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Ln-type estimators for the estimation of the population mean of a sensitive study variable using auxiliary information

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ABSTRACT

In this article, we offered two *ln*-type estimators for the population mean estimation of a sensitive study variable by using the auxiliary information under the design of basic probability sampling. The Taylor and log series were used to derive the expressions of mean square error and bias up to the first order. Improved classes of proposed estimators are obtained by using conventional parameters associated with the supplementary variable to obtained precise estimates. Mathematical comparisons of the estimators have been made with the usual mean and ratio estimators using theoretical equations of mean square error. A simulation study is conducted for the evaluation of proposed estimator's implementation using four artificial populations generated through *R*-software with different choices of mean vectors and variance-covariance matrices. The demonstration of proposed *ln*-type estimators was implemented through the real data application.

1. Introduction

In many real-life surveys, the main problem arises when our concerned variable is confidential in direct and natural observation is not possible. The answer may not be true but fabricated or even the rejection by the respondent whenever a sensitive question is asked of the respondent. To reduce the response error problem [1], first gave the used the randomized response technique (*RRT*) which supports interrogators to trustworthy information from sensitive questions while sustaining the respondent secrecy. The *RRT* permits the respondent to mark the real response by providing a knotted response, where there is a facility for researchers to decode at a comprehensive level (and not on an individual level). Many other researchers modified, extended, and improved the *RRTs* in different ways. Before the last decade, various authors assessed the average of the confidential variable without having the auxiliary data which includes [2–6]; and [7].

The use of supplementary information in sample surveys is very effectual for the substantial decline in the equation of *MSE* of the estimators, in particular, for the situation of high correlation between the support and interest variables. Nowadays, researchers usually might be inclined to get information on many variables (which may be connected with the main variable of variable) rather

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than just the main variable of interest. The variables collected along with the variable of main study to obtain precise estimates are known as the auxiliary variables. Various examples of auxiliary information along with the sensitive survey variable have been described in the literature. For instance, in business surveys, the purchase order (Y) is considered the confidential variable for enterprises, and the supporting variable (X) may be the revenue of each enterprise. Illegal income is a confidential study variable (Y) whereas household expenditures, savings, or bills may be considered non-sensitive auxiliary variables. The use of drugs is known as a sensitive variable (Y) while gender or age is may be treated as the assisting variable (X).

Several authors have provided numerous ratio and product type estimators when both auxiliary and research variables are readily observable. . These comprise [8–15] and many more. For the estimation of quantitative sensitive variables, traditional ratio, linear regression and regression-cum-ratio type estimators were respectively proposed by Refs. [16,17]; and [18]. Later on, [19–28]; and [29] have proposed numerous estimators using different *RRMs*.

In this article, we have provided two generalized *ln*-type estimators for the estimation of population mean for a sensitive variable in the context of the non-sensitive auxiliary variable under *SRS* design. The sampling procedure, basic notations, formulas, and some significant estimators are given in Section 2. We derive the expressions of approximate *MSE* and bias of the proposed estimators using well-known log and Taylor expansions in Section 3 and we got some new cases of the proposed estimators using different choices of the scalar constants. The proposed estimators are compared against the existing estimators mathematically in Section 4 to identify the ideal conditions. An extensive study of simulation is managed in Section 5 to judge the application of proposed *ln*-type estimators for the scrambled responses estimation. Real data application is also used in Section 6 to validate the findings obtained from simulation study. Comments and closing remarks are presented in Section 7.

2. Methodology, Notations and Associated Estimators

Let Y be the primary variable of the study, which is sensitive and cannot observe directly whereas X is the supporting variable correlated with the concerned study variable. Consider a finite population, say P, which involves of N distinct units, $P=(P_1, P_2 \cdots P_N)$ and numbered from 1 to N units. Let 'S' be a scrambling variable with mean zero and constant variance and assumed to be independent from Y an X. The reported response is indicated by Z obtained by the model $Z = Y + S$ given by Ref. [30]. As the mean, μ_s is zero, we have $\mu_z = \mu_y$. Let y_i, z_i and x_i are the observed values of Y, Z and X respectively. Assume that the pairs of (z_i, x_i) $(i = 1, 2, \ldots, n)$ values are taken from n units chosen from N population units using the scheme of simple random sampling without replacement (*SRSWOR*).

The population means and variances for Y, Z, X and S are respectively denoted by:

$\mu_y = (1/N)\sum_{j=1}^{N} y_j$, $\mu_z = (1/N)\sum_{j=1}^{N} z_j$, $\mu_x = (1/N)\sum_{j=1}^{N} x_j$ and $\mu_s = (1/N)\sum_{j=1}^{N} S_j$. $\sigma_y^2 = (N-1)^{-1}\sum_{j=1}^{N}(y_j-\mu_y)^2$, $\sigma_z^2 = (N-1)^{-1}\sum_{j=1}^{N}(z_j-\mu_z)^2$, $\sigma_x^2 = (N-1)^{-1}\sum_{j=1}^{N}(x_j-\mu_x)^2$ and $\sigma_s^2 = (N-1)^{-1}\sum_{j=1}^{N}(s_j-\mu_s)^2$.

Similarly, the sample means and variances for y, z, x and $s$are respectively denoted by:

$\bar{y} = (1/n)\sum_{j=1}^{n} y_j \bar{z} = (1/n)\sum_{j=1}^{n} z_j, \bar{x} = (1/n)\sum_{j=1}^{n} x_j$ and $\bar{s} = (1/n)\sum_{j=1}^{n} s_j$.

$$s_y^2 = (n-1)^{-1}\sum_{j=1}^{n}(y_j-\bar{y})^2,\ s_z^2 = (n-1)^{-1}\sum_{j=1}^{n}(z_j-\bar{z})^2,\ s_x^2 = (n-1)^{-1}\sum_{j=1}^{n}(x_j-\bar{x})^2 \text{ and } s_s^2 = (n-1)^{-1}\sum_{j=1}^{n}(s_j-\bar{s})^2$$

Let $C_y(=\sigma_y/\mu_y)$, $C_z(=C_y+\sigma_s/\mu_y)$, and $C_x(=\sigma_x/\mu_x)$ be the coefficients of variation of the subscripts and $\rho_{zx} = \rho_{yx}/\sqrt{1+(\sigma_s^2/\sigma_y^2)}$ and $\sigma_{zx} = (N-1)^{-1}\sum_{j=1}^{N}(z_j-\mu_z)(x_i-\mu_x)$ be the coefficient of correlation and covariance between the subscripts respectively.

The relative sampling errors are defined to derive the mathematical expressions of *MSE* and the bias of the proposed estimators as

$$\xi_z = \frac{\bar{z}}{\mu_y} - 1,\ \ \xi_x = \frac{\bar{x}}{\mu_x} - 1, \xi_x^{/} = \frac{s_x^2}{\sigma_x^2} - 1,\ \text{ and }\ \xi_{zx} = \frac{s_{zx}}{\sigma_{zx}} - 1,$$

We have

$$E(\xi_z) = E(\xi_x) = E(\xi_{zx}) = E(\xi_x^{/}) = 0,$$

$$E(\xi_z^2) = \theta C_z^2, E(\xi_x^2) = \theta C_x^2, E(\xi_x^{/2}) = \theta\delta_{04}^{*}, E(\xi_z\xi_x) = \theta\rho_{zx}C_zC_x, E(\xi_z\xi_x^{/}) = \theta\lambda_{12}C_x, \tag{1}$$

$$E(\xi_x\xi_x^{/}) = \theta\lambda_{03}C_x, H_{zx} = \rho_{zx}(C_z/C_x), \beta_{zx} = \sigma_{zx}/\sigma_x^2 f = n/N, \theta = n^{-1} - N^{-1},$$

$$\lambda_{pq} = \frac{\mu_{pq}}{\mu_{20}^{p/2}\mu_{02}^{q/2}}, \delta_{pq}^{*} = (\lambda_{pq}-1), \mu_{pq} = (N-1)^{-1}\sum_{i=1}^{N}(z_i-\mu_z)^p(x_i-\mu_x)^q.$$

where μ_{20} and μ_{02}be the second order moments and $\lambda_{pq}$is the momens ratio.

i. The fundamental approximation for estimating the population mean of a sensitive study variable is defined in Eq. (2) by the sample mean as

$$\widehat{\mu}_y = n^{-1}\sum_{j=1}^{n} z_j = \bar{z}. \tag{2}$$

Eq. (2) is the variance of $\widehat{\mu}_y$ is

$$MSE(\widehat{\mu}_y) = \theta\mu_y^2 C_z^2. \tag{3}$$

ii. Classical ratio estimator for estimating the population mean of sensitive study variable using auxiliary data suggested by Ref. [16] is defined in Eq. (4) as

$$\widehat{\mu}_r = \overline{z}\left(\frac{\mu_x}{\overline{x}}\right). \tag{4}$$

The expressions of approximate bias and *MSE* of $\widehat{\mu}_r$ are respectively mentioned in Eq. (5) and Eq. (6)

$$Bias(\widehat{\mu}_r) \approx \theta\mu_y C_x^2(1 - H_{zx}). \tag{5}$$

and

$$MSE(\widehat{\mu}_r) \approx \theta\mu_y^2\left(C_z^2 + C_x^2(1 - 2H_{zx})\right). \tag{6}$$

iii. For estimating the population mean of a confidential variable, the *Ln*-type modified ratio estimator is defined in Eq. (7) as

$$\widehat{\mu}_{lr} = ln\left(\frac{\alpha\sigma_x^2 + \delta}{\alpha s_x^2 + \delta}\right)^{\overline{z}}. \tag{7}$$

The approximate bias and *MSE* of $\widehat{\mu}_{lr}$ are expressed in Eq. (8) and Eq. (9) respectively

$$Bias(\widehat{\mu}_{lr}) \approx -\mu_y\left(1 + \theta\psi_j\left(0.5\psi_j\delta_{04}^* + \lambda_{12}C_x\right)\right). \tag{8}$$

and

$$MSE(\widehat{\mu}_{lr}) \approx \mu_y^2\left(1 + 2\theta\psi_j\left(\psi_j\delta_{04}^* + \lambda_{12}C_x\right)\right). \tag{9}$$

iv. *Ln*-type regression-cum-modified ratio estimator for the population mean estimation of sensitive study variable is given in Eq. (10)

$$\widehat{\mu}_{lR} = ln\left(\frac{\alpha\sigma_x^2 + \delta}{\alpha s_x^2 + \delta}\right)^{\left(\overline{z} + \frac{b_{zx}(\mu_x - \overline{x})}{\mu_x}\right)}. \tag{10}$$

The final expression of approximate bias and *MSE* of $\widehat{\mu}_{lR}$ are mentioned in Eq. (11) and Eq. (12)

$$Bias(\widehat{\mu}_{lR}) \approx \mu_y\left(1 + 0.5\theta\psi_j\left(2\lambda_{12} - \mu_y^{-1}\lambda_{03}\beta_{zx}\right)\psi_j\delta_{04}^*\right). \tag{11}$$

and

$$MSE(\widehat{\mu}_{lR}) \approx \mu_y^2\left(1 - 2\theta\psi_j\mu_y^{-1}C_x\left(\mu_y\lambda_{12} - \lambda_{03}\beta_{zx}\right)\right). \tag{12}$$

3. Proposed *ln*-type Estimators

The *ln*-function and its properties are very effective to obtain the precise estimates of the estimators in survey sampling [31]. In this article, we have improved ratio and regression-cum-ratio type estimators (named as PE-I and PE-II) using *ln*-function for mean estimation of confidential variable using concomitant variable under *SRS* design. We applied *ln*-function on the auxiliary variable of proposed estimators, PE-I and PE-II, defined as

$$\widehat{\mu}_{pi}^1 = ln\left(\frac{\alpha\sigma_x^2 + \delta}{\alpha s_x^2 + \delta} + 3\right)^{\overline{z}/3}. \tag{13}$$

and

$$\widehat{\mu}_{pi}^2 = ln\left(\frac{\alpha\sigma_x^2 + \delta}{\alpha s_x^2 + \delta} + 3\right)^{\left(\overline{z} + \frac{b_{zx}(\mu_x - \overline{x})}{\mu_x}\right)\Big/3}. \tag{14}$$

3.1. The derivation for approximate bias and MSE of PE-I (t_{pi}^1)

We re-write Eq. (13) in term of errors to get the *MSE* and the bias of the PE-I (t_{pi}^1) as

$$t_{pi}^1 = ln\left[\frac{\alpha\sigma_x^2+\delta}{\alpha\sigma_x^2\left(1+\xi_x^/\right)+\delta}+3\right]^{\frac{\mu_y(1+\xi_z)}{3}}. \tag{15}$$

On simplification of Eq. (15), we have

$$t_{pi}^1 = \frac{\mu_y(1+\xi_z)}{3}ln\left[\left\{1+\frac{\alpha\sigma_x^2\xi_x^/}{\alpha\sigma_x^2+\delta}\right\}^{-1}+3\right]. \tag{16}$$

Expanding and simplifying Taylor series up to first-order in Eq. (16), we have

$$t_{pi}^1 \cong \frac{\mu_y(1+\xi_z)}{3}ln\left(4-\psi_j\xi_x^/\right), \tag{17}$$

where $\psi_j = \alpha\sigma_x^2/(\alpha\sigma_x^2+\delta)$..

Applying *ln*-function on Eq. (17), we have

$$t_{pi}^1 \cong \frac{\mu_y(1+\xi_z)}{3}\left[\gamma+ln\left\{1-\frac{\psi_j\xi_x^/}{4}\right\}\right]. \tag{18}$$

where ln 4 = γ.

Note that $ln\left(1-\frac{\xi_x}{4}\right) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(-\frac{\xi_x^/}{4}\right)^n$. $|\xi_x^/| < 1$. So, we have

$$ln\left(1-\frac{\psi_j\xi_x^/}{4}\right) = -\frac{\psi_j\xi_x^/}{4}-\frac{\psi_j^2\xi_x^{/2}}{32}.$$

Using expression given in Eq. (18), we have

$$t_{pi}^1 \cong \frac{\mu_y(1+\xi_z)}{3}\left[\gamma-\frac{\psi_j\xi_x^/}{4}-\frac{\psi_j^2\xi_x^{/2}}{32}\right]. \tag{19}$$

On simplification of Eq. (19), we have

$$\left(t_{pi}^1-\mu_y\right) \cong \frac{\mu_y}{3}\left[(\gamma-3)+\gamma\xi_z-\frac{\psi_j\xi_x^/}{4}-\frac{\psi^2}{32}\xi_x^{/2}-\frac{\psi_j\xi_x^/\xi_z}{4}\right]. \tag{20}$$

Using the notation given in Eq. (1), we get the approximate bias of PE-I (t_{pi}^1) by applying expectations on both sides of Eq. (20) and summarized it in Eq. (21)

$$Bias\left(t_{pi}^1\right) \cong \frac{\mu_y}{3}\left[(\gamma-3)-0.03125\theta\psi_j\left\{\psi_j\delta_{04}^*+8\lambda_{12}C_x\right\}\right]. \tag{21}$$

To get the *MSE* of PE-I (t_{pi}^1), squaring on Eq. (20) and recalling the terms up to the first-order, we get

$$\left(t_{pi}^1-\mu_y\right)^2 \cong \frac{\mu_y^2}{9}\left[(\gamma-3)^2+\frac{\psi_j^2}{16}\xi_x^{/2}+\gamma^2\xi_z^2-(\gamma-3)\frac{\psi_j^2}{16}\xi_x^{/2}-(\gamma-3)\frac{\psi_j}{2}\xi_z\xi_x^/-\frac{\psi_j}{2}\gamma\xi_z\xi_x^/\right]. \tag{22}$$

The approximate *MSE* of the PE-I (t_{pi}^1) given in Eq. (23) is obtained by applying the expectations on both sides of Eq. (22) as

$$MSE\left(t_{pi}^1\right) \cong \frac{\mu_y^2}{9}\left[(\gamma-3)^2+\theta\left\{\gamma^2C_z^2-\frac{\psi_j}{16}\{8(2\gamma-3)\lambda_{12}C_x+\psi_j(\gamma-2)\delta_{04}^*\}\right\}\right]. \tag{23}$$

3.2. The derivation for approximate MSE and bias of PE-II (t_{pi}^2)

We re-write Eq. (14) in term of errors to get the bias and *MSE* of the PE-II (t_{pi}^2) as

$$t_{pi}^2 = ln\left(\frac{\alpha\sigma_x^2+\delta}{\alpha\sigma_x^2\left(1+\xi_x^/\right)+\delta}+3\right)^{\frac{1}{3}\left[\mu_y(1+\xi_z)+\frac{\sigma_{zx}(1+\xi_{zx})}{\sigma_x^2\left(1+\xi_x^/\right)}(-\xi_x)\right]}. \tag{24}$$

Simplifying and expanding Taylor series on Eq. (24) and recalling the terms up to first-order, we have

$$t_{pi}^2 \cong \frac{1}{3}\left[\mu_y\left(1+\xi_z\right)+\beta_{zx}(1+\xi_2)(1-\xi_1)(-\xi_x)\right]ln\left(4-\psi_j\xi_x^/\right). \tag{25}$$

On solving Eq. (25), we get

$$t_{pi}^2 \cong \frac{1}{3}\left[\mu_y\left(1+\xi_z\right)-\beta_{zx}\xi_x-\beta_{zx}(\xi_x\xi_2-\xi_x\xi_1)\right]\left[\gamma+ln\left\{1-\frac{\psi_j\xi_x^/}{4}\right\}\right]. \tag{26}$$

Applying ln-expansion on Eq. (26), we get

$$t_{pi}^2 \cong \frac{1}{3}\left[\mu_y\left(1+\xi_z\right)-\beta_{zx}\xi_x-\beta_{zx}(\xi_1\xi_x-\xi_2\xi_x)\right]\left[\gamma-\frac{\psi_j\xi_x^/}{4}-\frac{\psi_j{}^2\xi_x^{/2}}{32}\right]. \tag{27}$$

On simplification of Eq. (27), we have

$$\left(t_{pi}^2-\mu_y\right)\cong\frac{\mu_y}{3}\left[\begin{array}{l}(\gamma-3)-0.25\psi_j\xi_x^/-\frac{\psi_j^2}{32}\xi_x^{/2}+\gamma\xi_z-0.25\psi_j\xi_z\xi_x^/\\ -\gamma\mu_y^{-1}\beta_{zx}\xi_x+0.25\mu_y^{-1}\beta_{zx}\psi_j\xi_x\xi_x^/-\gamma\mu_y^{-1}\beta_{zx}(\xi_x\xi_2-\xi_x\xi_1)\end{array}\right]. \tag{28}$$

Using the notation given in Eq. (1), we get the approximate bias of PE-II (t_{pi}^2), stated in Eq. (29), by applying expectations on both sides of Eq. (28).

$$Bias\left(t_{pi}^2\right)\cong\frac{\mu_y}{3}\left[(\gamma-3)-\theta\left(0.25\mu_y^{-1}\beta_{zx}\left\{4\gamma\left(\mu_{12}\mu_{11}^{-1}-\mu_{03}\mu_{02}^{-1}\right)-\psi_j\lambda_{03}C_x\right\}-0.03125\psi_j\left\{\psi_j\delta_{04}^*+8\lambda_{12}C_x\right\}\right)\right]. \tag{29}$$

By squaring on Eq. (28) and keeping the terms up to the first order, we get the MSE of PE-II (t_{pi}^2) as

$$\left(t_{pi}^2-\mu_y\right)^2\cong\frac{\mu_y^2}{9}\left[\begin{array}{l}(\gamma-3)-0.25\psi_j\xi_x^/-0.03125\psi_j^2\xi_x^{/2}+\gamma\xi_z-0.25\psi_j\xi_z\xi_x^/\\ -\mu_y^{-1}\beta_{zx}\gamma\xi_x+0.25\mu_y^{-1}\beta_{zx}\psi_j\xi_x^/\xi_x-\mu_y^{-1}\gamma\beta_{zx}\left(\left(\xi_x\xi_{zx}-\xi_x\xi_x^/\right)\right)\end{array}\right]^2. \tag{30}$$

Simplifying and taking expectation on Eq. (30), we have the MSE of PE-II (t_{pi}^1), given in Eq. (31) as

$$MSE\left(t_{pi}^2\right)\cong\frac{\mu_y^2}{9}\left[(\gamma-3)^2+\frac{\theta}{16}\left\{\left(\psi_j^2\delta_{04}^*+16\gamma^2K_0+\mu_y^{-1}(\gamma-3)K_1\right)\right\}\right], \tag{31}$$

where

$$K_0=\left(C_z^2+\mu_y^{-2}C_x\left\{\beta_{zx}C_x\left(\beta_{zx}-2\mu_yH_{zx}\right)+0.5\psi_j\mu_y\gamma\left(\beta_{zx}\lambda_{03}-\mu_y\lambda_{12}\right)\right\}\right).$$

and

$$K_1=\left(8\left\{\beta_{zx}\lambda_{03}-\mu_y\lambda_{12}\right\}-\mu_yC_x\right)\psi_jC_x-\gamma\beta_{zx}\left(\mu_{12}\mu_{11}^{-1}-\mu_{03}\mu_{02}^{-1}\right).$$

Note that, we may obtain many special cases of the proposed ln-type estimators by setting different choices of α and δ such as coefficient of kurtosis ($\beta_1(x)$), coefficient of skewness ($\beta_1(x)$), and coefficient of variation (C_x) of the auxiliary variable. The classes of proposed ln-type ratio and regression-cum-ratio estimators are presented in Table-1 and Table-2 respectively.

4. Mathematical Comparisons

In this part, we have made some mathematical comparisons of EP-I and PE-II with the unbiased mean estimator and classical ratio estimator.

Table 1
Proposed Class of Estimator-I on Various Choices of α and δ.

Estimators	α	δ	Estimators	α	δ
$t_{p1}^1=ln\left(\frac{\sigma_x^2}{s_x^2}+3\right)^{\bar{z}/3}$	1	0	$t_{p2}^1=ln\left(\frac{\sigma_x^2+C_x}{s_x^2+C_x}+3\right)^{\bar{z}/3}$	1	C_x
$t_{p3}^1=ln\left(\frac{\sigma_x^2+\beta_1(x)}{s_x^2+\beta_1(x)}+3\right)^{\bar{z}/3}$	1	$\beta_1(x)$	$t_{p4}^1=ln\left(\frac{\sigma_x^2+\beta_2(x)}{s_x^2+\beta_2(x)}+3\right)^{\bar{z}/3}$	1	$\beta_2(x)$
$t_{p5}^1=ln\left(\frac{\beta_1(x)\sigma_x^2+\beta_2(x)}{\beta_1(x)s_x^2+\beta_2(x)}+3\right)^{\bar{z}/3}$	$\beta_1(x)$	$\beta_2(x)$	$t_{p6}^1=ln\left(\frac{\beta_2(x)\sigma_x^2+\beta_1(x)}{\beta_2(x)s_x^2+\beta_1(x)}+3\right)^{\bar{z}/3}$	$\beta_2(x)$	$\beta_1(x)$
$t_{p7}^1=ln\left(\frac{\beta_1(x)\sigma_x^2+C_x}{\beta_1(x)s_x^2+C_x}+3\right)^{\bar{z}/3}$	$\beta_1(x)$	C_x	$t_{p8}^1=ln\left(\frac{\beta_2(x)\sigma_x^2+C_x}{\beta_2(x)s_x^2+C_x}+3\right)^{\bar{z}/3}$	$\beta_2(x)$	C_x
$t_{p9}^1=ln\left(\frac{C_x\sigma_x^2+\beta_1(x)}{C_xs_x^2+\beta_1(x)}+3\right)^{\bar{z}/3}$	C_x	$\beta_1(x)$	$t_{p10}^1=ln\left(\frac{C_x\sigma_x^2+\beta_2(x)}{C_xs_x^2+\beta_2(x)}+3\right)^{\bar{z}/3}$	C_x	$\beta_2(x)$

Table 2
Proposed Class of Estimator-II on Various Choices of α and δ.

Estimators	α	δ	Estimators	α	δ
$t^1_{p1} = ln\left(\frac{\sigma_x^2}{s_x^2}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	1	0	$t^1_{p2} = ln\left(\frac{\sigma_x^2+C_x}{s_x^2+C_x}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	1	C_x
$t^1_{p3} = ln\left(\frac{\sigma_x^2+\beta_1(x)}{s_x^2+\beta_1(x)}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	1	$\beta_1(x)$	$t^1_{p4} = ln\left(\frac{\sigma_x^2+\beta_2(x)}{s_x^2+\beta_2(x)}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	1	$\beta_2(x)$
$t^1_{p5} = ln\left(\frac{\beta_1(x)\sigma_x^2+\beta_2(x)}{\beta_1(x)s_x^2+\beta_2(x)}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	$\beta_1(x)$	$\beta_2(x)$	$t^1_{p6} = ln\left(\frac{\beta_2(x)\sigma_x^2+\beta_1(x)}{\beta_2(x)s_x^2+\beta_1(x)}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	$\beta_2(x)$	$\beta_1(x)$
$t^1_{p7} = ln\left(\frac{\beta_1(x)\sigma_x^2+C_x}{\beta_1(x)s_x^2+C_x}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	$\beta_1(x)$	C_x	$t^1_{p8} = ln\left(\frac{\beta_2(x)\sigma_x^2+C_x}{\beta_2(x)s_x^2+C_x}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	$\beta_2(x)$	C_x
$t^1_{p9} = ln\left(\frac{C_x\sigma_x^2+\beta_1(x)}{C_xs_x^2+\beta_1(x)}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	C_x	$\beta_1(x)$	$t^1_{p10} = ln\left(\frac{C_x\sigma_x^2+\beta_2(x)}{C_xs_x^2+\beta_2(x)}+3\right)^{\left(\bar{z}+\frac{b_{zx}(\mu_x-\bar{x})}{\mu_x}\right)}\Big/3$	C_x	$\beta_2(x)$

- The PE-I performs more efficiently than the unbiased mean estimator if

$$MSE\left(t^1_{pi}\right) < MSE\left(\widehat{\mu}_y\right)$$

$$\frac{\mu_y^2}{9}\left[(\gamma-3)^2+\theta\left\{\gamma^2C_z^2-\frac{\psi_j}{16}\{8(2\gamma-3)\lambda_{12}C_x+\psi_j(\gamma-2)\delta^*_{04}\}\right\}\right]<\theta\mu_y^2C_z^2$$

which implies

$$\frac{\theta}{9}\left[\theta^{-1}A_1+A_2\right]<\theta C_z^2.$$

Or,

$$A_1<\theta\left(9C_z^2-A_2\right),$$

where $A_1=(\gamma-3)^2$, $A_2=\left\{\gamma^2C_z^2-\frac{\psi_j}{16}\{8(2\gamma-3)\lambda_{12}C_x+\psi_j(\gamma-2)\delta^*_{04}\}\right\}$..

- The PE-II performs more efficiently than the unbiased mean estimator if

$$MSE\left(t^2_{pi}\right) < MSE\left(\widehat{\mu}_y\right)$$

$$\frac{\mu_y^2}{9}\left[(\gamma-3)^2+\frac{\theta}{16}\left\{\left(\psi_j^2\delta^*_{04}+16\gamma^2K_0+\mu_y^{-1}(\gamma-3)K_1\right)\right\}\right]<\theta\mu_y^2C_z^2.$$

which implies

$$\frac{\theta}{9}\left[\theta^{-1}A_1+A_3\right]<\theta C_z^2.$$

Or,

$$A_1<\theta\left(9C_z^2-A_3\right),$$

where $A_3=\frac{1}{16}\{(\psi_j^2\delta^*_{04}+16\gamma^2K_0+\mu_y^{-1}(\gamma-3)K_1)\}$..

- The PE-I performs more efficiently than the classical ratio estimator if

$$MSE\left(t^1_{pi}\right) < MSE\left(\widehat{\mu}_r\right)$$

$$\frac{\mu_y^2}{9}\left[(\gamma-3)^2+\theta\left\{\gamma^2C_z^2-\frac{\psi_j}{16}\{8(2\gamma-3)\lambda_{12}C_x+\psi_j(\gamma-2)\delta^*_{04}\}\right\}\right]<\theta\,\mu_y^2\left(C_z^2+C_x^2(1-2H_{zx})\right).$$

which implies

$$\frac{\theta\mu_y^2}{9}\left[\theta^{-1}A_1+A_2\right]<\theta\mu_y^2\left(C_z^2+C_x^2(1-2H_{zx})\right).$$

Or,

$$A_1<\theta\left[9\left(C_z^2+C_x^2(1-2H_{zx})\right)-A_2\right].$$

- The PE-II performs more efficiently than the classical ratio estimator if

$$MSE\left(t_{pi}^2\right)<MSE(\widehat{\mu}_r)$$

$$\frac{\mu_y^2}{9}\left[(\gamma-3)^2+\frac{\theta}{16}\left\{\left(\psi_j^2\delta_{04}^*+16\gamma^2K_0+\mu_y^{-1}(\gamma-3)K_1\right)\right\}\right]<\theta\mu_y^2\left(C_z^2+C_x^2(1-2H_{zx})\right).$$

which implies

$$\frac{\theta\mu_y^2}{9}\left[\theta^{-1}A_1+A_3\right]<\theta\mu_y^2\left(C_z^2+C_x^2(1-2H_{zx})\right).$$

Or,

$$A_1<\theta\left[9\left(C_z^2+C_x^2(1-2H_{zx})\right)-A_3\right].$$

5. Simulation Study

An extensive study of simulation is conducted in this section to judge the application of the PE-I and PE-II over the existing estimators. Four different bi-variate populations of size 1000 are generated using *R* software and the 300-sample size is taken from each population.

The means, variances, covariance and correlation coefficient of simulated populations are.

Population 1. :

$$\mu_y=2,\quad \mu_x=2,\quad \sigma_y^2=9,\quad \sigma_x^2=4,\quad \sigma_{yx}=1.9 \text{ and } \rho_{yx}=0.3209$$

Population 2. :

$$\mu_y=2,\quad \mu_x=2,\quad \sigma_y^2=9,\quad \sigma_x^2=4,\quad \sigma_{yx}=3.2 \text{ and } \rho_{yx}=0.5154$$

Population 3. :

$$\mu_y=16,\mu_x=5,\sigma_y^2=9,\sigma_x^2=5,\sigma_{yx}=6.25 \text{ and } \rho_{yx}=0.7203$$

Population 4. :

$$\mu_y=2,\mu_x=2,\sigma_y^2=6,\sigma_x^2=2,\sigma_{yx}=3.00 \text{ and } \rho_{yx}=0.8684$$

To calculate the MSE's and PRE's of all the estimators taken into consideration in this study, the following processes have been coded in the R language.

- **Step 1:** we generated the populations (I-IV) and computed the population totals and the parameters associated with the auxiliary variable *x*.
- **Step 2:** The reported response Z is obtained by using additive *RRM*.
- **Step 3:** Different sample sizes are chosen from each population to produce the samples by operating *SRSWOR* and the values of all the estimators are computed.
- **Step 4:** The practice in Step-1 to Step-3 is iterated 20,000 times and the scores of *MSE's* and *PRE*'s are reported in Tables 3–6.

The formulae of percentage relative efficiency and mean square error are respectively given by

$$MSE\left(t^{j}_{pi}\right) = \frac{1}{R}\sum_{k=1}^{R}\left(t^{j}_{pi} - \mu_y\right)^2.$$

and

$$PRE\left(t^{j}_{pi}\right) = \frac{var\left(\widehat{\mu}_y\right)}{MSE\left(t^{j}_{pi}\right)} \text{x } 100.$$

$$\text{where } \begin{matrix} j = 1 \text{ and } 2. \\ i = 1, 2, ..., 10. \end{matrix}$$

5.1. Results and discussion

The results of *PRE* and *MSE* show that the PE-I and EP-II are more proficient than the usual mean per unit and traditional ratio estimators for all populations at different levels of correlation. The *MSEs* of PE-I and EP-II are found to be least and the *PREs* of the PE-I and PE-II are higher than the competing estimators. It is also noticed that the class of PE-II perform slightly better than the class of PE-I as shown in Tables 3–6. The provided estimator t^2_{P6} outperforms all other estimators taken into consideration in this paper.

6. Real Data Illustration

To support the theoretical findings obtained in Section 4, we compared the *MSE*s of the PE-I and PE-II with the competing estimators using real data taken from Ref. [32]; which was recently used by Ref. [24]. The sensitive variable (y) is the reported percent of alumni who donate while the non-sensitive concomitant variable (x) is the student to faculty ratio. We considered the scrambling variable to be normal with zero mean and variance equal to 1/2. To assess the performance of the intended estimators , four distinct sample sizes are chosen. The population parameters are

$$N = 777, \overline{Y} = 22.74, \overline{X} = 14.08,$$

$$S_y = 12.39, S_x = 3.95, \sigma_{YX} = 19.7641, \text{and } \rho_{yx} = 0.40.$$

The *MSE* (empirical and theoretical) and the *PRE* of all the estimators are respectively given in Tables 7–10.

6.1. Results and discussion

In real data application, the results summarized in Table-7 to Table-10 clearly show that the classes of PE-I and PE-II are useful than the challenging estimators even for the moderate correlation. The findings of *MSE* of proposed classes of estimators found to be relatively smaller than the competing estimators. We also observed that the small differences among the *MSE* of the sub-cases of the PE-I and PE-II. As the size of the sample increases, the *PRE* of all the estimators' increases, while the *MSE* decreases, which is the expected finding. Consequently, the *ln*-type proposed estimators and their sub-cases performed excellent over the unbiased mean estimator and the classical ratio estimator.

Table-3
The amount of *MSE* and *PRE* of all the estimators for Population-I.

	MSE				MSE		
Estimators	Empirical	Theoretical	PRE	Estimators	Empirical	Theoretical	PRE
t_y	1.11287	1.12542	100.0000	t_r	1.03370	1.01300	102.1626
t_{lr}	0.47388	0.56282	234.8401	t_{lR}	0.34238	0.36374	311.4612
t^1_{P1}	0.24095	0.25299	461.8530	t^2_{P1}	0.24082	0.28360	462.1079
t^1_{P2}	0.24034	0.25968	463.0385	t^2_{P2}	0.24020	0.28760	463.2977
t^1_{P3}	0.24083	0.33989	462.0905	t^2_{P3}	0.24070	0.32967	462.3462
t^1_{P4}	0.23862	0.29333	466.3626	t^2_{P4}	0.23848	0.26569	466.6370
t^1_{P5}	0.23762	0.30853	468.3212	t^2_{P5}	0.23748	0.28218	468.6147
t^1_{P6}	0.24091	0.25632	461.9314	t^2_{P6}	0.24078	0.30915	462.1865
t^1_{P7}	0.23813	0.27736	467.3252	t^2_{P7}	0.23749	0.24077	467.6059
t^1_{P8}	0.24073	0.26820	462.2761	t^2_{P8}	0.24060	0.28251	462.5324
t^1_{P9}	0.24067	0.31667	462.4008	t^2_{P9}	0.24053	0.26737	462.6576
t^1_{P10}	0.23797	0.23939	467.6465	t^2_{P10}	0.23782	0.24530	467.8298

Table-4
The amount of *MSE* and *PRE* of all the estimators for Population-II.

	MSE				MSE		
Estimators	Empirical	Theoretical	PRE	Estimators	Empirical	Theoretical	PRE
t_y	0.15284	0.15777	100.0000	t_r	0.10394	0.10416	115.2854
t_{lr}	0.06702	0.06641	256.7463	t_{lR}	0.06402	0.06344	289.6880
t_{P1}^1	0.03281	0.03594	465.7814	t_{P1}^2	0.03253	0.03301	469.7702
t_{P2}^1	0.03275	0.03887	466.6389	t_{P2}^2	0.03247	0.03512	470.6822
t_{P3}^1	0.03280	0.03345	465.8739	t_{P3}^2	0.03252	0.04165	469.8683
t_{P4}^1	0.03270	0.03474	467.3420	t_{P4}^2	0.03242	0.03703	471.4385
t_{P5}^1	0.03264	0.03322	468.2390	t_{P5}^2	0.03235	0.03248	472.4433
t_{P6}^1	0.03281	0.03478	465.8137	t_{P6}^2	0.03253	0.03393	469.8045
t_{P7}^1	0.03265	0.03283	468.0730	t_{P7}^2	0.03232	0.03237	472.4478
t_{P8}^1	0.03278	0.03476	466.1418	t_{P8}^2	0.03250	0.03849	470.1525
t_{P9}^1	0.03280	0.03438	465.8695	t_{P9}^2	0.03252	0.03282	469.8636
t_{P10}^1	0.03270	0.03314	467.3063	t_{P10}^2	0.03242	0.03674	471.3999

Table-5
The amount of *MSE* and *PRE* of all the estimators for Population-III.

	MSE				MSE		
Estimators	*Empirical*	*Theoretical*	*PRE*	*Estimators*	*Empirical*	*Theoretical*	*PRE*
t_y	0.11418	0.11442	100.0000	t_r	0.08796	0.08664	125.7644
t_{lr}	0.04796	0.04583	305.7644	t_{lR}	0.03753	0.03653	451.1286
t_{P1}^1	0.02490	0.02626	458.4785	t_{P1}^2	0.02379	0.02453	479.9086
t_{P2}^1	0.02476	0.02845	460.9957	t_{P2}^2	0.02366	0.02981	482.5722
t_{P3}^1	0.02491	0.03104	458.2442	t_{P3}^2	0.02380	0.02444	479.6604
t_{P4}^1	0.02462	0.02594	463.6576	t_{P4}^2	0.02352	0.02506	485.3800
t_{P5}^1	0.02435	0.02703	468.8736	t_{P5}^2	0.02326	0.02676	490.8329
t_{P6}^1	0.02490	0.02657	458.4012	t_{P6}^2	0.02379	0.02624	479.8267
t_{P7}^1	0.02428	0.02843	470.2852	t_{P7}^2	0.02319	0.02487	492.2833
t_{P8}^1	0.02485	0.02586	459.4876	t_{P8}^2	0.02374	0.02743	480.9771
t_{P9}^1	0.02491	0.03108	458.2286	t_{P9}^2	0.02380	0.02506	479.6439
t_{P10}^1	0.02461	0.03293	463.8146	t_{P10}^2	0.02351	0.02813	485.5452

Table-6
The amount of *MSE* and *PRE* of all the estimators for Population-IV.

	MSE				MSE		
Estimators	Empirical	Theoretical	PRE	Estimators	Empirical	Theoretical	PRE
t_y	0.10497	0.10645	100.0000	t_r	0.07658	0.07734	131.5597
t_{lr}	0.04158	0.04234	331.5597	t_{lR}	0.02449	0.02563	470.6578
t_{P1}^1	0.00770	0.00777	453.9538	t_{P1}^2	0.00621	0.00648	562.3705
t_{P2}^1	0.00761	0.00850	459.5673	t_{P2}^2	0.00613	0.00613	570.1274
t_{P3}^1	0.00771	0.00931	453.1928	t_{P3}^2	0.00623	0.00625	561.3087
t_{P4}^1	0.00752	0.00773	464.8337	t_{P4}^2	0.00605	0.00613	577.1654
t_{P5}^1	0.00746	0.01015	468.5021	t_{P5}^2	0.00601	0.00619	581.4913
t_{P6}^1	0.00770	0.00881	453.7065	t_{P6}^2	0.00622	0.00634	562.0257
t_{P7}^1	0.00745	0.01212	468.9507	t_{P7}^2	0.00601	0.00605	581.5144
t_{P8}^1	0.00766	0.00813	456.3927	t_{P8}^2	0.00618	0.00619	565.7593
t_{P9}^1	0.00772	0.00916	452.8598	t_{P9}^2	0.00623	0.00626	560.8436
t_{P10}^1	0.00750	0.01076	465.7566	t_{P10}^2	0.00604	0.00606	578.3438

7. Conclusion

The major objective of this research is to offer enhanced *ln*-type estimators for mean estimation of confidential variable using concomitant variable. We have used some known parameters of the auxiliary variable to get the sub-families of the *ln*-type estimators. We derived the properties of the proposed estimators using well-known Taylor and log expansions. We applied proposed estimators to four artificial datasets generated by *R*-software using different parameters. The simulation results showed that the *ln*-type estimators and their sub-cases are more efficient than the challenging estimators considered in this paper. Real data application also evident that the *ln*-type estimators are very effective and beneficial. It is notice that the *ln*-type estimators can be relatively distinctive for the

Table 7
The *MSE* and *PRE* of all the Estimators for sample size 50.

Estimators	MSE Empirical	MSE Theoretical	PRE	Estimators	MSE Empirical	MSE Theoretical	PRE
t_y	1.22600	1.18782	100.0000	t_r	1.02303	1.01155	116.4263
t_{lr}	0.91404	0.93435	124.2390	t_{lR}	0.88574	0.87645	132.5742
t^1_{P1}	0.71740	0.71557	379.6944	t^2_{P1}	0.76566	0.76588	388.7605
t^1_{P2}	0.77092	0.77067	382.8534	t^2_{P2}	0.75921	0.75929	387.3749
t^1_{P3}	0.77362	0.77345	381.5183	t^2_{P3}	0.76192	0.76184	384.1085
t^1_{P4}	0.78006	0.78023	378.3697	t^2_{P4}	0.76840	0.76878	376.5688
t^1_{P5}	0.79535	0.79519	371.0961	t^2_{P5}	0.78379	0.78329	366.4319
t^1_{P6}	0.81690	0.81672	361.3041	t^2_{P6}	0.80547	0.80557	348.1788
t^1_{P7}	0.74383	0.74312	396.7997	t^2_{P7}	0.73192	0.73187	403.2523
t^1_{P8}	0.85890	0.85846	343.6376	t^2_{P8}	0.84770	0.84723	391.8799
t^1_{P9}	0.76492	0.76433	385.8582	t^2_{P9}	0.75316	0.75394	395.5562
t^1_{P10}	0.78723	0.78701	374.9246	t^2_{P10}	0.77562	0.77595	380.5363

Table 8
The *MSE* and *PRE* of all the Estimators for sample size 100.

Estimators	MSE Empirical	MSE Theoretical	PRE	Estimators	MSE Empirical	MSE Theoretical	PRE
t_y	1.01540	1.04591	100.0000	t_r	0.86714	0.87450	212.2612
t_{lr}	0.800404	0.79834	214.7016	t_{lR}	0.76404	0.77834	219.7643
t^1_{P1}	0.34492	0.34458	380.7635	t^2_{P1}	0.33966	0.33958	386.6599
t^1_{P2}	0.34231	0.34311	383.6753	t^2_{P2}	0.33705	0.33715	389.6590
t^1_{P3}	0.34526	0.34536	380.2709	t^2_{P3}	0.34011	0.34012	386.1527
t^1_{P4}	0.34925	0.34810	380.1389	t^2_{P4}	0.34023	0.34018	386.0167
t^1_{P5}	0.33374	0.33728	393.5473	t^2_{P5}	0.32847	0.32866	399.8311
t^1_{P6}	0.33756	0.33644	389.0669	t^2_{P6}	0.33231	0.33243	395.2137
t^1_{P7}	0.54114	0.54454	342.6985	t^2_{P7}	0.30571	0.30539	445.1591
t^1_{P8}	0.33429	0.33484	389.0945	t^2_{P8}	0.44892	0.44893	392.5522
t^1_{P9}	0.34664	0.34427	378.8776	t^2_{P9}	0.34138	0.34104	384.7178
t^1_{P10}	0.34711	0.34125	378.3647	t^2_{P10}	0.34184	0.34197	384.1897

Table 9
The *MSE* and *PRE* of all the Estimators for sample size 200.

Estimators	MSE Empirical	MSE Theoretical	PRE	Estimators	MSE Empirical	MSE Theoretical	PRE
t_y	0.96490	0.95500	100.0000	t_r	0.85771	0.85600	215.7492
t_{lr}	0.77001	0.78070	217.0621	t_{lR}	0.74001	0.73995	225.0892
t^1_{P1}	0.19870	0.19510	390.6741	t^2_{P1}	0.14179	0.14075	398.1814
t^1_{P2}	0.14451	0.14011	393.0699	t^2_{P2}	0.14091	0.14115	400.6609
t^1_{P3}	0.14363	0.14771	389.9659	t^2_{P3}	0.14205	0.14024	397.4486
t^1_{P4}	0.14477	0.14107	387.7855	t^2_{P4}	0.14286	0.14340	395.1924
t^1_{P5}	0.14559	0.14559	417.3989	t^2_{P5}	0.13257	0.13568	425.8553
t^1_{P6}	0.13526	0.13444	392.8806	t^2_{P6}	0.14098	0.14924	400.4650
t^1_{P7}	0.14370	0.14147	347.4461	t^2_{P7}	0.15971	0.15707	353.5049
t^1_{P8}	0.14549	0.14510	382.2253	t^2_{P8}	0.14497	0.14351	389.4403
t^1_{P9}	0.14250	0.14014	388.0242	t^2_{P9}	0.14277	0.14741	395.4394
t^1_{P10}	0.14886	0.14155	379.2553	t^2_{P10}	0.14612	0.14211	386.3684

positive relationship between the benchmark and concerned variables. Moreover, the PRE increases by the increase of the sample sizes. We executed many estimators with different aspects and found the best results by adding 3 in the ratio component and divided the whole estimator by 3 as suggested by Ref. [31]. Thus, based on above-mentioned findings, we may infer that the *ln*-type estimators performed well and recommend for the precise estimation of scrambled responses.

In this research, we used *ln*-type ratio and regression-cum-ratio estimators for the estimation of finite population mean of confidential variable in the presence of an auxiliary variable using additive *RRM*. In the upcoming work, this study could be extended using different *RRMs* using multi-auxiliary variables under different sampling designs.

Table 10
The *MSE* and *PRE* of all the Estimators for sample size 400.

	MSE				*MSE*		
Estimators	*Empirical*	*Theoretical*	*PRE*	*Estimators*	*Empirical*	*Theoretical*	*PRE*
t_y	0.88994	0.87782	100.0000	t_r	0.74265	0.73550	226.4263
t_{lr}	0.65965	0.66550	233.4263	t_{lR}	0.55404	0.54415	239.2390
t_{P1}^1	0.04993	0.04757	481.0170	t_{P1}^2	0.04566	0.14501	489.1589
t_{P2}^1	0.04985	0.04276	483.4387	t_{P2}^2	0.04880	0.04364	491.6956
t_{P3}^1	0.04953	0.04665	481.6467	t_{P3}^2	0.04849	0.04906	489.8183
t_{P4}^1	0.04977	0.04750	480.8557	t_{P4}^2	0.04872	0.04991	488.9899
t_{P5}^1	0.04987	0.04393	478.7858	t_{P5}^2	0.04883	0.04709	486.8225
t_{P6}^1	0.05014	0.05017	435.2535	t_{P6}^2	0.04910	0.04646	441.3994
t_{P7}^1	0.05665	0.05122	407.1072	t_{P7}^2	0.05563	0.04724	416.5423
t_{P8}^1	0.05550	0.05132	483.1818	t_{P8}^2	0.04560	0.04172	491.8799
t_{P9}^1	0.04957	0.04413	480.4532	t_{P9}^2	0.04852	0.04732	491.4264
t_{P10}^1	0.04992	0.04970	491.6956	t_{P10}^2	0.04888	0.04780	488.5684

Data availability

The authors confirm that the data supporting the findings of this study are available within the article [and/or] its supplementary materials.

CRediT authorship contribution statement

Muhammad Nouman Qureshi: Writing - review & editing, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Yousaf Faizan:** Writing - original draft, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Amrutha Shetty:** Software, Investigation, Formal analysis, Data curation. **Marwan H. Ahelali:** Visualization, Validation, Resources, Methodology, Funding acquisition, Formal analysis. **Muhammad Hanif:** Writing - review & editing, Visualization, Validation, Supervision, Project administration, Methodology, Investigation, Conceptualization. **Osama Abdulaziz Alamri:** Visualization, Resources, Methodology, Investigation, Funding acquisition, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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