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Influence of nanoparticles on freezing inside container equipped with fins

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With loading of different shapes of nanoparticles, the solidification speed can be changed which was scrutinized in current work. Although the nanoparticles dispersion can decline the heat capacity, the conduction mode can be improved with such technique and changing the styles of nano-powders can alter the strength of conduction. The velocity terms were neglected in freezing, thus, the main equations include two equations with unsteady form for scalars of solid fraction and temperature. Grid adaption with position of ice front has been considered in simulations utilizing FEM. The upper sinusoidal and inner rectangular walls maintain cold temperature and freezing starts from these regions. Adding nanomaterial can expedite the process around 15.75% (for $m = 4.8$) and 29.8% (for $m = 8.6$). Also, utilizing particles with shapes of blade form can augment the freezing rate around 16.69%. The efficacy of m on freezing process rises around 4% with elevate of concentration of nanoparticles.

Obtaining the steadiness among the minimum and the maximum heat use, or even generation is an absorbing matter among the heat mechanism specialists^{1–5}. Within the previous years, scientists in various fields of analyses were made attempts to develop the heating use. The endeavors contain analyses on the increment of thermal procedure^{6–10}, promotion of solar unit applying great efficiency substances^{11–15} and etc. Energy storage mechanisms have a decent technique to that matter. Saving energy can happen as latent heat within the change of substance phases at constant temperature^{16–20}. The substances applied for this storage were called PCM. Compared to typical sensible unit, a high magnitude of heat can be saved in very lesser volumes of PCMs^{21–25}. Within the discharging and charging of heat, PCM can be installed in tinier temperature gradients^{26–30}. Nonetheless, the low heat conductivity of the PCMs is the key disadvantage of PCM substances in the performance of heat mechanisms^{31–36}. Therefore, increase in the heat conductivity can be assumed as key parameter for industrial uses^{37–43}. Cao et al.⁴⁴ expressed that the paraffin charging rate is soared by the addition of fins. The authors saw that the fins number is a main term for each wall temperature.

Authors of⁴⁵ inquired the impact of kind of substance of extend surface and nanomaterial on the efficiency of solar unit and expressed that the presence of such techniques can considerably increase the melting of PCM. Zeng et al.⁴⁶ inquired the impact of various sets of locations of a hole accumulated with paraffin. They saw that the speed of phase change has increased with changing the location to vertical style. Mehta et al.⁴⁷ presented a comparison between the horizontal and vertical containers and saw that the buoyancy force is operational in the vertical unit within the charging procedure, leading to a charging rate is approximately fixed in comparison to the horizontal units. Usman et al.⁴⁸ inquired various shapes of a thermal sink. They saw that installing fins with various arrangements reduce the most appropriate temperature. This was because of an increment in the efficient heat conductivity. Authors of⁴⁹ inquired the impact of Gr and the aspect ratio on the convection diffusion within freezing. Authors saw that speed of freezing relied on the heat and geometrical terms of mechanism. Chen et al.⁵⁰ inquired the impact of porous media and triangular double fins on charging of a vertical container. They saw that applying triangular fins including porous zone makes melting period decreases about 98%. Researchers of⁵¹ inquired the charging procedure of the paraffin wax as a PCM inside triangular enclosures and they reported the effective impact of apex angle on unsteady process. Mohamed et al.⁵² inquired RT44HC PCM to find the

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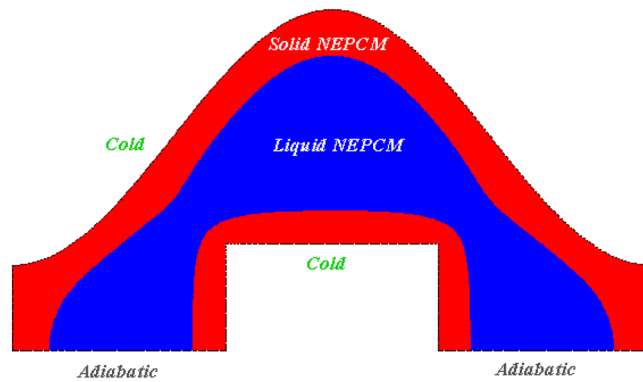


Figure 1. Freezing of water in attendance of nanoparticles through special container.

result of different amounts of wall thermal flux and saw that growing the input power decreases the required time by 42.10 percent.

It was mentioned in various published article that mixing the base PCM with nanoparticles can enhance the performance. So, to accelerate discharging process within the present container, nanoparticles with two styles have been utilized. The modeling procedure with assumption of neglecting velocity has been derived and such equations were solved via FEM. To portray the efficacy of two active factors, contours and plots have been reported and required time for solidification have been derived for all cases. Validation for this numerical approach was done according to previous article and good accommodation was reported.

Container with cold curved walls

Two curved walls with shapes of wavy and rectangular have been involved in present study as shown in Fig. 1. The water was mixed with two shapes of nanoparticle (CuO) and to incorporate the modeling, single phase technique was chosen⁵³. The curved walls are cold and there exist two adiabatic walls, too. The associated terms of velocity have been discarding in mathematic model because of its low efficacy in freezing. Thus, the related equations for modeling are⁵³:

$$(\rho C_p)_{nf} \frac{dT}{dt} - L_{nf} \frac{dS}{dt} = \nabla(k_{nf} \nabla T) \quad (1)$$

$$\begin{cases} S = 1 & AA < (-T_0) \\ S = 0 & AA > (-T_0) \\ S = (T_m + 0.5T_0 - T)/T_0 & (-T_0) < AA < T_0 \end{cases} \quad (2)$$

$$AA = (T - T_m)$$

There exist three terms in Eq. (1) which needs to calculate as features of nanomaterial and homogenous mixture approximation has been utilized for this purpose. The below formulas were utilized in current study⁵³:

$$\rho_{nf} = \rho_f(1 - \phi) + \rho_p \phi \quad (3)$$

$$(L\rho)_{nf} = (L\rho)_f(1 - \phi) \quad (4)$$

$$\frac{(C_p \rho)_{nf} - (C_p \rho)_f}{((\rho C_p)_p - (C_p \rho)_f)} = \phi \quad (5)$$

$$\frac{k_{nf}}{k_f} = \frac{-m BB \phi + k_p + mk_f - \phi BB + k_f}{k_f m + k_f + BB \phi + k_p}, \quad BB = (k_f - k_p) \quad (6)$$

In Eq. (6), there is term of shape factor to involve the various shapes of nanoparticles in simulations. Two shapes of cylinder ($m=4.8$) and blade ($m=8.6$) shapes were incorporated. The transient model need powerful method for modeling especially it should be combined with adaptive grid to increase the accuracy of modeling. Sheikholeslami⁵³ suggested to use finite element based approach in modeling of freezing process and He utilized various numerical approaches for designing the thermal storages units. In this work, the same method was utilized with involve of Galerkin method.

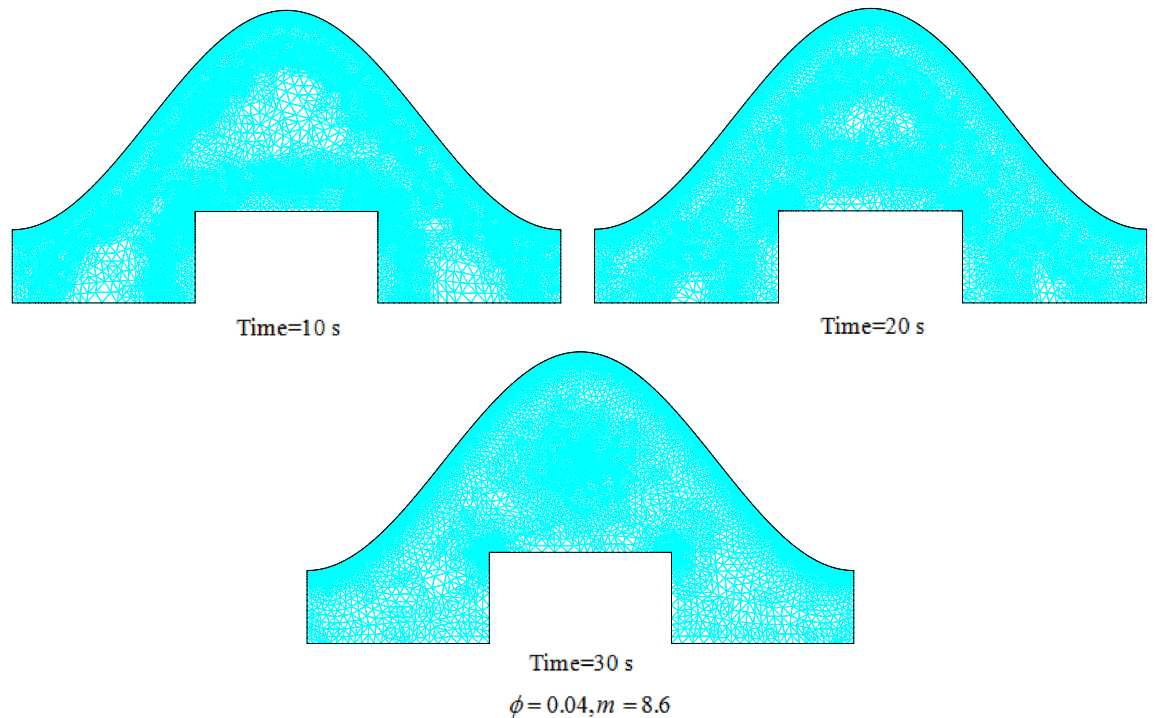


Figure 2. Grid resolution in different stages of process.

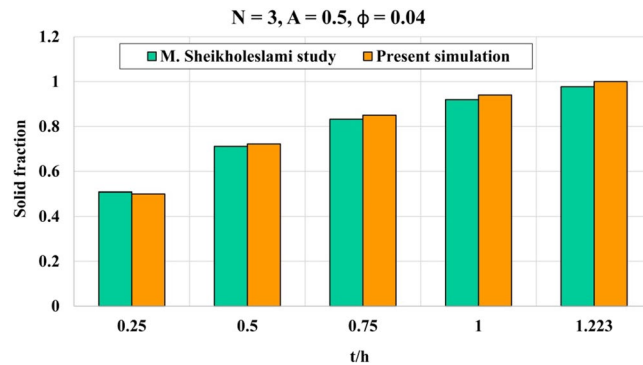


Figure 3. Evaluation of correctness of simulation⁵³.

Results and discussion

The tank including inner rectangular cylinder connecting to cold flow as well as outer sinusoidal cold wall has been analyzed in current study for heat release phenomena. The tank was filled with liquid water which is mixed with CuO nanoparticles to remove the limitation of its inherent low conductivity. Two different shapes of particles were involved to detect the efficacy of this factor on solidification rate. In scrutinized geometry, there exist two horizontal walls which are adiabatic while other wall has temperature of lower than 273 K. To gain the mathematical model, this fact that there is small value of velocity for freezing process has been considered. Thus, the equations of model include temperature equation and concentration of solid PCM. In temperature equation, the advection terms have been neglected and implicit technique for modeling process has been involved. Also, to include the freezing phenomena, there exist one transient term to represent converting liquid to solid. Efficacy of nanomaterial can be appeared in calculation of features of NEPCM which was done based on homogeneous model. The modeling was done by FEM approach and the style of mesh has been changed with augmenting time. Figure 2 portrays the resolution of grid at three levels of time and indicates the more number of elements have been applied in space near the ice front because the magnitude of temperature gradient is stronger than other places. The previous publication for validation purpose has been analyzed⁵³ and related outputs have been demonstrated in Fig. 3 which demonstrates good accommodation. So, the same modeling procedure has been utilized for present problem and single phase approach for modeling of NEPCM treatment was applied.

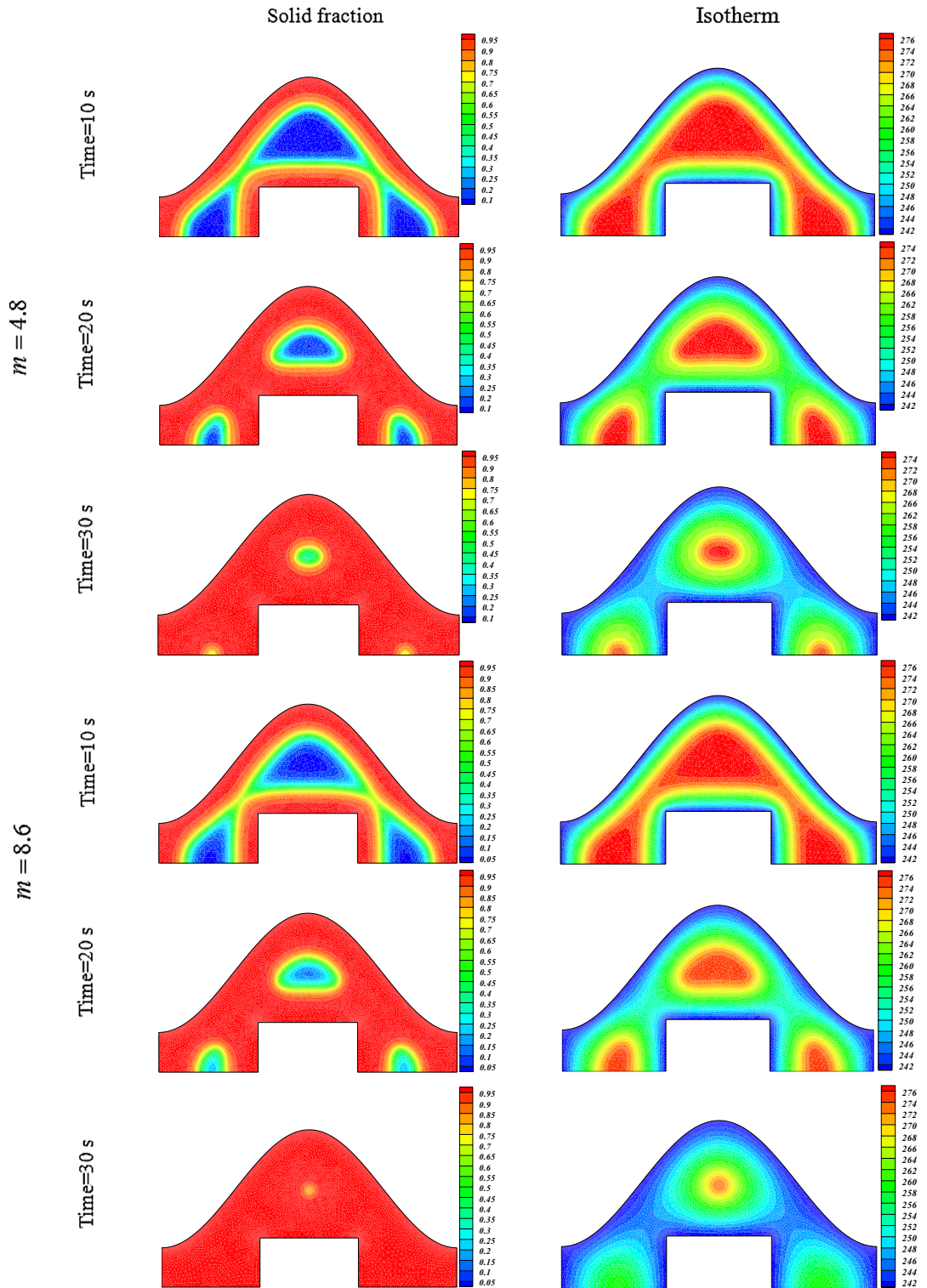


Figure 4. Impact of configuration of nano-powders on transient phenomena.

In this work, disperse of nanoparticles was applied as main method of enlightening the speed of freezing. Not only was the volume of particles but also configuration of powders assumed as active factor in modeling. Owing to low velocity of liquid phase, the main term in this phenomenon is thermal diffusion term which is main origin of conduction mode. Figures 4 and 5 illustrate the transient behavior of scalars of this work with applying various levels of active factors. The freezing time for pure water case is about 44.71 s and inclusion of nanoparticles with concentration of 0.02 leads to reduce of freezing time up to 40.1 s and 37.66 s for particles with shape factor of 4.8 and 8.6, respectively. Moreover, if the concentration of blade shape particles augments, the period declines

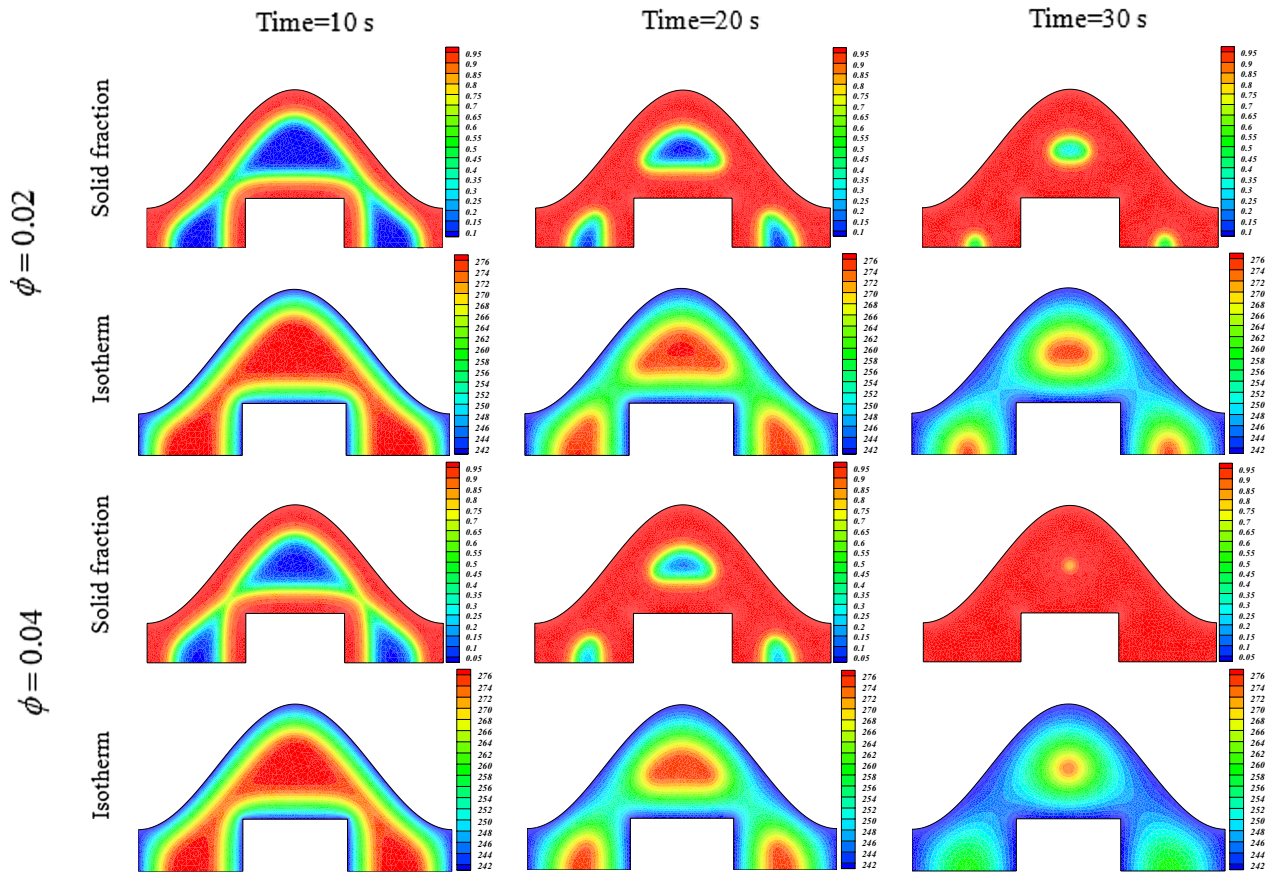


Figure 5. Roles of ϕ on transient phenomena.

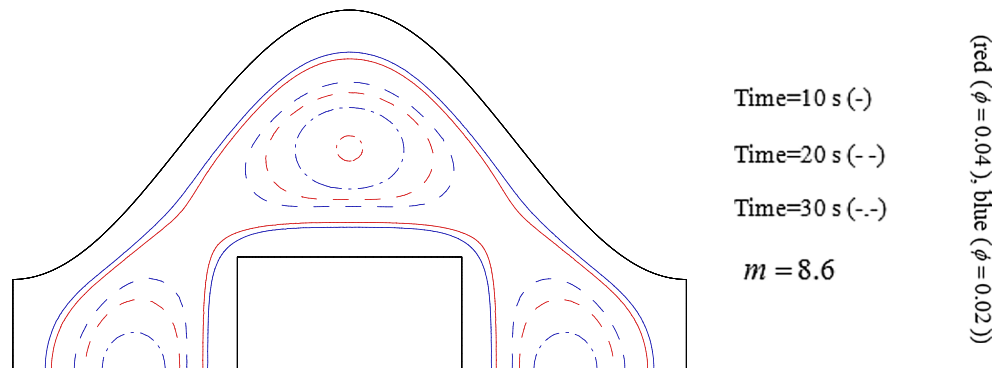


Figure 6. The role of ϕ on movement of ice front.

from 37.66 to 31.37 s. Increase of shape factor can augment the freezing rate and its amount for $\phi = 0.04$ is 4.5% superior than that of $\phi = 0.02$. With intensify of ϕ , the speed of process increases and the related value for blade shape is 4% greater than that of $m = 4.8$.

Increase of ϕ can change the movement of ice front as shown in Fig. 6 and average values of functions can be calculated in each time stages and related data were reported in Fig. 7. Reducing tendency of temperature and energy with progress of time is due to reduction of liquid PCM and more reduction can be observed if concentration of nanomaterial increases. The solid fraction enhances with grow of time and adding nanoparticles makes the magnitude of this scalar to increase. The comparison of blade and cylindrical shapes of nanoparticles in view of ice front and mean value of scalars were portrayed in Figs. 8 and 9. When the all domain convert to solid, the magnitude of SF extents to unity and minimum level of temperature can be reported. Both sensible and latent heat declines with promote of time, thus the energy of unit reduces. The amounts of energy for blade shape particles are lower than the other style of particle because of lower temperature. The significant factor of designing unit for discharging process is period of time and the associated data has been shown in Fig. 10. With augment of concentration of nano-powders with blade and cylindrical shapes, the needed time declines around

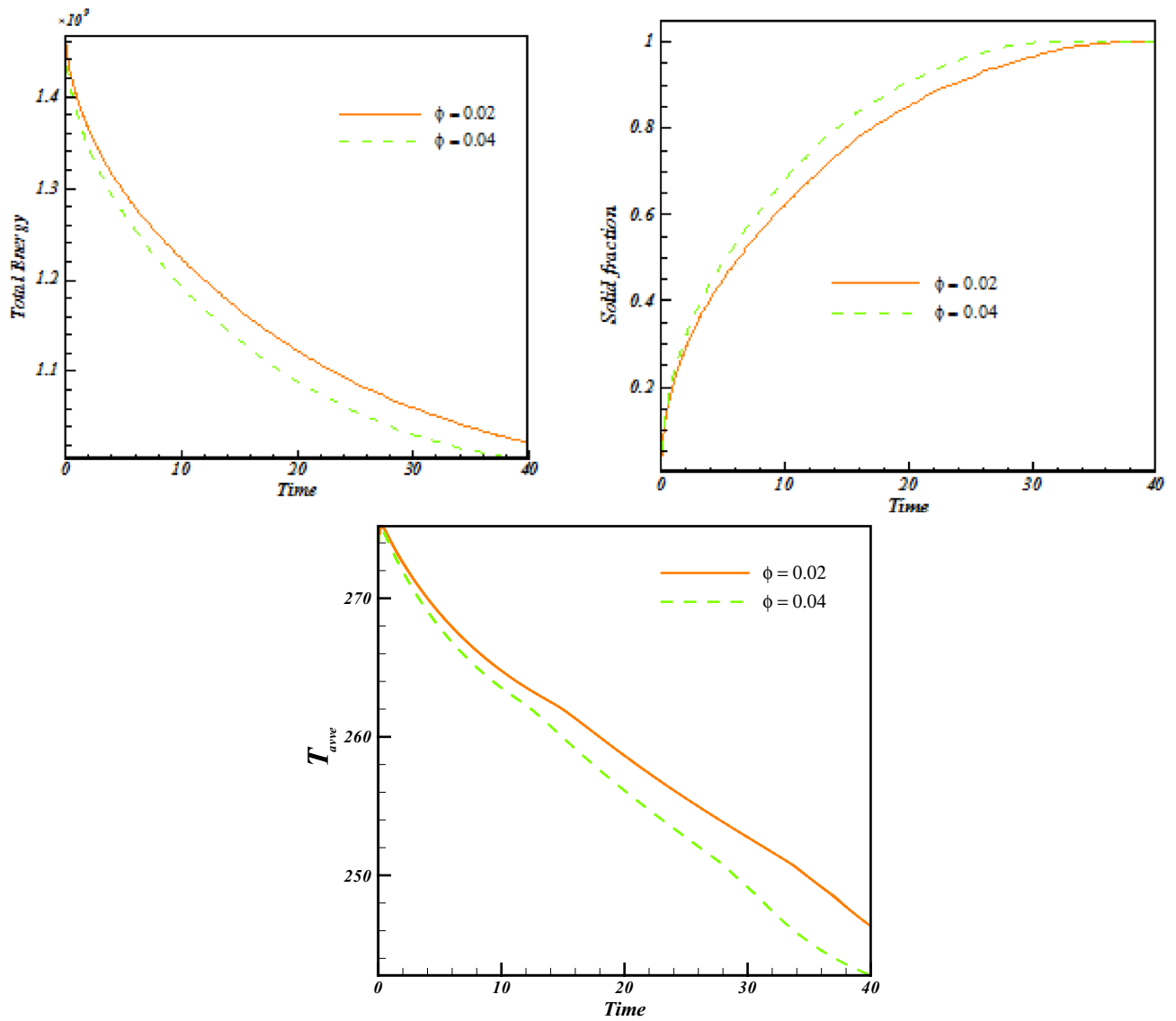


Figure 7. Progress of time change of parameters with later of ϕ .

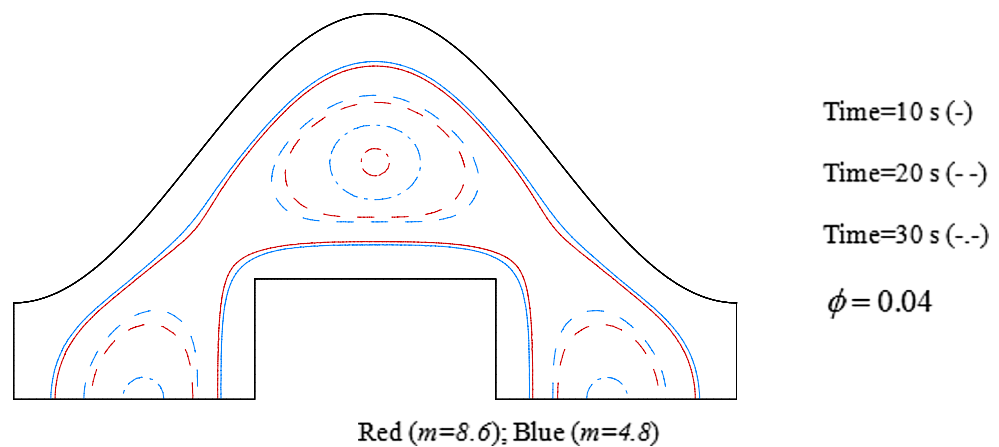


Figure 8. Role of m on movement of ice front.

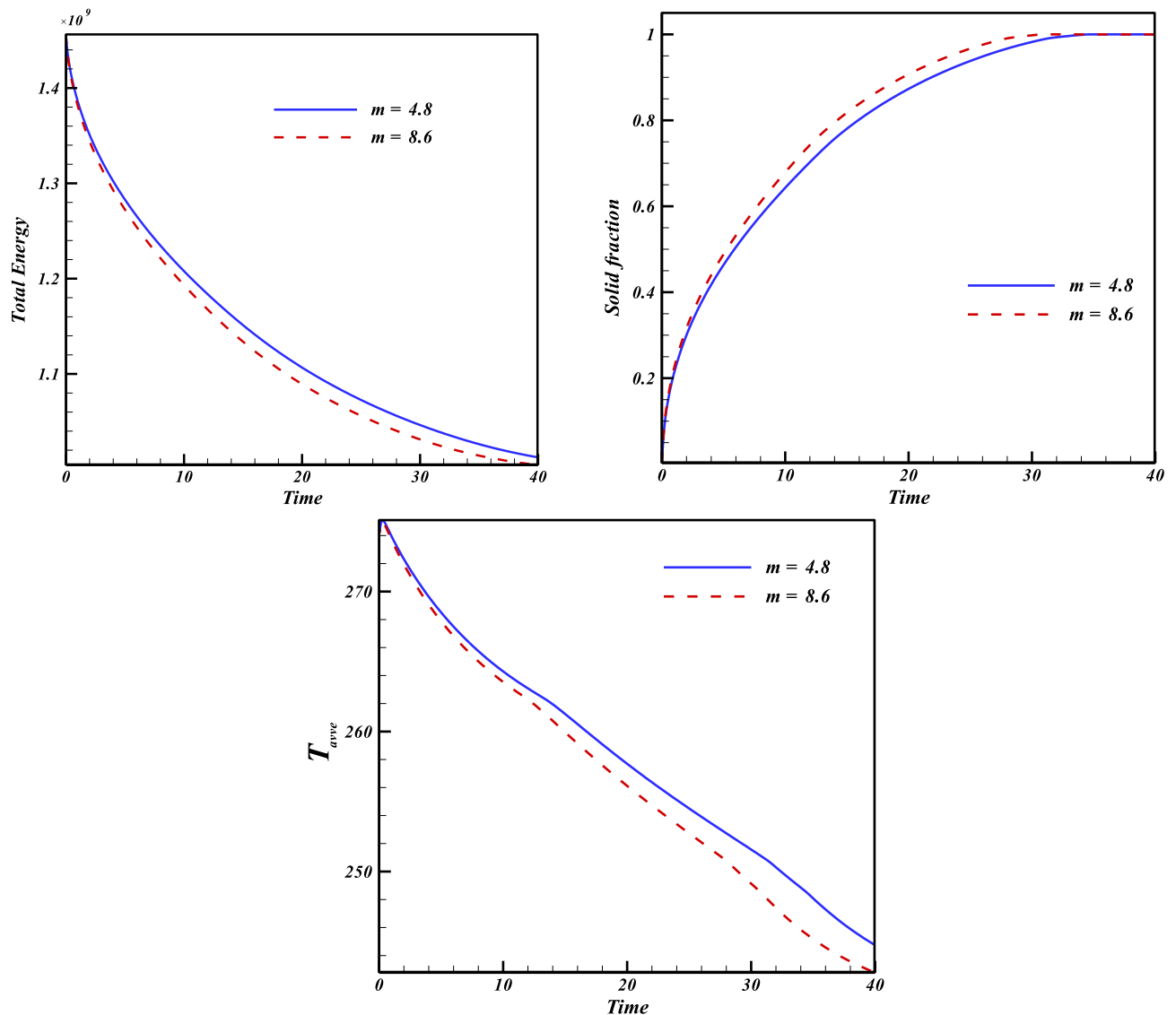


Figure 9. Progress of time change of parameters with later of m .

12.54% and 16.69%, respectively. Adding nano-powders can augment the freezing rate around 15.75% and 29.81%. Moreover, with change of style of particles from cylinder to blade shapes, the required time decreases around 6.07% and 10.53% when $\phi = 0.02$ and 0.04 , respectively.

Conclusion

Inclusion of nano-powders with two different shapes has been considered as passive technique of augmenting speed of phase change phenomena. The container has rectangular and sinusoidal walls which connected to cold flows and their temperature has been considered lower than 273 K. To find the new properties of water after dispersing nanoparticles, homogeneous mixture assumption has been applied. With appearance of nanoparticles, the penetration of cold region augments because of greater conduction mode. FEM technique was applied involving the time-dependent style of grid which helps to model the region of ice front more carefully. To verify the procedure of modeling, previous article has been validated and outputs depicted the good agreement. In calculation of conductivity of produced material, the influence of shape of nano-powders was applied and this factor has significant impact on freezing. The active factors of current study are concentration and shapes of nanoparticles and two levels have been utilized for reach of them. The augment of concentration makes the conduction to increase and penetration of cold flow increases which makes the solidification to accelerate. The temperature of zone reduces with rise of time and involving higher concentration of nanoparticles offers lower temperature. Also, the needed time can decline with soar of ϕ . The cylindrical and blade shapes were incorporated in this work and blade shape has greater conductivity which offers lower time for full solidification. Also, this shape has lower level of energy because of lower temperature of domain. With rise of ϕ with blade and cylindrical styles of particles, the required time decreases around 12.54% and 16.69%. Freezing time declines around 15.75% and 29.81% with adding nanoparticles depends the style of powders. Besides, with alter of style of particles from cylinder

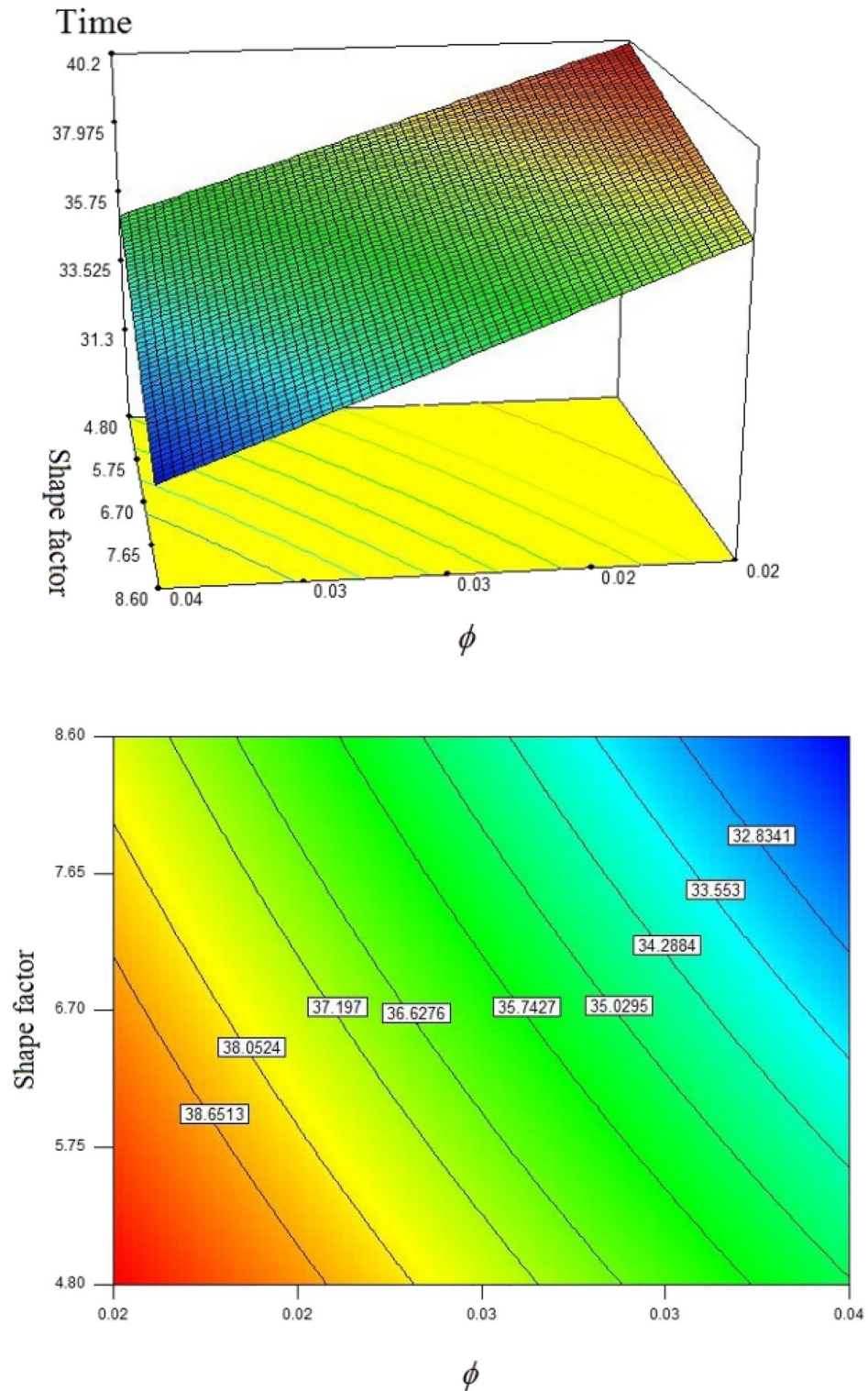


Figure 10. Change of factors and computed period of process.

to blade shapes, the needed time decreases by 6.07% and 10.53% at $\phi = 0.02$ and 0.04 . The solidification time for pure water case is about 44.71 s and with adding blade shapes nanoparticles, the needed time reaches to 31.37 s.

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References

- Li, F., Almarashi, A., Jafaryar, M., Hajizadeh, M. R. & Chu, Y.-M. Melting process of nanoparticle enhanced PCM through storage cylinder incorporating fins. *Powder Technol.* **381**, 551–560 (2021).
- Chu, Y.-M., Yadav, D., Shafee, A., Li, Z. & Bach, Q.-V. Influence of wavy enclosure and nanoparticles on heat release rate of PCM considering numerical study. *J. Mol. Liq.* **319**, 114121. <https://doi.org/10.1016/j.molliq.2020.114121> (2020).
- Xu, Y. H. *et al.* State of charge estimation of supercapacitors based on multi-innovation unscented Kalman filter under a wide temperature range. *Int. J. Energy Res.* <https://doi.org/10.1002/er.8334> (2022).
- Nazeer, M. *et al.* Theoretical study of MHD electro-osmotically flow of third-grade fluid in micro channel. *Appl. Math. Comput.* **420**, 126868. <https://doi.org/10.1016/j.amc.2021.126868> (2022).
- Chu, Y.-M., Hajizadeh, M. R., Li, Z. & Bach, Q.-V. Investigation of nano powders influence on melting process within a storage unit. *J. Mol. Liq.* **318**, 114321. <https://doi.org/10.1016/j.molliq.2020.114321> (2020).
- Zhou, J., Bai, J. & Liu, Y. Fabrication and modeling of matching system for air-coupled transducer. *Micromachines* **13**(5), 781. <https://doi.org/10.3390/mi13050781> (2022).
- Chu, Y.-M. *et al.* Combined impact of Cattaneo-Christov double diffusion and radiative heat flux on bio-convective flow of Maxwell liquid configured by a stretched nano-material surface. *Appl. Math. Comput.* **419**, 126883. <https://doi.org/10.1016/j.amc.2021.126883> (2022).
- Xiong, P.-Y. *et al.* Nanomaterial transportation and exergy loss modeling incorporating CVFEM. *J. Mol. Liq.* **330**(15), 115591. <https://doi.org/10.1016/j.molliq.2021.115591> (2021).
- Zhao, T.-H., Ijaz Khan, M. & Chu, Y.-M. Artificial neural networking (ANN) analysis for heat and entropy generation in flow of non-Newtonian fluid between two rotating disks. *Math. Methods Appl. Sci.* <https://doi.org/10.1002/mma.7310> (2021).
- Jiang, C. *et al.* Flexible interface design for stress regulation of a silicon anode toward highly stable dual-ion batteries. *Adv. Mater. (Weinheim)* **32**(17), e1908470. <https://doi.org/10.1002/adma.201908470> (2020).
- Zhao, T.-H., He, Z.-Y. & Chu, Y.-M. Sharp bounds for the weighted H^older mean of the zero-balanced generalized complete elliptic integrals. *Comput. Methods Funct. Theory* **21**(3), 413–426. <https://doi.org/10.1007/s40315-020-00352-7> (2021).
- Chu, Y. M., Bilal, S. & Hajizadeh, M. R. Hybrid ferrofluid along with MWCNT for augmentation of thermal behavior of fluid during natural convection in a cavity. *Math. Methods Appl. Sci.* **2020**, 1–12. <https://doi.org/10.1002/mma.6937> (2020).
- Xu, Y.-P. *et al.* Optimization of a biomass-driven Rankine cycle integrated with multi-effect desalination, and solid oxide electrolyzer for power, hydrogen, and freshwater production. *Desalination* **525**, 115486. <https://doi.org/10.1016/j.desal.2021.115486> (2022).
- Sheikholeslami, M., Jafaryar, M., Shafee, A. & Babazadeh, H. Acceleration of discharge process of clean energy storage unit with insertion of porous foam considering nanoparticle enhanced paraffin. *J. Clean. Prod.* **261**, 121206. <https://doi.org/10.1016/j.jclepro.2020.121206> (2020).
- Zhao, T.-H., Wang, M.-K. & Chu, Y.-M. Concavity and bounds involving generalized elliptic integral of the first kind. *J. Math. Inequal.* **15**(2), 701–724. <https://doi.org/10.7153/jmi-2021-15-50> (2021).
- Wu, N. *et al.* K-ion battery cathode design utilizing trigonal prismatic ligand field. *Adv. Mater. (Weinheim)* <https://doi.org/10.1002/adma.202101788> (2021).
- Xu, H.-Z., Qian, W.-M. & Chu, Y.-M. Sharp bounds for the lemniscatic mean by the one-parameter geometric and quadratic means. *Rev. R. Acad. Cienc. Exactas Físicas Nat. Ser. A Mat. RACSAM* **116**(1), 21. <https://doi.org/10.1007/s13398-021-01162-9> (2022).
- Li, J. *et al.* Heat recovery application of nanomaterial with existence of turbulator. *J. Mol. Liq.* **326**, 115268. <https://doi.org/10.1016/j.molliq.2020.115268> (2021).
- Karthikeyan, K., Karthikeyan, P., Baskonus, H. M., Venkatachalam, K. & Chu, Y.-M. Almost sectorial operators on Ψ -Hilfer derivative fractional impulsive integro-differential equations. *Math. Methods Appl. Sci.* <https://doi.org/10.1002/mma.7954> (2021).
- Li, Z. *et al.* In situ chemical lithiation transforms diamond-like carbon into an ultrastrong ion conductor for dendrite-free lithium-metal anodes. *Adv. Mater.* <https://doi.org/10.1002/adma.202100793> (2021).
- Sheikholeslami, M. Analyzing melting process of paraffin through the heat storage with honeycomb configuration utilizing nanoparticles. *J. Energy Storage* **52**, 104954. <https://doi.org/10.1016/j.est.2022.104954> (2022).
- Chu, Y.-M. *et al.* Nanoparticle enhanced PCM exergy loss and thermal behavior by means of FVM. *J. Mol. Liq.* **320**, 114457. <https://doi.org/10.1016/j.molliq.2020.114457> (2020).
- Chu, Y.-M., Nazir, U., Sohail, M., Selim, M. M. & Lee, J.-R. Enhancement in thermal energy and solute particles using hybrid nanoparticles by engaging activation energy and chemical reaction over a parabolic surface via finite element approach. *Fract. Fract.* **5**(3), 119. <https://doi.org/10.3390/fractalfract5030119> (2021).
- Zhang, Y. *et al.* Study on the suppression mechanism of (NH₄)₂CO₃ and SiC for polyethylene deflagration based on flame propagation and experimental analysis. *Powder Technol.* **399**, 117193. <https://doi.org/10.1016/j.powtec.2022.117193> (2022).
- Rashid, S., Sultana, S., Karaca, Y., Khalid, A. & Chu, Y.-M. Some further extensions considering discrete proportional fractional operators. *Fractals* **30**(1), 2240026. <https://doi.org/10.1142/S0218348X22400266> (2022).
- Valipour, P., Jafaryar, M., Moradi, R. & Aski, F. S. Two phase model for nanofluid heat transfer intensification in a rotating system under the effect of magnetic field. *Chem. Eng. Process.-Process Intensif.* **123**, 47–57 (2018).
- Chu, Y.-M. & Moradi, R. Computational investigation of non-uniform magnetic field on thermal characteristic of nanofluid stream inside 180 degree elbow pipe. *Mod. Phys. Lett. B* <https://doi.org/10.1142/S0217984921501578> (2020).
- Sheikholeslami, M. Numerical investigation of solar system equipped with innovative turbulator and hybrid nanofluid. *Sol. Energy Mater. Sol. Cells* **243**(15), 111786. <https://doi.org/10.1016/j.solmat.2022.111786> (2022).
- Zhao, T.-H., Qian, W.-M. & Chu, Y.-M. Sharp power mean bounds for the tangent and hyperbolic sine means. *J. Math. Inequal.* **15**(4), 1459–1472. <https://doi.org/10.7153/jmi-2021-15-100> (2021).
- Chu, Y.-M., Almusawi, M. B., Hajizadeh, M. R., Yao, S.-W. & Bach, Q.-V. Hybrid nanomaterial treatment within a permeable tank considering irreversibility. *Int. J. Mod. Phys. C* <https://doi.org/10.1142/S0129183121500613> (2020).
- Xu, Y. *et al.* Experimental investigation of pneumatic motor for transport application. *Renew. Energy* **179**, 517–527. <https://doi.org/10.1016/j.renene.2021.07.072> (2021).
- Sheikholeslami, M., Said, Z. & Jafaryar, M. Hydrothermal analysis for a parabolic solar unit with wavy absorber pipe and nanofluid. *Renew. Energy* **188**, 922–932. <https://doi.org/10.1016/j.renene.2022.02.086> (2022).
- Rashid, S., Abouelmagd, E. I., Sultana, S. & Chu, Y.-M. New developments in weighted n-fold type inequalities via discrete generalized h -proportional fractional operators. *Fractals* **30**(2), 2240056. <https://doi.org/10.1142/S0218348X22400564> (2022).
- Xu, Y.-P., Liu, R.-H., Tang, L.-Y., Wud, H. & She, C. Risk-averse multi-objective optimization of multi-energy microgrids integrated with power-to-hydrogen technology, electric vehicles and data center under a hybrid robust-stochastic technique. *Sustain. Cities Soc.* **79**, 103699. <https://doi.org/10.1016/j.scs.2022.103699> (2022).
- Chu, Y.-M., Bashir, S., Ramzan, M. & Malik, M. Y. Model-based comparative study of magnetohydrodynamics unsteady hybrid nanofluid flow between two infinite parallel plates with particle shape effects. *Math. Methods Appl. Sci.* <https://doi.org/10.1002/mma.8234> (2022).
- Babazadeh, H., Muhammad, T., Shakeriaski, F., Ramzan, M., Hajizadeh, M. R., Nanomaterial between two plates which are squeezed with impose magnetic force. *J. Therm. Anal. Calorim.* 1–7 (2020).
- Wang, T. *et al.* Approaches for expedition of discharging of PCM involving nanoparticles and radial fins. *J. Mol. Liq.* <https://doi.org/10.1016/j.molliq.2020.115052> (2020).

38. Zhang, X., Tang, Y., Zhang, F. & Lee, C. A novel aluminum-graphite dual-ion battery. *Adv. Energy Mater.* **6**(11), 1502588. <https://doi.org/10.1002/aenm.201502588> (2016).
39. Sheikholeslami, M. & Ebrahimpour, Z. Thermal improvement of linear fresnel solar system utilizing Al₂O₃-water nanofluid and multi-way twisted tape. *Int. J. Therm. Sci.* **176**, 107505. <https://doi.org/10.1016/j.jthermalsci.2022.107505> (2022).
40. Qian, W.-M., Chu, H.-H., Wang, M.-K. & Chu, Y.-M. Sharp inequalities for the Toader mean of order s - 1 in terms of other bivariate means. *J. Math. Inequal.* **16**(1), 127–141. <https://doi.org/10.7153/jmi-2022-16-10> (2022).
41. Yu, A., Pan, Q., Zhang, M., Xie, D. & Tang, Y. Fast rate and long life potassium-ion based dual-ion battery through 3D porous organic negative electrode. *Adv. Func. Mater.* **30**(24), 2001440. <https://doi.org/10.1002/adfm.202001440> (2020).
42. Chu, Y.-M., Salahshoor, Z., Shahraki, M. S., Shafee, A. & Bach, Q.-V. Annulus shape tank with convective flow in a porous zone with impose of MHD. *Int. J. Mod. Phys. C* <https://doi.org/10.1142/S0129183120501685> (2020).
43. Xiong, P.-Y. *et al.* Nanoparticles for phase change process of water utilizing FEM. *J. Mol. Liq.* <https://doi.org/10.1016/j.molliq.2021.116096> (2021).
44. Cao, X., Yuan, Y., Xiang, Bo., Sun, L. & Xingxing, Z. Numerical investigation on optimal number of longitudinal fins in horizontal annular phase change unit at different wall temperatures. *Energy Build.* **158**, 384–392. <https://doi.org/10.1016/j.enbuild.2017.10.029> (2018).
45. Elarem, R. *et al.* Numerical study of an evacuated tube solar collector incorporating a nano-PCM as a latent heat storage system. *Case Stud. Therm. Eng.* **24**, 100859. <https://doi.org/10.1016/j.csite.2021.100859> (2021).
46. Kant, K., Shukla, A., Sharma, A. & Biwole, P. H. Melting and solidification behaviour of phase change materials with cyclic heating and cooling. *J. Energy Storage* **15**, 274–282 (2018).
47. Mehta, D. S., Solanki, K., Rathod, M. K. & Banerjee, J. Thermal performance of shell and tube latent heat storage unit: Comparative assessment of horizontal and vertical orientation. *J. Energy Storage* **23**, 344–362 (2019).
48. Usman, H. *et al.* An experimental study of PCM based finned and un-finned heat sinks for passive cooling of electronics. *Heat Mass Transf.* **54**(12), 3587–3598. <https://doi.org/10.1007/s00231-018-2389-0> (2018).
49. Bouteldja, M., Mezaache, E. H. & Aouer, A. Numerical study of the solidification of phase change materials in a rectangular cavity: Effects of convection and aspect ratio. *Ann. Chim. Sci. Mater.* **43**(1), 1–9. <https://doi.org/10.18280/acsm.430101> (2019).
50. Chen, S.-B. *et al.* Combined effect of using porous media and nano-particle on melting performance of PCM filled enclosure with triangular double fins. *Case Stud. Therm. Eng.* **25**, 100939. <https://doi.org/10.1016/j.csite.2021.100939> (2021).
51. Elsayed, A. O. Numerical investigation on PCM melting in triangular cylinders. *Alex. Eng. J.* **57**(4), 2819–2828. <https://doi.org/10.1016/j.aej.2018.01.005> (2018).
52. Fadl, M. & Eames, P. C. An experimental investigations of the melting of RT44HC inside a horizontal rectangular test cell subject to uniform wall heat flux. *Int. J. Heat Mass Transf.* **140**, 731–742. <https://doi.org/10.1016/j.ijheatmasstransfer.2019.06.047> (2019).
53. Sheikholeslami, M. Numerical simulation for solidification in a LHTESS by means of Nano-enhanced PCM. *J. Taiwan Inst. Chem. Eng.* **86**, 25–41 (2018).

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Author contributions

A.A and A.M H supervised and examined the verification and M.M modeled and simulated the problem. C.Z wrote the revised manuscript and H.A.S edited the version grammatically.

Competing interests

The authors declare no competing interests.

Additional information

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