



# Creativity in mathematics performance: The role of divergent and convergent thinking

Isabelle C. deVink\* , Robin H. Willemsen, Ard. W. Lazonder   
and Evelyn H. Kroesbergen 

Behavioural Science Institute, Radboud University, Nijmegen, The Netherlands

**Background.** Creativity requires both divergent and convergent thinking. Previous research established that *divergent* thinking relates to mathematics performance, but generally ignored the role of *convergent* thinking and, hence, leaves it unclear how both might interact when children work on mathematical tasks. This study addressed this paucity in the research literature, with the goal of improving our understanding of the role of creative thinking in primary school mathematics.

**Aims.** This study examined how divergent and convergent thinking contribute to mathematics performance, both directly and jointly, on single- and multiple-solution tasks.

**Sample.** The study was conducted with 229 Dutch fifth graders of 12 primary schools.

**Method.** Divergent and convergent thinking were measured with a visual and verbal task. Path analysis was used including verbal and visual divergent and convergent thinking tasks in relation to single- and multiple-solution mathematics task performance. Working memory was included as a covariate.

**Results.** Verbal convergent thinking positively predicted single- and multiple-solution task performance. Verbal divergent and convergent thinking interacted in relation to single-solution task performance, while visual divergent and convergent thinking interacted in relation to multiple-solution task performance.

**Conclusions.** Children's mathematics performance mainly relies on convergent thinking. The role of divergent thinking is twofold: it complements convergent thinking on multiple-solution tasks and compensates convergent thinking on single-solution tasks.

The ability to solve mathematical problems lies at the heart of primary school mathematics education (Schoenfeld, 2014; Sriraman & English, 2010). When students solve a mathematics problem, creative thinking allows them to connect problem elements and find different ways to arrive at a solution (Hadamard, 1996; Mann, 2005). Therefore, creativity is pivotal to teach students. However, teachers struggle to incorporate creativity in mathematics education (Kaufman & Baer, 2004). To effectively support teachers to incorporate creativity, it is crucial that we gain more insight into creativity by investigating

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\*Correspondence should be addressed to Isabelle C. de Vink, Behavioural Science Institute, Radboud University, Montessorilaan 3, P.O. Box 9104, 6500 HE Nijmegen, The Netherlands (email: [isabelle.devink@ru.nl](mailto:isabelle.devink@ru.nl)).

its constituent skills. Creativity can be defined as the production of novel and useful products in a social context (Plucker, Beghetto, & Dow, 2004) and involves two modes of thinking: divergent and convergent thinking (Brophy, 2001; Guilford, 1973). Research found positive associations between divergent thinking and mathematics performance (Jeon, Moon, & French, 2011; Kroesbergen & Schoevers, 2017; Leikin, 2009). Convergent thinking is also argued to aid mathematics performance (Leikin, Koichu, & Berman, 2009; Tabach & Levenson, 2018; Tan & Sriraman, 2017), but empirical evidence of a relation is scarce. This study examined how divergent and convergent thinking are related to mathematics performance, independently and in interaction, in a sample of fifth-grade students.

The theoretical foundation of divergent and convergent thinking is rooted in the work of Guilford (1973), who postulated that both modes of thought are necessary for problem-solving. Divergent thinking is defined as the generation of problem definitions and solutions – that is, from a specific starting point multiple approaches to a problem are generated. Convergent thinking concerns the selection and development of ideas in working towards the best possible solution to the problem (Brophy, 2001). Research supports a conceptual distinction between divergent and convergent thinking, as factor analyses point to two different factors with low intercorrelations (Barbot, Besançon, & Lubart, 2016; Lee & Theriault, 2013; Storme et al., 2015). Divergent and convergent thinking help to solve a mathematical problem because the interplay between the two allows children to implement the most suitable solution from a range of options, which is especially useful when no learned solution to a problem is available (Assmus & Fritzlar, 2018; Leikin, 2009; Mann, 2005; Tabach & Levenson, 2018). Divergent and convergent thinking can thus be theoretically associated with problem-solving, including mathematical problem-solving (Brophy, 2001; Guilford, 1973). Previous research on creativity and mathematics has mainly focussed on divergent thinking, because novel ideas and solutions to problems are thought to stem from divergent thinking while convergent thinking is generally associated with more conventional ideas and solutions (Cropley, 2006). However, creativity theory emphasizes that in order to generate ideas that are not merely novel, but also useful (following Plucker et al.'s definition of creativity) both divergent and convergent thinking are important. In order to produce an effective creative idea to solve a mathematical problem, it seems pivotal that the ideas generated through divergent thinking are also evaluated and selected via convergent thinking (Brophy, 2001; Cropley, 2006).

Children's mathematics performance can be assessed by different types of tasks. A coarse-grained distinction can be made between tasks with problems that require one solution (single-solution task; SST) or with problems that require multiple solutions (multiple-solution task; MST). While SSTs are binary-scored for correctness, performance on MSTs is usually evaluated in terms of fluency (number of correct solutions), flexibility (diversity of solutions), and originality (novelty of solutions) (Assmus & Fritzlar, 2018). Former research indicated that creative thinking is more strongly related to MST than SST performance (Schoevers, 2019). However, little is known about differences in divergent and convergent thinking. Therefore, it is interesting to take a closer look at how divergent and convergent thinking can be applied in both types of tasks.

### ***Mechanisms of divergent and convergent thinking***

Based on leading creativity theory (Brophy, 2001; Guilford, 1973), we expect divergent and convergent thinking to be beneficial to mathematics performance because both

modes of thinking enable children to solve a mathematical problem by generating and evaluating ideas. Indeed, empirical evidence confirms that divergent thinking is positively related to performance on SSTs and MSTs in fourth to eighth graders (Jeon et al., 2011; Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2013; Schoevers, Kroesbergen, & Kattou, 2018). This corroborates the theory that divergent thinking facilitates finding suitable approaches to a problem (Leikin, 2013). Research has shown that considering a problem from different angles helps a student to pose related problems or questions and apply different solutions and techniques, benefitting mathematical problem-solving (Van Harpen & Sriraman, 2013). Therefore, divergent thinking can contribute to the production of different and original solutions on MSTs (Assmus & Fritzlar, 2018; Leikin, 2009). In solving SSTs, divergent thinking helps to find a single correct answer to a problem by applying different strategies, which is especially useful when there is no available learned solution to a problem (Leikin, 2009). Specifically, if a student has no access to a conventional solution to a problem (as taught in textbooks), divergent thinking facilitates finding different possible strategies.

As regards convergent thinking, theory supports a relationship with mathematics performance, because it can help students to evaluate ideas and find a good solution to the problem (Brophy, 2001; Cropley, 2006; Guilford, 1973). However, convergent thinking has received little attention in primary school mathematics research. Following Leikin et al. (2009) and Tan and Sriraman (2017), we argue that convergent thinking is beneficial to mathematics performance as it facilitates evaluation of the used strategies and mathematical reasoning. Besides, convergent thinking is closely related to the construction of mathematical knowledge and fact retrieval and automatization processes (Tabach & Levenson, 2018). Specifically, convergent thinking allows students to connect different elements and choose a fitting strategy to solve the problem. On an MST, convergent thinking can be used to apply mathematical knowledge, resulting in the selection of an appropriate strategy or solution (Assmus & Fritzlar, 2018; Tabach & Levenson, 2018). For an SST, we argue that convergent thinking is actually more important than divergent thinking as it facilitates a more narrow thinking towards one solution by applying memorized knowledge (Cropley, 2006; Tan & Sriraman, 2017). For example, to find the answer to  $100 \div \frac{1}{4}$ , it helps if one uses convergent thinking to apply the knowledge that dividing by a fraction means multiplying by the reciprocal. Thus, we expect positive associations between divergent and convergent thinking and mathematics performance on SSTs and MSTs, but expect that convergent thinking will be more strongly associated with SST performance than divergent thinking.

### ***Interaction between divergent thinking and convergent thinking***

There is reason to assume that divergent and convergent thinking affect mathematics performance both independently and together. Theoretically, the two might interact in different ways. For example, Brophy (2001) argued that divergent and convergent thinking are complementary because the most successful creative output is produced by children who can generate many ideas *and* evaluate them well. Alternatively, divergent and convergent thinking might compensate each other such that a strength in divergent thinking can offset weaknesses in convergent thinking and vice versa. To illustrate, students who find it hard to evaluate ideas might make up for this deficiency by generating many ideas to create creative output (Cropley, 2006).

Based on these theories, we argue that divergent and convergent thinking interact in relation to mathematics performance and that these interactive mechanisms are different

for SSTs and MSTs. Specifically, we expect that divergent and convergent thinking complement each other while solving MSTs, while compensating one another on SSTs. On an MST, both divergent and convergent thinking are necessary because these tasks require children to produce and evaluate different solutions (Tabach & Levenson, 2018). Thus, children who are able to think of many solutions but cannot evaluate these, or can evaluate their solutions well but are not able to generate different ones, are expected to perform worse on an MST than children who are good at both. In contrast, for an SST we hypothesize that finding different possible strategies (i.e., divergent thinking) is less important than evaluative thinking and applying memorized knowledge (i.e., convergent thinking) to come to one correct answer (Cromptley, 2006; Tan & Siraman, 2017). However, when there is no learned solution, divergent thinking helps to generate different solution strategies (Leikin, 2009). Thus, children who have trouble applying memorized knowledge and evaluating their thinking might compensate this weakness by finding different strategies (i.e., divergent thinking) that lead to a correct solution.

### ***The present study***

This study aimed to expand existing knowledge on the relationship between creativity and mathematics performance by studying direct associations and interaction effects of divergent and convergent thinking with mathematics performance on an SST and MST in a sample of fifth graders. We hypothesized that (1) divergent and convergent thinking are positively related to SST and MST performance, (2) convergent thinking is more strongly related to SST performance than divergent thinking, and (3) divergent and convergent thinking interact by complementing each other on MSTs while compensating each other on SSTs. That is, we expect the highest MST scores from children who score high on both divergent and convergent thinking (i.e., a complementary effect). We expect the highest SST scores from children who score high on convergent thinking. Besides, in the group of children that score low on convergent thinking, we expect the highest SST scores from children that score the highest on divergent thinking (i.e., a compensatory effect). These hypotheses were tested through path analysis, controlled for children's working memory. We controlled for working memory because we wanted to get insight into specific relations of divergent and convergent thinking with mathematics performance, independent of any pre-existing differences between children in working memory. Previous research has shown that working memory is one of the strongest cognitive predictors of both mathematics performance (Friso-van den Bos, Van der Ven, Kroesbergen, & Van Luit, 2013), as well as divergent and convergent thinking (Benedek, Jauk, Sommer, Arendasy, & Neubauer, 2014; Lee & Therriault, 2013; Stolte, García, Van Luit, Oranje, & Kroesbergen, 2020) across development.

## **Method**

### ***Participants***

In this study, 229 grade 5 students from 12 regular Dutch primary schools participated. The researchers contacted schools in the central east of the Netherlands by phone, explaining the purpose of the research. When the principal and class teacher agreed to participate (19.7% of schools approved participation after contact via telephone, e-mail or personal communication), schools were included in the sample. Children were included in the sample if a parent or caregiver signed informed consent. Fifth-grade students were

selected because the mathematics curriculum is more complex in upper-primary school (Noteboom, Aartsen, & Lit, 2017), requiring more problem-solving skills. Besides, relations between creative thinking and mathematics performance are stronger in upper-primary school as a result of increased mathematical and creative thinking skill (Bahar & Maker, 2011). Children in the sample (52.4% boys, 47.6% girls) were between 9 and 13 years old ( $M_{\text{age}} = 10.66$  years,  $SD_{\text{age}} = 0.55$ ). Socioeconomic status data provided by parents indicated that children's parents were predominantly of Dutch nationality (94.5%) and highly educated (i.e., 66.5% earned a (applied) university degree). Prior to data collection, ethical approval was granted by the local Ethics Committee.

### **Procedure**

Data were gathered by the first and second author, accompanied by a trained student assistant from October 2019 until January 2020. In total, administration of tasks took 1 hr and 45 min in each classroom, including a short break. All paper-and-pencil tasks were administered in class to ensure a familiar environment. First, the set of divergent thinking tasks was administered, then the set of convergent thinking tasks was administered as it has been found that in general, performance on a more open task (i.e., a divergent thinking task) is impaired when such a task follows a more closed task (i.e., a convergent thinking task; Moreau & Engeset, 2016). Within these sets, the order of tasks (verbal and visual) was counterbalanced. Finally, the MST was administered. Computerized working memory tasks were administered by the teacher in the weeks around the visit, in a quiet area of the school.

### **Measurements**

For divergent and convergent thinking, a verbal and visual task were used as creativity researchers advise using multiple measures to increase task validity (Barbot et al., 2016; Cropley, 2010). Similarly, for working memory a verbal and visuo-spatial task were used, and for mathematics performance an SST and MST. Each task is a widely used task that was shown to measure the intended construct reliably.

#### *Divergent thinking*

Subtest 1 (form A) of the Evaluation of Creative Potential (EPoC; Lubart, Besançon, & Barbot, 2011) is a visual task that intends to measure divergent thinking.

The EPoC has good construct validity and internal consistency for the divergent and convergent thinking indexes (correlations between indexes .11–.47, indicating separate constructs) (Lubart et al., 2011). In this task, children were presented with an abstract c-shape and asked to create as many different drawings including this shape as possible within 10 min. Children used a pencil for this task. The score for this subtask is a fluency score, that is, the number of correct answers generated by the student (in this case the number of drawings including the shape).

Furthermore, activity 5 (unusual uses) from the Torrance Test of Creative Thinking (Torrance, 2008) is a verbal task intended to measure divergent thinking. Children were asked to write down as many different uses for a cardboard box as possible. The task yielded three different scores. Fluency referred to number of correct answers. Flexibility was the number of different categories of answers. Examples of categories are 'toy use' or 'animal use'. Originality was based on how often a specific answer was given. The test

manual specified a score of 0 or 1 for non-original and original answers, respectively, and a total originality score of all answers was computed by adding all originality scores. Next,  $z$ -scores for fluency, flexibility, and originality were computed. These scores were added and divided by three to compute a standardized total score. Answers were scored by the first and second author, resulting in good interrater reliability ( $ICC = .96-1.00$ ).

### *Convergent thinking*

Subtest 3 (form A) of the EPoC is a visual task that intends to measure convergent thinking. Children were presented with eight shapes and had to make one drawing that included at least four of these shapes in 15 min. Children used three differently coloured felt-tip pens and were instructed to title their drawing after the 15 min had passed. Children's drawings were scored according to the manual and received a score on a 7-point scale based on use and integration of different elements. Drawings were scored by the first and second author, resulting in good interrater reliability ( $ICC = .76$ ).

In addition, a Dutch version of the Remote Associates Task (RATje) for primary school was used as this verbal task intends to measure convergent thinking. The RATje was adapted from the original version of the task and demonstrated good reliability and validity (Lazonder, Willemsen, de Vink, & Kroesbergen, unpublished manuscript; Bowden & Jung-Beeman, 2003; Mednick, 1962). Children received three stimulus words and were asked to find a remote associate: a solution word that fits with all three stimulus words. An example is worm, case, mark, for which book is a match (making bookworm, bookcase, and bookmark). Children were given 6 min and 40 s to complete the 10 items (40 s per item). Internal consistency of the RAT in this study was sufficient ( $\alpha = .72$ ).

### *Single-solution task*

Test scores from the Dutch Central Institute for Test Development (CITO) (Janssen, Scheltens, & Kraemer, 2007) were obtained from the school administration. The CITO test contains questions with a single solutions and intends to measure mathematics performance. The CITO test includes multiple-choice questions on different mathematics strands like arithmetic, geometry, fractions, and proportions. All questions have a single correct answer. The CITO test is administered twice a year as part of the national progress monitoring system. The national average CITO score is between 214.9 and 264.5. A new version of the CITO test (CITO 3.0) was implemented recently. However, one of the participating schools used a prior version (2.0). To correct for differences between versions, equipercentile equating was performed (see Finch, French, & Immekus, 2016; Shea & Norcini, 1995). The CITO mathematics test demonstrated good internal consistency, as shown by various indicators:  $KR-20 = .95$ , greatest lower bound =  $.97$  (Hop, Scheltens, Janssen, & Engelen, 2016).

### *Multiple-solution task*

The Mathematical Creativity Test (MCT) (Kattou et al., 2013; Dutch translation by Schoevers et al., 2018) is an MST intended to measure mathematics performance. The test consisted of three MSTs for which children had to generate as many different answers as possible. An example of a task is to indicate for a set of two different triangles and one square which shape is different from the others. Different arguments as to which shape is different and why can be named. Children were given 30 min to complete the task.

Answers were scored for fluency (number of correct answers), flexibility (number of different answer categories used, e.g. answers referring to the size of the shape or to invisible qualities of the shape) and originality (based on the number of children who gave a certain answer) (see also Kattou et al., 2013). For originality, the following scores could be given: 0.2, 0.4, 0.6, 0.8, or 1, if one or more of the answers appeared in >20, 11–20, 6–10, 1–5, or <1% of the sample, respectively. The first author and two trained student-assistants scored the answers to the three questions, which yielded good interrater reliability (ICC = .82–1.00). The internal consistency for the MCT was found to be high ( $\alpha = .80$ ; Schoevers et al., 2018).

### *Working memory*

The lion game is a visuo-spatial complex span task intended to measure visuo-spatial working memory (Van de Weijer-Bergsma, Kroesbergen, Prast, & Van Luit, 2015). The lion game takes 10 min. Children were shown lions in different colours in a 4x4 matrix and were asked to remember in which cell a lion of a certain colour last appeared. Difficulty increased throughout the task as children were asked to remember an additional colour each level. The monkey game is a verbal backwards span task intended to measure verbal working memory (Van de Weijer-Bergsma, Kroesbergen, Jolani, & Van Luit, 2016). The test took approximately 10 min, and children were asked to remember series of words and click on these words in reverse order in a 3 × 3 matrix. Like the lion game, the monkey game increased in difficulty as children were asked to remember an increasing number of words. For the lion and monkey game, a proportional correctness score was calculated per level and then averaged over the whole task. Internal consistency for the lion and monkey game was found to be good, respectively, .86–.90 and .89 (Van de Weijer-Bergsma, Kroesbergen, Prast, et al., 2015; Van de Weijer-Bergsma et al., 2016).

### **Analyses**

Descriptive statistics were examined using SPSS 25. Skewness and kurtosis scores indicated that fluency of task 2 of the MST and visuo-spatial working memory score were not normally distributed (i.e., values > ±3.29). Eight univariate ( $z$ -score > ±3.29) and two multivariate outliers (Mahalanobis distance >  $\chi^2(6) = 22.46$ ) were detected. The outliers were not removed, as they had realistic values and the influence of the multivariate outliers was minimal (Cook's distance < 0.03) (Cousineau & Chartier, 2010). Forty-six children showed missing data on one or more of the variables. Little's MCAR test showed that data were missing at random ( $\chi^2 = 212.29, p = .14$ ).

The analyses proceeded in three steps. First, Spearman correlations between all variables were checked. Second, a confirmatory factor analysis was performed on the MCT subscales to determine whether a single latent factor could be constructed. Third, a path model was tested in which divergent and convergent thinking were related to SST and MST performance. The model included two interactions: verbal divergent with verbal convergent thinking, and visual divergent with visual convergent thinking. Working memory was included in this model as a covariate. Structural Equation Models (SEM) were analysed with R-software (version 3.5.1), using the lavaan package (Rosseel, 2012). To account for a non-normal distribution, outliers and missing data, full information maximum likelihood estimation was used with robust standard errors. Goodness of fit for both SEM models was evaluated using chi-square ( $p > .05$ ), the ratio between chi-square and degrees of freedom (< 3), CFI and TLI ( $\geq .95$ ) and RMSEA ( $\leq .06$ ).

## Results

Prior to the main analyses, descriptive statistics for all observed variables were checked, see Table 1. Next, variables were standardized to aid interpretation.

Table 2 shows Spearman correlations between all variables, including MST performance as a latent factor. Visual divergent thinking was weakly to moderately correlated with verbal divergent thinking and verbal convergent thinking. Verbal convergent thinking was also weakly to moderately correlated with SST, MST, and working memory task performance. The two working memory tasks were moderately correlated to each other and to SST and MST performance. SST and MST performance were also moderately correlated. Note that partial correlations, controlling for verbal or visuo-spatial working memory, showed a very similar result.

A confirmatory factor analysis was performed to construct a latent factor for MST performance. A model with MST performance as a first-order latent factor and each of the three tasks as second-order latent factor fitted well, consistent with findings by Stolte et al. (2020). Herein, each of the three MST tasks significantly contributed to MST performance (Figure 1). Initially, a small negative variance (i.e.,  $-0.392$ ) was obtained for flexibility of question 2. As this variance was likely caused by outliers, it was scaled to zero (Bollen, 1987). Fit statistics of the final model were  $\chi^2 = 47.81$ ,  $p < .01$ ,  $\chi^2/df = 1.91$ , CFI = .98, TLI = .97, RMSEA = .06.

Next, a path model was tested in which verbal and visual divergent and convergent thinking were related to SST and MST performance, including direct effects of divergent and convergent thinking and interactions between them (Figure 2). Verbal and visuo-spatial working memory were included as covariates in this model. All significant correlations between exogenous variables were also included. This model fit the data well:  $\chi^2 = 167.77$ ,  $p < .01$ ,  $\chi^2/df = 1.41$ , CFI = .96, TLI = .95, RMSEA = .04. Within this model, we found a significant positive association between verbal convergent thinking

**Table 1.** Descriptive statistics of all variables included in the path models

	N	Mean	SD	Range	Skewness	Kurtosis
Verbal DT	222	0.00	0.94	-1.86-2.10	0.44	-0.74
Visual DT	221	8.86	3.87	2-24	0.72	0.84
Verbal CT	221	3.99	2.18	0-10	0.28	-0.24
Visual CT	222	4.21	1.10	0-7	0.07	0.89
SST	218	246.62	24.26	171-316	-0.23	-0.11
MST 1 - fluency	218	2.21	1.34	0-7	0.81	0.57
MST 1 - flexibility	218	1.70	0.78	0-3	-0.11	-0.39
MST 1 - originality	218	0.62	0.31	0-1	-0.37	-1.15
MST 2 - fluency	222	15.60	14.57	0-84	1.89	5.06
MST 2 - flexibility	222	2.65	1.25	0-5	-0.09	-1.19
MST 2 - originality	222	0.42	0.22	0-1	0.53	-0.19
MST 3 - fluency	213	2.23	1.43	0-7	0.70	0.21
MST 3 - flexibility	213	1.73	0.94	0-4	0.32	-0.32
MST 3 - originality	213	0.61	0.29	0-1	-0.53	-0.65
Verbal WM	198	0.59	0.15	0.09-0.98	-0.73	1.08
Visuo-spatial WM	201	0.75	0.14	0.05-0.99	-1.65	4.45

CT = convergent thinking, DT = divergent thinking, MST = multiple-solution task performance (task 1, 2 & 3), SST = single-solution task performance, WM = working memory.



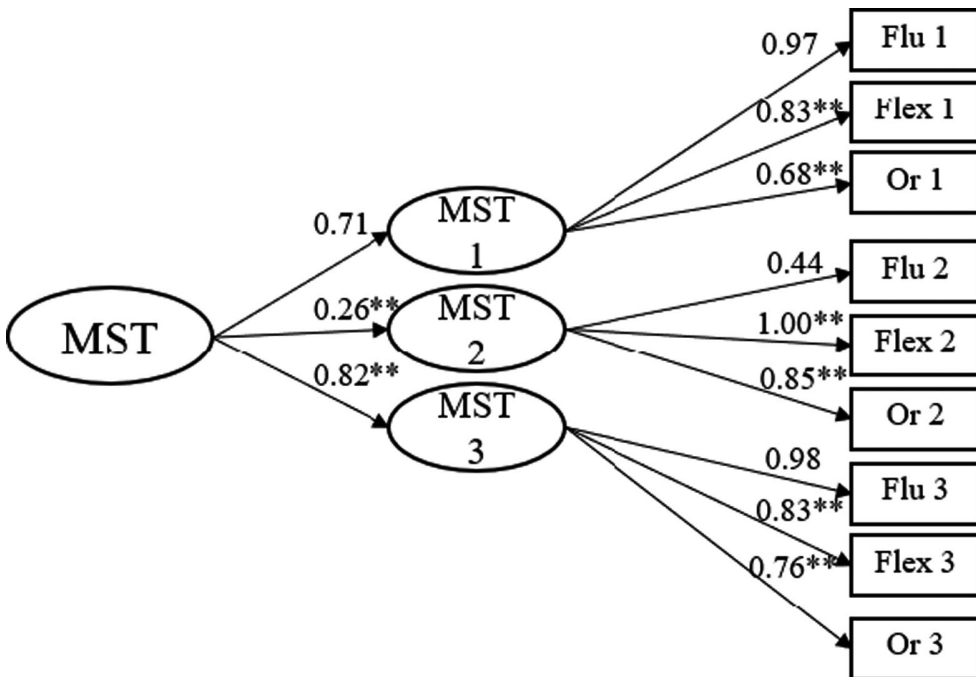
**Table 2.** Spearman correlations between all variables included in the path models

	1	2	3	4	5	6	7	8
1. Verbal DT	–							
2. Visual DT	.36**	–						
3. Verbal CT	.08	.14*	–					
4. Visual CT	.09	–.02	.10	–				
5. SST	.07	.01	.24**	.07	–			
6. MST (factor)	.07	.00	.26**	.06	.30**	–		
7. Verbal WM	.05	.00	.25**	.09	.39**	.28**	–	
8. Visuo-spatial WM	.12	.05	.28**	.02	.35**	.36**	.49**	–

Note. All variables are standardized.

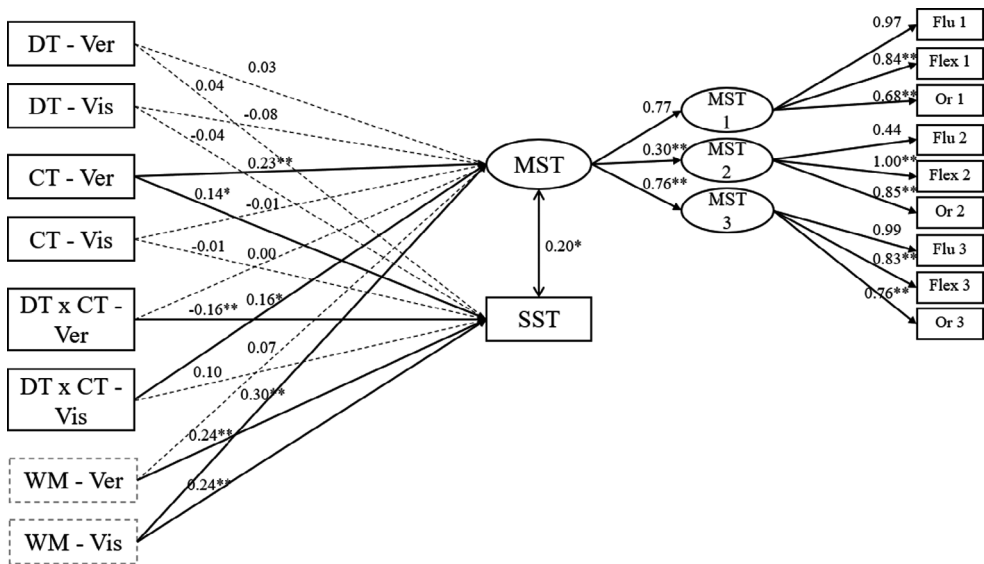
CT = convergent thinking, DT = divergent thinking, MST = multiple-solution task performance, SST = single-solution task performance, WM = working memory.

\*\* $p < .01$ ; \* $p < .05$ .



**Figure 1.** Standardized factor loadings for the confirmatory factor analysis. Note. MST = multiple-solution task performance (task 1, 2 & 3), flu = fluency, flex = flexibility, or = originality. Note. \* $p < .05$ , \*\* $p < .01$ . Since each first factor loading is scaled to 1 (standardized values can vary from this value) to aid interpretation, no significance can be determined for these loadings.

and SST and MST performance. SST and MST performance showed a positive covariance. Verbal working memory as a covariate showed significant positive associations with SST performance, while visual working memory showed a positive association with SST and MST performance. Furthermore, we found a significant interaction effect between verbal convergent thinking and verbal divergent thinking for SST performance, and a significant

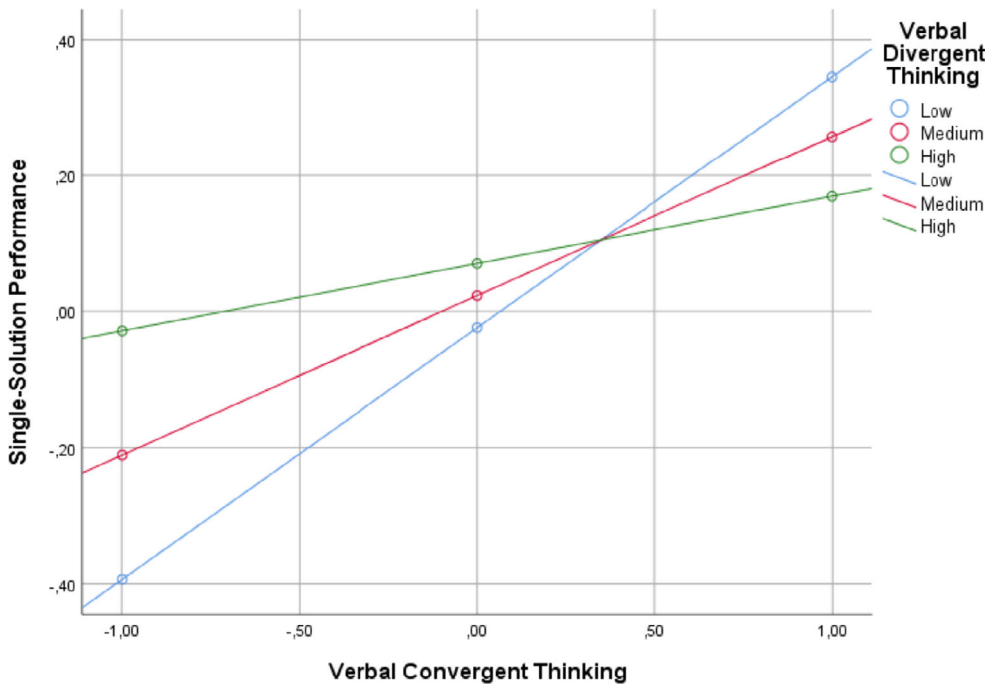


**Figure 2.** Standardized path coefficients and factor loadings for the path model, including direct and interaction effects for divergent and convergent thinking and working memory as a covariate. For clarity, covariances between exogenous variables are excluded from this picture. Note. DT = divergent thinking, CT = convergent thinking, Ver = verbal, Vis = visual, WM = working memory, MST = multiple-solution task performance (task 1, 2 & 3), SST = single-solution task performance, flu = fluency, flex = flexibility, or = originality. \* $p < .05$ , \*\* $p < .01$ .

interaction effect between visual convergent thinking and visual divergent thinking for MST performance (Figure 2). These interaction effects are illustrated in Figures 3 and 4. We divided the students into three groups (low, medium, and high divergent thinking) and plotted the relation between convergent thinking and single- or multiple-solution mathematics performance for each group (indicated by the coloured lines). Specifically, for the SST, children who showed above average verbal divergent thinking demonstrated the highest SST scores when verbal convergent thinking performance was low. However, these children were outperformed by children who showed low divergent thinking but high convergent thinking, as these children demonstrated the highest overall SST scores. For MST performance a different pattern was observed. When visual divergent thinking was low, the highest MST performance was demonstrated by children who also showed low visual convergent thinking performance. The highest overall MST performance was demonstrated by children who showed high divergent and convergent thinking performance.

## Discussion

This study was one of the first studies to investigate the association between divergent thinking, convergent thinking and mathematics performance on an SST and MST, including working memory as a covariate. A path model was tested, including direct relationships of verbal and visual divergent and convergent thinking with SST and MST performance, and interactions between divergent and convergent thinking. In line with our hypotheses, children who demonstrated good verbal convergent thinking skill also

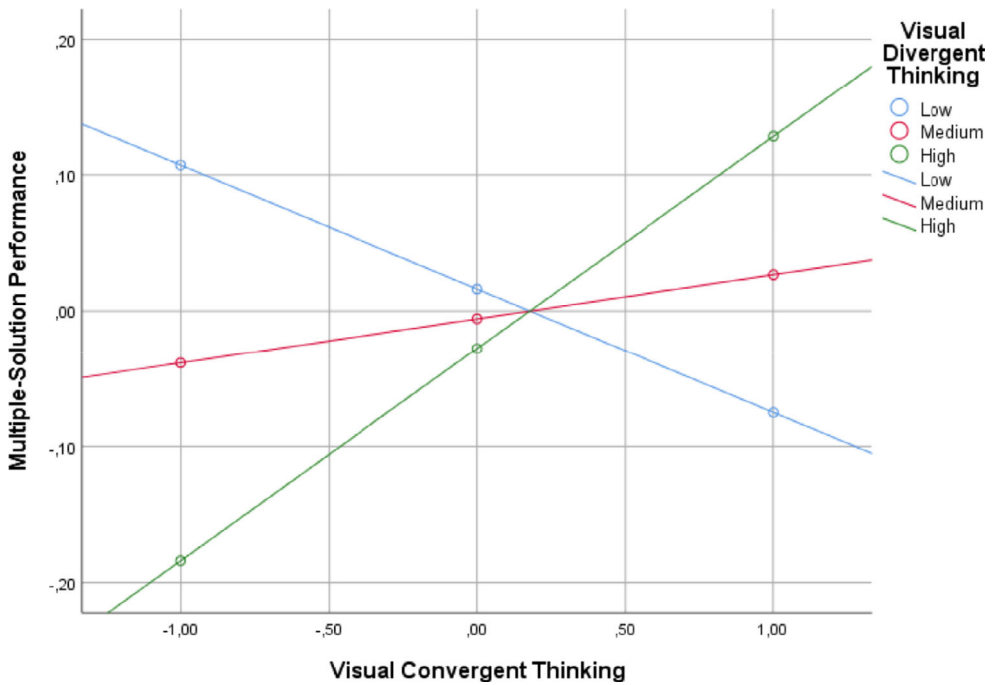


**Figure 3.** Interaction effect for verbal divergent and convergent thinking with SST performance. The red line indicates mean divergent thinking performance, the blue line indicates performance one standard deviation below the mean, and the green line performance one standard deviation above the mean. SST = single-solution task performance.

performed significantly better on the SST and MST. We found no such direct association for divergent thinking. Furthermore, we found an interaction effect for verbal divergent and convergent thinking on the SST, and visual divergent and convergent thinking on the MST.

Regarding convergent thinking, this study is one of the first to demonstrate a positive association of *verbal* convergent thinking with SST and MST performance, supporting our first hypothesis. This finding is in line with the theory that convergent thinking fosters mathematical problem-solving (Assmus & Fritzlär, 2018; Leikin et al., 2009; Tan & Sriraman, 2017). Presumably, convergent thinking processes like being logical, recognizing the familiar and reapplying set techniques, facilitate applying memorized knowledge and automatized mathematical processes and thus help children to find one or more fitting solutions (Cropley, 2006; Tabach & Levenson, 2018). No such association was found for *visual* convergent thinking. Although visual skills like visuo-spatial WM or visual perception are related to mathematics performance (Pieters, Desoete, Roeyers, Vanderswalmen, & Van Waelvelde, 2012; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015), it might be that visual convergent thinking skill is generally less called upon in mathematics tasks. This is in line with research that has shown verbal creativity tasks to be more strongly associated with academic achievement than visual creativity tasks (Gajda, Karwowski, & Beghetto, 2017).

Contrary to our first hypothesis and previous research (Jeon et al., 2011; Kattou et al., 2013; Schoevers et al., 2018), divergent thinking was not associated with SST and MST



**Figure 4.** Interaction effect for visual divergent and convergent thinking with MST performance. The red line indicates mean divergent thinking performance, the blue line indicates performance one standard deviation below the mean, and the green line indicates performance one standard deviation above the mean.

performance. Theoretically, divergent thinking is part of the creative problem-solving process (Brophy, 2001; Guilford, 1973). Therefore, we expected divergent thinking to facilitate mathematical problem-solving. Specifically, divergent thinking is proposed to enhance performance on the MST because it enables children to generate different original solutions (Assmus & Fritzlar, 2018; Leikin, 2009). For the SST, divergent thinking was expected to be especially useful when there is no available learned solution to a problem, as it can help children in applying different strategies to solve a problem (Leikin, 2009). The tasks we used in this study might explain why we found no effect of divergent thinking. For visual divergent thinking, we used a subtask from the EPoC. To our knowledge, this task has not been related to mathematics achievement previously. However, a study in which a similar visual divergent thinking task was used, also showed no relation with mathematics performance (Huang, Peng, Chen, Tseng, & Hsu, 2017). As with visual convergent thinking, it might be that in line with a stronger association between verbal creativity tasks and academic achievement (Gajda et al., 2017), visual divergent thinking is less called upon in the two mathematics tasks. For verbal divergent thinking, we used activity 5 (unusual uses) from the TTCT in line with Jeon et al. (2011) and Schoevers et al. (2018). However, a notable difference between our study and those studies is that both used other activities from the TTCT in addition to activity 5 and calculated a total score or general divergent thinking factor score (note: activity 5 also had the lowest factor loading in Schoevers et al.). Thus, associations of divergent thinking with mathematics achievement might be task-specific, and it could be that the way divergent

thinking is measured in the other activities more closely resembles the way divergent thinking can be applied on a mathematics task. As divergent thinking and SST performance were unrelated, it cannot be determined whether convergent thinking contributes *more* to SST performance than divergent thinking does (i.e., our second hypothesis).

Besides direct associations between creative thinking and mathematics, we tested interaction effects to determine how divergent and convergent thinking together affect mathematics performance. The relation of convergent thinking and SST or MST performance was compared between groups of children with different divergent thinking ability, which confirmed our hypothesis that divergent and convergent thinking compensate each other on an SST, while they complement each other on an MST, for verbal and visual measures respectively. Specifically, children with high convergent thinking but low divergent thinking skill performed best on the SST. Amongst children who showed low convergent thinking, those with high divergent thinking performance did better on the SST than those with low divergent thinking scores. Thus, it seems likely that those children could compensate a lack of learned solutions or idea evaluation (for which convergent thinking is used) by generating different strategies and finding alternative routes to a correct solution (for which divergent thinking is used). A possible explanation for this finding might be the familiarity of the problem. Most children might know how to solve the SST problems using familiar procedures and are thus able to do well using convergent thinking only, while children for whom the problem is less familiar might rely more on finding alternative solutions and applying divergent thinking (Schoenfeld, 2014).

A different pattern emerged on the MST, where children with both high visual divergent and convergent thinking skill showed the highest MST performance. This outcome supports our hypothesis that divergent thinking and convergent thinking complement each other. This is in line with theory that proposes that a combination of divergent and convergent thinking is needed to solve mathematical problems (Leikin et al., 2009; Tabach & Levenson, 2018; Tan & Sriraman, 2017). Besides, research has shown that measures of creative thinking are more strongly connected to open mathematics tasks with multiple solutions (Schoevers, 2019). This might explain why children who score high on convergent thinking only perform the best on the SST, while for the MST children seem to rely on both modes of thinking to do well. In the group of low convergent thinkers, children who also showed low divergent thinking outperformed children who showed high divergent thinking scores. Although children who score high on both divergent and convergent thinking perform the best on the MST, it might be that divergent thinking does not aid performance as much when children are not good at convergent thinking. Tabach and Levenson (2018) argued that excessive divergent thinking on an MST can lead to random and ineffective solutions. Thus, when children are unable to evaluate their ideas, generating many different ideas might be less helpful. This could be different for an MST than for an SST, as the MST is a new task for children which is scarcely used in primary math education (Kolovou, 2011). Therefore, the task might be more complex, probably making it harder to generate effective novelty through divergent thinking (Siswono, 2010).

Although not the focus of this study, our results point to a possible difference in verbal and visual skills related to mathematics performance. Only verbal convergent thinking was related to mathematics performance directly, which might reflect the verbal nature of the mathematics tasks. This was corroborated by the finding that children can compensate verbal convergent thinking with verbal divergent thinking, and a positive

correlation of verbal working memory with SST performance. Although this seems to point to a dominance of verbal skills, visuo-spatial working memory was also positively correlated with SST performance, indicating that visual skills can play a role too. For the MST, we found only visuo-spatial working memory to be positively correlated and found a complementary mechanism of visual divergent and convergent thinking. Thus, although only verbal convergent thinking is directly related to performance, combining visual divergent and convergent thinking can also be helpful on an MST. Visuo-spatial working memory and visual attention have been related to learning new math skills and generating multiple solutions, respectively (Van de Weijer-Bergsma, Kroesbergen, Prast, et al., 2015; Zmigrod, Zmigrod, & Hommel, 2015), which might explain why visual skills seem to play a slightly larger role on a novel task like the MST.

### ***Future studies and limitations***

Future studies have to shed more light on these issues, as the cross-sectional design of this study makes it hard to pinpoint direction or causality of the included associations. An interesting direction could be to look at how interactions between divergent and convergent thinking skill affect mathematics performance over time. Such a study can be more sensitive to hypothesized age differences in divergent and convergent thinking (Alfonso-Benlliure & Santos, 2016) than our study, which was limited to fifth-grade students. Furthermore, the present study aimed to disentangle the associations of divergent and convergent thinking with mathematics performance, but more research is needed to understand these mechanisms. Specifically, purity of measurement might be an issue, as researchers have argued that convergent thinking processes might play a role on divergent tasks and vice versa (Cortes, Weinberger, Daker, & Green, 2019). Although this does not seem to have affected our study too much, as we found low correlations between divergent and convergent measures, it cannot be ruled out completely that the positive association we found between convergent thinking and mathematics performance in part reflects divergent thinking processes too. More fine-grained (qualitative) research that focuses on how divergent and convergent thinking alternate each other in problem-solving could shed light on subtle differences between the two.

### ***Implications***

This study advanced the understanding of the relationship between creativity and mathematics, specifically convergent thinking, and has implications for the implementation of creativity in mathematics education. In line with previous literature, the results support the conceptual distinction between divergent and convergent thinking (Barbot et al., 2016; Lee & Therriault, 2013; Storme et al., 2015) because different effects were found for each. Thus, it is important for teachers to be aware of the diversity within creative thinking skill, which might mean that divergent and convergent thinking need to be incorporated in mathematics education in different ways. Convergent thinking has a direct positive relation with performance on both mathematics tasks, whereas divergent thinking does not. Based on this finding, we recommend primary math teachers to promote convergent thinking in their lessons. Previous research in the field of marketing and innovation has investigated techniques to promote convergent thinking, like analysing the strengths and weaknesses of an idea (Licuanan, Dailey, & Mumford, 2007). However, mathematics teachers struggle to promote creativity (Kaufman & Baer, 2004), and there is a paucity of research that investigates how teachers can support creative

thinking in education (Davies et al., 2014). Therefore, more research is needed to fully understand how creative thinking and mathematics performance are related and how creative thinking can be promoted. This study was one of the first to simultaneously study divergent and convergent thinking in mathematics. The interaction effects we found show it is important for educators to be aware of individual differences in creative thinking and their possible influence on mathematical achievement.

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## Conflict of interest

All authors declare no conflict of interest.

## Author contributions

**Isabelle C de Vink:** Conceptualization (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Writing – original draft (equal). **Robin H Willemsen:** Conceptualization (equal); Investigation (equal); Writing – review & editing (equal). **Ard W Lazonder:** Conceptualization (equal); Funding acquisition (equal); Supervision (equal); Writing – review & editing (equal). **Evelyn H Kroesbergen:** Conceptualization (equal); Funding acquisition (equal); Supervision (equal); Writing – review & editing (equal).

## Data availability statement

Research data are not shared.

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