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Determination of the best geographic weighted function and estimation of spatio temporal model – Geographically weighted panel regression using weighted least square



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REVIEW HIGHLIGHTS

- The method used was the development of a spatio temporal Geographically Weighted Panel Regression model with within estimators and geographic weights that contain elements of location, time and correlation between the two.
- The geographic weights used were the Gaussian kernel function, the Bisquare kernel function and the exponential kernel function, then the best weight was determined based on the optimal bandwidth value and the lowest CV.
- This paper determined the spatial classification and mapping of 34 provinces based on significant predictor variables.

ARTICLE INFO

Method name:

Geographically Weighted Panel Regression Keyword:

Geographically weighted panel regression Geographic weighted function National food security index Spatio temporal model Statistical modeling Weighted least square

ABSTRACT

This study proposes the development of a spatio-temporal model with geographic weights containing elements of location, time and the correlation between the two. The spatio-temporal model is a spatial regression model that combines geographic information and time series simultaneously. The model can overcome the problem of spatial heterogeneity and spatial effects. The spatial temporal model used is the Geographically Weighted Panel Regression (GWPR) model with a within estimator. Therefore, it is necessary to determine the best geographic weighting with the optimal bandwidth value and the lowest Cross Validation (CV). The geographic weights used were the Gaussian kernel function, the Bisquare kernel function and the exponential kernel function. Estimation of spatio-temporal model parameters using Weighted Least Square (WLS). The GWPR model was applied to food security index data in 34 Indonesian provinces. The problem of food security is an important problem to be solved in Indonesia, one way is to find the factors that influence the food security index through spatio-temporal modeling. This study consists of data exploration, descriptive statistics, spatial mapping distribution, selection of geographic weights and GWPR modeling. The results showed that the spatio temporal statistical model of GWPR was more accurate with a good model of 92.78 % and a Root mean Square Error value of 3.41. Some highlights of the proposed approach are:

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Specifications table

Subject area:	Mathematics and Statistics
More specific subject area:	Statistics: Regression, Spatial Regression
Name of your method:	Geographically Weighted Panel Regression
Name and reference of original	D. Yu, Exploring spatiotemporally varying regressed relationships: The geographically weighted panel regression analysis, Int.
method:	Arch. Photogramm. Remote Sens. Spat. Inf. Sci. Arch. 38, Part II. (2010) 134–139.
Resource availability:	The dependent variable used in this study is the Food Security Index (y) in Indonesia 2019–2021. The predictor variables (x_i)
	used in this study are Rice Production, Red Chili Production, Shallot Production, Palm Oil Production, Beef Production,
	Production of Laying Chicken Meat, Average Monthly Food Expenditure per capita, Percentage of Poor Population, Percentage
	of Population According to Food Consumption Insufficiency Status and Percentage of Population with Food Insecurity. The
	data was obtained through the publication of the Agricultural Data Center and Information System of the Secretariat General
	of the Ministry of Agriculture, 2(1) of 2022.

Method details

Introduction

This research presents innovations in the field of statistics and statistical modeling, especially geographically weighted models. The spatio-temporal model is a model that represents observed natural phenomena in spatial and temporal dimensions [1–3]. Data analysis on the spatio-temporal model considers the spatial dependency between observation areas and the correlation between one or several time lags [4,5]. One of the spatio temporal models is the Geographically Weighted Panel Regression (GWPR) model [6,7]. This research has the aim of studying and developing the theory in the field of statistical modeling, especially the spatio temporal Geographically Weighted Panel Regression model.

The first aim of this research is to determine the best geographic weight by using the Gaussian kernel function, the Bisquare kernel function and the exponential kernel function. The selection of geographic weighting uses cross validation and the root mean square error of each geographic weighting function. The aim of further theoretical development is to estimate the spatio-temporal model parameters using Weighted Least Square (WLS). The model obtained is then implemented on food security index data in 34 provinces in Indonesia. There are ten predictor variables that will be analyzed for their effect on the response variable. Observational data is influenced by geographic spatial and time elements, which are suitable for the analysis of the Geographically Weighted Panel Regression model.

Food is a basic human need that is most essential to sustain life. The importance of food so that every individual and even the state must be able to maintain food. Food security in Indonesia has been stipulated in Law no. 18 of 2012 concerning food which states provisions regarding the determination of government food reserves. There are four dimensions to achieving food security, namely (1) food availability, (2) economic and physical affordability of food, (3) food use which includes food quality and safety, and (4) stability in the other three dimensions. Indonesia is one of the developing countries in the world which is ranked 69th in 2017 to 2021 out of 113 countries with a global food security index of 59.2 [8].

This study will present statistical descriptions and spatial mapping distribution based on research data for each variable. The research results obtained are the spatio temporal statistical model of GWPR and the factors that influence the food security index in 34 provinces in Indonesia. Classification and spatial mapping of 34 provinces were also obtained based on significant food security indicators.

Materials and model specifications

Spatio-Temporal models

The Spatio Temporal Model is a spatial regression model that combines geographic information and time series simultaneously [9–11]. This model can overcome the problem of spatial heterogeneity and spatial effects [12–14]. One of the spatio-temporal models is the Geographically Weighted Panel Regression (GWPR) model which was built from panel regression, especially the Fixed Effect Model (FEM) and Geographically Weighted Regression (GWR) models [15,16].

Fixed effect model panel regression

Panel regression is a method for modeling the effect of predictor variables on response variables using observational data in the form of panel data. Panel data is a combination of cross section data with time series data [17]. One of the models in panel regression is the Fixed Effect Model (FEM). Panel regression with the FEM approach is a linear regression model that assumes that each individual model has a different intercept value [18,19]. The general form of FEM is stated in Eq. (1).

$$y_{it} = \beta_{0,i} + \beta_1 x_{it1} + \beta_2 x_{it2} + \dots + \beta_k x_{itk} + \epsilon_{it}, t = 1, 2, \dots, T$$
(1)

 y_{ii} is the observed value of the response variable i; i = 1, 2, ..., n. The predictor variable x_{iik} is the *i* th observation value and the *t*-th period for the *k*-th predictor variable, β_{0_i} in Eq. (1) shows that the intercept of each unit cross section is different. β_k is the coefficient of the regression parameter of the *k*-th predictor variable. ϵ_{ii} is the error value of the *i* th observation; i = 1, 2, ..., n; t = 1, 2, ..., T.

Spatio temporal model with geographically weighted panel regression

The GWPR model is a local regression model from FEM, with repeated data at each observation location, at different times, and is spatial data [16]. Based on the FEM model with the within estimator, the GWPR model at the *i* th observation location and *t*-time [6] is stated in Eq. (2) as follows

$$y_{it}^* = \beta_1 (u_i, v_i) x_{it1}^* + \beta_2 (u_i, v_i) x_{it2}^* + \dots + \beta_k (u_i, v_i) x_{itk}^* + \epsilon_{it}^*, i = 1, 2, \dots, n, t = 1, 2, \dots, T$$
(2)

The coordinates at each observation location are known by the coordinates of the *i* th observation location (u_i, v_i) where u_i represents the location of the latitude and v_i represents the location of the longitude. y_i is the observed value of the response variable for the *i* th location; i = 1, 2, ..., n. x_{itk} is the observed value of the *k*-th predictor variable at the *t*-th time for the *i* th observation location. $\beta_k(u_i, v_i)$ is the regression coefficient of the *k*-th predictor variable at the *i* th observation location. (u_i, v_i) is the geographic coordinate point (latitude and longitude) of the *i* th observation location. ε_i is the error value of the *i* th observation.

Estimation of GWPR model parameter with weighted least square approach

GWPR model parameter estimation can be carried out using the Weighted Least Square (WLS) approach, which is a form of Ordinary Least Square (OLS) method development by considering the spatial weighting at each observation location. Based on the WLS method, the GWPR model parameter estimator is obtained by minimizing the sum of the squared errors [20] of Eq. (2) with spatial weighting so that Eq. (3) is obtained.

$$\sum_{t=1}^{T} \sum_{i=1}^{n} w_{it}(u_i, v_i) \varepsilon_{it}^2 = \sum_{t=1}^{T} \sum_{i=1}^{n} w_{it}(u_i, v_i) [y_{it}^* - \beta_1(u_i, v_i) x_{111}^* + \beta_2(u_i, v_i) x_{212}^* + \dots + \beta_1(u_i, v_i) x_{itk}^*]$$
(3)

Eq. (3) can be expressed in matrix form so that Eq. (4) is obtained.

$$\boldsymbol{\varepsilon}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \boldsymbol{\varepsilon}^{*} = \left[\mathbf{y}^{*} - \mathbf{X}^{*} \boldsymbol{\beta}(u_{i}, v_{i}) \right]^{\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \left[\mathbf{y}^{*} - \mathbf{X}^{*} \boldsymbol{\beta}(u_{i}, v_{i}) \right] = \mathbf{y}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{y}^{*} - \mathbf{y}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{y}^{*} + \boldsymbol{\beta}^{\mathrm{T}}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{y}^{*} - \boldsymbol{\beta}^{\mathrm{T}}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{y}^{*} - 2\boldsymbol{\beta}^{\mathrm{T}}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*} \boldsymbol{\beta}(u_{i}, v_{i}) \right]$$

$$(4)$$

where

$$\boldsymbol{\beta}(u_i, v_i) = \left[\beta_1(u_i, v_i) \ \beta_2(u_i, v_i) \ \dots \ \beta_p(u_i, v_i)\right]^T$$

and

 $\mathbf{W}(u_i, v_i) = diag[w_{i11}, w_{i21}, \dots, w_{in1}, w_{i12}, w_{i22}, \dots, w_{in2}, \dots, w_{i1T}, w_{i2T}, \dots, w_{inT}]$

Derivating first order Eq. (4) with respect to $\beta^{T}(u_{i}, v_{i})$ and equating to zero will obtain the Eq. (5) as follows.

$$-2\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_{i}, v_{i})\mathbf{y}^{*} + 2\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_{i}, v_{i})\mathbf{X}^{*}\boldsymbol{\beta}(u_{i}, v_{i}) = 0$$

$$2\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_{i}, v_{i})\mathbf{X}^{*}\boldsymbol{\beta}(u_{i}, v_{i}) = 2\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_{i}, v_{i})\mathbf{y}^{*}$$

$$\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_{i}, v_{i})\mathbf{X}^{*}\boldsymbol{\beta}(u_{i}, v_{i}) = \mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_{i}, v_{i})\mathbf{y}^{*}$$
(5)

Both sides in Eq. (6) are multiplied by $(\mathbf{X}^{*T}\mathbf{W}(u_i, v_i)\mathbf{X}^*)^{-1}$ to get $\hat{\boldsymbol{\beta}}^*(u_i, v_i)$ such that

$$\left(\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_i, v_i)\mathbf{X}^*\right)^{-1}\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_i, v_i)\mathbf{X}^*\boldsymbol{\beta}(u_i, v_i) = \left(\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_i, v_i)\mathbf{X}^*\right)^{-1}\mathbf{X}^{*\mathrm{T}}\mathbf{W}(u_i, v_i)\mathbf{y}^*$$
(6)

Thus, we obtain the parameter $\hat{\beta}^*(u_i, v_i)$ in Eq. (7)

 $\hat{\boldsymbol{\beta}}^{*}(u_{i}, v_{i}) = (\mathbf{X}^{*T} \mathbf{W}(u_{i}, v_{i}) \mathbf{X}^{*})^{-1} \mathbf{X}^{*T} \mathbf{W}(u_{i}, v_{i}) \mathbf{y}^{*}$

where $\hat{\boldsymbol{\beta}}^*(u_i, v_i)$ is an estimator of GWPR model parameter and

$$\mathbf{X}_{it}^{*T} = \begin{bmatrix} x_{i11}^{*} & x_{i12}^{*} & \cdots & x_{i1p}^{*} \\ x_{i21}^{*} & x_{i22}^{*} & \cdots & x_{i2p}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ x_{iT1}^{*} & x_{iT2}^{*} & \cdots & x_{iTp}^{*} \end{bmatrix}$$
 is the matrix for the *i*-th observation at each time unit from matrix \mathbf{X}^{*} .

Geographically weighted panel regression model fit test

The model fit test aims to examine whether there is a difference between the panel regression model and the GWPR model [21]. The model fit test hypothesis is as follows.

$$H_0: \beta_k(u_i, v_i) = \beta_k, i = 1, 2, \dots, n; k = 1, 2, \dots, p$$

*H*₁:At least one $\beta_k(u_i, v_i) \neq \beta_k$

(7)

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(9)

(10)

The test statistic of model fit F_{GWPR} is given by the Eq. (8).

$$F_{GOF} = \frac{JKG(H_0)/db_1}{JKG(H_1)/db_2}$$
(8)

 $JKG(H_0)$ obtained based on Eq. (9) below.

$$JKG(H_0) = \mathbf{y}^{*T}(\mathbf{I} - \mathbf{H}^*)\mathbf{y}^*$$

with
$$\mathbf{H}^* = \mathbf{X}^* (\mathbf{X}^{*T} \mathbf{X}^*)^{-1} \mathbf{X}^{*T}$$

While $JKG(H_1)$ obtained in Eq. (10) below.

$$JKG(H_1) = \mathbf{y}^{*T}(\mathbf{I} - \mathbf{L}^*)^{\mathbf{I}}(\mathbf{I} - \mathbf{L}^*)\mathbf{y}^*$$

where **I** is an identity matrix of size $nT \times nT$ and **L**^{*} is the projection matrix of the GWPR model of size $nT \times nT$.

The test statistic F_{GOF} follows the distribution $F_{db_1;db_2}$ where critical area H_0 is rejected at the significance level α if $F_{GOF} > F_{db_1;db_2}$

where $db_1 = \frac{\delta_1^2}{\delta^2}$ with $\delta_i = tr([(\mathbf{I} - \mathbf{L}^*)^{\mathbf{T}}(\mathbf{I} - \mathbf{L}^*)]^i)$, i = 1, 2 and $db_2 = (nT - p - 1)$ or if $p_{value} < \alpha$.

GWPR model parameter significance test

The significance test of the parameters in the GWPR model was carried out partially with the aim of knowing which predictor variables had an effect on the *i* th location, where i = 1, 2, 3, ..., n [21,6]. The hypothesis for testing the GWPR model parameters is given as follows.

$$H_0: \beta_k(u_i, v_i) = 0, i = 1, 2, 3, \dots, n; k = 1, 2, 3, \dots, p$$

$$H_1: \beta_k(u_i, v_i) \neq 0, \ i = 1, 2, 3, \dots, n; \ k = 1, 2, 3, \dots, p$$

The test statistic used in testing the significance of the GWPR parameter is shown in Eq. (11) below.

$$T_{GWPR} = \frac{\beta_k(u_i, v_i)}{\hat{\sigma}\sqrt{C_{kk}}}$$
(11)

where \mathbf{C}_{kk} is the k-th diagonal element of matriks $\mathbf{C}_{it}^T \mathbf{C}_{it}$ with $\mathbf{C}_{it} = (\mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_i, v_i) \mathbf{X}^{*\mathrm{T}})^{-1} \mathbf{X}^{*\mathrm{T}} \mathbf{W}(u_i, v_i)$ and $\hat{\sigma} = \sqrt{\frac{JKG(H_1)}{\delta_1}}$. The test statistic *T* follows the distribution *t* with predictor degree $\frac{\delta_1^2}{\delta_2}$ where $\delta_i = tr([(\mathbf{I} - \mathbf{L}^*)^{\mathbf{T}}(\mathbf{I} - \mathbf{L}^*)]^i)$, i = 1, 2. Critical area with H_0 is rejected at the significance level α if $|T_{GWPR}| \ge t$ $(\frac{\alpha}{2}; \frac{\delta_1^2}{\delta_2})$ or if $p_{value} < \alpha$.

Spatial effect testing

Spatial effect testing aims to determine the observed data identified spatial heterogeneity. The statistical test used is the Gleiser test, with the following hypothesis formulation.

 $H_0:\sigma_{1,1}^2 = \sigma_{2,1}^2 = \dots = \sigma_{n,T}^2 = \sigma^2$ (Homoscedasticity) $H_1:$ At least one $\sigma_{i,t}^2 \neq \sigma^2$, $i = 1, 2, \dots, n; t = 1, 2, \dots, T$ (Heteroscedasticity) Spasial effect Gleiser test statistic is written in Eq. (12).

$$F_{Glejser} = \frac{\left(\hat{\boldsymbol{\varphi}}^{\mathrm{T}} \mathbf{X}^{*\mathrm{T}} \boldsymbol{\varepsilon}^{*} - n(\boldsymbol{\varepsilon}^{*})^{2}\right) / p}{\left(\boldsymbol{\varepsilon}^{*\mathrm{T}} \boldsymbol{\varepsilon}^{*} - \hat{\boldsymbol{\varphi}}^{\mathrm{T}} \mathbf{X}^{*\mathrm{T}} \boldsymbol{\varepsilon}^{*}\right) / (nT - n - p)}$$
(12)

The test statistic $F_{Glejser}$ follows the distribution $F_{(p; nT-n-p)}$ where n is the number of observation points, T is the amount of observation time and p is the number of predictor variables. The critical area of the Glejser test where H_0 is rejected at the significance level α if $F_{Glejser} > F_{(\alpha; p; nT-n-p)}$ or if $p_{value} < \alpha$ [6,18,22].

Determination of the spatial weighting function in the GWPR model

The GWPR model considers the location aspect of each observed data. Spatial weighting is used to estimate the GWPR model. The study used geographic weighting of the Gaussian kernel function, the Bisquare kernel function and the tricube kernel function [1,23,24].

(1) The Gaussian kernel function is stated in Eq. (13).

$$w_{ij} = exp\left(-\frac{1}{2}\left(\frac{d_{ij}}{b}\right)^2\right), if \ d_{ij} \le b$$
(13)

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(2) The bisquare kernel function is expressed in Eq. (14).

$$w_{ij} = \begin{cases} \left(1 - \left(\frac{d_{ij}}{b}\right)^2\right)^2 & , if d_{ij} \le b \\ 0 & , otherwise \end{cases}$$
(14)

(3) The tricube kernel function is stated in the Eq. (15).

$$w_{ij} = \begin{cases} \left(1 - \left(\frac{d_{ij}}{b}\right)^3\right)^3 & , ifd_{ij} \le b \\ 0 & , otherwise \end{cases}$$
(15)

where w_{ij} is a weighting function between location *i* and location *j*, and d_{ij} is the distance between location i and location j obtained from the Euclidean distance which can be calculated by the Eq. (16).

$$d_{ij} = \sqrt{\left(u_i - u_j\right)^2 + \left(v_i - v_j\right)^2}$$
(16)

b is the bandwidth for the GWPR model estimator at the i th location. The method for selecting the optimum bandwidth is Cross Validation (CV) which is shown in Eq. (17) below.

$$CV = \sum_{i=1}^{n} \left[y_i - \hat{y}_{\neq i}(b) \right]^2$$
(17)

with $\hat{y}_{\neq i}(b)$ is the estimator value y_i where the *i* th location observation is omitted from the estimation process [23].

GWPR model goodness-of-fit testing

The goodness and accuracy of the model will be calculated using the coefficient of determination and Root Mean Square Error (RMSE).

Coefficient of determination

The coefficient of determination includes a measure of the goodness of the model which shows the level of accuracy of the model. The coefficient of determination indicates how much influence the predictor variable has on the response variable. The higher the value of the coefficient of determination, indicating that the resulting model is getting better. The coefficient of determination is expressed by the following Eq. (18).

$$R^{2} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T} \left(\hat{y}_{it}^{*} - \bar{y}^{*} \right)^{2}}{\sum_{i=1}^{n} \sum_{t=1}^{T} \left(y_{it}^{*} - \bar{y}^{*} \right)^{2}}$$
(18)

with the value of the coefficient of determination is in the range $0 \le R \le 1$ [19,25].

Root mean square error (RMSE)

Root Mean Square Error (RMSE) is a tool to select a model based on the error of the estimation results. The error indicates how much the estimation result differs from the value to be estimated. This value is used to determine the best model [15,26]. The RMSE formula can be expressed by the following Eq. (19).

$$RMSE = \sqrt{\frac{1}{nT} \sum_{i=1}^{n} \sum_{t=1}^{T} \left(y_{it}^* - \hat{y}_{it}^* \right)^2}$$
(19)

where *n* is the number of observation locations and *T* is the number of observation times [20,27,28].

Research methods

Research data and variables

Research variable and observational data is described in Table 1.

Description of research Variabel and data sources.

Variable	Symbol	Variable Description	Unit	Source	Scale
Respond	у	Food Security Index	Index	National Food Agency [29]	34 Provinces in Indonesia Year 2019 - 2022
Predictor	<i>x</i> ₁	Rice Production	Ton(s)	Ministry of Agriculture, Statistics Indonesia, and Department of Agriculture throughout Indonesia [29]	34 Provinces in Indonesia Year 2019 - 2022
	<i>x</i> ₂	Red Chili Production	Ton(s)	Statistics Indonesia and Directorate General of Horticulture [30]	34 Provinces in Indonesia Year 2019 - 2022
	<i>x</i> ₃	Shallot Production	Ton(s)	Statistics Indonesia and Directorate General of Horticulture [30]	34 Provinces in Indonesia Year 2019 - 2022
	x_4	Palm Oil Production	Ton(s)	Directorate General of Plantations [29,30]	34 Provinces in Indonesia Year 2019 - 2022
	<i>x</i> ₅	Beef Production	Ton(s)	Directorate General of Livestock and Animal Health [29,30]	34 Provinces in Indonesia Year 2019 - 2022
	<i>x</i> ₆	Production of chicken meat	Ton(s)	Directorate General of Livestock and Animal Health [29,30]	34 Provinces in Indonesia Year 2019 - 2022
	<i>x</i> ₇	Expenditure For Food	Rupiah(s)	National Socioeconomic Survey on March by Statistics Indonesia [29,30]	34 Provinces in Indonesia Year 2019 - 2022
	x_8	Percentage of Poor Population	Percent	National Socioeconomic Survey on March by Statistics Indonesia [29,30]	34 Provinces in Indonesia Year 2019 - 2022
	<i>x</i> ₉	Percentage of Population According to Inadequate Consumption Status	Percent	National Socioeconomic Survey on March by Statistics Indonesia [29,30]	34 Provinces in Indonesia Year 2019 - 2022
	x_{10}	Percentage of Population with Food Insecurity	Percent	National Socioeconomic Survey on March by Statistics Indonesia [29–31]	34 Provinces in Indonesia Year 2019 - 2022

Research stages

The data analysis technique used is descriptive statistical analysis, Fixed Effect Model (FEM) modeling stages and Geographically Weighted Panel Regression (GWPR) modeling. The program used for data analysis is the R program. The stages of data analysis are as follows:

- (1) Descriptive Statistical Analysis.
- (2) Spatial Mapping based on Variable characteristics and observational data.
- (3) Multicollinearity Detection.
- (4) Transform data into *demean* data with the *within estimator* according to the equation $y_{it}^* = (y_{it} \bar{y}_i)$, $x_{itk}^* = (x_{itk} \bar{x}_{ik})$, and $\varepsilon_{it}^* = (\varepsilon_{it} \overline{\varepsilon_i})$
- (5) Panel regression modeling uses the Fixed Effect Model.
- (6) GWPR Modelling.
- (7) Calculate the Euclidean distance between observation locations based on geographic location.
- (8) Determine the optimum bandwidth based on the CV value of the Gaussian kernel function, the Bisquare kernel function and the tricube kernel function for each observation location.
- (9) GWPR Model Parameter Estimation.
- (10) Testing the suitability of the GWPR model.
- (11) Testing the significance of the parameters of the GWPR model.
- (12) Determine the accuracy value of the model based on the coefficient of determination in Eq. (18) and RMSE in Eq. (19).

Results and discussion

Characteristics of Indonesia's food security index

The condition of the Food Security Index (Indeks Ketahanan Pangan/IKP) in 34 Provinces in Indonesia from 2020 to 2022 is presented by the graph in Fig. 1. In 2022 the highest index was achieved by the Province of Bali with a value of 83.82 and the lowest index was owned by the province of Papua with index value of 35.48 [8]. The distribution of the spatial mapping of the index for 34 Provinces is shown in Fig. 2.

Statistical description of food security indices and predictor variables

Descriptive statistical data on the research variables consist of the average, minimum value, maximum value, and standard deviation. The results of descriptive statistical calculations are presented in Table 2.

Based on Table 2, it is known that the average Food Security Index in 34 provinces in Indonesia in 2020 is 66.84 with a standard deviation of 13.18. The lowest Food Security Index occurred in the province of Papua at 25.13 and the highest occurred in the

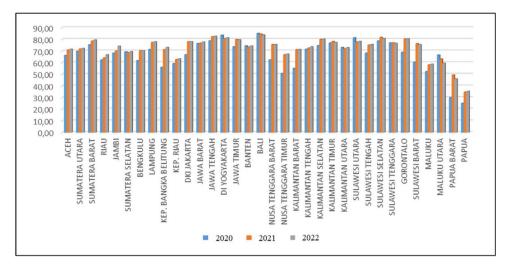


Fig. 1. Food security index in 34 provinces in Indonesia 2020–2022.

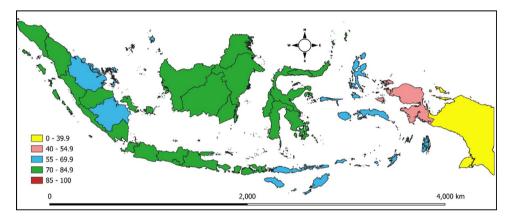


Fig. 2. Spatial distribution of Indonesia's food security index for 2020-2022.

province of Bali at 85.15. The average Food Security Index in 34 provinces in Indonesia in 2021 has increased to 72.11 with a standard deviation of 9.95, compared to 2019's. The lowest Food Security Index occurred in the province of Papua at 34.79 and the highest occurred in the province of Bali at 84.54. The average Food Security Index in 34 provinces in Indonesia in 2022 has increased to 72.43 with a standard deviation of 10.15, compared to 2020's. The lowest Food Security Index occurred in the province of Papua at 35.48 and the highest occurred in the province of Bali at 83.82.

The Food Security Index and variables affecting 34 provinces in Indonesia in 2021 and 2022 can be described through a distribution map which can be seen in Figs. 3–5 below.

Panel regression models

The FEM panel regression model with the within estimator for food security index data with the influencing predictor variable x, is given in Eq. (20). After the multicollinearity test process was carried out, there were two variables that detected multicollinearity, so that the model to be formed consisted of 8 predictor variables.

$$y_{ii}^{*} = \beta_1 x_{ii1}^{*} + \beta_2 x_{ii2}^{*} + \beta_3 x_{ii3}^{*} + \beta_4 x_{ii4}^{*} + \beta_5 x_{ii5}^{*} + \beta_6 x_{ii6}^{*} + \beta_7 x_{ii7}^{*} + \beta_8 x_{ii8}^{*} + \epsilon_{ii}^{*}; i = 1, 2, \dots 34; t = 1, 2, 3$$
(20)

Estimation of the FEM panel regression parameters and variable significant statements based on the T test are given in Table 3. The panel regression model with the resulting parameter estimator values is written in Eq. (21).

$$\hat{y}_{it}^* = 4,2842 \times 10^{-5} x_{it1}^* + 3,7692 \times 10^{-5} x_{it2}^* + 2,0883 \times 10^{-6} x_{it3}^* + 2,4960 \times 10^{-4} x_{it4}^* + 2,0538 \times 10^{-5} x_{it5}^* - 5,4685 x_{it6}^* + 0,8169 x_{it7}^* + 0,1284 x_{it8}^*; i = 1,2,...34; t = 1,2,3$$
(21)

Based on Table 3, it is found that variable RPM (x_5) , PPM (x_6) , and PPSKKP (x_7) have effects on the Food Security Index in Indonesia and for variables PCM (x_1) , PBM (x_2) , PKS (x_3) , PDARP (x_4) , PPKPSB (x_8) have no effect on the Food Security Index in

Table 2

Descriptive statistics of research data.

Variable	Year	Mean	Min	Max	Standard Deviation
Food Security Index (y)	2020	66,84	25,13	85,15	13,18
	2021	72,11	34,79	84,54	9,95
	2022	72,43	35,48	83,82	10,15
Rice Production (x_1)	2020	1.606.001	1.151	9.655.654	2.674.447
	2021	1.607.329	853	9.944.538	2.682.974
	2022	1.600.450	855	9.789.588	2.699.403
Red Chili Production (x_2)	2020	35.718	1,00	263.949	60.527,55
	2021	37.182	1,00	266.067	63.131,96
	2022	40.017	1,00	343.067	73.779,46
Shallot Production (x_3)	2020	46.477,90	1,00	481.890	112.454,50
	2021	53.395,00	1,00	611.165	133.370,90
	2022	58.958,60	2,00	564.255	135.779,30
Palm Oil Production (\mathbf{x}_4)	2020	1.385.890	1,00	9.513.208	2.411.896
	2021	1.375.459	1,00	9.887.675	2.447.530
	2022	1.378.073	1,00	8.785.327	2.402.367
Beef Production (x_5)	2020	13.859	583	103.292	22.977,11
	2021	13.336	685	91.028	21.270,09
	2022	12.876	627	93.303	19.796,28
Production of Laying Chicken Meat	2020	2.943,30	6,00	46.040	4.464,35
x ₆)	2021	4.493,00	10,00	37.926	8.386,89
0	2022	4.301,90	10,00	38.874	8.177,78
Average Monthly Food Expenditure	2020	565.670	429.471	877.538	136.371,20
per capita (\mathbf{x}_7)	2021	615.954	442.700	944.687	114.871,20
	2022	634.229	453.031	923.933	107.170,30
Percentage of Poor Population (x_8)	2020	10,46	3,47	27,53	5,68
U I U	2021	10,43	3,78	26,64	5,44
	2022	10,76	4,53	26,86	5,40
Percentage of Population According	2020	10,23	1,43	38,21	9,13
o Food Consumption Insufficiency	2021	11,02	1,94	35,55	8,55
Status (\mathbf{x}_9)	2022	11,32	1,78	37,37	8,03
Percentage of Population with Food	2020	6,88	2,68	14,99	3,05
nsecurity (\mathbf{x}_{10})	2021	6,15	1,84	15,46	3,07
	2022	6,04	2,87	15,31	2,73

Table 3

Estimation value and significance test of the parameters of the FEM panel regression model.

Variable	Parameter	Parameter Estimated Value	$ T_{FEM} $	P_{value}	Decision
PCM (\mathbf{x}_1)	β_1	$4,2842 \times 10^{-5}$	0,7728	0,4427	Not Significant
PBM (\mathbf{x}_2)	β_2	$3,7692 \times 10^{-5}$	1,1071	0,2727	Not Significant
PKS (x_3)	β_3	$2,0883 \times 10^{-6}$	0,6950	0,4897	Not Significant
PDARP (\mathbf{x}_4)	β_4	$2,4960 \times 10^{-4}$	1,4818	0,1436	Not Significant
RPM (x_5)	β_5	$2,0538 \times 10^{-5}$	2,4779	0,0160	Significant
PPM (\mathbf{x}_6)	β_6	-5,4685	2,8023	0,0068	Significant
PPSKKP (x_7)	β_7	0,8169	2,5859	0,0122	Significant
PPKPSB (x_8)	β_8	0,1284	0,2630	0,7934	Not Significant

Table 4 Spatial effect testing.				
F_{count} $F_{(0,05;8;60)}$		Pvalue	Decision	
2,3093	2,0970	0,0314	Heteroscedasticity Spatial	

Indonesia. Since several variables were declared insignificant, the spatial effect testing stage would be carried out to proceed to the spatio-temporal GWPR model.

Spatial effect testing

Spatial effect testing was carried out to determine the error variance for all locations where homoscedasticity or heteroscedasticity were observed. The results of the spatial effects testing analysis are shown in Table 4. Spatial effect hypothesis formulation

$$H_0: \sigma_{1,1}^2 = \sigma_{2,1}^2 = \dots = \sigma_{34,3}^2 = \sigma^2$$
 (Homoscedasticity)
 H_1 : At least one $\sigma_{i,t}^2 \neq \sigma^2; i = 1, 2, \dots 34; t = 1, 2, 3$ (Heteroscedasticity)
Spatial effect statistical test

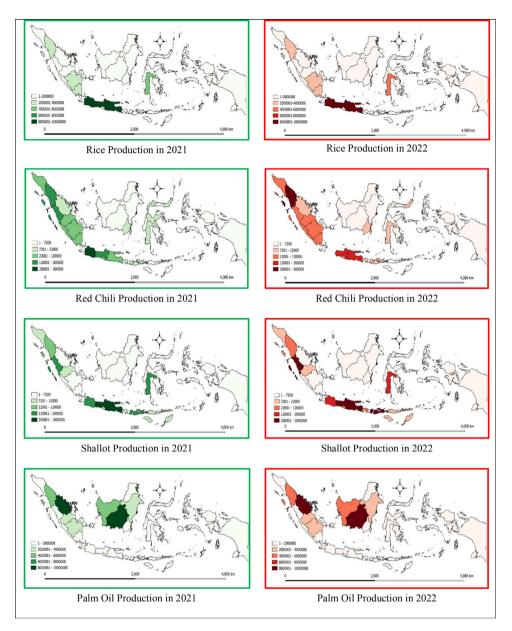


Fig. 3. Spatial distribution map of rice production variables, red chili production, shallot production, palm oil production in 2021-2022.

Based on Table 4, the error variance is not constant at all observation locations. Thus, the assumption of FEM homoscedasticity is not fulfilled. This causes the resulting FEM model to be less suitable for modeling the Food Security Index in 34 provinces in Indonesia. Then it will be continued with the Geographically Weighted Panel Regression (GWPR) model.

Geographically weighted panel regression model (GWPR)

The general model of GWPR at the i th location at the t-th time for the Food Security Index data with 8 predictor variables is written in Eq. (22).

$$\hat{y}_{it}^{*} = \beta_{1}(u_{i}, v_{i})x_{it1}^{*} + \beta_{2}(u_{i}, v_{i})x_{it2}^{*} + \beta_{3}(u_{i}, v_{i})x_{it3}^{*} + \beta_{4}(u_{i}, v_{i})x_{it4}^{*} + \beta_{5}(u_{i}, v_{i})x_{it5}^{*} + \beta_{6}(u_{i}, v_{i})x_{it6}^{*} + \beta_{7}(u_{i}, v_{i})x_{it7}^{*} + \beta_{8}(u_{i}, v_{i})x_{it8}^{*} + \varepsilon_{it}^{*}; i = 1, 2, ... 34; t = 1, 2, 3$$
(22)

(23)

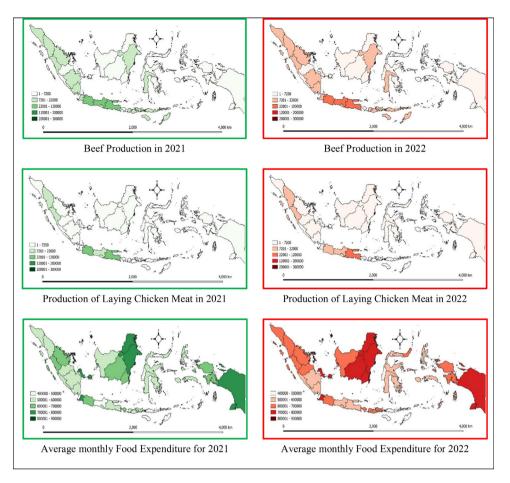


Fig. 4. Map of spatial distribution of beef production variables, laying hens, average per capita food expenditure for 2021–2022.

Optimum bandwidth and CV value of weighting function.					
Kernel	Bandwidth	CV Value			
Kernel Gaussian Function Kernel Bisquare Function Kernel Tricube Function	1,755,476 13,51,204 13,51,221	561,2135 1.119,3610 1.152,2390			

Determination geographic weighted function and estimation of spatio temporal model geographically weighted panel regression using weighted least square

The step taken before estimating the parameters of the GWPR model is to calculate the Euclidean distance between observation locations using Eq. (16), then proceed to calculate the weighting function according to Eqs. (13)–(15). Determining the optimum bandwidth at each observation location uses Cross Validation (CV) based on Eq. (17). Optimum bandwidth and CV values for the weighting function can be seen in Table 5 and bandwidth exploration for each region in Fig. 6.

The bandwidth and CV values presented in Table 5 find the best weighting function to be the Gaussian weighting function. The results of analysis and optimal bandwidth, resulting in the GWPR model can be formulated in Eq. (23). Given the following GWPR model for the province of South Sulawesi.

$$\hat{y}_{27t}^* = -0,00128x_{1t1}^* + 0,00016x_{1t2}^* - 0,000003x_{1t3}^* + 0,00382x_{1t4}^* + 0,00001x_{1t5}^* - 11,92535x_{1t6}^* + 4,23953x_{1t7}^* + 1,81443x_{1t8}^*; t = 1,2,3$$

The next stage of analysis is testing the suitability of the model and testing the significance of parameters.

GWPR model fit test

The hypothesis of GWPR model fit test is as follows.

Table 5

 H_0 : $\beta_k(u_i, v_i) = \beta_k, \ k = 1, 2, \dots, 8; i = 1, 2, \dots, 34$

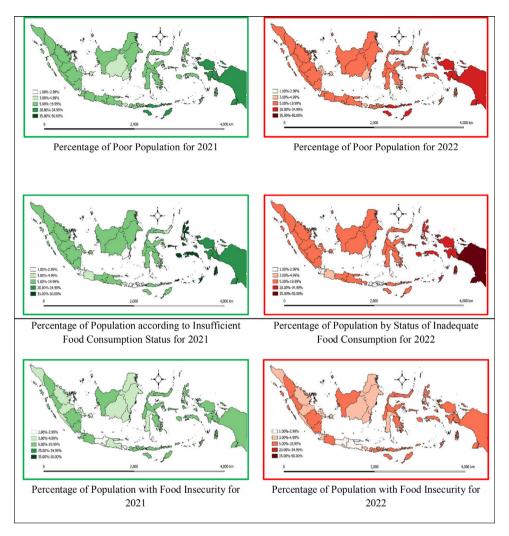


Fig. 5. Map of the spatial distribution of the percentage of the population according to the status of inadequate food consumption, the percentage of the population with food insecurity, the percentage of the Poor in Indonesia, 2020–2021.

Table 6

Model fit test.

F _{count}	F _(0,05;8;93)	Pvalue	Decision
13,48,100	2,03,950	$8,7785 \times 10^{-13}$	H_0 is rejected

(There is no difference between the panel regression model and the GWPR model)

*H*₁ : At least one $\beta_k(u_i, v_i) \neq \beta_k$, k = 1, 2, ..., 8; i = 1, 2, ..., 34

(There is a difference between the panel regression model and the GWPR model)

The value of the test statistic F_{GOF} and p_{value} can be seen in Table 6 below.

Based on Table 6, we obtained that $F_{count} = 13,4810 > F_{(0,05;8;93)} = 2,03950$ or $p_{value} = 8,7785 \times 10^{-13} < \alpha = 0,05$, then we decided to refuse H_0 so it can be concluded that there is a difference between the panel regression model and the GWPR model. Furthermore, testing the significance of model parameters spatially will be carried out.

Partial significance test of GWPR parameters

The hypothesis of testing the significance of the parameters of the GWPR model is as follows.

 H_0 : $\beta_k = 0; k = 1, 2, \dots 8$

(Variable x_k has no effect on the variable Food Security Index in Indonesia)

 H_1 : $\beta_k \neq 0, \ k = 1, 2, \dots, 8$

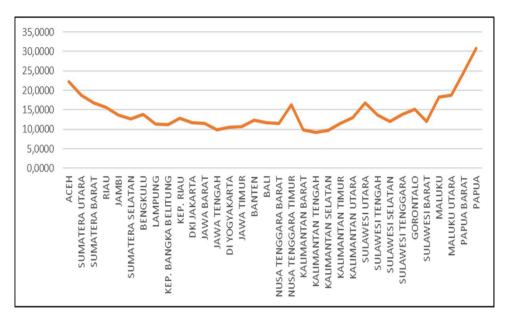


Fig. 6. Bandwidth exploration for each region.

Table 7
Partial significance test of the GWPR model.

Location (i)	Parameter	Estimation Value	Pvalue	
South Sulawesi (27)	β_1	-0,00,128	0,10,900	
	β_2	0,00,016	0,02,900*	
	β_3	0,00,000	0,63,800	
	β_4	0,00,382	0,00,500*	
	β_5	0,00,001	0,81,900	
	β_6	-11,92,535	0,00,500*	
	β_7	4,23,953	0,00,000*	
	β_8	1,81,443	0,03,000*	

(Variable x_k has effect on the variable Food Security Index in Indonesia)

Test statistics based on T_{GWPR} which is given by Eq. (11). The results of testing the significance of the parameters of the GWPR model partially for one of the observation locations in the province of South Sulawesi can be seen in Table 7 below.

Based on Table 7, the results are obtained that β_2 , β_4 , β_6 , β_7 , β_8 have $p_{value} < \alpha = 0,05$, then decided to refuse H_0 so that it can be seen that the variables that influence the Food Security Index in the province of South Sulawesi are Shallot Production, Laying Chicken Meat Production, Percentage of Poor People, Percentage of Population according to Inadequate Food Consumption Status, and Percentage of Population with Moderate or Severe Food Insecurity, Experience Scale Food Insecurity.

Province classification and mapping based on significant variables

The GWPR model classification based on significant variables is divided into 16 groups which can be seen in Table 8 below. The GWPR model classification mapping can be seen in Fig. 7 below.

GWPR model goodness-of-fit and accuracy measures

The measure of the goodness-of-fit and accuracy of the model used in this study is the coefficient of determination and *Root Mean Square Error* (RMSE) the results of which can be seen in Table 9 below.

Based on Table 9, the FEM Panel Regression determination coefficient of 24.02 % is obtained, which indicates that FEM can explain the diversity of the Food Security Index in 34 provinces in Indonesia by 24.02 % with an RMSE value of 4.28790. The coefficient of determination for the GWPR model is 92.78 %, which indicates that the GWPR model can explain the diversity of the Food Security Index in 34 provinces in Indonesia, which is 92.78 % with an RMSE value of 3.41..

Based on the results above, it can be seen that the coefficient of determination of the GWPR model is greater than the coefficient of determination of the FEM and the RMSE value generated by the GWPR model is smaller than the FEM so it can be concluded that the GWPR model is better at modeling the Food Security Index.

Table 8

GWPR model classification based on influential variables.

Group	Influential Variables	Province
1	-	Aceh, North Sumatera, West Sumatera, Riau, Central Java, DI Yogyakarta, Central Kalimantan, South Kalimantan, North
		Kalimantan, West Papua
2	<i>x</i> ₁	North Maluku
3	x5	East Java and Maluku
4	x ₆	DKI Jakarta, West Java, Banten, and North Sulawesi
5	x ₇	Lampung and Southeast Sulawesi
6	x_8	Bengkulu and East Nusa Tenggara
7	x_4 dan x_7	Jambi and Riau Islands
8	x_5 dan x_8	West Nusa Tenggara
9	$x_6 \operatorname{dan} x_7$	East Kalimantan and Gorontalo
10	x_7 dan x_8	South Sumatera
11	$x_2, x_6, dan x_7$	Central Sulawesi
12	$x_4, x_7, dan x_8$	Bangka Belitung Islands
13	$x_5, x_6, dan x_8$	Bali
14	$x_2, x_4, x_6, dan x_7$	West Sulawesi
15	$x_4, x_5, x_6, dan x_7$	West Kalimantan
16	$x_2, x_4, x_6, x_7 \text{ dan } x_8$	South Sulawesi

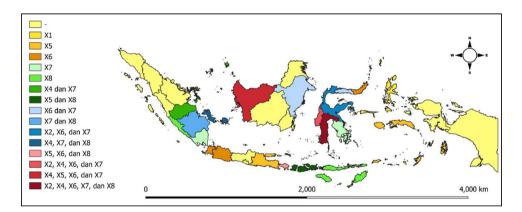




Table 9

Model goodness-of-fit and accuracy measures.

Model	\mathbf{R}^2 Value	RMSE
FEM Panel Regression	24,02	4,28,790
GWPR	92,78	3,41,044

Conclusion

- (1) Determination of the best geographic weighting in the GWPR model for modeling the national food security index with optimal bandwidth and CV values is the Gaussian weighting function.
- (2) GWPR model parameter estimation using the Weighted Least Square optimization method obtained a good model of 92.78 % with a Root mean Square Error value of 3.41. The GWPR model is better than the FEM Panel Regression model.
- (3) The factors that influence the national food security index are red chili production, red onion production. oil palm production, laying hen meat production, average per capita food expenditure per month, percentage of poor people, percentage of population according to insufficient food consumption status, and percentage of population with food insecurity.
- (4) West Sulawesi, West Kalimantan and South Sulawesi are the provinces most affected by significant variables, this is in accordance with the results of the analysis and classification of the GWPR model in Table 8 and Fig. 7. The results of this study are in accordance with the conditions of the regions which are the largest food suppliers in Indonesia.

Ethics statement

The dependent variable used in this study is the Food Security Index in Indonesia 2019–2021. The predictor variables used in this study are Rice Production, Red Chili Production, Shallot Production, Palm Oil Production, Beef Production, Production of Laying

Chicken Meat, Average Monthly Food Expenditure per capita, Percentage of Poor Population, Percentage of Population According to Food Consumption Insufficiency Status and Percentage of Population with Food Insecurity. The data was obtained through the publication of the Agricultural Data Center and Information System of the Secretariat General of the Ministry of Agriculture, 2(1) of 2022.

Data availability

Data will be made available on request.

Institutional review board statement

Not applicable.

Informed consent statement

Not applicable.

Declaration of competing interest

The authors devlare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Sifriyani: Conceptualization, Methodology, Formal analysis, Data curation, Investigation, Resources, Writing – original draft, Project administration. **I Nyoman Budiantara:** Validation, Writing – review & editing, Supervision. **M. Fariz Fadillah Mardianto:** Software, Visualization, Data curation. **Asnita:** Investigation, Resources, Writing – original draft, Project administration.

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