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Research article

Simulating wood companies development considering the effect of ethical marketing

Zhenhuan Chen [a], Hongge Zhu [a,*], Fei Liu [b,**]

[a] College of Economics and Management, Northeast Forestry University, Harbin 150040, China
[b] School of Public Administration, Hengshui University, Hengshui 05300, China

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ABSTRACT

Dedicating oneself to a greater good is one of the secret ingredients to business success in an evolving and highly interconnected markets environment. For wood companies, this is even more needed to face the overwhelming ongoing environmental crisis affecting business practices and customers' behavior in general. To build solid reputation and increase one's presence on competing markets, wood companies should not only care about products competitiveness, but also customers' perception, environmental awareness and ethical marketing. In this article, we simulated the effect of ethical commerce on wood companies' reputation using a modified competitive and cooperative model based on differential equations considering wood type, products categories and companies brand score or reputation as function of their commitment to social and environmental causes. The qualitative analysis and results of numerical experiment show that, focusing on sharing values, and building strong relationship with customers are highly important for long-term development. Moreover, reducing environmental impact by shifting production strategy towards man-made materials could meet today customers' demands for ethically and environmentally aware manufactured wood products.

1. Introduction

Sustaining stable development is difficult to achieve for wood companies in our fast changing markets environment submitted to uncontrollable factors such as human perception and environmental awareness. In such complex environment, ensuring customers' satisfaction is challenging as many factors related to human mind, human society, human perception or values, overall social responsibility and environmental cause's benefits of the consumed woods products have to be taken into account for long-term development plan. Although, production and processing of wood are known to be energy efficient or low carbon footprint compares to any other construction materials, it remains highly important to execute woodworking projects by including social and environmental ethics in the decision-making process. Particularly when it comes to categories and types of wood to work with, whether they are natural soft and hard wood or composite man-made materials [1–5].

Wood industries, as well as any other industries are bounded to give special attention to customer's perceptions by shifting reputation management to focus on social media and ORM (Online Reputation Management) according to many published related research works and current management practices. Increasing one's company online presence and managing customers perception are

* Corresponding author.
** Corresponding author.
E-mail addresses: honggebill@nefu.edu.cn (H. Zhu), liufei937087578@126.com (F. Liu).

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the best available methods for monitoring brand reputation in today fast moving society characterized by the proliferation of unpredictable contents with potential damage to given brand reputation. Suppressing negative information and exchanging with customers to clear up false allegations and other bad disinformation are parts of the strategy of knowing customers perception for effective marketing development [6–9].

With the ongoing environmental crisis, people are more likely to be shifting toward more green or environment friendly products. However, for wood industry, this is not defined very well for the public, as all wood products are made of wood coming from trees. Relying on online activities remains the only effective way to reach larger audience by choosing the right content marketing strategy for example. This approach is well known to be effective in providing informative contents and improving customers overall satisfaction and prospects general perception of a brand [10–13].

The rising awareness of environmental and social issues in online virtual communities is driving more attention. Many sensitive related topics and discussions could be generated and could ignite negative sentiment for respective brands as competitors could use this fertile terrain to launch negative propaganda and hidden agenda with the ultimate goal of damaging competitors' reputation. For example, people are more attracted to hard wood constructed furniture, bookshelves and so forth, say mostly made from pines, maples and cherry; but we know that these trees take much longer to grow. This could be a supply problem for larger markets with intensive demands. The environmental impact of companies continuously providing hard wood products to such markets could trigger hot online discussions. It is then crucial to take into consideration what the public is discussing about in online forums and talk rooms [14–17].

Being socially responsible is not easy as it appears for small wood businesses for example, as social and environmental responsibilities refer to how companies market their goods and what are their philosophy and ethical values when it comes to honesty, relationships with customers and morality. From this perspective, for wood companies, demand could move away from wood intensive furniture as end-users and businesses could undergo profound changes in the coming years with respect to environmental awareness and ethical marketing, as well as the ongoing digitization of the economy and financial infrastructures [18–20].

The overall objective of this investigation is to provide a holistic approach of customer relationships optimization for wood industries based on evolving human consumption behavior, human appreciation variation over time of wood products, social values evolution and shift and multilayered wood and other connected markets ongoing crisis. Particularly, our work aims to provide solid evidences on the importance of including social values shift, environmental awareness and ethical marketing practices in short and long term development plan for both small and big wood businesses. To do that, in this article, we analyze the dynamics of wood companies' development considering the effects of reputation, social and ethical marketing and customers' perceptions variations with respect to the ongoing environmental crisis based on three-dimensional differential equations system characterized by interaction, cooperation and competition. We classify all wood made products into two categories taking into account the overall green footprint of respective wood materials. We model companies' reputation as function of ethical commerce practices and the willingness and endeavor of the companies to dedicate themselves to social and environmental issues that are profoundly affecting people perceptions of wood products. In this approach, respective companies reputation may increase or decrease in function of positive feedback coming from satisfied or unsatisfied consumers (via social media such as Twitter or dedicated platform), with respect to social and environmental issues [21–23]. The study conducted by the authors in [24] pointed out that the preservation of the environment has became evident in ecoconscious market places, leading to the emergence of green marketing concept widely adopted nowadays. They particularly investigated consumers' attitude in general towards environment friendly products and its' impact on consumers' purchases intentions. They show that green marketing influences consumer's buying behavior, attitude and intentions in long run. While in [25], the author stated that the public environmental awareness reflects many aspects of environmental status, such as people's knowledge, personal consideration and behavior and human attitude towards sustainable society as a whole. The results of this investigation indicate that people are willing to pay for environmental improvement and to purchase environment friendly products. It emphasizes the importance of promoting green products, labels, recycling and processing awareness in the dynamics of purchasing decisions of consumers. For wood industries, one can use existing tools and methods such as sentiment analysis based on data mining and social network analysis to determine customers' sentiments toward specific wood products for instance and orient the right marketing content tailored toward more aware today wood products consumers.

We performed qualitative analyses of the proposed system and found that the positive solutions of the dynamical system could reach local and global stability at certain conditions, depending on parameters values or the performance of competing wood products, respective companies' reputation score and customers' sentiment, as the simulation results show. The implication on wood companies development is that substantial effort should be dedicated to strengthen customers' awareness of respective companies' ethical manners via content marketing on social media and other available platforms, with the overall goal of raising the company positive perception in customers mind, and indirectly tackling false allegations or negative sentiments reflected in the company long-term development.

2. Model formulation

In this section, we describe the proposed system differential equations mathematical implications and assumptions.

Let P_1 and P_2 be respectively the first and second category of wood products market penetration rate at time t, and F the company reputation score or perception value in the eyes of the public. In this analysis, we consider P_1 as all consumer products manufactured using hard or soft natural wood, considered as coming from trees with long or short maturation period, say with respect to carbon footprint. This category of raw materials is well known for its low carbon footprint and energy efficiency with respect to harvesting, treatment, processing and storage compares to P_2 category. We group all non-natural wood-based products in P_2. This could be all

composite and manufactured wood materials made from fibers mixed with plastic, plywood or foam boards and so forth, known to be flexible, lightweight and easy to work with in numerous projects. We assume F is related to the overall brand value in terms of reputation score. This score or value is related and submitted to respective companies' performance as a whole, and to the feedback coming from satisfied consumers who share their opinion via comments, multimedia contents etc. The value of F may be impacted negatively in function of negative comments, allegations or disinformation orchestrated by some individuals with the goal to damage target companies reputation for marketing purpose for example. To take into consideration this observation, we set up in the companies' reputation differential equation a decay or harvesting factor. A model-system [1] is given by:

$$\begin{aligned}
&\frac{dP_1}{dt} = P_1(t)[\alpha - P_1(t) - aP_2(t) + uF(t)] = f_1(P_1, P_2, F),\\
&\frac{dP_2}{dt} = P_2(t)[\beta + bP_1(t) - P_2(t) + vF(t)] = f_2(P_1, P_2, F),\\
&\frac{dF}{dt} = F(t)[F + uP_1(t) + vP_2(t) - \theta] = f_3(P_1, P_2, F),\\
&a, b, \alpha, \beta, u, v, \theta > 0,\\
&P_1(0), P_2(0), F(0) \geq 0,\\
&P_1(t), P_2(t), F(t) > 0.
\end{aligned} \tag{1}$$

where α and β are respectively, the per capita growth rate of P_1 and P_2 per unit time, a and b represent respective interaction coefficients of P_1 and P_2. u and v capture the positive effect coming from companies reputation in improving each category of products sales, overall good sentiment and positive feedback on social media or dedicated platforms from consumers. θ is acting as both a decay factor and a control parameter formalizing the decrease in reputation for respective companies due say to negative information, propaganda, social crisis or social values shift.

Clearly, for $(P_1,P_2,F) > (0,0,0)$, it is logical to demand $dP_1/dt > 0, dP_2/dt > 0$ and $dF/dt > 0$. Meaning $P1$ and $P2$ sales and positive feedback from consumers should be maintained at decent level for wood companies to sustain their development.

This implies from Ref. [1] that

$$\begin{aligned}
&dP_1/dt > 0 \rightarrow P_1(t) < \alpha - aP_2(t) + uF(t),\\
&P_2(t) < \frac{1}{a}[\alpha + uF(t)], F(t) > \frac{1}{u}[aP_2(t) - \alpha],
\end{aligned} \tag{2}$$

$$P_2 / dt > 0 \rightarrow P_2(t) < \beta bP_1(t) + vF(t), \tag{3}$$

and

$$\begin{aligned}
&dF/dt > 0 \rightarrow F(t) > \theta - uP_1(t) - vP_2(t),\\
&0 < u, v \ll 1.
\end{aligned} \tag{4}$$

If [2–4] could be satisfied, then all trajectories of the positive solutions of the system lying on the phase space would be bounded and more likely be uniquely defined. Based on [4], maintaining and improving the reputation of a wood company involved in commercializing both categories of products would require to have u and v very small. From practical stand point, this signifies that wood companies should not rely heavily and solely on their current reputation to increase sales. They must dedicate substantial effort in reinforcing customer relationship and overall satisfaction instead of relying on current reputation status.

Computing the divergence and Laplacian of the system, we obtain

$$\begin{aligned}
&\nabla(f_1,f_2,f_3) = \alpha + \beta + (2 + u + v)F(t) + (b + u)P_1(t) + vP_2(t)\\
&-[2P_1(t) + (2 + a)P_2(t) + \theta] \neq 0,\\
&\nabla^2(f_1,f_2,f_3) = -2 < 0.
\end{aligned} \tag{5}$$

Based on the divergence having non-definite sign and the Laplacian being negative definite as shown in Ref. [5], different scenarios with wide range of possibilities in terms of dynamical behavior could exist. The vectors field of the system could be converging or diverging at given equilibrium points or nodes, depending on parameters value and sign of the divergence. Furthermore, if one positive equilibrium point of the system is stable, then based on the Laplacian, we can conclude that this equilibrium point would be a local maximum for traveling trajectories initiating at the origin.

3. Steady state equilibrium

Let us say P_1 and P_2 have inundated a market, and consumers are reacting sharing their experiences and thoughts on companies dedicated platforms or any other social media, then F value would be fluctuating submitted to what is being said about the brand in these online forums and many other factors related to the company's prestige, practices and social engagement. Solving the differential equations of the system using matrix method gives

$$\begin{pmatrix} -1 & -a & u \\ b & -1 & v \\ u & v & 1 \end{pmatrix} = \begin{pmatrix} -\alpha \\ -\beta \\ \theta \end{pmatrix} \tag{6}$$

The determinant of [6] is computed as $\Delta = u^2 + v^2 + ab + uv(b-a) + 1 > 0$.
For $0 < a \leq 1, \alpha, \theta \geq 1$,

$$\Delta_{P_1} = \alpha\left(1 + v^2\right) + \theta u - (a\beta + a\theta v + \beta uv) > 0.$$

For $0 < a, \beta \ll 1$,

$$\Delta_{P_2} = \theta v + \beta + \theta bu + \alpha b + u(\beta u - \alpha v) > 0$$

For $\beta u > \alpha v$,

$$\Delta_F = ab\theta + a\beta u + \theta - v(\beta - \alpha b) - \alpha u > 0$$

One sufficient condition for the solutions to exist and be real numbers is
For $\theta > v\beta, \alpha < \beta, 0 < b \ll 1, 0 < v \ll 1$.
It follows, the system [1] admits a unique positive equilibrium point given by

$$\begin{aligned} &(P_1{}^*, P_2{}^*, F^*) = \left(\frac{\Delta_{P_1}}{\Delta}, \frac{\Delta_{P_2}}{\Delta}, \frac{\Delta_F}{\Delta}\right), \\ &P_1{}^* = \frac{\alpha(1+v^2) + \theta u - (a\beta + a\theta v + \beta uv)}{u^2 + v^2 + ab + uv(b-a) + 1}, \\ &P_2{}^* = \frac{\theta v + \beta + \theta bu + \alpha b + u(\beta u - \alpha v)}{u^2 + v^2 + ab + uv(b-a) + 1}, \\ &F^* = \frac{ab\theta + a\beta u + \theta - v(\beta - \alpha b) - \alpha u}{u^2 + v^2 + ab + uv(b-a) + 1}. \end{aligned} \tag{7}$$

The system would admit, in case of coexistence on the market of both categories of products $(P_1{}^*, P_2{}^*, F^*)$ as unique positive equilibrium point [7] lying on the phase space of the system. Based on Poincaré recurrence theorem, if we apply small perturbations at the vicinity of any point of this space, the system will return to a steady state and all trajectories would converge to $(P_1{}^*, P_2{}^*, F^*)$.

4. Effects of reputation

In this section, we evaluate the effects of reputation on market penetration of P_1 and P_2 by varying respective parameters in $(P_1{}^*, P_2{}^*, F^*)$ components using variational calculus and partial derivative/methods. In this way, we can evaluate how the occurring interactions evolve or behave in function of time and respective given model parameters value.

For P_1

$$\begin{aligned} &\frac{dP_1{}^*}{du} = \frac{(\theta - \beta v)q - (2u + bv - av)q_1}{q^2}, \\ &q_1 = \alpha\left(1 + v^2\right) + \theta u - (a\beta + a\theta v + \beta uv), \end{aligned} \tag{8}$$

After expanding, we find that the sign of [8] depends on

$$\begin{aligned} &b_0u^2 + b_1u + b_2, b_0 = -(\theta - \beta v), b_1 = -2\left(\alpha + \alpha v^2 - a\beta - a\theta v\right), \\ &b_2 = (\theta - \beta v)\left(1 + ab + v^2\right) - v(b-a)\left(\alpha + \alpha v^2 - a\beta - a\theta v\right). \end{aligned} \tag{9}$$

Solving system [9] for u, we get

$$u_{1,2} = \frac{1}{2b_0}\left[-b_1 \pm \sqrt{b_1{}^2 - 4b_0b_2}\right], b_1{}^2 \geq 4b_0b_2, \tag{10}$$

One possible outcome for the roots [10] to stay in $\mathbb{R}$ is to restrain parameters value such that $u^* = -b_1/2b_0 < 0$ to have

$$u \in \left]0, u_1\right[, \frac{dP_1{}^*}{du} < 0, u \in \left]u_1, 1\right[, \frac{dP_1{}^*}{du} > 0, \tag{11}$$

We can conclude that, respective companies' perception in the mind of customers would impact negatively the sales of P_1 for $u \in]0, u_1[$ [11]. It would have the opposite effect if $u \in]u_1, 1[$ [11].

Similarly, based on [7] we have

$$\frac{dP_1^*}{dv}=\frac{(dq_0)_v q-(dq)_v q_0}{q^2},$$
$$(dq_0)_v=-\beta u^3+v^3+[(b-a)-\beta]uv^2+2u^2v-a\theta\left(v^2-u^2\right)$$
$$+a\theta(b-a)uv+(ab+1)(2v-\beta u)-a\theta(ab+1),$$
$$(dq)_v=2v+u(b-a),$$
$$(dq_0)_v q-(dq)_v q_0=v^3+\rho_0 v^2+\rho_1 v+\rho_2,$$
$$\rho_0=[(b-a)-\beta]u-a\theta<0,\rho_1=2u^2+a\theta(b-a)u+2(ab+1)>0,$$
$$\rho_2=-\beta u^3+a\theta u^2-(ab+1)\beta u-a\theta(ab+1)<0, \tag{12}$$

It follows, if we take v_1 and v_2 as roots of [12] i.e. $(dq_0)_v q-(dq)_v q_0=0$ when solving for v, we find

$$v\in\left]0,v_1\right[\cup]v_2,\infty\left[,\frac{dP_1^*}{dv}>0,v\in\right]v_1,v_2\left[,\frac{dP_1^*}{dv}<0, \tag{13}$$

The brand value or reputation would be affecting negatively the sales of P_1 for $v\in]v_2,1[$. It would have the opposite effect if $v\in]0,v_1[$ according to [13]

For P_2

$$\frac{dP_2^*}{du}=\frac{f(u)}{q^2},$$
$$f(u)=[(\beta(b-a)+2\alpha)v-(\theta b+\alpha v)]u^2$$
$$+\left[\alpha(b-a)v^2+2\beta(ab+1)+v^2-2(\theta v+\alpha b+\beta)\right]u$$
$$+(\theta b+\alpha v)(v^2+ab+1)-(b-a)(\theta v+\beta+\alpha b)-\theta b(b-a)v,$$
$$\frac{df(u)}{du}=2[(\beta(b-a)+2\alpha)v-(\theta b+\alpha v)]u-2(\theta v+\alpha b+\beta)$$
$$+[\alpha(b-a)+2\beta(ab+1)+1]v^2,$$
$$\frac{df(u)}{du}=0\rightarrow u^*=\frac{2(\theta v+\alpha b+\beta)-[\alpha(b-a)+2\beta(ab+1)+1]v^2}{2[(\beta(b-a)+2\alpha)v-(\theta b+\alpha v)]}, \tag{14}$$

To solve [14], we demand $2(\theta v+\alpha b+\beta)<[\alpha(b-a)+2\beta(ab+1)+1]v^2$ and $(\beta(b-a)+2\alpha)v<\theta b+\alpha v$ to have $u^*>0$. It follows $df(u)/du$ is positive before reaching u^* and negative after, meaning

$$u\in\left]0,u^*\right[,\frac{dP_2^*}{du}>0,u\in\left]u^*,1\right[,\frac{dP_2^*}{du}<0, \tag{15}$$

Companies ethical commerce practices are impacting negatively the sales of P_2 for $u\in]u^*,1[$, and have positive effect for $u\in]0,u^*[$ based on [15].

Similarly,

$$\frac{dP_2^*}{dv}=\frac{g(v)}{q^2},$$
$$g(v)=(\alpha u-\theta)v^2-2\left(\beta+\theta bu+\alpha b+\beta u^2\right)v$$
$$+(\theta b-\alpha u)(u^2+ab+1)-(b-a)\left(\beta+\theta bu+\alpha b+\beta u^2\right)u,$$
$$g(v)=n_0v^2+n_1v+n_2=0,n_0=\alpha u-\theta<0,n_1=-2\left(\beta+\theta bu+\alpha b+\beta u^2\right)<0,$$
$$n_2=(\theta b-\alpha u)\left(u^2+ab+1\right)-(b-a)\left(\beta+\theta bu+\alpha b+\beta u^2\right)u<0,$$
$$\frac{dg(v)}{dv}=2n_0v+n_1=0,\rightarrow v^*=-\frac{n_1}{2n_0}<0, \tag{16}$$

Solving [16], we need $g(v)=0$, this yields

$$v_{1,2}=\frac{1}{2n_0}\left[-n_1\pm\sqrt{{n_1}^2-4n_0n_2}\right],{n_1}^2>4n_0n_2, \tag{17}$$

It follows

$$v\in\left]v_2,v_1\right[,\frac{dP_2^*}{dv}>0,v\in\left]0,v_2\right[\cup]v_1,1\left[,\frac{dP_2^*}{dv}<0, \tag{18}$$

Companies reputation would be affecting positively P_2 sales for $v\in]v_2,v_1[$ and having the opposite effect if $v\in]0,v_2[\cup]v_1,1[$

according to Refs. [17,18].

For F

$$\begin{aligned}
&\frac{dF^*}{du} = \frac{h(u)}{q^2},\\
&\qquad h(u) = -(a\beta - \alpha)u^2 - 2[a\theta b + \theta - (\beta + \alpha b)v]u\\
&+(a\beta - \alpha)(v^2 + ab + 1) - (b - a)[a\theta b + \theta - (\beta + \alpha b)v],\\
&h(u) = (\alpha - a\beta)u^2 - 2t_0 u + t_1 = 0,\\
&t_0 = -[a\theta b + \theta - (\beta + \alpha b)v] > 0,\\
&t_1 = (a\beta - \alpha)\left(v^2 + ab + 1\right) - (b - a)[a\theta b + \theta - (\beta + \alpha b)v],\\
&u_{1,2} = \frac{1}{2(\alpha - a\beta)}\left[2t_0 \pm \sqrt{4{t_0}^2 + t_1(a\beta - \alpha)}\right],
\end{aligned} \tag{19}$$

Solving [19] requires $4{t_0}^2 + t_1(a\beta - \alpha) \geq 0$, to have at least one real (number) positive root.

$$\begin{aligned}
&h^{'}(u) = -2(a\beta - \alpha)u - 2[a\theta b + \theta - (\beta + \alpha b)v],\\
&u^* = \frac{\theta(ab + 1) - (\beta + \alpha b)v}{a\beta - \alpha}, \theta(ab + 1) > (\beta + \alpha b)v, a\beta > \alpha,
\end{aligned} \tag{20}$$

It follows

$$u \in \left]0, u_1\right[, \frac{dF^*}{du} < 0, u \in \left]u_1, 1\right[, \frac{dF^*}{du} > 0, \tag{21}$$

Companies reputation would be increasing for $u \in]u_1, 1[$ and decreasing for $u \in]0, u_1[$ based on [20,21].

Similarly

$$\begin{aligned}
&\frac{dF^*}{dv} = \frac{-(\beta + \alpha b)q - (2v + bu - au)p}{q^2} \ll 0,\\
&p = ab\theta + a\beta u + \theta - v(\beta - \alpha b) - \alpha u > 0,
\end{aligned}$$

Companies reputation would be decreasing when we vary v.

5. Effects of competition and cooperation

Based on the components of the unique positive equilibrium point, when we vary the decay parameter θ we get

For P_1

$$\frac{dP_1{}^*}{d\theta} = -\frac{av}{q} < 0, \tag{22}$$

For P_2

$$\frac{dP_2{}^*}{d\theta} = \frac{v + bu}{q} > 0, \tag{23}$$

For F

$$\frac{dF^*}{d\theta} = \frac{(ab + 1)}{q} > 0, \tag{24}$$

Clearly, varying θ will end up boosting P_2 market [23] and the reputation of respective companies [24]. Varying the decay factor of F would be affecting negatively market dynamics of P_1 [22].

When we vary a and b, we get

$$\begin{aligned}
&\frac{dF^*}{da} = \frac{w_0 - w_1}{q^2},\\
&w_0 = (u\beta + b\theta)\left(u^2 + v^2 + buv + 1\right),\\
&w_1 = (b - uv)[\theta - \alpha u - v(\beta + \alpha b)],
\end{aligned} \tag{25}$$

It follows

$$w_0 > w_1 \cap b \leq uv, \frac{dF^*}{da} > 0, w_0 < w_1, \frac{dF^*}{da} < 0, \tag{26}$$

$$\begin{aligned}&\frac{dF^*}{db}=\frac{a(\theta-1)-v(u+\alpha)[b(uv+a)+u^2+v^2-auv+1]}{q^2},\\&\frac{dF^*}{db}<0,\rightarrow a\theta\ll(\alpha+u)v+a,\end{aligned}\tag{27}$$

a will be affecting respectively, companies reputation positively or negatively if $w_0>w_1\cap b\leq uv$ or $w_0<w_1$, see (25) and (26). Varying b will result in hurting given companies reputation as long as $a\theta\ll(\alpha+u)v+a$ hold (27).

6. Stability analysis

In this section of the article, we analyze the system local and global stability based on the determinant equation, Lyapunov method and Taylor approximation. The polynomial characteristic equation is computed based on the following determinant equation

$$\begin{vmatrix}\lambda-(m_0-m_1) & -aP_1^* & uP_1^*\\ bP_2^* & \lambda-(m_2-2P_2^*) & vP_2^*\\ uF^* & vF^* & \lambda-(m_3-\theta)\end{vmatrix}=0,\tag{28}$$

where.

$m_0=\alpha+uF^*, m_1=2P_2^*+aP_2^*, m_2=\beta+bP_1^*+vF^*, m_3=2F^*+uP_1^*+vP_2^*$. It follows (28) could be written in cubic equation (29) form as follows

$$\lambda^3-T\lambda^2+\Omega\lambda-D=0,\tag{29}$$

where

$$\begin{cases}T=m_0+m_2+m_3-(2P_2^*+m_1+\theta),\\ \Omega=(m_2-2P_2^*)(m_3-\theta)-\left(v^2P_2^*+u^2P_1^*\right)F^*+abP_1^*P_2^*\\ +(m_0-m_1)(m_2+m_3-2P_2^*-\theta),\\ D=(m_0-m_1)\left[(m_2-2P_2^*)(m_3-\theta)+v^2P_2^*F^*\right]+u^2(m_2-2P_2^*)P_1^*F^*\\ -abP_1^*P_2^*(m_3-\theta)+(b-a)uvP_1^*P_2^*F^*,\end{cases}\tag{30}$$

Based on Routh Hurwitz stability criteria, solutions of the system would be locally asymptotically stable when approaching the unique positive equilibrium state in case $T<0,\Omega>0,D<0,|T|\Omega>|D|$, that is based on (30)

$$\begin{aligned}&m_0<m_1,m_2<2P_1^*,m_3<\theta,0<u,v\ll1,\\&m_0+m_2+m_3<(2P_2^*+m_1+\theta),\\&(m_2-2P_2^*)(m_3-\theta)>0,\\&(m_0-m_1)(m_2+m_3-2P_2^*-\theta)>0,\\&\Omega-\left(v^2P_2^*+u^2P_1^*\right)F^*\gg0,\end{aligned}\tag{31}$$

If (31) could be satisfied, then (P_1^*,P_2^*,F^*) would be attracting all curves lying on the phase space at relative speed, depending on parameters values and initial conditions. Wood companies good reputation could be sustained as long as the status quo is maintained, customers are satisfied and respective perception values are not challenged by new social trends or other related or similar phenomena.

Furthermore, (P_1^*,P_2^*,F^*) global behavior on the phase space could be analyzed using Lyapunov stability theorem if we take $V(P_1,P_2,F)$ as a Lyapunov candidate function, which is generally considered as a scalar function established on the phase space of the system that can be used to show an equilibrium point's stability. Suppose $V(P_1,P_2,F)$ is defined and positive definite $\forall t\rightarrow\infty,P_1>0,P_2>0,F>0$, then

$$V(P_1,P_2,F)=\varepsilon_0P_1^2+\varepsilon_1P_2+\varepsilon_2(P_2-F)^2,\tag{32}$$

clearly

$$V(0,0,0)=0,V(P_1,P_2,F)>0\forall(P_1,P_2,F)>(0,0,0),\varepsilon_0,\varepsilon_1,\varepsilon_2>0,$$

(32) is continuous and derivable on the phase plane as long as $P_1>0,P_2>0,F>0$. Furthermore, $V(P_1,P_2,F)$ is radially unbounded as $\lim_{t\rightarrow\infty}V(P_1,P_2,F)\rightarrow\infty$.

The time derivative of $V(P_1,P_2,F)$ is given by

$$\begin{aligned}
&\frac{dV}{dt} = 2\varepsilon_0 {P_1}^2[\alpha - P_1 - aP_2 + uF] + 2\varepsilon_2 F[F + uP_1 + vP_2 - \theta] \\
&+P_2(\varepsilon_1 + 2\varepsilon_2(P_2 - F))[\beta + bP_1 - P_2 + vF], \\
&\quad \frac{dV}{dt} = 2\varepsilon_0 {P_1}^2[{P_1}^* - P_1 + a({P_2}^* - P_2) + u(F - F^*)] \\
&+P_2(\varepsilon_1 + 2\varepsilon_2(P_2 - F))[b(P_1 - {P_1}^*) + {P_2}^* - P_2 + v(F - F^*)] \\
&+2\varepsilon_2 F[F - F^* + u(P_1 - {P_1}^*) + v(P_2 - {P_2}^*)], \\
&\frac{dV}{dt} = 2\varepsilon_0 {P_1}^2\rho_0 + P_2(\varepsilon_1 + 2\varepsilon_2(P_2 - F))\rho_1 + 2\varepsilon_2 F\rho_2,
\end{aligned} \tag{33}$$

$$\begin{aligned}
&\rho_0 \gg 0, 0 < \varepsilon_0 \ll 1, \rho_1 < 0, \\
&\varepsilon_1 + 2\varepsilon_2(P_2 - F) > 0, P_2 < F, \\
&P_2 - F > \frac{\varepsilon_1}{2\varepsilon_2}.
\end{aligned} \tag{34}$$

Based on Lyapunov theorem, if we could ensure (33) by choosing the right parameters value, then $({P_1}^*, {P_2}^*, F^*)$ would be locally and globally stable and all positive interior trajectories would be converging to this node. Meaning, as far as (34) holds, the evolution of P_1 and P_2 markets could be sustained and the company reputation would be fluctuating in function of social values and customers perception of P_1 and P_2 benefits in addition to the overall ethical practices of respective companies. However, this dynamic is subject to drastic changes that could be triggered by very small perturbations. For example, a social, marketing or any unpredictable event could drive more attention toward one category of wood product available on the market, say P_2. This will end up affecting the dynamics of the system, as more people will purchase P_2 products. P_1 and F will be affected differently according to the model parameters values and theoretical assumptions.

7. Results and discussion

In this section, we first present the results of computer simulations carried out to test the proposed system behavior around the steady state equilibrium for different initial conditions using Matlab. Then, we analyze the obtained results and discuss practical implications for predicting and improving wood companies reputation evolution in function of social and environmental engagements of the managing board. Model parameters value are computer generated based on different scenarios presented in theoretical analysis and real reputation management observations in wood industries. We tested the case P_1 and P_2 are continuously competing on a segment of the wood market, and consumers are interacting on the manufacturer official platform, posting comments, images, videos and so forth. We assume they have also the possibility to share their experience and thoughts on each purchased good quality, benefits etc freely on any other existing platforms.

F could be chosen according to respective companies market share, leadership or prestige for instance on a scale or score as in the well-known General Electric resource allocation model. Furthermore, to be realistic, one would carefully monitor customers' online discussions to prevent any abuse that would end up damaging the reputation of the brand.

Fig. 1 displays an interaction dynamic scenario in which the reputation of the company is strongly bounded to P_1 market evolution. This could be the case where traditional or iconic items are from this category, and customers' perception of these items is shifting or

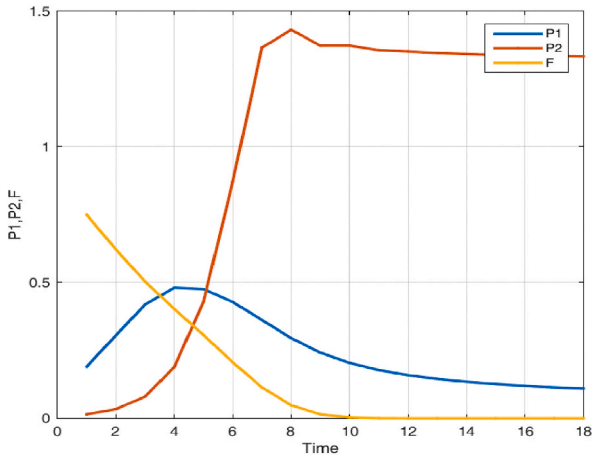

Fig. 1. Interaction dynamics around the steady state equilibrium when P2 is highly competitive for: P1 (0) = 0.19; P2 (0) = 0.014; F (0) = 0.75; α = 1.1; β = 2.3; a = 0.021; b = 0.3; u = 0.93; v = 0.05; θ = 0.098.

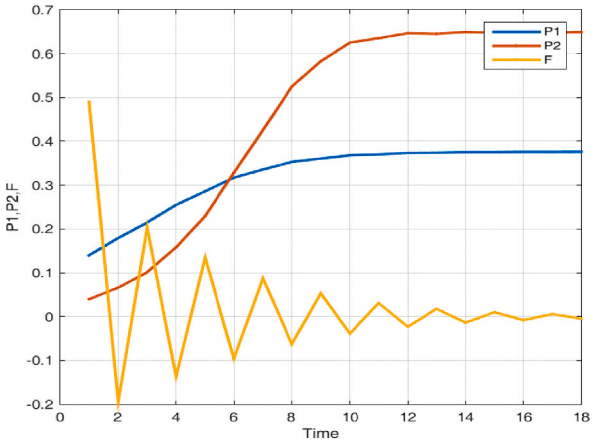

Fig. 2. Interaction dynamics around the steady state equilibrium when both P1 and P2 are relatively competitive for: P1 (0) = 0.14; P2 (0) = 0.04; F (0) = 0.49; α = 1.39; β = 1.6; a = 0.021; b = 0.13; u = 0.07; v = 0.17; θ = 0.9.

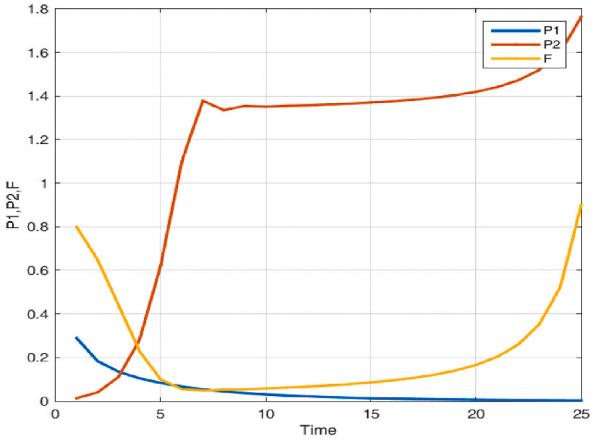

Fig. 3. Interaction dynamics when customer perception shift and rise the company reputation for: P1 (0) = 0.29; P2 (0) = 0.014; F (0) = 0.8; α = 0.9; β = 2.31; a = 0.021; b = 0.02; u = 0.03; v = 0.75; θ = 0.008.

decreasing due to social or environmental crisis for example.

P_2 is winning a large portion of the market, however this is not helping in strengthening the company reputation for significantly larger $u = 0.93$ compared to $v = 0.05$. To overcome this issue, one could use existing tools and methods to influence people perception of respective brand by focusing on rising social or environmental engagements awareness related to raw materials treatment for manufacturing P_2 for example. The overall goal is to provoke perception shift towards P_1 as shown in Fig. 2, where P_1 is doing better for $u = 0.07$ and $v = 0.17$. This outcome confirms theoretical analysis presented in Ref. [13] for adjusting v to boost $P1$ sales; and [21] for adjusting u to enhance the reputation of the company. F is fluctuating, submitted to the eigenvalues, roots of the polynomial characteristic equation nature and sign for this set of parameters value. In this situation, traditional marketing is less effective to attract and retain targeted public and especially drive customer positive action on social media. This result suggests providing truly relevant content to customer (improve content marketing) and help to solve prospects issues to reduce the negative effect of fluctuating reputation on overall wood companies development in complex interconnected markets and highly saturated social environment. In Fig. 3, for almost equally competitive products, $a = 0.021$ and $b = 0.02$, the product with larger value for the public P_2 is positively affecting respective public perception of the producer or brand reputation. This is an ideal scenario where better marketing strategies turn out to increase only one category of product market share, while the other category diminishes at relative speed for the benefit of good reputation and business success. This result shows that reputation monitoring is highly important for wood companies in continuously evolving markets environment. The public decision to purchase a good is a complex process submitted to a variety of external and internal influences. Wood companies managers should grab each opportunity to boost their reputation by monitoring and influencing people attitude, perception and sentiments through discussions, interactions and other online activities. For example, the managerial board could focus on two marketing keys: cost savings and better customer loyalties.

Figs. 4 and 5, show how small perturbations could result in significant changes in the system behavior. By varying only the initial conditions of F, $F(0) = 0.08$ for Fig. 4 and $F(0) = 0.8$ for Fig. 5, the outcome of interactions is significantly affected. In Fig. 4, we have a

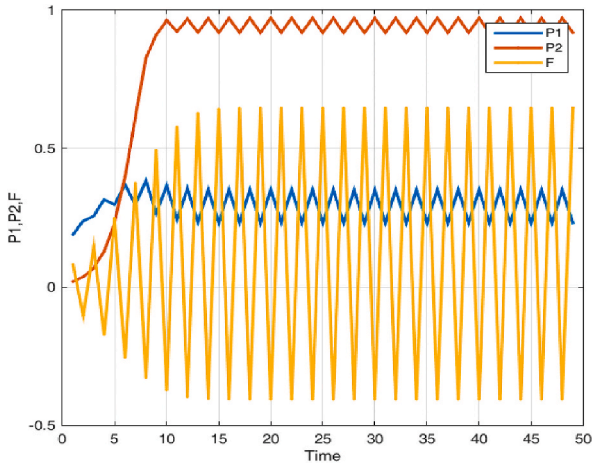

Fig. 4. Interaction dynamics when periodic orbits exist around the steady state equilibrium for: P1 (0) = 0.19; P2 (0) = 0.019; F (0) = 0.08; α = 1.39; β = 1.9; a = 0.1; b = 0.13; u = 0.7; v = 0.07; θ = 1.5.

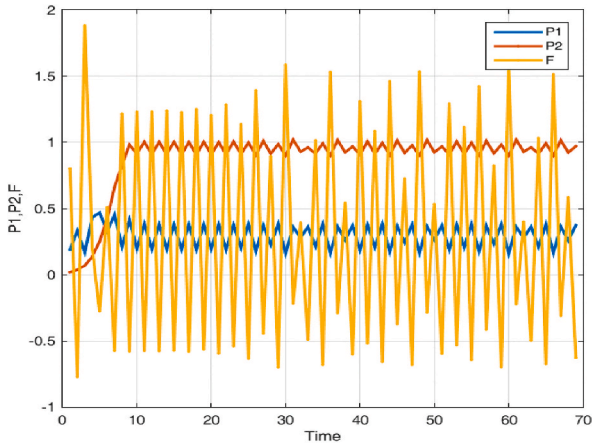

Fig. 5. Interaction dynamics when chaotic orbits exist around the state equilibrium for: P1 (0) = 0.19; P2 (0) = 0.019; F (0) = 0.8; α = 1.39; β = 1.9; a = 0.1; b = 0.13; u = 0.7; v = 0.07; θ = 1.9.

desired dynamic where all trajectories are orbiting around the steady state equilibrium considered as a spiral node. One could expect to see a long lasting status quo with higher and lower seasons for P_1, P_2 sales for example, and fluctuating F. While in Fig. 5, chaotic curves arise when approaching the steady state equilibrium for a very high reputation score. This implies, it is better to keep the status quo when it comes to the company reputation or public perception of a brand and not be aggressive in marketing competing products with high potential benefits for the customers.

In the reputation management and ethical marketing processes for wood companies, a well organized and highly effective marketing content is required to orient the public towards the most important points. The managerial board social responsibility influence on market performance of competing products is critical. We introduced a holistic approach taking into account internal and external factors to investigate the underlying complex relationship determining wood companies markets and customer sentiment shifting in highly volatile today environment submitted to multiple crises. The results of simulated relevant situations related to reputation management, customer cares and content marketing strategies are compatible with the results presented in "Online reputation management" by Peter Markovič, Peter Dorčák and František Pollák; "Content Marketing Strategy. Definition, Objectives and Tactics" by Simona Vinerean and "Wood-based Entrepreneurs Toolkit: Strategic Marketing" by Eric Hansenand.

The reputation function is one of the innovative approaches of our investigation in addition to the system thinking that is lacking in current reputation management literature.

The omnipresence of communication tools such as social media in today market environment may affect dramatically or positively any company reputation. Competing iconic wood products pricing or manufacturer non respect of ethical values for example may decrease people's willingness to share good comments on dedicated platform.

Environmental issues, social values shift and perhaps the lack of sufficient information on the public favorite brand may cause significant market disruption or severe damage to wood companies' reputation. This research entails better content and reputation

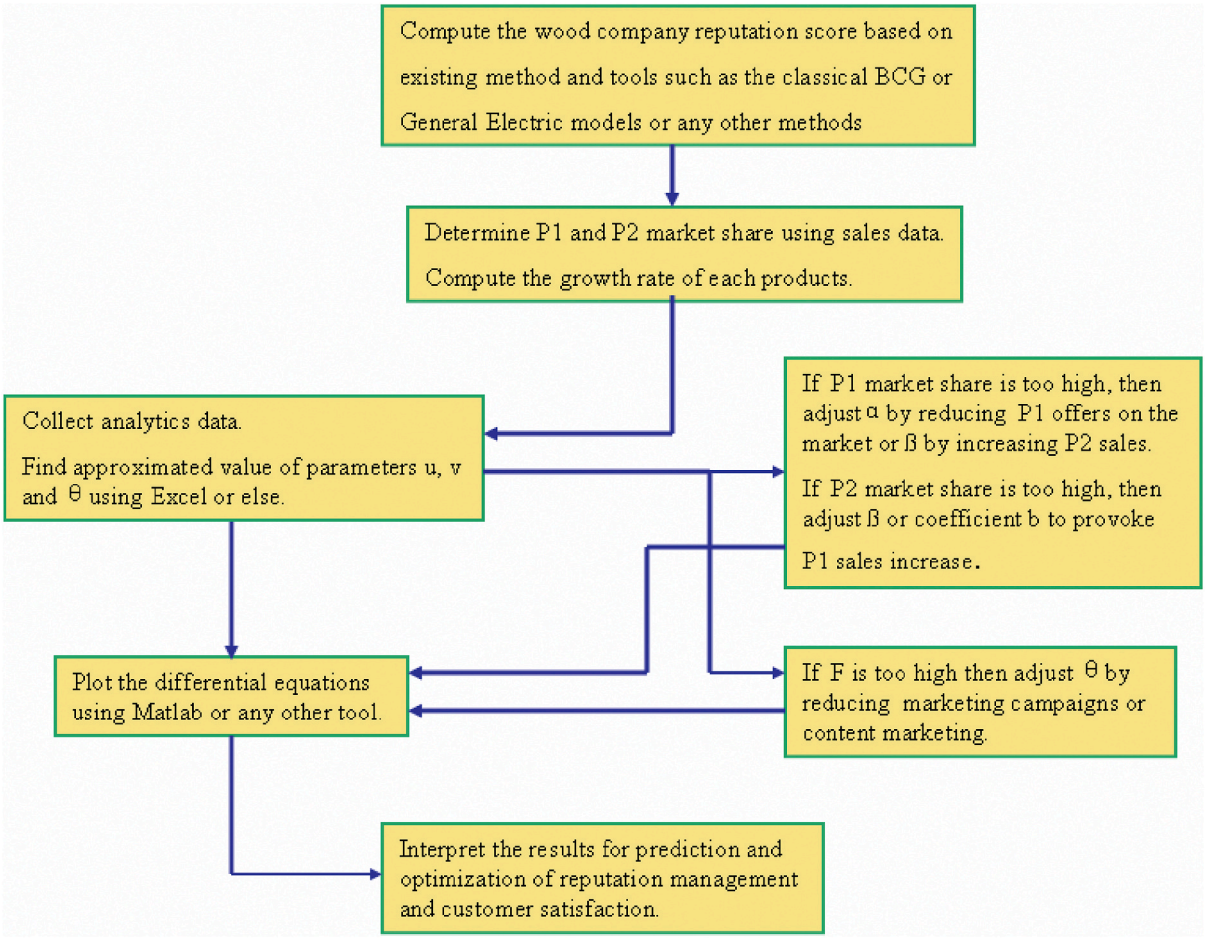

Fig. 6. Diagram of wood companies' reputation management in function of market share and environmental awareness of consumers for optimal overall development and customer satisfaction.

marketing strategies for wood companies and provides a method of measuring the underlying interactions occurring when respective wood companies' markets share and reputation fluctuate in function of the public awareness of environmental issues as summarized in Fig. 6. However, there are some limitations in the method we use in this study. Firstly, we only use simulation results to verify theoretical assumptions. With real data, the outcome may be slightly different depending on the size of the companies, the quality and relevance of the collected data and the simulation method. Secondly, we neglected some factors in the mathematical modeling for analysis and practical purposes. For example, in cooperative and competitive terms, using Holling Type formulation would be more realistic as positive or negative feedback could be slow or fast depending on competing products attractiveness or real value to the public (translated into assimilation efficiency or abilities of competitors). Also, there must be a threshold to reach for marketed products or companies sales objectives in short or long terms. Lastly, the company reputation could be affected by events that are not related to the company ethical practice or social responsibility.

8. Conclusion

In this article, a three dimensional interactive model is proposed to analyze wood companies development and reputation management considering evolving and highly interconnected markets and public perception of the brand. Customers' interactions and online discussions are considered as main tools of measuring and evaluating the company value taking into account the sensitivity of online circulating information that need to be carefully monitored. We show that the proposed model-system solutions could be locally and globally asymptotically stable under certain conditions, as long as marketing and pricing policies are decided in function of social and environmental engagements of the companies and their ethical values with respect to customer's perception. The results suggest that focusing on informative and useful content that helps in solving customer real issues is crucial for building strong relationships with customers, optimizing wood companies' reputation and enhancing sustainable development in today highly volatile environment. Moreover, reducing environmental impact by shifting production strategies could meet today customers' demand for ethically and environmentally aware treated and manufactured wood products.

Author contribution statement

Z. Chen and H. Zhu conceived and supervised the study. F. Liu helped with the methodology and software. Z. Chen and F. Liu validated the study. Z. Chen performed the formal analysis. F. Liu investigated the study. H. Zhu helped with resources. F. Liu curated the data. Z. Chen and F. Liu prepared the original draft of the manuscript and reviewed and edited the manuscript. Z. Chen and F. Liu carried out visualization.

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Data availability statement

No data was used for the research described in the article.

Declaration of interest's statement

The authors declare no competing interests.

Additional information

No additional information is available for this paper.

References

[1] C.R. Hennigar, D.A. MacLean, L. Amos-Binks, A novel approach to optimize management strategies for carbon stored in both forests and wood products, For. Ecol. Manag. 256 (2008) 786–797.
[2] C.E. Keegan III, T.A. Morgan, K.A. Blatner, J.M. Daniels, Trends in lumber processing in the Western United States. Part I: board foot Scribner volume per cubic foot of timber, For. Prod. J. 60 (2) (2010) 133–139.
[3] D.B. McKeever, J.L. Howard, Solid Wood Timber Products Consumption in Major End Uses in the United States, 1950–2009: A Technical Document Supporting the Forest Service 2010 RPA Assessment. FPL-GTR- 199, USDA Forest Service, Forest Products Laboratory, Madison, Wisconsin, 2011.
[4] O.C. Ferrell, A framework for understanding organizational ethics, in: Business Ethics: New Challenges for Business Schools and Corporate Leaders. R.A. Peterson and O.C. Ferrell, Armonk, New York: M.E., 2005, pp. 3–17. Sharpe.
[5] K. Skog, Sequestration of carbon in harvested wood products for the United States, For. Prod. J. 58 (6) (2008) 56–72.
[6] N.A. Gardberg, C.J. Fombrun, For better or worse – the most visible American corporate reputations, Corp. Reput. Rev. 4 (4) (2002) 385–391.
[7] E. Black, T. Carnes, V. Richardson, The market value of corporate reputation, Corp. Reput. Rev. 3 (1) (2000) 31–42.
[8] J. Doorley, F.H. Garcia, Reputation Management: the Key to Successful Public Relations and Corporate Communication, Routledge, New York, 2015, 2015. 442 s. ISBN 9781134749027.
[9] E. Kline, Gate-to-gate life cycle inventory of oriented strandboard production, Wood Fiber Sci. 37 (CORRIM Special Issue) (2005) 74–84.
[10] R. Bergman, S. Bowe, Environmental impact of producing hardwood lumber using life-cycle inventory, Wood Fiber Sci. 40 (3) (2008) 448–458.
[11] K.L. Keller, Conceptualizing, measuring and managing customer based brand equity, J. Market. 54 (1) (1993) 1–22.
[12] M. Morrison, F. Crane, Building the service brand by creating and managing an emotional brand experience, J. Brand Manag. 14 (2007) 410–421.
[13] C. Page, P. Herr, An investigation of the process by which product design & brand strength interact to determine initial affect & quality judgment, J. Consum. Psychol. 12 (2) (2002) 133–147.
[14] P.N. Howard, New Media Campaigns and the Managed Citizen, New Media Campaigns and the Managed Citizen, 2005.
[15] E. Ferrara, Z. Yang, Measuring Emotional Contagion in Social Media, Plos One (2015).
[16] Xu S. Computational Propaganda Poses Challenge, Global Times, 2017.
[17] Andini anindya Prameswari and Gayes Mahestui "studies the effect of brand image on customer satisfaction", Int. J. Scient. Technol. Res. ISSN 8 (8) (2019) 2277–8616.
[18] B. Renee, Kim and Yan choa ( ), "effects of brand experience, brand image and brand trust on brand building process", J. Int. Stud. 12 (3) (2019) 10–12.
[19] N.J. McCullen, Multiparameter models of innovation diffusion on complex networks, SIAM J. Appl. Dyn. Syst. 12 (1) (2013) 515–532.
[20] D.J. Watts, P.S. Dodds, Influentials, Networks, and Public Opinion Formation, J. Cons. Res. 34 (2007) 441–458.
[21] K.-S. Chiu, T. Li, Oscillatory and periodic solutions of differential equations with piecewise constant generalized mixed arguments, Math. Nachr. 292 (2019) 2153–2164.
[22] M. José, Juan Carcione, E Santos, Claudio Bagaini, Ba Jing, A simulation of a covid-19 epidemic based on a deterministic seir model, Front. Public Health 8 (2020) 230.
[23] P. Mehta, H.S. Chahal, Consumer attitude towards green products:revisiting the profile of green consumers using segmentation approach, Manag. Environ. Qual. 32 (5) (2021) 902–928, https://doi.org/10.1108/MEQ-07-2020-0133.
[24] S. Ingrid, Attitude of customer towards green purchasing, environmental awareness and green products, Holist. Approach Environ. 7 (2) (2017) 115–122.
[25] Y. Saira, G. Abdul, S.L. Abdul, A. Manzoor, K. Rukhsana, Green marketing: a study of consumers' attitude towards environment friendly products, Lasbela Univ. J. Sci. Technol. (4) (2015) 109–116.