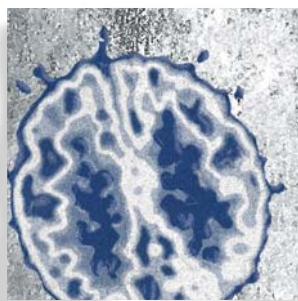


A history of chaos theory

Christian Oestreicher, PhD



Whether every effect can be precisely linked to a given cause or to a list of causes has been a matter of debate for centuries, particularly during the 17th century, when astronomers became capable of predicting the trajectories of planets. Recent mathematical models applied to physics have included the idea that given phenomena cannot be predicted precisely, although they can be predicted to some extent, in line with the chaos theory. Concepts such as deterministic models, sensitivity to initial conditions, strange attractors, and fractal dimensions are inherent to the development of this theory. A few situations involving normal or abnormal endogenous rhythms in biology have been analyzed following the principles of chaos theory. This is particularly the case with cardiac arrhythmias, but less so with biological clocks and circadian rhythms.

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Science enables an interpretation of nature and, in most cases, this is done on the basis of *models*, particularly mathematical ones. Thus, equations constructed by the human brain are considered to be adequate representations of reality, and this concordance between thinking and the environment is a gift quite specific to humanity.

Scientific theories are characterized by the fact that they remain open to refutation through experimental studies, while mathematical models are noncontradictory (in the sense of mathematical logic) and deduced from a list of axioms. Physicists, biologists, and medical researchers have the mission of understanding whether events from the world follow given mathematical laws or not. Indeed, there are three successive steps in constructing such a model: first the observation of the phenomenon, then its translation into equations, then the solving of these equations. Obviously, researchers in biological sciences and in medicine place emphasis on the first step, that of the observation of the phenomena, in order to understand components of a phenomenon, ie, to establish a body of knowledge that could then be translated into models and equations.

Chaos theory is a mathematical theory, and it is still in development. It enables the description of a series of phenomena from the field of dynamics, ie, that field of physics concerning the effect of forces on the motion of objects. The archetype of all theories of dynamics is that of Newton, concerning celestial motions.

When employing mathematical theorems, one should remain careful about whether their *hypotheses are valid* within the frame of the questions considered. Among such hypotheses in the domain of dynamics, a central one is the *continuity of time and space* (ie, that an infinity of

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points exists between two points). This hypothesis, for example, may be invalid in the cognitive neurosciences of perception, where a finite time threshold often needs to be considered.

This article presents the major historical steps in the acquisition of knowledge in physics that led to chaos theory. Since these steps were made in fields other than biology or medicine, these will be referred to, in particular astronomy. Some readers might not be familiar with physics or mathematics; therefore explanations using the language of equations have been kept to a minimum. It is, however, necessary to use the appropriate terms and concepts, and *Table I* provides a list of definitions according to the concepts of physics.

The review is focused on those strictly deterministic dynamic systems that present the peculiarity of *being sensitive to initial conditions*, and, when they have a propriety of recurrence, cannot be predicted over the long term. Chaos theory has a few applications for modeling endogenous biological rhythms such as heart rate, brain functioning, and biological clocks.

The roots of modern science

Newton and causality

Johannes Kepler published the three laws of planetary motion in his two books of 1609¹ and 1618,² and Galileo

Table I. Definitions of concepts related to the history of chaos theory.*

• Causality principle. Every effect has an antecedent, proximate cause.
• Determinism. A philosophical proposition that every event is physically determined by an unbroken chain of prior occurrences.
• Predictability. This refers to the degree that a correct forecast of a system's state can be made either qualitatively or quantitatively.
• Model. A pattern, plan, representation, or description designed to show the structure or workings of an object, system, or concept.
• Dynamical system. A system that changes over time in both a causal and a deterministic manner, ie, its future depends only on phenomena from its past and its present (causality) and each given initial condition will lead to only one given later state of the system (determinism). Systems that are noisy or stochastic, in the sense of showing randomness, are not dynamical systems, and the probability theory is the one to apply to their analysis.
• Phase space. An abstract space in which all possible states of a system are represented which, each possible state of the system corresponding to one unique point in the phase space.
• Sensitivity to initial conditions. This is when a change in one variable has the consequence of an exponential change in the system.
• Integrable system. In mathematics, this refers to a system of differential equations for which solutions can be found. In mechanics, this refer to a system that is quasiperiodic.
• Linear system. A system is said to be linear when the whole is exactly equal to the sum of its components.
• Attractor. A set to which a dynamical system evolves after a long enough time.
• Characteristic Lyapunov time. The characteristic time of a system is defined as the delay when changes from the initial point are multiplied by 10 in the phase space.
• Feedback. A response to information, that either increases effects (positive feedback), or decreases them (negative feedback), or induces a cyclic phenomenon.
• Self-similarity. This means that an object is composed of subunits and sub-subunits on multiple levels that (statistically) resemble the structure of the whole object. However, in every day life, there are necessarily lower and upper boundaries over which such self-similar behavior applies.
• Fractal. Is a geometrical object satisfying two criteria: self-similarity and fractional dimensionality.
• Fractal dimension. Let an object in a n-dimensions space be covered by the smallest number of open spheres of radius r. The fractal dimension is $\log(N)/\log(1/r)$ when r tends towards 0.

Table I. Definitions of concepts related to the history of chaos theory.*

*Some of the terms are used with different meanings in fields other than physics. For example, the adjective linear pharmacokinetics describes a body clearance of a constant fraction per unit of time of the total amount of a substance in the body, while a nonlinear pharmacokinetics describes the elimination of a constant quantity of compound per unit of time. Also, feedback is a well-known term in biology or medicine, while its use in physics is less familiar to nonphysicists.

Galilei wrote, in 1623³:

Philosophy is written in this vast book, which continuously lies open before our eyes (I mean the universe). But it cannot be understood unless you have first learned to understand the language and recognize the characters in which it is written. It is written in the language of mathematics, and the characters are triangles, circles, and other geometrical figures.

The *principle of causality* (Table I), perhaps the most basic of all principles of physics, is directly derived from the philosophy of René Descartes in his 1641 Third Meditation.⁴ The principle of causality is nonrefutable, ie, not confirmed by experience, since it is an axiom that precedes experiences. For example, this principle is accepted a priori in physics. In a simple form, it reads: “Every effect has a cause.”

In 1687 Isaac Newton then consolidated the causality principle by asserting that the two concepts of initial conditions and *law of motion* had to be considered separately.⁵ In order to calculate the planets' trajectories, Newton simplified the model and assumed that each planet was singly related to the sun, and his calculation was concordant with Kepler's laws.

Newton, having developed differential calculus and written the gravitational law, can be seen as the researcher who launched the development of classical science, ie, physics up to the beginning of the 20th century (before relativity and quantum mechanics). Newton wrote his manuscript on differential calculus in 1669,⁶ but it remained unpublished by his publisher until 1711. Gottfried Wilhelm von Leibniz had another point of view on this theme, and he published his book in 1684.⁷ A conflict occurred between the two men; Newton took Leibniz to court, accusing him of having stolen his ideas. Yet, later it was Leibniz' ideas that were used. These steps in the acquisition of human knowledge are described in Arthur Koestler's books on astronomy⁸ and mentioned in science dictionaries.^{9,10}

Laplace and determinism

Determinism is predictability based on scientific causality (Table I). One distinguishes schematically between local and universal determinism. Local determinism concerns a finite number of elements. A good illustration would be ballistics, where the trajectory and the site of impact of a projectile can be precisely predicted (on the basis of the propulsive force of the powder, the angle of

shooting, the projectile mass, and the air resistance). Local determinism raises no particular problem. In contrast, universal determinism, also called in French “*déterminisme laplacien*” remains problematic: how can one consider the universe in its totality as a deterministic system? Obviously, one cannot. The French philosopher d'Holbach, coauthor of the *Encyclopédie de Diderot et d'Alembert*, was the first to include, in chapter IV of his 1770 book *Le système de la nature*,¹¹ a deterministic statement about the feasibility of calculating the effects of a given cause.

In a whirlwind of dust, raised by elemental force, confused as it appears to our eyes, in the most frightful tempest excited by contrary winds, when the waves roll high as mountains, there is not a single particle of dust, or drop of water, that has been placed by chance, that has not a cause for occupying the place where it is found; that does not, in the most rigorous sense of the word, act after the manner in which it ought to act; that is, according to its own peculiar essence, and that of the beings from whom it receives this communicated force. A geometrician exactly knew the different energies acting in each case, with the properties of the particles moved, could demonstrate that after the causes given, each particle acted precisely as it ought to act, and that it could not have acted otherwise than it did.

However, it was the mathematician and astronomer Pierre-Simon Laplace who most clearly stated the concept of universal determinism shortly after d'Holbach, in 1778¹²:

We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the motions of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Laplace is also famous for his exchange with Napoleon asking about his work: “You have written this huge book on the system of the world without once mentioning the author of the universe.” To this Laplace responded: “Sire, I had no need of that hypothesis.”¹³ These words attest to the self-confidence of this man.

The creativity of Laplace was tremendous. He demonstrated that the totality of celestial body motions (at his time, the sun and the planets) could be explained by the

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law of Newton, reducing the study of planets to a series of differential equations.

Urbain Jean Joseph Le Verrier discovered the planet Neptune in 1848, only through calculation and not through astronomical observation. He then developed further Laplace's methods (by, for example, approximating solutions to equations of degree 7) and concluded¹⁴:

It therefore seems impossible to use the method of successive approximations to assert, by virtue of the terms of the second approximation, whether the system comprising Mercury, Venus, Earth, and Mars will be stable indefinitely. It is to be hoped that geometricians, by integrating the differential equations, will find a way to overcome this difficulty, which may well just depend on form.

In the middle of the 19th century, it became clear that the motion of gases was far more complex to calculate than that of planets. This led James Clerk Maxwell and Ludwig Boltzmann to found *statistical physics*. One of their main postulates was the following: an isolated system in equilibrium is to be found in all its accessible microstates with equal probability. In 1859, Maxwell described the viscosity of gases as a function of the distance between two collisions of molecules and he formulated a *law of distribution* of velocities.

Boltzmann assumed that matter was formed of particles (molecules, atoms) an unproven assumption at his time, although Democritus had already suggested this more than 2000 years previously. He postulated that these particles were in perpetual random motion. It is from these considerations that Boltzmann gave a mathematical expression to entropy. In physical terms, entropy is the measure of the uniformity of the distribution of energy, also viewed as the quantification of randomness in a system. Since the particle motion in gases is unpredictable, a probabilistic description is justified.

Changes over time within a system can be modeled using the a priori of a continuous time and differential equation(s), while the a priori of a discontinuous time is often easier to solve mathematically, but the interesting idea of discontinuous time is far from being accepted today.

Birth of the chaos theory

Poincaré and phase space

With the work of Laplace, the past and the future of the solar system could be calculated and the precision of

this calculation depended on the capacity to know the initial conditions of the system, a real challenge for “geometricians,” as alluded to by d’Holbach and Le Verrier. Henri Poincaré developed another point of view,¹⁵ as follows: in order to study the evolution of a physical system over time, one has to construct a model based on a choice of laws of physics and to list the necessary and sufficient parameters that characterize the system (differential equations are often in the model). One can define the state of the system at a given moment, and the set of these system states is named *phase space* (see Table I).

The phenomenon of *sensitivity to initial conditions* (Table I) was discovered by Poincaré in his study of the *n-body problem*, then by Jacques Hadamard using a mathematical model named geodesic flow, on a surface with a nonpositive curvature, called Hadamard’s billiards. A century after Laplace, Poincaré indicated that randomness and determinism become somewhat compatible because of the long-term unpredictability.

A very small cause, which eludes us, determines a considerable effect that we cannot fail to see, and so we say that this effect is due to chance. If we knew exactly the laws of nature and the state of the universe at the initial moment, we could accurately predict the state of the same universe at a subsequent moment. But even if the natural laws no longer held any secrets for us, we could still only know the state approximately. If this enables us to predict the succeeding state to the same approximation, that is all we require, and we say that the phenomenon has been predicted, that it is governed by laws. But this is not always so, and small differences in the initial conditions may generate very large differences in the final phenomena. A small error in the former will lead to an enormous error in the latter. Prediction then becomes impossible, and we have a random phenomenon.

This was the birth of chaos theory.

Kolmogorov and the statistics of dynamical systems

Andreï Nicolaïevitch Kolmogorov is surely one of the most important mathematicians of the 20th century, his name being associated with the probability theory, turbulence, information theory, and topology, among other achievements. When Kolmogorov, in 1954,¹⁶ revisited the work of Poincaré (before Jürgen K. Moser in 1962, and Vladimir Igorevitch Arnold in 1963), he showed further that a quasiperiodic regular motion can persist

in an integrable system (*Table I*) even when a slight perturbation is introduced into the system. This is known as the KAM (Kolmogorov-Arnold-Moser) theorem which indicates limits to integrability. The theorem also describes a progressive transition towards chaos: within an integrable system, all trajectories are regular, quasiperiodic; introducing a slight perturbation one still has a probability of 1 to observe a quasiperiodic behavior (within a point chosen arbitrarily in the phase space). When a more significant perturbation is introduced, the probability of a quasiperiodic behavior decreases and an increasing proportion of trajectories becomes chaotic, until a completely chaotic behavior is reached. In terms of physics, in complete chaos, the remaining constant of motion is only energy and the motion is called ergodic. Kolmogorov led the Russian school of mathematics towards research on the statistics of dynamical complex system called the *ergodic theory*.¹⁷

In a *linear system* (*Table I*), the sum of causes produces a corresponding sum of effects and it suffices to add the behavior of each component to deduce the behavior of the whole system. Phenomena such as a ball trajectory, the growth of a flower, or the efficiency of an engine can be described according to linear equations. In such cases, small modifications lead to small effects, while important modifications lead to large effects (a necessary condition for reductionism).

The *nonlinear equations* concern specifically discontinuous phenomena such as explosions, sudden breaks in materials, or tornados. Although they share some universal characteristics, nonlinear solutions tend to be individual and peculiar. In contrast to regular curves from linear equations, the graphic representation of nonlinear equations shows breaks, loops, recursions - all kinds of turbulences. Using nonlinear models, one can identify critical points in the system at which a minute modification can have a disproportionate effect (a sufficient condition for holism).

The above observations from the field of physics have been applied in other fields, in the following manner: in the terms of reductionism, the whole can be analyzed by studying each of its constituents, while in holism, the whole is more than the sum of its constituents, and therefore cannot be deduced from its parts. When should one analyze rhythmic phenomena with reductionist versus holistic models? This is a question that one can ask in the field of chronobiology.

Rebirth of chaos theory

Lorenz and the butterfly effect

Edward Lorenz, from the Massachusetts Institute of Technology (MIT) is the official discoverer of chaos theory. He first observed the phenomenon as early as 1961 and, as a matter of irony, he discovered by chance what would be called later the chaos theory, in 1963,¹⁸ while making calculations with uncontrolled approximations aiming at predicting the weather. The anecdote is of interest: making the same calculation rounding with 3-digit rather than 6-digit numbers did not provide the same solutions; indeed, in nonlinear systems, multiplications during iterative processes amplify differences in an exponential manner. By the way, this occurs when using computers, due to the limitation of these machines which truncate numbers, and therefore the accuracy of calculations.

Lorenz considered, as did many mathematicians of his time, that a small variation at the start of a calculation would induce a small difference in the result, of the order of magnitude of the initial variation. This was obviously not the case, and all scientists are now familiar with this fact. In order to explain how important sensitivity to the initial conditions was, Philip Merilees, the meteorologist who organized the 1972 conference session where Lorenz presented his result, chose himself the title of Lorenz's talk, a title that became famous: "Predictability: does the flap of a butterfly's wing in Brazil set off a tornado in Texas?"¹⁹ This title has been cited and modified in many articles, as humorously reviewed by Nicolas Witkowski.²⁰ Lorenz had rediscovered the chaotic behavior of a nonlinear system, that of the weather, but the term chaos theory was only later given to the phenomenon by the mathematician James A. Yorke, in 1975.²¹ Lorenz also gave a graphic description of his findings using his computer. The figure that appeared was his second discovery: the *attractors*.

Ruelle and strange attractors

The Belgian physicist David Ruelle studied this figure and he coined the term *strange attractors* in 1971.²² The clearly recognizable trajectories in the phase space never cut through one another, but they seemed to form cycles that are not exactly concentric, not exactly on the same plan. It is also Ruelle who developed the thermodynamic

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formalism.²³ The strange attractor is a representation of a chaotic system in a specific phase space, but attractors are found in many dynamical systems that are nonchaotic. There are four types of attractors. *Figure 1* describes these types: fixed point, limit-cycle, limit-torus, and strange attractor.

According to Newton's laws, we can describe perfectly the future trajectories of our planet. However, these laws may be wrong at the dimension of the universe, because they concern only the solar system and exclude all other astronomical parameters. Then, while the earth is indeed to be found repetitively at similar locations in relation to the sun, these locations will ultimately describe a figure, ie, the strange attractor of the solar system.

A chaotic system leads to amplification of initial dis-

tances in the phase space; two trajectories that are initially at a distance D will be at a distance of 10 times the value of D after a delay of once the value of characteristic Lyapunov time (*Table I*). If the characteristic Lyapunov time of a system is short, then the system will amplify its changes rapidly and be more chaotic. However, this amplification of distances is restricted by the limits of the universe; from a given state, the amplification of the system has to come to an end. It is within the amplification of small distances that certain mathematicians, physicists, or philosophers consider that one can find randomness. The solar system characteristic Lyapunov time is evaluated to be in the order of 10 000 000 years.

The terms of negative and positive *feedback* (*Table I*) concern interactions that are respectively regulations and amplifications. An example of negative feedback is the regulation of heat in houses, through interactions of heating apparatus and a thermostat. Biology created negative feedback long ago, and the domain of endocrinology is replete with such interactions. An example of positive feedback would be the Larsen effect, when a microphone is placed close to a loudspeaker. In biology, positive feedbacks are operative, although seemingly less frequent, and they can convey a risk of amplification. Negative and positive feedback mechanisms are ubiquitous in living systems, in ecology, in daily life psychology, as well as in mathematics. A feedback does not greatly influence a linear system, while it can induce major changes in a nonlinear system. Thus, feedback participates in the frontiers between order and chaos.

The golden age of chaos theory

Feigenbaum and the logistic map

Mitchell Jay Feigenbaum proposed the scenario called *period doubling* to describe the transition between a regular dynamics and chaos. His proposal was based on the *logistic map* introduced by the biologist Robert M. May in 1976.^{24,25} While so far there have been no equations this text, I will make an exception to the rule of explaining physics without writing equations, and give here a rather simple example. The logistic map is a function of the segment $[0,1]$ within itself defined by:

$$x_{n+1} = rx_n(1 - x_n)$$

where $n = 0, 1, \dots$ describes the discrete time, the single dynamical variable, and $0 \leq r \leq 4$ is a parameter. The

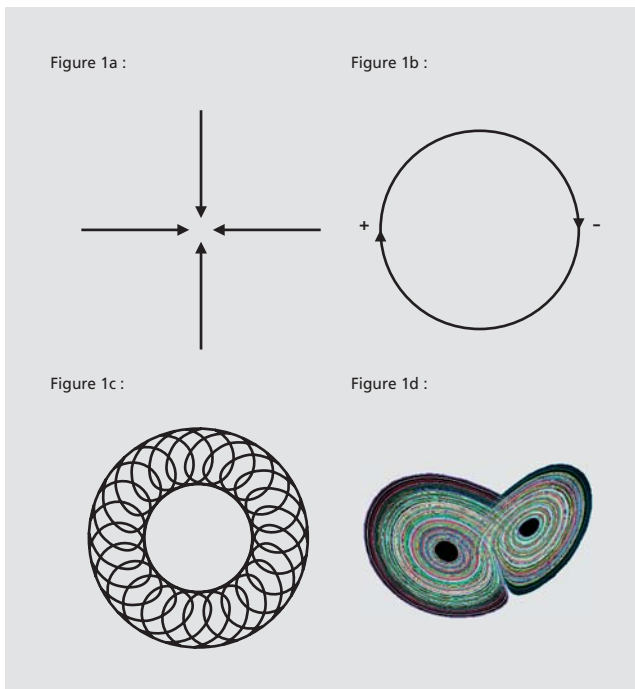


Figure 1. a. Fixed point: a point that a system evolves towards, such as the final states of a damped pendulum.

b. Limit cycle: a periodic orbit of the system that is isolated. Examples include the swings of a pendulum clock and the heart-beat while resting.

c. Limit-torus: there may be more than one frequency in the periodic trajectory of the system through the state of a limit cycle. If two of these frequencies form an irrational ratio, the trajectory is no longer closed, and the limit cycle becomes a limit torus.

d. Strange attractor: it characterizes the behavior of chaotic systems in a phase space. The dynamics of satellites in the solar system is an example. This figure shows a plot of Lorenz's attractor.

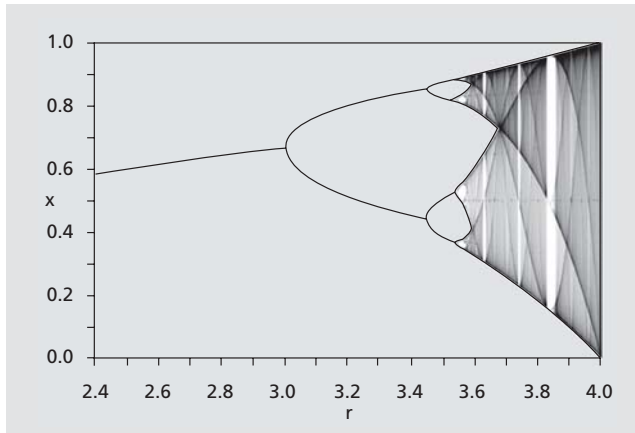


Figure 2. The horizontal axis shows the values of the parameter r while the vertical axis shows the possible long-term values of x .

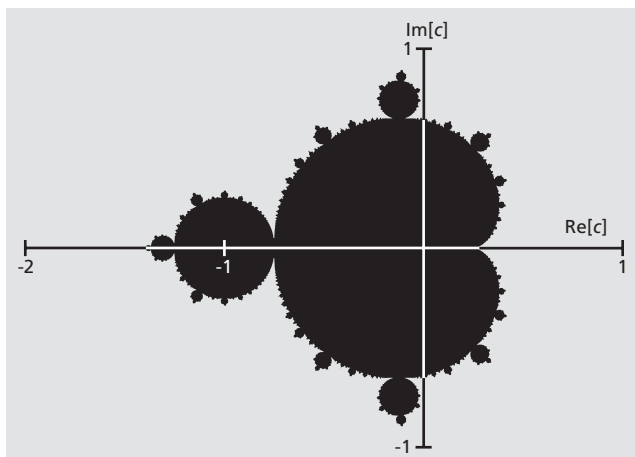


Figure 3. The Mandelbrot set, a point c is colored black if it belongs to the set, and white if not.

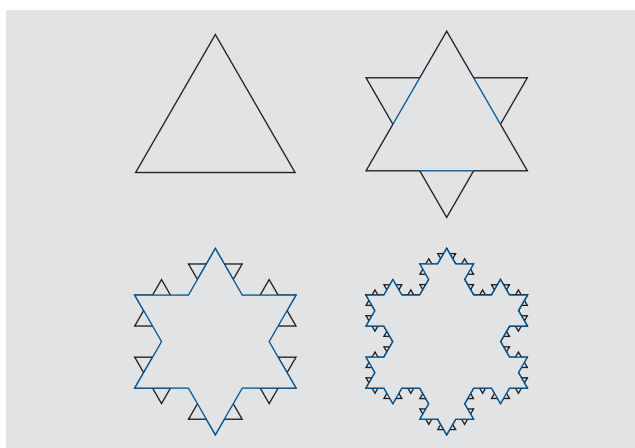


Figure 4. The first four iterations of the Koch snowflake.

dynamic of this function presents very different behaviors depending on the value of the parameter r :

For $0 \leq r < 3$, the system has a fixed point attractor that becomes unstable when $r=3$.

Pour $3 \leq r < 3,57\dots$, the function has a periodic orbit as attractor, of a period of 2^n where n is an integer that tends towards infinity when r tends towards $3,57\dots$

When $r=3,57\dots$, the function then has a Feigenbaum fractal attractor.

When over the value of $r=4$, the function goes out of the interval $[0,1]$ (Figure 2).

This function of a simple beauty, in the eyes of mathematicians I should add, has numerous applications, for example, for the calculation of populations taking into account only the initial number of subjects and their growth parameter r (as birth rate). When food is abundant, the population increases, but then the quantity of food for each individual decreases and the long-term situation cannot easily be predicted.

Mandelbrot and fractal dimensions

In 1973, Benoît Mandelbrot, who first worked in economics, wrote an article about new forms of randomness in science.²⁶ He listed situations where, in contrast to the classical paradigm, incidents do not compensate for each other, but are additive, and where statistical predictions become invalid. He described his theory in a book,²⁷ where he presented what is now known as the Mandelbrot set. This is a fractal defined as the set of points c from the complex plane for which the recurring series defined by $z_{n+1} = z_n^2 + c$, with the condition $z_0 = 0$, remains bounded (Figure 3).

A characteristic of fractals is the repetition of similar forms at different levels of observation (theoretically at all levels of observation). Thus, a part of a cloud looks like the complete cloud, or a rock looks like a mountain. Fractal forms in living species are for example, a cauliflower or the bronchial tree, where the parts are the image of the whole. A simple mathematical example of a fractal is the so-called Koch curve, or Koch snowflake.²⁸ Starting with a segment of a straight line, one substitutes the two sides of an equilateral triangle to the central third of the line. This is then repeated for each of the smaller segments obtained. At each substitution, the total length of the figure increased by $4/3$, and within 90 substitutions, from a 1-meter segment, one obtains the distance from the earth to the sun (Figure 4).

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Fractal objects have the following fundamental property: the finite (in the case of the Koch snowflake, a portion of the surface) can be associated with the infinite (the length of the line). A second fundamental property of fractal objects, clearly found in snowflakes, is that of *self-similarity*, meaning that parts are identical to the whole, at each scaling step.

A few years later, Mandelbrot discovered fractal geometry and found that Lorenz's attractor was a fractal figure, as are the majority of strange attractors. He defined fractal dimension (*Table 1*). Mandelbrot quotes, as illustration of this new sort of randomness, the French coast of Brittany; its length depends on the scale at which it is measured, and has a fractal dimension between 1 and 2. This coast is neither a one-dimensional nor a two-dimensional object. For comparison the dimension of Koch snowflake is 1.26, that of Lorenz's attractor is around 2.06, and that of the bifurcations of Feigenbaum is around 0.45.

Thom, Prigogine, and determinism again

René Thom is the author of *catastrophe theory*.²⁹ This theory is akin to chaos theory, but it was constructed from the study of singularities, ie, continuous actions that produce discontinuous results. Catastrophe theory is interesting in that it places much emphasis on explanation rather than measurement. Thom was at the origin of a renewed debate on the issue of determinism. In a 1980 article "Stop chance, silence the noise,"³⁰ that was then received with much controversy, he said:

I'd like to say straight away that this fascination with randomness above all bears witness to an unscientific attitude. It is also to a large degree the result of a certain mental confusion, which is forgivable in authors with a literary training, but hard to excuse in scientists experienced in the rigors of rational enquiry. What in fact is randomness? Only a purely negative definition can be given: a random process cannot be simulated by any mechanism or described by any formalism. Asserting that "chance exists" is tantamount to the ontological position that there are natural phenomena that we will never be able to describe, nor therefore to understand.

Ilya Prigogine, author of a theory of dissipative structures in thermodynamics, considers that the universe is neither totally deterministic nor totally stochastic.³¹ He speaks of a generalization of dynamics at the level of statistics that has no equivalent in terms of trajectories. Initial condi-

tions can no longer be assimilated to a point in the phase space, but they correspond to a region described by a probability distribution. It is a nonlocal description, a new paradigm.

Turing and self-organization

The mathematician Alan Turing, famous for his work on cybernetics and artificial intelligence, showed that the synergy between reaction and diffusion could lead to spontaneous modes of concentrations.³² He proposed that such mechanisms might explain the occurrence of structured rules in the ontogenesis of living species. A morphogen is a substance participating in reactions generating forms. Morphogens (growth factors, transcription factors, or other endogenous compounds) influence in a spatial and temporal manner, the expression of series of genes; this influence is very precise, possibly because morphogens are rapidly synthesized, but diffuse more slowly, and this discrepancy would lead to periodical maximal values of concentration. This model proposed by Turing enables to explain several phenomena: stationary structures, oscillations, chemical waves.

The phenomenon of spontaneous exchange of information was used by biologists such as Meinhardt and Gierer³³ in their explanation of the periodic structure of leaves. This kind of self-organized chemical reactions could also explain the emergence of the zebra skin or a quantity of biological phenomena that illustrate self-organizing structures.

Discussion

Poincaré showed that some dynamical nonlinear systems had unpredictable behaviors. A century later, deterministic chaos, or the chaos theory, is much debated. Biologists, economists, specialists in social sciences, and researchers in medicine call themselves chaoticists. Moreover, debates on the chaos theory are no longer limited to groups of scientists having an extended knowledge in mathematics, but is widely found through the media, with participation from philosophers, psychoanalysts, journalists, or movie makers. This suggests that the concepts of chaos theory find some resonance in present social and philosophical preoccupations.

A superficial analysis might lead to the conclusion that the success of the chaos theory has only a semantic origin: the term deterministic chaos being constructed as an oxy-

moron, as are the successful terms of “virtual reality” or “artificial intelligence.” Moreover, terms such as equilibrium, unpredictability, and strange attractors that are at the core of the chaos theory, can easily be used in many other contexts and meanings.

Some researchers in the field of social sciences even propose that the chaos theory offers a revolutionary new paradigm, away from the materialistic utopia, and that social system should be maintained *at the edge of chaos*, between too much and too little authoritarian control. This comment concerns politics rather than physics.

The specificity of present time physics, with entropy, chaos, and fractal dimensions, confers reality to phenomena as we can perceive and measure them, and it somehow invalidates the idea of a fundamental, or true, reality that might be explained by an elegant model. The use of such models entails too many simplifications, and may lead for instance to the reversibility of time that is imposed by the mathematical structure of mechanics.

The initial conditions of the universe with mass, charge of particles, size of atoms, fundamental forces, speed of light, combination of carbon and oxygen, and many others happened to be organized in such a way that life could appear, and with it consciousness. This could suggest that the destiny of the universe is not towards an inevitable and generalized chaos. On the contrary, this destiny might be oriented towards complexity.

Chaos theory and medicine

Many discoveries in medicine can be seen as indications that organs function in a linear and deterministic manner, and that the causality principle applies to normal or abnormal physiology: for example, pressure increases when arteries constrict, and obstruction of an artery from the heart leads to angina pectoris. In chronobiology, destruction of the suprachiasmatic nucleus alters circadian rhythms, and genetic crossing of insects strains with different circadian clock gene modifies the period of circadian rhythms in a predictable manner, etc.

These obvious findings are numerous and they might hide, to some extent, the fact that bodily functions and their temporal coordination are probably under laws that are inherently complex. Indeed, living species are capable of increasing their complexity, to organize orderly functions from disorder (in terms of physics, not medicine), and they do this without external informational input. Thus, living species exhibit some complex chaotic systems.

However, in order to confirm this, it is necessary to have access to a huge number of data points; only then does it become possible to describe a biological phenomenon in its phase space and to study its evolution over time, using the methods of nonlinear dynamic systems. These methods have shed light on a few aspects of organ functioning, in particular the cardiovascular system³⁴⁻³⁶ and the brain,^{37,38} but also the respiratory system.^{39,40}

Indeed, the cardiac rhythm is sensitive to initial conditions and to the fractal dimension of its attractor, and it was found that when the heart rate becomes highly regular, the heart is less capable of adaptation to demands, and that this condition predisposes to arrhythmias and myocardial infarction. This chaotic behavior of the cardiac rhythm raises the essential question of the role of chaos in biology in general. In fact, the cardiac system could not function without chaos, since the power of self-organization participates in the capacity of the heart to adapt to physiological demands.

Dynamical models of the brain are also a domain of research, notably into the artificial neural networks. There is, however, a risk linked to self-reference stating that we try to understand the brain using our brains, a somewhat problematic circular approach.¹⁴ However, it might be that the functional assembly of many researchers' brains could in itself lead to more than the sum of the constituents of a model brain.

Brain disorders are accompanied by measurable changes in the electroencephalogram (EEG), most obvious in a series of epileptic fits, where high-amplitude synchronized waves are observed. The various types of EEG patterns present different attractors and different fractal dimensions. When a healthy person stands with his or her eyes open, the EEG shows low-amplitude high-frequency alpha waves, and the corresponding attractor has a high dimension. When the eyes are shut, EEG wave amplitude increases, frequency decreases, and the corresponding fractal dimension is lower. It is small during slow-wave sleep, and even more so during an epileptic fit or a coma. Thus one can conclude that the cognitive power, defined as the capacity to perceive and analyze information, parallels the fractal dimension of the EEG.⁴² In rabbits, electrophysiological measurements in the olfactory lobe show chaotic behavior when the animal is in a resting condition, while the presentation of odors leads to different patterns of electrical neuronal activity, less chaotic and nearly periodic.⁴³

Arnold Mandell was the first psychiatrist to combine

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abstract mathematical models and quantitative experimental finding in an effort to approach clinical questions, for example that of the structure of personality.⁴⁴ Others are now applying models of a sequentially altered architecture to describe psychiatric disorders. For example, it was proposed that schizophrenia was characterized by nonlinear phenomena alternating with pure randomness in the brain function architecture, a proposal in line with the early theoretical work of Prigogine.⁴⁵⁻⁴⁸

Finally, mathematical models have been used to describe biological rhythms and to explore the biological clocks functional rules, eg, to explore how biological clocks interact.⁴⁹⁻⁵⁵ The success of the chaos theory seems to be, in my impression, due to epistemology: the fact that a phenomenon obeying deterministic laws could be unpredictable can be seen as a sign of the defeat of the causality principle. In several cases, this conclusion seems to apply to chronobiology. □

Una historia acerca de la teoría del caos

Por centurias el hecho que cada efecto se pueda relacionar con precisión a una causa determinada o a una lista de causas, ha sido tema de debate, en especial durante el siglo XVII cuando los astrónomos fueron capaces de predecir la trayectoria de los planetas. La aplicación reciente de modelos matemáticos a la física ha incorporado la idea que determinados fenómenos no pueden predecirse con precisión; sin embargo, de alguna forma sí se pueden predecir de acuerdo con la teoría del caos. Los conceptos de modelos deterministas, sensibilidad a las condiciones iniciales, atractores extraños y dimensiones fractales son esenciales para el desarrollo de esta teoría. En biología son pocas las situaciones que incluyendo ritmos endógenos normales o anormales, se hayan analizado según los principios de la teoría del caos. Esto ha ocurrido principalmente con las arritmias cardíacas, pero en menor medida con los relojes biológicos y los ritmos circadianos.

Un historique de la théorie du chaos

Décider si chaque effet est lié ou non précisément à une cause donnée ou à une liste de causes a été l'objet de débat pendant des siècles, notamment au cours du XVII^e siècle, lorsque les astronomes ont été capables de prévoir les trajectoires des planètes. Des modèles mathématiques récents appliqués à la physique retiennent l'idée qu'un phénomène donné ne peut être prévu exactement, bien qu'il puisse l'être dans une certaine mesure, en accord avec la théorie du chaos. Des concepts tels que les modèles déterministes, la sensibilité aux conditions initiales, les attracteurs étranges et les dimensions fractales sont inhérents à cette théorie. Quelques situations concernant les rythmes endogènes normaux ou anormaux en biologie ont été analysées suivant les principes de la théorie du chaos. C'est surtout le cas des arythmies cardiaques, mais moins celui des horloges biologiques.

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