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ORIGINAL PAPER

Innovative Approach of EOQ Structure for Decaying Items with Time Sensitive Demand, Cash- Discount, Shortages and Permissible Delay in Payments

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Abstract
Growing business process and rising aggressive conditions are encouraged to use the inventory control scheme and components in an ideal way. Cash discount and permissible delay are beneficial for vendor and buyer both. This study considers an EOQ model through demand rate depends on the time. A lower or higher time leads to lower or higher demand after feedback vice versa. In this paper deterioration, cash- discount, shortages and permissible delay are also considered. Mathematical models are discussed under four different states of affair. Solution method is given for finding the finest answer. The main aim is to maximize total profit. Numerical examples are provided for all four dissimilar situations. Optimal values with strictures are calculated to analyze the sensitivity investigation of optimal strategy concerning the parameters of the system. It is revealed that the total income is concave by means of cycle time.

Keywords Cash- discount; Inventory; Deterioration · Shortages · Demand · Trade credit

Mathematics Subject Classification 90B05

Introduction

The production of commodities is the first stage of manufacture. Three stage system, a manufacturer, seller and market are considered in the process. The retailer's term represents for part of the system works as a bridge between market and manufacturer. EOQ models begin by taking into account that the demand rate was stable along with cycle time. This assumption was a serious restriction because in real life, demand of a commodity can depend on such multiple issue at the time, stock – level, quantity, selling price, lead time, advertisings, rebate, exponential etc.

Silver and Peterson [1] pointed out that the demand rate of some goods may be influenced by the stock stage. Indeed, huge heaps of supplies exhibited in a superstore sometimes guides

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buyers to purchase more. There are two common rules for the inventory manager always keep high stock- level to make best use of profit of the inventory administration and depleting the order a new order begins. It might be profitable to raise the order level in each cycle and request a fresh order previous two stock runs out. Baker and Urban [2] designed an EOQ model with a store- linked, where the aim was profit maximization. Tripathi [3] analyzed an EOQ structure of items whose demand is a decreasing association of trade cost. Alfares [4] published the inventory model containing stock- echelon sensitive demand and a storage time- connected, carrying cost taking two times – linked holding cost functions. Tripathi and Mishra [5] described a production inventory model for time – associated demand. Time induced demand EOQ systems are established by Dave [6] and Maiti et al. [7]. Yang [8] described a structure under store- linked demand rate and stock- connected carrying cost under shortages. Several research papers published in this direction by Chang et al. [9], Soni and Shah [10], Tripathi [11],Teng et al. [12], Hsieh and Dye [13], Wee and Wang [14], Zhou et al. [15], Shah et al. [16] etc.

Deterioration of items is a major problem in the universe. Generally almost all items deteriorate with time. Deterioration means freshness decay of commodities. A normal common man declines his activity after passing the time and at last comes to the end. Tripathi and Pandey [17] studied an EOQ model to find the optimal total cost for price sensitive demand with Weibull distribution. Geetha and Kumar [18] addressed a model in which inventory cost will be lowered, if the seller can effectively reduce deterioration by improving the storage facility. Jaggi et al. [19] offered a model, including selling price dependent demand under deterioration. Mishra and Talati [20] proposed a single set- up multiple deliveries for fading stuff with fixed life time.Jani et al. [21] have studied an inventory policy for the item which has expiry date with two levels of trade credit depending on the quantity of order. It is considered that a supplier is ready to give a mutually agreed credit period to retailer only if the order quantity purchased by retailer is more than the predetermined quantity of ordered. Some researches in this in this direction are Chaudhury et al. [22], Duan et al. [23], Pervin et al. [24], Teng and Chang [25], Dye [26], Ghiami and Williams [27], Shah et al. [28] and others.

In the world economics, inflation and time value of capital cannot be unnoticed because of uncertainty of demand, weather, climate change abruptly, storm lock down (for example COVID-19), labor strike, flood etc. Buzacott [29] presented first an EOQ system by assuming inflationary effect on costs. Yang et al.[30] designed a variety of EOQ models with time – unstable demand pattern under inflation. Sarkar and Moon [31] explored a manufacture EOQ model for random demand through cause of inflation.Chaudhari et al. [32] designed a single retailer and single product which deteriorates continuously for time dependent deteriorating item with seasonal demand, quadratic demand is debated here which is suitable for the items whose demand with starting of the season increases initially and after end of the season, it starts decreases. Reduction of deterioration is reduced by preservation technology. Some researchers like Sarkar et al. [33], Yang [34], Dey et al. [35], Tripathi and Chaudhary [36] consequences special type of structures under inflation and time – reduction.

Trade credits and shortages both play an important role in any type of business. Tripathi [37] published at an EOQ system for spoilage products with curvilinear time – linked demand, shortages under traffic credits. Pal and Chandra [38] established a sporadic review EOQ model under traffic credits and price discounts. Chern et al. [39] extended an EOQ system to include particulars that (i) advertising cost is considerably superior to unit acquiring cost and (ii) interest rate charged by the trader is not essentially advanced than the seller's savings return rate under permitted delay. Jiangtao et al. [40] addressed a multi- commodity system for unpreserved substances where demand rates of goods are stored- linked two- echelon traffic credits. Chern and Teng [41] designed an EOQ system for trader for finding his/ her best possible replenishment cycle time, included the fact that (i) failing foodstuffs decline constantly and having utmost life time and (ii) a seller frequently proposes an allowed delay in payments to draw additional purchasers.

The remainder of the work designed as follows. In the subsequent Sect. 2, assumptions and notation are mentioned. Mathematical formulation is argued in Sect. 3. The most beneficial explanation is renowned in part 4. After that, numerical examples are offered of all four cases to display hypothetical fallouts. Sensitivity exploration by means of distinct parameters is conversed in segment 6. We present conclusion and future research in the last.

Notations and Assumptions

Notations

K cost of ordering (in \$)
c,p,h and s unit purchasing, selling, carrying and shortage cost/item
Q_1 and Q_2 highest inventory level and maximum shortage quantity
Q Lot – size
$D(t)$ demand rate
I_p and I_e unit interest paid and earned/\$
$Q(t)$ level of inventory at moment 't'
ϕ deterioration rate.
r cash reduction rate, 'r' lies between zero and one
t_1 time to end up inventory
M_1 and M_2 stage of cash reduction and allowable delay ($M_2 > M_1$)
T cycle time.
C_H,C_D and C_S carrying, deterioration and shortages cost
S_R sales profits
$IP_1$and IP_3 interest payable (in \$) (cases 1 and 3)
IE_i interest earned (in \$), i = 1- 4
T_i* optimal T
$P_i(T)$ total profit /yr (in \$)

Assumptions

1. Demand rate is time- sensitive, i.e. $D(t) = \alpha + \beta.t$, α is positive, $0 \le \beta \le 1$, β is not zero.
2. Shortages are permitted.
3. Vendor suggests cash discount, if payment is ready in t_1, or as a well full expense is charged. Inside credit period M_2.
4. Replenishment arises immediately at endless pace.
5. Deterioration rate is steady, $0 \le \phi < 1$.

Mathematical Models

Reduction of originality is a shapeless and natural phenomenon for items with passing time. Some preservation technologies can maintain freshness for some time, but they cannot continue for a long time. The level of inventory *Q (t)* for some moment *'t'* is ruled by subsequent equations are:

$$\frac{dQ(t)}{dt}=\begin{cases}-\phi Q(t)-\ D(t),\ 0\ \le\ t\ \le\ t_1\\ -D(t),\ t_1\le t\ \le\ T\end{cases} \tag{1}$$

Solution of (1) and (2) with $Q(t_1)=0$ is:

$$Q(t)=\begin{cases}\frac{1}{\phi}\left\{(\alpha\phi-\beta)\left(\frac{e^{\phi(t_1-t)}-1}{\phi}\right)+\beta\left(t_1e^{\iota(t_1-t)}-t\right)\right\}\\ -(t-t_1)\left\{\alpha+\frac{\beta(t_1+t)}{2}\right\}\end{cases} \tag{2}$$

$$Q=\mathrm{Q}_1+Q_2 \tag{3}$$

where Q_1 and Q_2 is obtained by substituting t = 0 and t = T in (4) and (5) respectively?

$$C_H=\frac{h}{\phi}\left\{\left(\alpha-\frac{\beta}{\phi}\right)\left(\frac{e^{\phi t_1}-1-\phi t_1}{\phi}\right)+\beta t_1\left(\frac{e^{\phi t_1}-1}{\phi}-\frac{t_1}{2}\right)\right\} \tag{4}$$

$$C_S=s\int_{t_1}^{T}-Q(t)dt=\frac{s(T-t_1)^2}{6}\{3\alpha+\beta(T+2t_1)\} \tag{5}$$

Since seller offers permitted delay of cash reduction and payment. As a result, two situations may occur (i) payment is pleased at M_1 with reduction and (ii) payment is paid at M_1, lacking the cash diminish.

Case 1: $M_1\le t_1\le T$

In nearby learning, $T\ge M_1$. Since supplier presents allowed delay of cash concession, interest paid is nil. Figures of all mentions cases are as follows:

Therefore,

$$IP_1=\frac{cI_p}{\phi}\left[\left(\alpha-\frac{\beta}{\phi}\right)\left\{\frac{e^{\phi(t_1-M_1)}-1-\phi(t_1-M_1)}{\phi}\right\}+\beta\left\{\frac{t_1\left(e^{\phi(t_1-M_1)}-1\right)}{\phi}-\frac{\left(t_1^2-M_1^2\right)}{2}\right\}\right] \tag{6}$$

$$IE_1=pI_c\int_0^{M_1}(\alpha+\beta t).t.dt=pI_eM_1^2\left(\frac{\alpha}{2}+\frac{\beta M_1}{3}\right) \tag{7}$$

And,

$$S_R=p\int_0^{T}(\alpha+\beta t)dt=pT\left(\alpha+\frac{\beta T}{2}\right) \tag{8}$$

The trader gets a cash allowance from supplier, due to payment is remunerated at M_1. Thus

$$C_D=rp\left[\frac{1}{\phi}\left\{\left(\alpha-\frac{\beta}{\phi}\right)(e^{\phi t_1}-1)+\beta t_1e^{\phi t}\right\}+\frac{(T-t_1)\{2\alpha+\beta(t_1+T)\}}{2}\right] \tag{9}$$

Thus,

$$P_1(T)\ =\{S_R-(A\ +C_H+C_S+IP_1-IE_1-C_D)\}/T \tag{10}$$

Case 2: $M_1>T$

In this situation, $M_1>T$, therefore $IC_2=0$ and

$$IE_2=pI_et_1\left\{\alpha\left(M_1-\frac{t_1}{2}\right)+\frac{\beta t_1}{2}\left(M_1-\frac{t_1}{3}\right)\right\} \tag{11}$$

Thus,

$$P_2(T) = \{S_R - (A + C_H + C_S - IE_2 - C_D)\}/T \tag{12}$$

Payment is Compensated at Credit Phase M_2

Case 3: M2 < T

In such case, $T > M_2$, the seller has no cash price cut, thus

$$IP_3 = cI_p \int_{M_2}^{t_1} Q(t)dt \tag{13}$$

$$IE_3 = pI_e \int_0^{M_2} (\alpha + \beta t)tdt = pI_e M_2^2 \left(\frac{\alpha}{2} + \frac{\beta M_2}{3}\right) \tag{14}$$

As a result

$$P_3(T) = \{S_R - (A + C_H + C_S + IP_3 - IE_3)\}/T \tag{15}$$

Case 4: M2 ≥ T

In this situation, T ≤ M_2, the vendor has no cash price cut & $IP_4 = 0$, thus

$$IE_4 = pI_e t_1 \left\{\alpha\left(M_2 - \frac{t_1}{2}\right) + \frac{\beta t_1}{2}\left(M_2 - \frac{t_1}{3}\right)\right\} \tag{16}$$

$$P_4(T) = \{S_R - (A + C_H + C_S - IE_4)\}/T \tag{17}$$

For small decline rate, we can presume (Figs. 1, 2, 3 and 4).

$$e^{\phi T} \approx 1 + \phi T + \frac{\phi^2 T^2}{2} \text{ etc}, \phi T < 1 \tag{18}$$

Hence, the entirety income of case 1–4 is falling too:

$$\begin{aligned} P_1 =& \frac{(1+r)p(2\alpha + \beta T)}{2} - \frac{K}{T} - \frac{h t_1^2\{\alpha(3+\phi t_1) + t_1(2+\theta T_1)\beta\}}{6T} - \\ & \frac{s(T-t_1)^2\{3\alpha + \beta(2t_1+T)\}}{6T} - \frac{cI_p(t_1 - M_1)^2}{6T} \\ & [\alpha\{3 + (t_1 - M_1)\phi\} + \beta\{M_1 + 2t_1 + \phi(t_1 - M_1)t_1\}] \\ & + \frac{pI_e M_1^2}{6T}(3\alpha + 2\beta M_1) + \frac{rp\phi t_1^2(\alpha + \beta t_1)}{2T} \end{aligned} \tag{19}$$

$$\begin{aligned} P_2 =& p(1+r)\left(\alpha + \frac{\beta T}{2}\right) - \frac{K}{T} - \frac{h t_1^2\{a(3+\phi t_1) + t_1(2+\phi t_1)\beta\}}{6T} - \frac{s(T-t_1)^2\{3\alpha + (T+2t_1)\beta\}}{6T} \\ & + \frac{pI_e t_1}{6T}\{3\alpha(2M_1 - t_1) + \beta t_1(3M_1 - t_1)\} + \frac{rp(2\alpha+\beta T)}{2} + \frac{rpt_1^2(\alpha+\beta t_1)\phi}{2T} \end{aligned} \tag{20}$$

$$\begin{aligned} P_3 =& p\left(\alpha + \frac{\beta T}{2}\right) - \frac{K}{T} - \frac{ht_1^2\{\alpha(3+\phi t_1) + \beta(2+\phi t_1)t_1\}}{6T} - \frac{s(T-t_1)^2\{3\alpha + \beta(2t_1+T)\}}{6T} \\ & - \frac{cI_p(t_1 - M_2)^2[\alpha\{3 + \phi(t_1 - M_2)\} + \beta\{M_2 + 2t_1 + \phi(t_1 - M_2)t_1\}]}{6T} + \frac{pI_e M_2^2}{T}\left(\frac{\alpha}{2} + \frac{\beta M_2}{3}\right) \end{aligned} \tag{21}$$

$$P_4 = p\left(\alpha + \frac{\beta T}{2}\right) - \frac{K}{T} - \frac{ht_1^2}{6T}\{\alpha(3+\phi t_1) + (2+\phi t_1)\beta t_1\} - \frac{s(T-t_1)^2\{3\alpha + \beta(T+2t_1)\}}{6T}$$

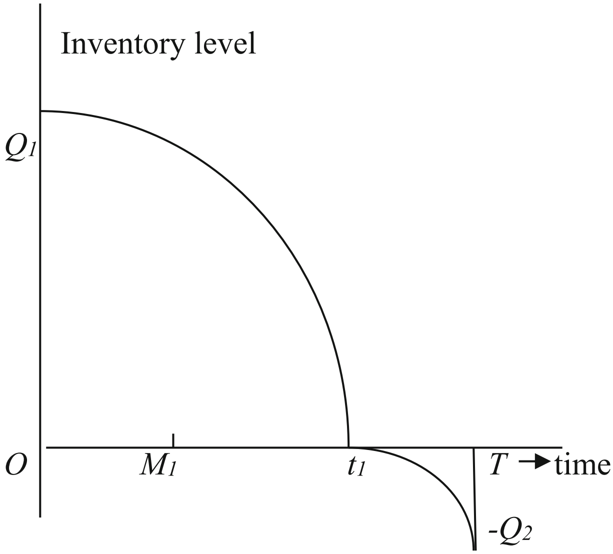

Fig.1 $M_1 < T$

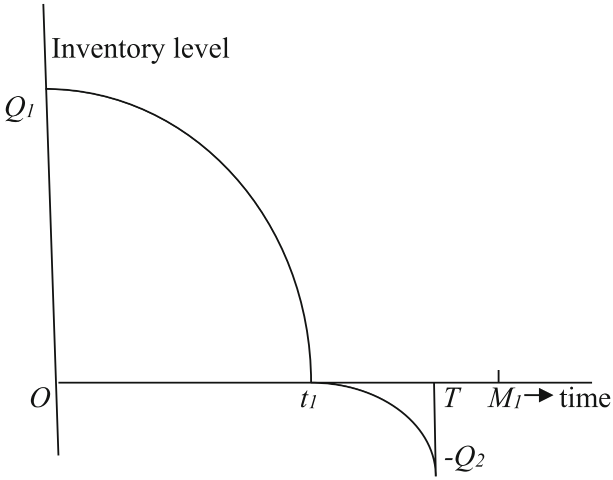

Fig. 2 $M_1 \geq T$

$$+\frac{pI_e t_1}{6T}\{3\alpha(2M_2 - t_1) + \beta(3M_2 - t_1)t_1\} \tag{22}$$

Since $t_1 < T$, taking, $t_1 = \gamma T$, γ is stable ($0 < \gamma < 1$). Equations (19) – (22) become:

$$P_1(T) = \frac{p(1+r)(2\alpha + \beta T)}{2} - \frac{K}{T} - \frac{h\gamma^2\{\alpha(3 + \gamma\phi T) + \beta(2 + \phi\gamma T)\gamma T\}T}{6}$$
$$- \frac{s(1-\gamma)^2\{3\alpha + (1+2\gamma)\beta T\}T}{6}$$
$$- \frac{cI_p(T\gamma - M_1)^2\left[\alpha\{3 + \phi(\gamma T - M_1)\} + \beta\{M_1 + 2T\gamma + \gamma\phi(T\gamma - M_1)\}T\right]}{3T}$$

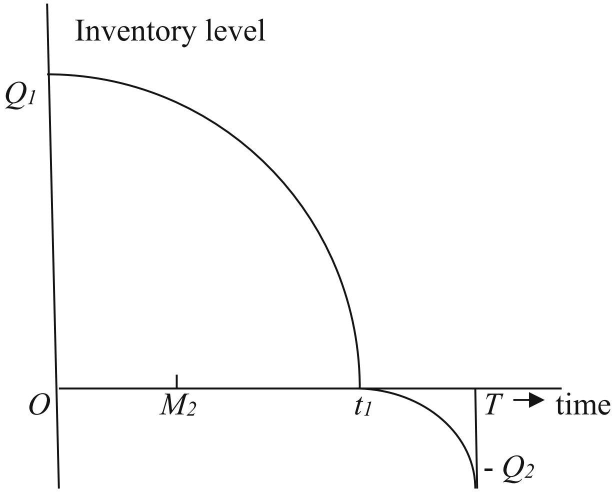

Fig. 3 $M_2 < T$

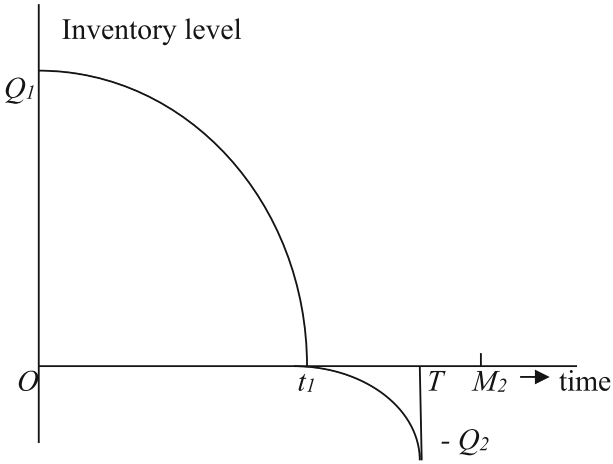

Fig. 4 $M_2 \geq T$

$$+\frac{pI_e(3\alpha+2\beta M_1)M_1^2}{6T}+\frac{rp\gamma^2(\alpha+\beta\gamma T)T}{2} \tag{23}$$

$$\begin{aligned} P_2(T) =& \frac{(1+r)p(2\alpha+\beta T)}{2}-\frac{K}{T}-\frac{h\gamma^2\{\alpha(3+\phi\gamma T)+\beta\gamma(2+\gamma\phi T)T\}T}{6} \\ &-\frac{s(1-\gamma)^2\{3\alpha+\beta T(1+2\gamma)\}T}{6} \\ &+\frac{pI_e\gamma\{3\alpha(2M_1-\gamma T)+\beta\gamma T(3M_1-\gamma T)\}}{6}+\frac{r\phi\gamma^2(a+\beta\gamma T)pT}{2} \end{aligned} \tag{24}$$

$$\begin{aligned} P_3(T) =& p\left(\alpha+\frac{\beta T}{2}\right)-\frac{K}{T}-\frac{h\gamma^2\{a(3+\phi\gamma T)+\beta(2+\gamma\phi T)\gamma T\}T}{6} \\ &-\frac{s(1-\gamma)^2\{3\alpha+\beta(1+2\gamma)T\}T}{6} \end{aligned}$$

$$-\frac{cI_p(\gamma T-M_2)^2}{6T}\left[\alpha\{3+(\gamma T-M_2)\phi\}+\beta\{M_2+2\gamma T+\gamma\phi(\gamma T-M_2)T\}\right]$$
$$+\frac{pI_e(3\alpha+2\beta M_2)M_2^2}{6T} \tag{25}$$

$$P_4(T)=\frac{p(2\alpha+\beta T)}{2}-\frac{K}{T}-\frac{h\gamma^2T\{\alpha(3+\phi\gamma T)+\beta\gamma(2+\phi\gamma T)T\}}{6}$$
$$-\frac{s(1-\gamma)^2\{3\alpha+\beta(2\gamma+1)T\}T}{6}$$
$$+\frac{p\,I_e\,\gamma\{3\alpha(2M_2-\gamma T)+\beta\gamma T(3M_2-\gamma T)\}}{6} \tag{26}$$

Optimal Solution

Necessary and sufficient circumstances for maximization are: $\frac{dP_i}{dT}=0$, and $\frac{d^2P_i}{dT^2}<0$. P_i,for $i=1$–4.

Putting first derivative of (23) – (26) w.r.t. T, to zero, we find

$$3\beta(1+r)pT^2+6K-h\gamma^2T^2\{\alpha(3+2\gamma\phi T)+\beta(4+3\gamma\phi T)\gamma T\}-s(1-\gamma)^2\{3\alpha+2(1+2\gamma)\beta T\}T^2$$
$$-cI_p\left\{(\gamma T)^2-M_1^2\right\}\left[\alpha\{3+\phi(T\gamma-M_1)\}+\{M_1+2T\gamma+\phi\gamma(\gamma T-M_1)T\}\beta\right]$$
$$-cI_p\gamma(\gamma T-M_1)^2T\left[\alpha\phi+\beta\{2+(2T\gamma-M_1)\phi\}\right]-pI_e(3\alpha+2\beta M_1)M_1^2+rp\gamma^2\phi(\alpha+2\beta\gamma T)T^2=0 \tag{27}$$

$$6\beta(1+r)pT^2+6K-h\gamma^2T^2\{\alpha(3+2\gamma\phi T)+\gamma\beta(4+3\phi\gamma T)T\}-s(1-\gamma)^2\{3\alpha+2\beta(1+2\gamma)T\}T^2$$
$$\_p\,I_e\gamma^2\{3\alpha-\beta(3M_1-2\gamma T)\}T^2+rp\phi(\gamma T)^2(\alpha+2\gamma\beta T)=0 \tag{28}$$

$$3\beta pT^2-h\gamma^2T^2\{\alpha(3+2\gamma\phi T)+\beta\gamma T(4+3\gamma\phi T)\}-s(1-\gamma)^2\{3\alpha+2\beta(1+2\gamma)T\}T^2$$
$$-cI_p\left(T^2\gamma^2-M_2^2\right)\left[\alpha\{3+\phi(\gamma T-M_2)\}+\beta\{M_2+2\gamma T+\phi\gamma(\gamma T-M_2)T\}\right]$$
$$-cI_p\gamma T(\gamma T-M_2)^2\left[\alpha\phi+\beta\{2+\phi\,(2\gamma T-M_2)\}\right]-pI_eM_2^2(3\alpha+2\beta M_2)$$
$$-3cI_p(t_1-M_2)^2(\alpha+\beta t_1)-pI_eM_2^2(3\alpha+2\beta M_2)+6K=0 \tag{29}$$

and

$$3\beta pT^2+6\,K-h\gamma^2\{\alpha(3+2\gamma\phi T)+\beta\gamma(4+3\gamma\phi T)T\}T^2-s(1-\gamma)^2\{3\alpha+2(1+2\gamma)\beta T\}T^2$$
$$+pI_e\gamma^2T^2\{3\alpha-(3M_2-2\gamma T)\beta\}=0 \tag{30}$$

Also

$$\frac{d^2P_1}{dT^2}=-\frac{2K}{T^3}-\frac{h\gamma^3\{\alpha\phi+\beta(2+3\phi\gamma T\}}{3}-\frac{s\beta(1-\gamma)^2(2\gamma+1)}{3}$$
$$-\frac{cI_p\left(\gamma^2T^2-M_1^2\right)\left[\alpha\phi+\beta\{2+\phi(2\gamma T-M_1)\}\right]}{3}$$
$$\frac{cI_pM_1^2}{T^3}\left[\alpha\left\{1+\frac{\phi(\gamma T-M_1)}{3}\right\}+\{M_1+2\gamma T+\phi\gamma(\gamma T-M_1)T\}\right]+\frac{cI_p\lambda^2\beta\phi}{T^3}+r\beta p\phi\gamma^3 \tag{31}$$

$$\frac{d^2P_2}{dT^2}=-\left[\frac{2K}{T^3}+\frac{h\,\gamma^3\{\alpha\phi+\beta(2+3\phi\gamma T\}}{3}+\frac{s\beta\,(1-\gamma)^2(1+2\gamma)}{3}+\frac{\beta pI_e\gamma^3}{3}-r\beta p\phi\gamma^3\right]<0 \tag{32}$$

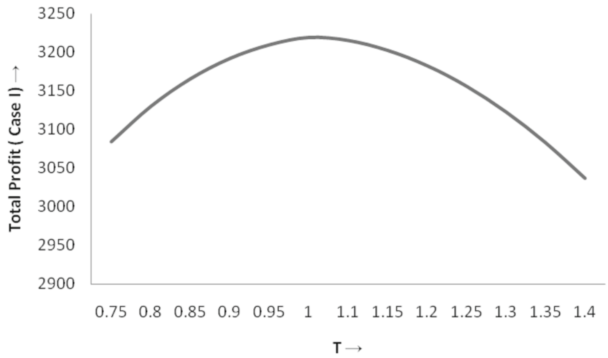

Fig. 5 Figure between T & P_1

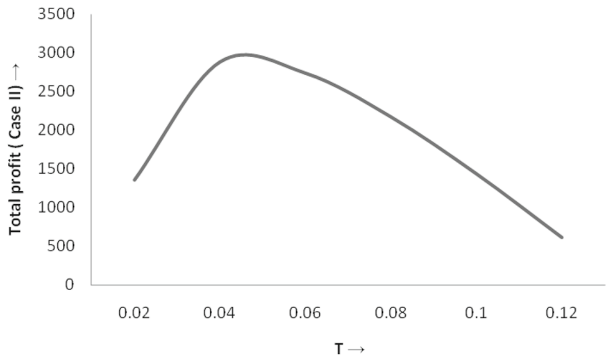

Fig. 6 Illustrative depiction connecting T & P_2

$$
\begin{aligned}
\frac{d^2P_3}{dT^2} &= -\frac{2K}{T^3} - \frac{h\gamma^3\{\alpha\phi + \beta(2+3\phi\gamma T\}}{3} - \frac{s\,\beta(1-\gamma)^2(1+2\gamma)}{3} \\
&- \frac{c\,I_p\left(\gamma^2T^2 - M_2^2\right)\left[\alpha\phi + \beta\{2+\phi(2\gamma T - M_2)\}\right]}{3} \\
&- \frac{cI_pM_2^2}{T^3}\left[\alpha\left\{1 + \frac{(\gamma T - M_2)\phi}{3}\right\} + \{M_2 + 2\gamma T + \gamma\phi T(\gamma T - M_2)\}\right] \\
&- \frac{cI_p\gamma^2\beta\phi}{3T} + \frac{2pI_eM_2^2}{T^3}\left(\frac{\alpha}{2} + \frac{\beta\,M_2}{3}\right) < 0
\end{aligned} \tag{33}
$$

$$
\frac{d^2P_4}{dT^2} = -\left[\frac{2K}{T^2} + \frac{h\gamma^3\{\alpha\phi + \beta(2+3\gamma\phi T)\}}{3} - \frac{s(1-\gamma)^2(1+2\gamma)}{3} + \frac{\beta pI_e\gamma^3}{3}\right] < 0 \tag{34}
$$

Since $\frac{d^2P_i}{dT^2} < 0$, i = 1 - 4. P_i is maximum at T_i*. We have also shown by graphs in numerical examples.

Algorithm

In this section, we provide a solution procedure and flow diagram for finding an optimal resolution.

Step 1 locate T_i* by resolve (27)–(30), $i = 1$–4.

Step 2 if $T_1* \geq M_1$, come across P_1* by (23).

Step 3 if $T_2* < M_1$, discover P_2* by (24).

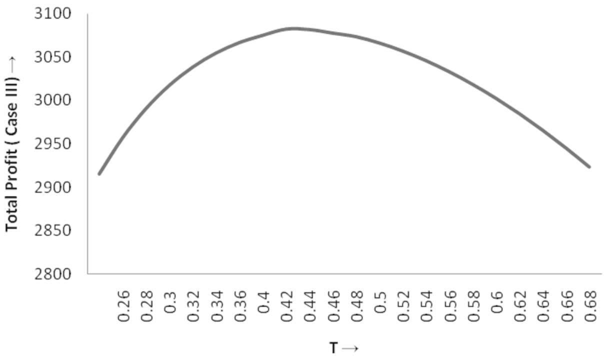

Fig. 7 Pictographic demonstration connecting T and P_3

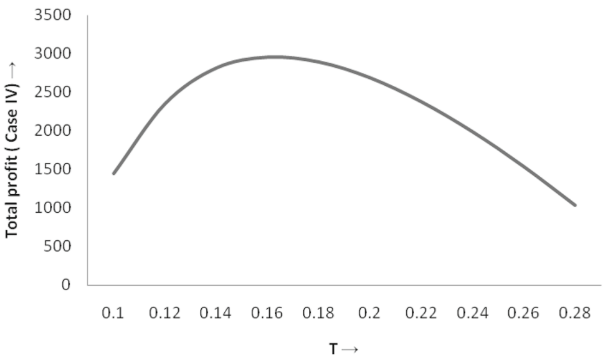

Fig. 8 Graphical depiction T vs. P_4

Step 4 if $T_3{*} \geq M_2$, find $P_3{*}$ by (25).
Step 5 if $T_4{*} < M_2$, locate $P_4{*}$ by (26).
Step 6 Obtain most favorable income $P_i = \max P_i$.
Step 7 end.

Numerical Examples

Examples are supplied to make obvious conclusions of structure discussed in each case:

Example 1 ($M_1 \leq \mathbf{T}$).

Bearing in mind subsequent constrains in proper component:
$\alpha = 1.5 \times 10^3$, $\beta = 150$, $\phi = 1/100$, $s = 100$, $K = 1.0 \times 10^3$, $p = 250$, $h = 10$, $I_e = 13/100$, $I_p = 3/20$, $M_1 = 1/5$ yr, $c = 100$, $r = 1/50$ & $\gamma = 3/5$. Putting these in (27), and resolving for T, we find, $T_1{*} = 0.65989$ yr, that validate case 1, cprresponding $Q{*} = 1023.7$ and $P_1{*} = \$\ 2973.7$.

Example 2 ($M_1 > \mathbf{T}$).

Considering following strictures in their suitable units:
$\alpha = 1.5 \times 10^3$, $\beta = 150$, $\phi = 1/100$, s = 500, $h = 10$, $K = 100$, $p = 300$, $I_e = 13/100$, $I_p = 3/20$, $M_1 = 1/5$ yr, $c = 50$, $r = 1/50$ & $\gamma = 3./5$. Replacement of those in (28) and solving for T, we gain, $T_2{*} = 0.1157$ yr, which proves case 2, accordingly $Q{*} = 174.57$ & $P_2{*} = \$\ 794.37$.

Table 1 Deviation of *T, Q* and P_i by *s, K, c, p, h,* ϕ, α and β

s	$T*$	$Q*$	P_1*	K	$T*$	$Q*$	P_1*
105	0.55973	863.97	2448.7	1100	0.69855	1085.8	2882.8
110	0.48786	750.29	2020.7	1200	0.73295	1141.2	2793.4
115	0.43481	666.92	1717.6	1300	0.76408	1191.6	2705.4
120	0.39437	603.65	1347.1	1400	0.79263	1237.8	2619.0
125	0.36259	554.11	1069.4	1500	0.81908	1280.8	2533.9
c	$T*$	$Q*$	P_1*	p	$T*$	$Q*$	P_1*
105	0.63346	981.41	2903.7	260	0.80909	1264.6	3860.7
110	0.61039	944.57	2842.4	270	1.01837	1608.3	4993.1
115	0.59018	912.36	2788.4	280	1.27802	2044.3	6417.3
120	0.57241	884.10	2740.8	290	1.56867	2544.8	8148.3
125	0.55671	859.18	2698.5	300	1.87570	3088.0	9976.4
h	$T*$	$Q*$	P_1*	ϕ	$T*$	$Q*$	P_1*
12	0.57088	881.67	2505.4	0.02	0.65874	1023.2	2969.2
14	0.50394	775.66	2113.6	0.03	0.65771	1022.7	2964.8
16	0.45278	695.11	1777.9	0.04	0.65664	1022.1	2960.4
18	0.41281	632.46	1483.8	0.05	0.65559	1021.6	2956.0
20	0.38084	582.53	1221.2	0.06	0.65454	1021.2	2951.7
1250	1.8827	2533.8	8242.99	200	2.24652	3890.5	13,933.5
α	$T*$	$Q*$	P_1*	β	$T*$	$Q*$	P_1*
1000	2.9219	3581.6	14,854.0	160	0.92394	1456.7	4323.9
1050	2.6585	3337.9	12,968.5	170	1.26014	2029.8	6151.1
1100	2.3968	3080.9	11,238.0	180	1.60881	2654.0	8405.0
1200	2.1378	2812.2	9662.94	190	1.94021	3279.6	11,017.0
1250	1.8827	2533.8	8242.99	200	2.24652	3890.5	13,933.5

Example 3 ($\boldsymbol{M_2 < T}$).

Let us choose following constraints in proper entities:
$\alpha = 1.1 \times 10^3$, $\beta = 150$, $\phi = 1/100$, $s = 100$, $K = 1.0 \times 10^3$, $p = 250$, $h = 10$, $I_e = 13/100$, $I_p = 3/20$, $M_1 = 1/4$ yr, $c = 100$, $r = 1.50$, and $M_2 = 140$ days. On putting these (29) and solving for T, we find, $T_3* = 0.42588$ yr, which confirms case 3, related $Q* = 652.92$ & $P_3* = \$3082.5$.

Example 4 ($\boldsymbol{M_2 \geq T}$).

Following constraints are taken in suitable units:
$\alpha = 1.5 \times 10^3$, $\beta = 150$, $\phi = 1/100$, $s = 100$, $K = 1.0 \times 10^3$, $p = 500$, $h = 10$, $I_e = 13/100$, $I_p = 3/20$, $c = 50$, $r = 1/100$, and $M_2 = 1/4$ yr. On substituting those in (30), and solving for T, we obtain, $T_4* = 0.19933$ years, that verifies case 4, resultant $Q* = 338.0$ & $P_4* = \$2328.2$.

Using above algorithm Case 3 gives the optimal (maximum) solution (Figs. 5, 6, 7 and 8).

Table 2 Disparity of T, Q and P_i with s, K, p, h, α, and β

s	T^*	Q^*	$P_2{}^*$	K	T^*	Q^*	$P_2{}^*$
510	0.10770	162.46	995.48	105	0.11837	178.65	639.13
520	0.10111	152.47	1146.3	110	0.12098	182.60	467.84
530	0.09557	144.07	1260.5	115	0.12352	186.46	340.12
540	0.09083	136.89	1347.3	120	0.12599	190.22	195.83
s	T^*	Q^*	$P_2{}^*$	K	T^*	Q^*	$P_2{}^*$
p	T^*	Q^*	$P_2{}^*$	h	T^*	Q^*	$P_2{}^*$
280	0.09515	143.43	1044.4	12	0.11192	168.85	891.37
285	0.09929	149.71	1025.6	14	0.10848	163.62	976.45
290	0.10440	156.84	982.16	16	0.10532	158.84	1032.2
295	0.10941	165.04	907.87	18	0.10241	154.43	1117.2
310	0.15204	199.42	397.41	20	0.09972	150.36	1175.4
α	T^*	Q^*	$P_2{}^*$	β	T^*	Q^*	$P_2{}^*$
1600	0.20466	330.71	2508.0	120	0.07868	118.40	1933.9
1700	0.19034	326.41	2584.1	125	0.08253	124.24	1874.5
1800	0.17865	324.06	2865.0	130	0.08699	130.99	1771.0
1900	0.16886	323.07	3171.3	135	0.09222	138.93	1663.0
2000	0.16051	323.05	3501.8	140	0.09848	148.42	1524.6

Sensitivity Analysis

Case 1

Considering identical data as in Ex. 1, sensitivity study is conversed. Fallouts are reviewed in Table 1.

Case 2

Using the same figures as in Ex. 2, sensitivity scrutiny is conversed in Table 2.

Case 3

With parallel information as in Ex 3, sensitivity inquiry id discussed below:

Case 4

By means of alike data as design in Ex. 4, sensitivity inspection is as follows:

Following judgment can be finished from Table 1:

1. Enlarge of s, K, c and it will cause a drop of P_1.
2. Elevate of 'p' and ϕ will lead augment in P_1.

Following submission can be equipped from Table 2.

1. Lift of s, K and h will cause weakening in P_2.

Table 3 Dissimilarity of T, Q and P_i by s, K, c, p, h, α. β and ϕ

s	T^*	Q^*	$P_3{}^*$	K	T^*	Q^*	$P_3{}^*$
75	1.67627	2733.5	6987.5	1100	0.49783	766.03	2866.4
80	1.31673	2110.2	5093.2	1200	0.55300	853.29	2676.3
85	1.00272	1582.4	4340.7	1300	0.59869	925.92	2502.7
90	0.74353	1158.3	3786.5	1400	0.63815	988.91	2341.1
95	0.55171	857.21	3383.2	1500	0.67315	1045.0	2186.6
c	T^*	Q^*	$P_3{}^*$	p	T^*	Q^*	$P_3{}^*$
105	0.42468	651.04	3082.9	260	0.52326	806.18	3576.3
110	0.42375	649.59	3082.4	270	0.60849	941.53	3847.7
115	0.42302	648.50	3082.4	280	0.72163	1123.0	4148.2
120	0.42342	647.51	3082.4	290	0.85594	1340.9	4487.9
125	0.42193	646.74	3082.4	300	1.00290	1582.7	4874.5
h	T^*	Q^*	$P_3{}^*$	α	T^*	Q^*	$P_3{}^*$
4	0.88331	1385.7	4114.6	1050	2.4952	2838.8	9524.6
5	0.77881	1215.4	3882.6	1100	2.2308	2563.1	8170.0
6	0.68476	1063.6	3679.2	1150	1.9681	2407.9	6982.0
7	0.60240	1024.3	3501.0	1200	1.7079	1978.6	5959.8
8	0.53225	820.04	3344.5	1250	1.4518	1675.3	5101.8
β	T^*	Q^*	$P_3{}^*$	ϕ	T^*	Q^*	$P_3{}^*$
160	0.68667	1019.1	3016.7	0.03	0.42441	651.61	3080.5
170	1.05000	1671.9	4298.9	0.04	0.42368	650.97	3079.4
160	1.41903	2315.8	5859.2	0.05	0.42296	650.33	3078.5
190	1.76202	2947.4	7735.1	0.06	0.42239	649.69	3077.5
200	2.07512	2556.9	9809.1	0.07	0.42153	649.06	3076.5

2. Raise of p consequences enhances P_2.

Following proposition can be finished from Table 3.

1. Boost of s, K, and h will direct diminish P_3.
2. Augment of will direct decline in P_3.
3. Amplification of p causes moves up in P_3.

Deductions made from Table 4 are as follows:

1. Boost of s and p will express make bigger in P_4.
2. Improve of A and h lead, turn down in P_4.

Managerial Insights

These above deviations have the following managerial implications.

- Higher values of s, c, h, ϕ and α implies lower values of cycle time, order quantity and total profit for case I and III.

Table 4 Discrepancy of *T, Q & P_i* by *s, K, c, p* and *h*

s	*T*	*Q**	P_4*	*K*	*T**	*Q**	P_4*
460	0.21621	327.94	2178.7	950	0.21711	329.32	2657.3
470	0.21024	318.80	3026.5	960	0.21824	331.06	2590.9
480	0.20475	310.39	1872.6	970	0.21837	332.80	2524.7
490	0.19968	302.62	1717.4	980	0.22050	334.52	2454.9
500	0.19498	285.42	1561.5	990	0.22162	336.24	2393.4
p	*T**	*Q**	P_4*	*h*	*T**	*Q**	P_4*
450	0.20427	306.45	700.10	12	0.21976	333.39	2261.4
460	0.20758	314.72	1022.0	14	0.21691	329.02	2194.0
470	0.21107	320.06	1361.2	16	0.21417	324.81	2126.1
480	0.21474	325.70	1687.2	18	0.21153	320.80	2052.7
490	0.21862	331.65	2009.7	20	0.20899	316.87	1989.0
α	*T**	*Q**	P_3*	β	*T**	*Q**	P_3*
1600	0.20466	330.71	2584.1	100	0.17645	266.32	727.25
1700	0.19034	326.41	2865.0	110	0.18345	277.12	1047.4
1800	0.17865	324.06	3171.4	120	0.19134	289.31	1369.4
1900	0.16886	323.07	3501.8	130	0.20032	303.20	1691.9
2000	0.16051	232.05	3854.8	140	0.21065	319.21	2012.7

- Higher values of p and β implies higher values of cycle time, order quantity and total profit for case I and III.
- Higher values of K implies higher values of cycle time and order quantity while lower values of total profit.
- Higher values of s, h, and α implies lower values of cycle time,order quantity and lower values total profit for case II and IV.
- Higher values of K, p, and β implies higher values of cycle time,order quantity and lower values total profit for case II and IV.

Conclusion

In this study, we have deliberated EOQ models under trade credits permit for four unlike conditions. We have attempted to locate characteristic of cash decline into the conventional model with permitted delay. Numerical examples are completed on credible attempt. Optimal explanation is acquired for finding optimal variables. Solution process is communicated to find most advantageous solution. Sensitivity reading of the clarification for dissimilar constraints has been conferring. This research is obliging for returning products since demand for continuing foodstuffs is usually time allied. It is seen that disparity in shortage, ordering, procure, carrying and selling costs, lead to momentous possessions on finest P_i, $i = 1$–4). Entire income is around stable with adjust in weakening rate. Outcomes came into view in sensitivity analysis is conflicting, like expand in cost fallouts reject of earnings whereas intensify in selling price argued lift in income.

A variety of likely extensions of the model that can be presented as like: (i) variable decay and Weibull deterioration (ii) to assume a variable carrying cost (iii) to comprise fall in the purchasing cost/ unit (iv) to study the case of inflation and shipment charges and (v) to study stock- sensitive demand.

References

1. Silver, E.S., Peterson, R.: Decision systems for inventory management and production planning, 2nd edn. Wiley, New York (1995)
2. Baker, R.C., Urban, T.L.: A deterministic inventory system with an inventory level- dependent demand. Euro. J. Oper. Res. **39**, 823–831 (1988)
3. Tripathi, R.P.: Economic order Quantity models for price dependent demand and different holding cost functions. Jor. J. Math. Stat. **12**(1), 15–33 (2019)
4. Alfares, H.K.: Inventory model with stock- level dependent demand rate and variable holding cost. Int. J. Prod. Eco. **108**, 259–265 (2007)
5. Tripathi, R.P., Mishra, S.: Innovative study of EPQ model with time – induced demand under decline release. Int. J. Sup. Chain. Oper. Manag. **6**(1), 264–275 (2019)
6. Dave, U.: An order level inventory model for deteriorating items with variable instantaneous and discrete opportunities for replenishment. Opsearch **23**, 244–254 (1986)
7. Maiti, A.K., Maiti, M.K., Maiti, M.: Inventory model with stochastic lead time and price dependent demand incorporating advance payment. App. Math. Mod. **33**, 2433–2443 (2009)
8. Yang, C.T.: An inventory model with both stock- dependent demand rate and stock- dependent holding cost rate. Int. J. Pro. Eco. **155**, 214–221 (2014)
9. Chang, C.T., Teng, J.T., Goyal, S.K.: Optimal replenishment policies for non-instantaneous deteriorating item with stock- dependent demand. Int. J. Pro. Eco. **123**, 62–68 (2010)
10. Soni, H., Shah, N.H.: Optimal Ordering policy for stock- dependent demand under progressive payment scheme. Eur. J. Oper. Res. **184**, 91–100 (2008)
11. Tripathi, R.P.: Inventory model with time – dependent demand rate under inflation when supplier credit linked to order quantity. Int. J. Bus. Inf. Tech. **1**(3), 174–18 (2011)
12. Teng, J.T., Min, J., Pan, Q.: Economic order quantity model with trade credit financing for non- decreasing demand. Omega **40**, 328–335 (2012)
13. Hsieh, T.P., Dye, C.Y.: A production – inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. J. App. Comp. Math. **239**, 25–36 (2003)
14. Wee, H.M., Wang, W.T.: A variable production scheduling policy for deteriorating item with time- varying demand. Com. Oper. Res. **26**(3), 237–254 (1999)
15. Zhau, Y.W., Lau, H.S., Yang, S.L.: A new variable production scheduling strategy for deteriorating item with time varying demand and partial lost sale. Com. Oper. Res **30**(21), 1753–1776 (2003)
16. Shah, N.H., Chaudhari, U., Jani, M.Y.: An inventory model with an expiration date of items and deterioration under two-level trade credit and preservation technology investment for time and price-sensitive demand: DCF approach. Int. J. Log. Sys. Manag. **27**(4), 420–437 (2017)
17. Tripathi, R.P., Pandey, H.S.: Optimal ordering policies for non- instantaneous Weibull deteriorating items with price linked demand under trade credits. Int. J. Sup. Cha. Inv. Mana. **3**(2), 77–92 (2020)
18. Geetha, K.V., Uttayakumar, R.: Economic design for an inventory policy for non- instantaneous deteriorating item under permissible delay in payments. J. App. Comp. Math. **233**(10), 2492–2505 (2020)
19. Jaggi, C.K., Tiwari, S., Goel, S.K.: Credit financing in economic ordering policies for non- instantaneous deteriorating item with price dependent demand and two storage facilities. Ann. Oper. Res. **248**(1–2), 253–280 (2017)
20. Mishra, P., Talati, I.: Quantity discount for integrated supply chain model with back order and controllable deterioration rate. Yugo. J. Oper. Res. **28**(5), 355–369 (2018)
21. Jani, MY., Shah, NH., and Chaudhari, UB.,: Inventory Control Policies for Time-Dependent Deteriorating Item with Variable Demand and Two Level Order Linked Trade Credit. In: Shah, NH., and Mittal, M., (eds.) Optimization and inventory management, (pp. 55-67). Springer, Singapore (2020)
22. Chaudhury, K.D., Karmakar, B., Das, M., Datta, T.K.: An inventory model for deteriorating items with stock- dependent demand, time varying holding cost and shortages. Opsearch **52**(1), 55–74 (2015)
23. Duan, Y., Li, G., Tien, J.M., Huo, J.: Inventory models for perishable items with inventory level dependent demand rate. Appl. Math. Model. **36**, 5015–5028 (2017)
24. Pervin, M., Roy, S.K., Weber, G.W.: A two – echelon inventory model with stock- dependent demand and variable holding cost for deteriorating item. Numer. Alg. Con. Opt. **7**(1), 21–50 (2017)
25. Teng, J.T., Chang, C.T.: Economic production quantity models for deteriorating items with price and stock-dependent demand. Comp. Oper. Res. **32**, 297–308 (2005)

26. Dye, C.Y.: The effect of preservation technology investment on a non- instantaneous deteriorating inventory model. Omega **41**, 872–880 (2013)
27. Ghiani, Y., Williams, T.: A two- level production inventory model for deteriorating items with multiple buyers. Int. J. Prod. Res. **159**, 233–240 (2015)
28. Shah, N.H., Jani, M.Y., Chaudhari, U.: Optimal replenishment time for a retailer under partial upstream prepayment and partial downstream overdue payment for quadratic demand. Math. Comp. Mod. Dyna. Sys. **24**(1), 1–11 (2018)
29. Buzacott, J.A.: Economic order quantities with inflation. Oper. Res. Quar. **26**, 553–558 (1975)
30. Yang, H.L., Teng, J.T., Chern, M.G.: Deterministic inventory lot- size models under inflation with shortages and deterioration for fluctuating demand. Nov. Res. Log. **48**, 144–158 (2001)
31. Sarkar, B., Moon, I.: An EPQ model with inflation in an imperfect production system. App. Math. Comp. **217**, 6159–6167 (2011)
32. Chaudhari, U.B., Shah, N.H., Jani, M.Y.: Inventory modelling of deteriorating item and preservation technology with advance payment scheme under quadratic demand. Opt. Inv. Mana. **3**(2), 69–79 (2020)
33. Sarkar, B.R., Jamal, A.M.M., Wang, J.: Supply chain model for perishable products under inflation and permissible delay in payments. Comp. Oper. Res. **27**, 59–75 (2000)
34. Yang, H.L.: Two warehouse inventory models for deteriorating items with shortages under inflation. Eur. J. Oper. Res. **157**, 344–356 (2004)
35. Dey, J.K., Mondal, S.K., Maity, M.: Two storage inventory problem with dynamic demand and interval valued lead time over finite time horizon under inflation and time value of money. Euro. J. Oper. Res. **185**, 170–194 (2008)
36. Tripathi, R.P., Chaudhary, S.K.: Inflationary induced EOQ model for Weibull distribution deterioration and trade credits. Int. J. App. Comp. Math. **3**(4), 3341–3353 (2017)
37. Tripathi, R.P.: Innovation of Economic Order quantity (EOQ) model for deteriorating items with time-linked quadratic demand under non- decreasing shortages. Int. J. App. Comp. Math. **5**(5), 1–13 (2019)
38. Pal, M., Chandra, S.: A periodic review inventory model with stock- dependent demand, permissible delay in payments and price discount in backorders. Yugo. J. Oper. Res. **24**(1), 99–110 (2014)
39. Chern, S.C., Cardenss- Barron, L.E., Teng, J.T.: Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments linked to order quantity. Int. J. Pro. Eco. **155**, 284–291 (2014)
40. Jiangtao, M., Guimei, C., Ting, F., Hong, M.: Optimal ordering policies for perishable multi- item under stock- dependent demand and two level trade credits. App. Math. Mod. **38**(9–10), 2422–2532 (2014)
41. Chen, S.C., Teng, T.: Retailer's optimal ordering policy for deteriorating items with maximum lifetime under supplier's trade credit financing. App. Math. Mod. **38**, 4049–4061 (2014)