movement of the state variables, and the relation between the state variables and the observed data, respectively.

As previously mentioned that particle filtering is a processor that approximates the PDF of the tracking parameters at each stage (time step). This processor is based on the point mass density (or probability mass function (PMF)) in order to assess the density of past iteration using the principles of sequential importance sampling (SIS) [32, 33]. The estimated posterior PDF can be written as

$$p(\mathbf{X}_k | \mathbf{Y}_k) \approx \sum_{i=1}^N w_k^i \delta(\mathbf{X}_k - \mathbf{X}_k^i) \quad (3)$$

and

$$\begin{aligned} w_k^i &\propto \frac{p(\mathbf{X}_k^i | \mathbf{Y}_k)}{q(\mathbf{X}_k^i | \mathbf{Y}_k)} \\ &= w_{k-1}^i \frac{p(\mathbf{y}_k | \mathbf{x}_k^i) p(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{y}_k^i)} \end{aligned} \quad (4)$$

where  $N$  is the number of particle,  $w_k^i$  is the weight of  $i^{th}$  particle at time  $k$ ,  $\delta$  is the Dirac data function, and  $q(\mathbf{X}_k^i | \mathbf{Y}_k)$  is the proposal density [34]. Since the estimation is done sequentially, therefore the results from the previous step are used. If we assume that the state and the observed data are statistically independent, hence

$$q(\mathbf{x}_k | \mathbf{X}_{k-1}, \mathbf{Y}_k) = q(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{y}_k). \quad (5)$$

As suggested by [35] that the following choice of the proposal density can be chosen to minimize the IS error in the prediction step, i.e., if

$$q(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{y}_k^i) = p(\mathbf{x}_k | \mathbf{x}_{k-1}), \quad (6)$$

then Eq. (4) can be written as

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{y}_k | \mathbf{x}_k^i) p(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{y}_k^i)}, \quad (7)$$

consequently, the weight of the particle can be updated by

$$w_k^i = p(\mathbf{y}_k | \mathbf{x}_k^i) w_{k-1}^i, \quad (8)$$

and thus the estimated posterior PDF can now be rewritten as

$$p(\mathbf{x}_k | \mathbf{x}_k) \approx \sum_{i=1}^N w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i). \quad (9)$$

Since the SIS mostly fails after a few iterations, resampling plays an important role in solving this problem. The resampling process creates a new set of particles from the high weigh particles and ignores those having negligible weights [35, 36, 37, 38]. The inclusion of a resampling into the SIS is called sequential importance resampling (SIR) [15, 39]. To decide if resampling is needed, the following quantity  $N^{eff}$  is used to make a judgment:

$$N^{eff} = \frac{1}{\sum_{i=1}^N (w_k^i)^2} \quad (10)$$

The illustration of SIRPF is shown in Fig. 1. Please be noted that the quantities  $\mathbf{x}_{0:k}$  and  $\mathbf{y}_{0:k}$  in the figure are referred to the state and observed data up to time  $k$ .

The resampling step reduces the effects of degeneracy, but new problems are presented [10]: limitation of the ability to parallelize the SIS algorithm is introduced, and statistical independence assumption is no longer valid after resampling. Consequently, convergence problems occur if resampling takes a serious loss of particle validity. This is well-known as sample impoverishment problem which is typically happened

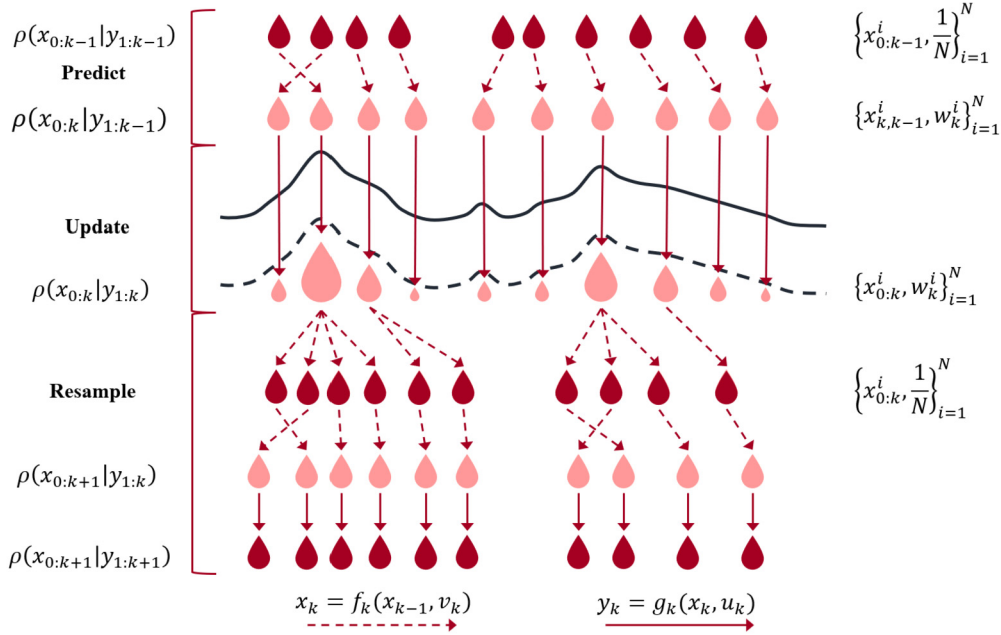


Fig. 1. The sequential importance sampling resampling particle filter (SIR PF).

when the noise level in the measured data is low. Currently, some developments of resampling for PFs [2, 10, 25, 26, 40, 41] have been proposed to tackle these problems.

In this work, we propose a new resampling algorithm which has little computational complexity for PF instead of using conventional resampling and those algorithms presented in literature. The computational cost is very low while the tracking results which will be seen in Section 5 are excellent as compared to the other existing resampling methods. The detail of the proposed technique is presented in the next section.

### 3. Percentile-based resampling particle filter (PBRPF)

This section provides the proposed technique that improves the performance of the sequential Bayesian framework in terms of lower complexity, yet computation time. The developed technique can be considered as a type of adaptive resampling which aimed mainly to reduce the complexity of the conventional SIR scheme while the tracking performance remains the same or better than the convention one. Percentile-based resampling can be considered as the selection process of the parent particles by accumulating the best parent particles according to the importance weights obtained from the update step in the PF. The top ten quantity is used in this work [42] since it sufficiently captures very important particles to represent the probability distribution of the states.

Consider a set of particle weights at time step  $k$ ,  $\{w_k^i\}_{i=1}^N$ , where  $N$  is number of particles. It should be emphasized that  $\sum_{i=1}^N w_k^i = 1$ . Without loss of generality, let  $\mathbf{W}_k = [w_{k,1} \ w_{k,2} \ \dots \ w_{k,N}]$  be a vector containing the particle weight in ascending order, i.e.,  $w_{k,1} > w_{k,2} > w_{k,1} \dots > w_{k,N} \geq 0$ . To formulate PBR-PF, let  $P$  be the required percentage (above 90% is preferable) of the sum of the weights of particles, this can be obtained by the following condition:

$$P \geq \sum_{j=1}^{N_p} w_{k,j}, \quad (11)$$

where  $N_p$  represents number of particle weights that their sum reaches  $P$ . It is clear that  $N_p$  is smaller than  $N$  in general. For the extreme case, if  $N_p = 1$ , which means that a single particle already occupies  $P$  percents of the whole distribution, and we will use this particle as a

parent particle to generate offspring particles. If  $N_p$  is greater than one, we will use those particles as a set of parent particles, then the offspring particles are created according to this set. Number of offspring particles depends on the individual parent weight, i.e., if  $w_{k,1} > w_{k,2}$ , particle  $\mathbf{x}_{k,1}$  will be duplicated in a greater number than that the copies of  $\mathbf{x}_{k,2}$ . After generating offspring particles, a total number of particle remains the same.

Now, let  $\mathbf{W}_{k,p} = \{w_{k,1}, w_{k,2}, \dots, w_{k,p}\}$  be a set of  $P$ -percentile weights of the parent particles, the offspring particles are constructed from the corresponding parent particle achieving  $n_1, n_2, \dots, n_p$  particles, respectively, where  $n_1 + n_2 + \dots + n_p = N$ . To this, it is intuitively seen that the number of new particles are proportionally generated according to the weight of each chosen parent, i.e.,

$$n_i = \left\lceil \frac{w_{k,i}}{\sum_{j=1}^{N_p} w_{k,j}} N \right\rceil. \quad (12)$$

Note that  $\lceil \cdot \rceil$  creates the roundup to the nearest integer.

Number of new particles that were generated depends on the weight of each parent particle. If the weight is high, it will be able to generate new offspring particles more than those with the low-weight ones. It must be noticed here that  $\sum_{j=1}^{N_p} n_j$  may not equal to  $N$  in practical implementation. Moreover, number of total particles for each iteration may vary. We have to eliminate or duplicate some offspring particles in order to retain total number of particle as  $N$ . The process of alteration in PBR is demonstrated in Fig. 2. The final result obtained from PBR contains a set of new particles that were generated from high quality parent particles, these new particles will be used in the next PF iteration.

### 4. Time-frequency representation of signal and particle filter implementation

We consider a noisy signal in the time-domain that is composed of multiples frequencies. In addition, the number of components evolves over time.

Standard Short-Time Fourier Transform (STFT) is utilized in the work for a TF representation of the time-varying noisy signal, the corresponding squared magnitude of the STFT is expressed by:

$$SG_x = \frac{1}{2\pi} \left| \int x(\tau) w(\tau - t) e^{-j\omega\tau} d\lambda \right|^2, \quad (13)$$

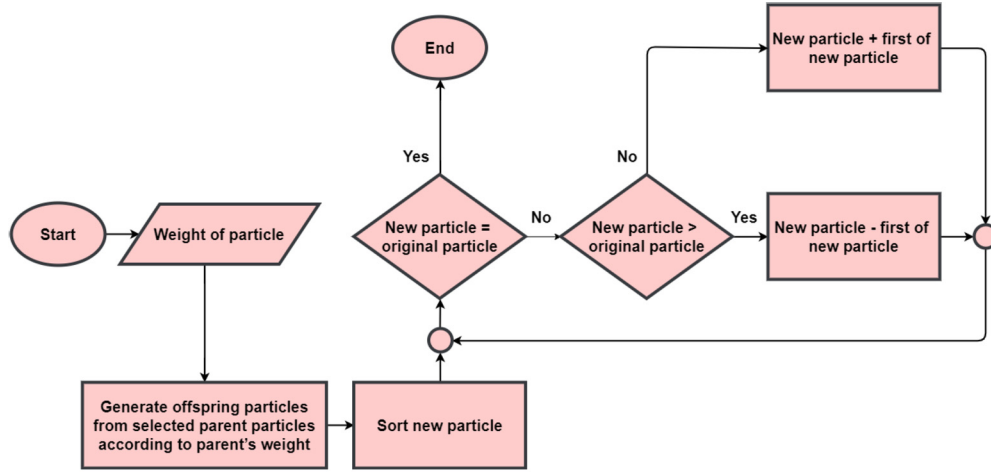


Fig. 2. Process of alteration in percentile-based resampling.

where  $w(t)$  is the window function applied for enhancing the signal information in STFT calculation.

In the frequency domain, the frequency components of the signal can be considered as a sum of squared *sinc* functions. With modeling by *sinc* function, the center of a *sinc* is therefore a position of a frequency component and it indeed evolves with time. The particle filter tracks this position along with the amplitude of each frequency component. Therefore, the state vector contains the frequencies of the signal (*sinc* position) and their amplitudes. In addition to this, since the signal is time-varying, we hence allow the dimension of the state vector to change with time. In other words, the filter also tracks the evolution of the frequency components by allowing the birth and death processes throughout tracking operation. For full detail describing this, the readers can consult [43, 44, 45, 46].

## 5. Experimental results

### 5.1. One-dimensional experiment

In this section, we illustrate via numerical example the performance of the PFs from the famous and commonly used in sequential Bayesian filtering [47, 48, 49]. This experimental system requires high performance of the filtering framework because it is highly nonlinear, and the likelihood function is bimodal. The system model is described as follows:

$$x_k = \frac{1}{2}x_{k-1} + \frac{25x_{k-1}}{1+x_{k-1}^2} + 8 \cos[1.2(k-1)] + v_k \quad (14)$$

and

$$y_k = \frac{1}{20}x_k^2 + \omega_k \quad (15)$$

where  $v_k \sim \mathcal{N}(0, \sigma_{v,k}^2)$  and  $\omega_k \sim \mathcal{N}(0, \sigma_{\omega,k}^2)$  represent the process and measurement noises, respectively. In the experiment, we set  $\sigma_{v,k} = 2$  and  $\sigma_{\omega,k} = 1$ . The initial particles are drawn from

$$x_0^j \sim \mathcal{N}(0.5, \sigma_{v,k}^2). \quad (16)$$

In this experiment, the number of particles was 200, and the time steps was 100 in one realization. The number of noisy realizations,  $N_r$ , was set as 200 in order to evaluate the performance of the PFs.

The metrics used for performance evaluation are Monte Carlo root mean squared error (*MCRMSE*), mean absolute error (*MAE*), and average running time of each iteration ( $T_{av}$ ). These quantities are defined as:

$$MCRMSE = \sqrt{\frac{1}{KN_r} \sum_{j=1}^{N_r} \sum_{k=1}^K (x_{k,j} - \hat{x}_{k,j})^2}, \quad (17)$$

Table 1. Performance comparison of the PF with different resampling schemes.

Method	MCRMSE	MAE	Tav
SIRPF	5.1217	3.0189	1.2532
AFRPF	4.8245	2.1549	1.3016
PBRPF	3.9156	1.9249	0.6395

$$MAE = \frac{1}{KN_r} \sum_{j=1}^{N_r} \sum_{k=1}^K |x_{k,j} - \hat{x}_{k,j}|, \quad (18)$$

and

$$T_{av} = \frac{1}{KN_r} \sum_{j=1}^{N_r} \sum_{k=1}^K T_{k,j}, \quad (19)$$

where  $K$  represents number of time steps.  $x_{k,j}$  and  $\hat{x}_{k,j}$  are the true and estimated state values.  $T_{k,j}$  is the running time at the time  $k$  in the  $j$ th experiment.

Presented in Table 1 the *MCRMSE*, *MAE*, and  $T_{av}$  of the proposed method and others. It can be observed that the proposed PBRPF delivers better results as compared to the other resampling techniques. Moreover, the computational time of the PBRPF is significantly lower than those from the other methods. These results are the evidences that the proposed resampling scheme could enhance estimating results as well as computation time, resulting in the feasibility of real-time usage for highly nonlinear systems.

Finally, to demonstrate the comprehensive understanding of the superior ability in state estimation of the proposed method over the other techniques, we show in Figs. 3, 4 and 5, with number of particle of 200, the estimates of the state with the true state values superimposed as provided by SIRPF [11], AFRPF [30], and our PBRPF, respectively. These estimation results emphasize that the PBRPF offers more consistent tracks with the real state values than those delivered by SIRPF and ARPF. The performance of both SIRPF and AFRPF are similar, while the proposed method appears to perform more accurately.

### 5.2. Frequency tracking from noisy signals

For this experiment, we generated two signals for performance evaluation, and their spectrograms of the clean signals were calculated according to what we discussed in Section 4. Shown in Figs. 6 and 7 are the spectrograms of both signals. For spectrogram calculation, Hamming window was used in this work [50]. We used these two signals for frequency tracking for all PFs with different resampling schemes. The existing resampling schemes used for performance evaluation in this work include conventional resampling [11] and existing adaptive

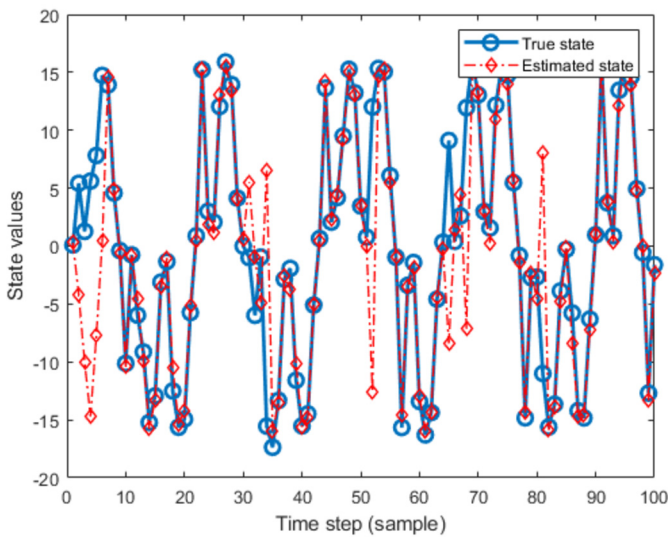


Fig. 3. State estimation results obtained by the SIRPF and true state values superimposed.

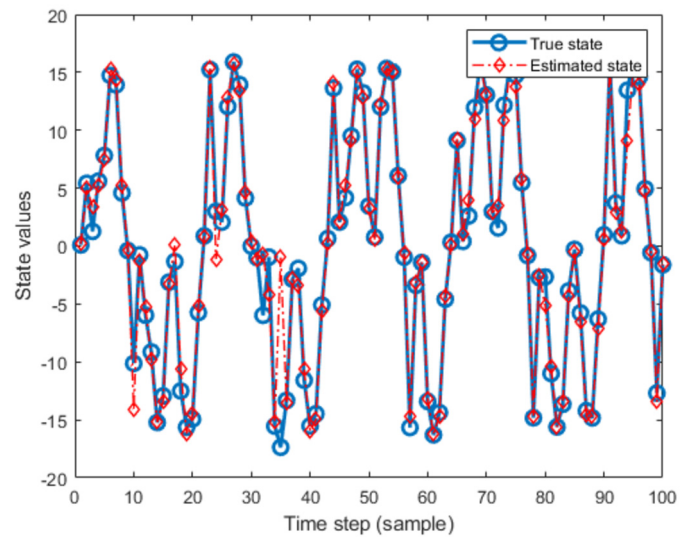


Fig. 5. State estimation results obtained by the PBRPF and true state values superimposed.

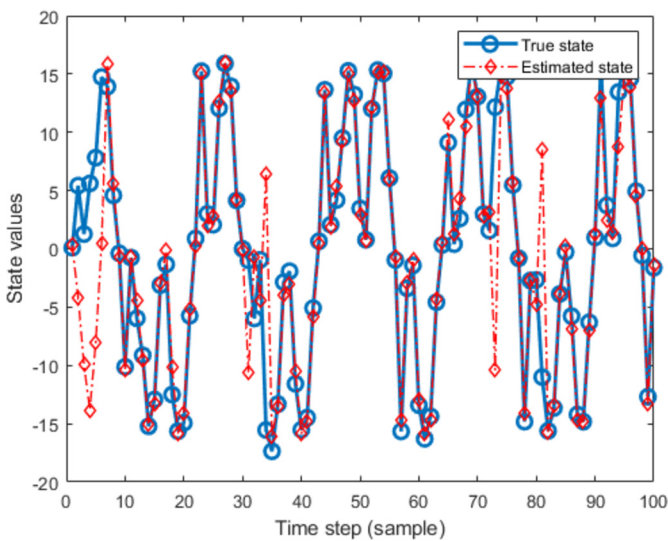


Fig. 4. State estimation results obtained by the AFRPF and true state values superimposed.

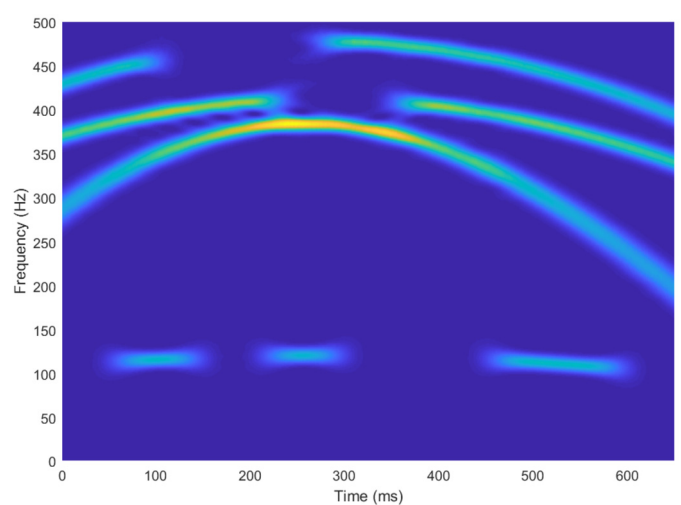


Fig. 6. Spectrogram of the clean signal 1.

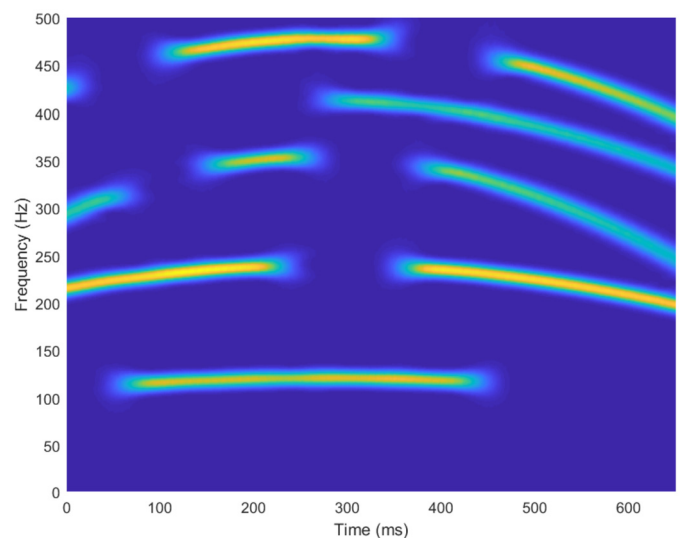


Fig. 7. Spectrogram of the clean signal 2.

resampling proposed in literature. In addition, three different SNR levels were considered to investigate the noise robustness of the proposed algorithm.

For signal 1, the tracking results demonstrate that the PBRPF provided better tracks than the conventional SIRPF [11, 51] and ARPF [52]. This can be seen in Fig. 8(a)-(c), where the SNR was 20 dB. The PBRPF is able to track the low frequency component of the signal, while the other two filters cannot. Only some periods of time that the PBRPF misses the tracks, these errors could be a result from low number of particles used in this experiment which was only 1000 particles. To further investigate the noise robustness of the filter, we applied each filter for different SNR levels.

We conducted more experiments for lower SNRs, 15 dB and 10 dB, and the frequency estimates are presented in Figs. 9 and 10, respectively. Shown in Figs. 9(a) and 10(a), the SIRPF cannot track the frequency trajectories precisely. Specifically, it misses identify the frequency component of 100 Hz at time 50-150 ms. Moreover, illustrated via Figs. 9(b) and 10(b), the ARPF always misses the 100 Hz component. On the other hand, the proposed PBRPF provides much better

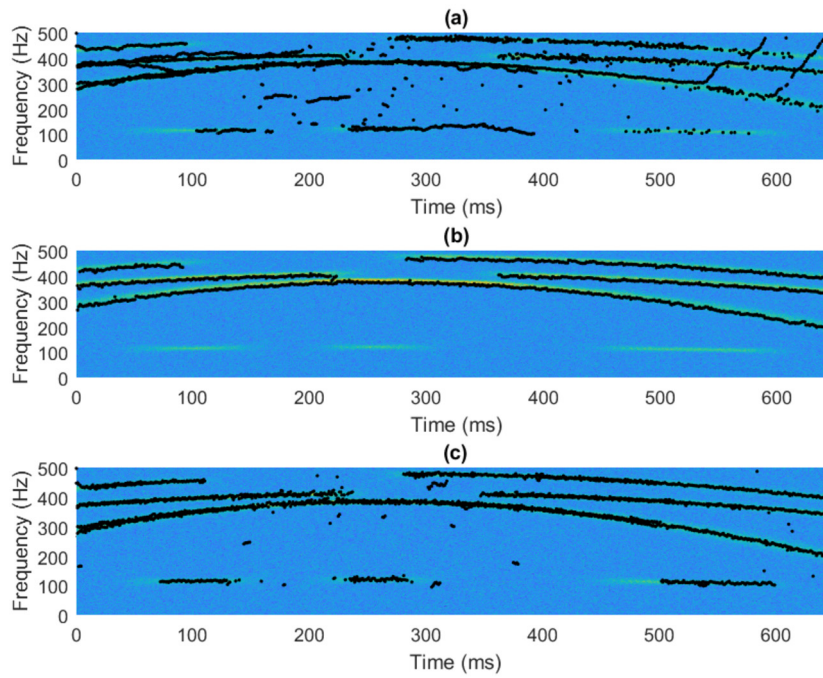


Fig. 8. Frequency estimates of signal 1 from the PFs: (a) SIRPF, (b) ARPF, and (c) PBRPF. The number of particles was 1,000 and SNR was 20 dB.

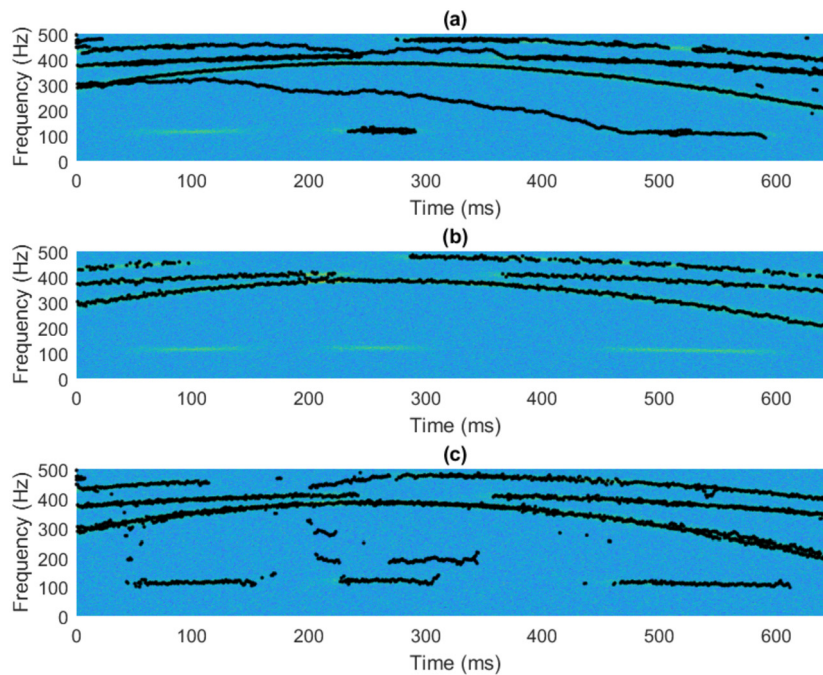


Fig. 9. Frequency estimates of signal 1 from the PFs: (a) SIRPF, (b) ARPF, and (c) PBRPF. The number of particles was 1,000 and SNR was 15 dB.

tracking results than those two filters, this can be seen in Figs. 9(c) and 10(c).

For the results shown in Figs. 8, 9 and 10, the reason that SIRPF and ARPF failed to track the low frequency components in the signal could be as follows. Since PF adapts itself by predicting and updating the state values as time evolves to formulate the best distribution of the estimating frequency, but this can be done effectively by using a good set of particles from the previous time step. Unfortunately, SIRPF and AFPF did not deliver a good set of particles, but the PBRPF. It can be seen in the figures that the low frequency components of the signal are quite apart from the others, the tracking results indicate that the ability to capture frequency components that are apart from the

others of SIRPF and ARPF is low, while the PBRPF can capture these components satisfactory.

To see more about the tracking capability of the proposed filter, we created signal 2 for further investigation. This signal contains more complication in terms of the appearance of the frequency trajectories. As expected, illustrated by three different SNR levels, 20 dB, 15 dB, and 10 dB, the PBRPF provides better tracking results than the SIRPF and ARPF. These experimental results confirm better tracking performance of the proposed method over the other two, the frequency estimates are displayed in Figs. 11, 12 and 13.

Finally, we present in Table 2 the computation time for all resampling schemes. It is obviously found that the proposed PBRPF takes

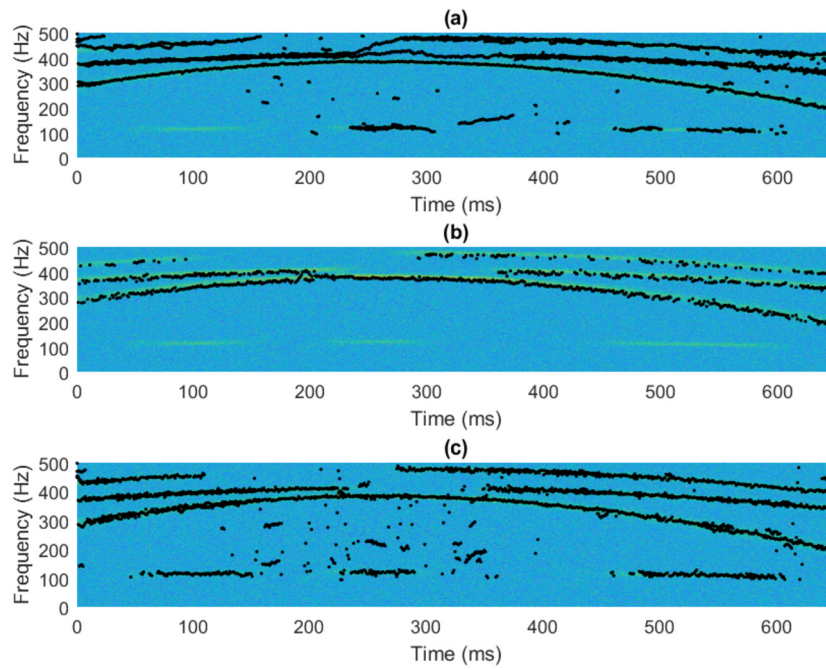


Fig. 10. Frequency estimates of signal 1 from the PFs: (a) SIRPF, (b) ARPF, and (c) PBRPF. The number of particles was 1,000 and SNR was 10 dB.

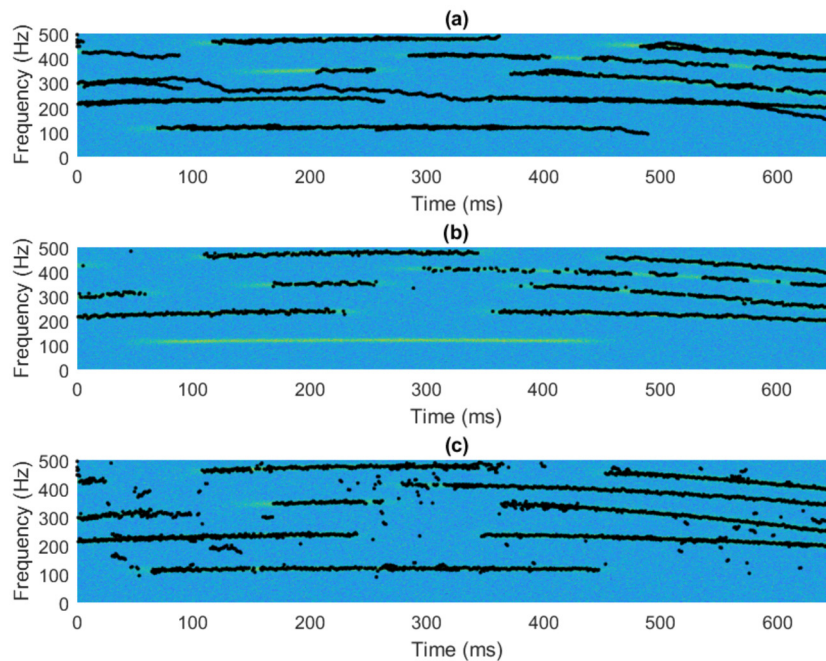


Fig. 11. Frequency estimates of signal 2 from the PFs: (a) SIRPF, (b) ARPF, and (c) PBRPF. The number of particles was 500 and SNR was 20 dB.

Table 2. Computation time of the PF with different resampling schemes (ms).

	Signal 1	Signal 2
SIRPF	427.93	358.70
ARPF	460.58	418.32
PBRPF	382.12	341.03

lowest computational time for both signals. Therefore, the utilization of the proposed method for online or realtime applications could be possible. This is the main advantage of the PBRPF in addition to tracking accuracy.

In summary, comparing with two existing resampling schemes, all experiments have confirmed that the proposed PBRPF offers much better tracking results than the other techniques with lower computation time.

### 6. Conclusions

We presented in this paper a new resampling algorithm called percentile-based resampling (PBR) that is one of the important parts in the particle filtering framework to replace the conventional resampling. This algorithm selects the most important weights of the parent particles by adding the most top weights until the pre-set percentile is

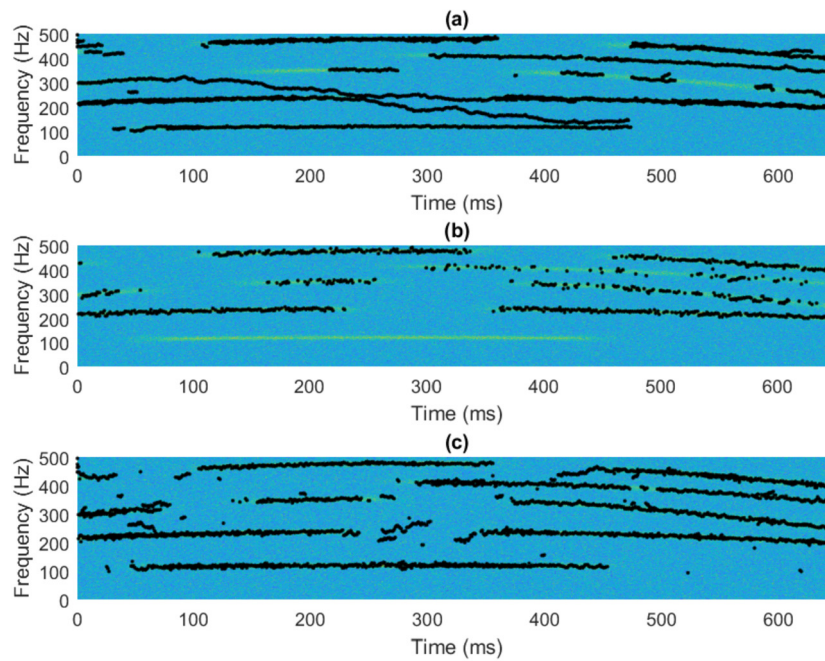


Fig. 12. Frequency estimates of signal 2 from the PFs: (a) SIRPF, (b) ARPF, and (c) PBRPF. The number of particle was 500 and SNR was 15 dB.

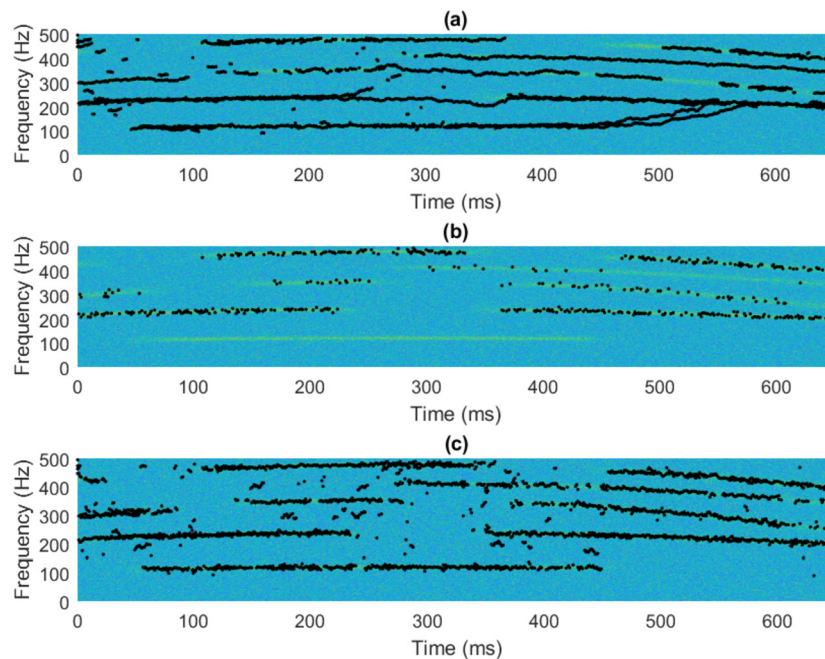


Fig. 13. Frequency estimates of signal 2 from the PFs: (a) SIRPF, (b) ARPF, and (c) PBRPF. The number of particles was 500 and SNR was 10 dB.

reached. The rest of the particles that have low weights are ignored, and a new set of particles is then generated according to the selected parents particles. The amount of the offspring is proportional to the weight of each parent particle. The number of particles in the filtering remains the same, no matter how many parent particles were selected.

The simulation results showed that the PBRPF could deliver better frequency estimates as compared to the conventional resampling (SIRPF), adaptive fission resampling (AFRPF), and another adaptive resampling (ARPF) technique. The computation time for each resampling method has been provided and the results dictated the advantage of the proposed method. This technique is easy to implement and it takes less computation time than other techniques. Therefore, the proposed

resampling scheme is plausible for realtime object and parameter tracking applications. Moreover, this work can be applied to ocean acoustics signals for modal frequency tracking and dispersion curves estimation, vibration applications, nondestructive testing for crops analysis and classification, and speech processing, etc.

**Declarations**

*Author contribution statement*

Nattapol Aunsri: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Con-



tributed reagents, materials, analysis tools or data; Wrote the paper. Kunrutai Pipatphol, Benjawan Thikeaw & Satchakorn Robroo: Performed the experiments; Analyzed and interpreted the data; Wrote the paper. Kosin Chamnongthai: Analyzed and interpreted the data; Wrote the paper.

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#### Data availability statement

Data will be made available on request.

#### Declaration of interests statement

The authors declare no conflict of interest.

#### Additional information

No additional information is available for this paper.

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