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An approach towards a perfect thermal diffuser

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A method for the most efficient removal of heat, through an anisotropic composite, is proposed. It is shown that a rational placement of constituent materials, in the radial and the azimuthal directions, at a given point in the composite yields a uniform temperature distribution in spherical diffusers. Such arrangement is accompanied by a very significant reduction of the source temperature, in principle, to infinitesimally above the ambient temperature and forms the basis for the design of a *perfect* thermal diffuser with maximal heat dissipation. Orders of magnitude enhanced performance, compared to that obtained through the use of a diffuser constituted from a single material with isotropic thermal conductivity has been observed and the analytical principles underlying the design were validated through extensive computational simulations.

The rapid dissipation of thermal energy from a heat source as well as an accurate control of the path of heat transfer is necessary for a wide variety of applications, and has been considered under the scope of efficient thermal management^{1,2}. At the very outset, it may be thought that the optimal method of dissipating thermal energy would involve omnidirectional transfer and could be mediated either through spherical or cylindrical symmetry, for point and line sources of heat, respectively. However, as we will show, materials design considering isotropic thermal conductivity is inefficient¹. Additionally, spherical/cylindrical thermal energy diffusers based on such considerations yield nonlinear temperature distribution and concomitant non-uniform heat flux. The staggering of the temperature throughout the volume of the heat-dissipating element in such cases may also result in the creation of undesirable *hot spots*³. As a solution to the above shortcomings we report on the fundamental notion of an optimal materials arrangement necessary for the *most* efficient removal of heat, in which a linear temperature profile along with the desirable characteristic of isotropic heat transfer would be obtained. We will show that, in the steady state, our design allows for a significant reduction of the source temperature.

We examine a typical situation where a given steady-state heat flux (q_{in}), produced by a thermal energy source (which releases heat uniformly in all directions), needs to be dissipated to the ambient (at a temperature of T_{amb}). For this purpose, a thermal diffuser of a certain shape/geometry needs to be placed between the heat source and the ambient. If any rectangular geometry, comprising a material with isotropic and constant thermal conductivity (κ), were chosen, the azimuthal asymmetry results in a non-uniform temperature gradient and anisotropic heat transfer. In comparison, isotropic heat transfer with *unequally* spaced isotherms (along the heat flux direction) would be obtained in circular geometries with constant thermal conductivity. This results in an undesired staggering of isotherms near the heat source: Fig. 1(A). A more desired diffuser configuration, wherein the isotherms would be equally spaced yielding a uniform temperature distribution, is indicated in Fig. 1(B).

The design of the desired heat diffuser proceeds through a conformal mapping^{4,5} from the rectangular geometry-shown in Fig. 2(A) to a circular configuration – shown in Fig. 2 (B), so as to obtain a linear temperature gradient concomitant with isotropic heat transfer. The space spanned in the rectangular geometry: $U = (x, y, z)$ is related to the space in the circular geometry: $V = (X, Y, Z)$ through a relation of the form:

$$X = y \sin\left(\frac{x}{R}\right), Y = y \cos\left(\frac{x}{R}\right), Z = z \quad (1)$$

The mapping of the (x, y, z) space to (X, Y, Z) space, indicated through Eqn. 1, is conformal as it preserves the parallelism of the isotherms in the two spaces. In these relations, R refers to a mean radius pertinent to the semi-circular strip of width: $2W$ (the inner and the outer radius of the circular strip, in Fig. 2(B), are R_i and R_o , and $R = [R_i + R_o]/2$ and $W = [R_o - R_i]/2$). For a correspondence to the (r, θ, z) system in this figure, $x = R \tan^{-1}(X/Y) = R\phi$, and $r = \sqrt{X^2 + Y^2}$. Symmetry considerations allow the probing of a semi-circular half space and a two-dimensional layout. The thermal conductivity of one coordinate system (say, the rectangular system, where

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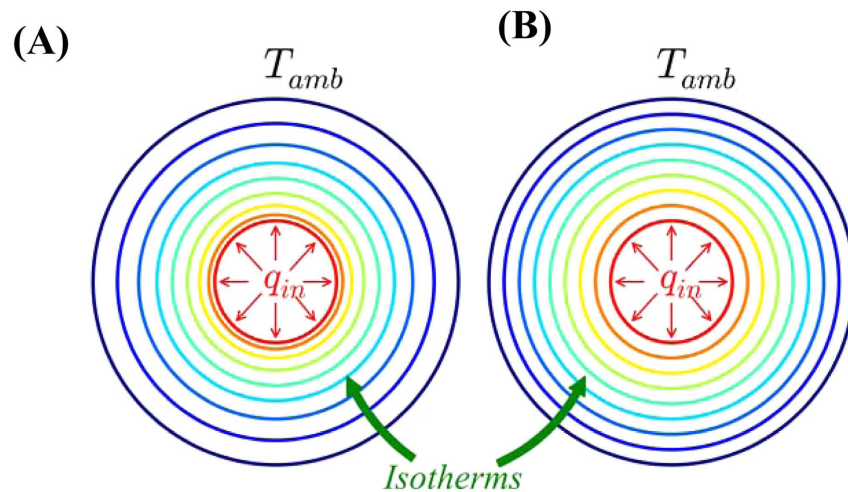


Figure 1. A comparison of diffusional heat flow in spherical geometry, in a thermal diffuser, constituted from (A) a material with isotropic and a constant thermal conductivity, with unequally spaced isotherms, and (B) an ideal configuration with uniformly spaced isotherms and lower source temperature. The latter is feasible through the optimized arrangement of the constituent materials in a composite thermal diffuser.

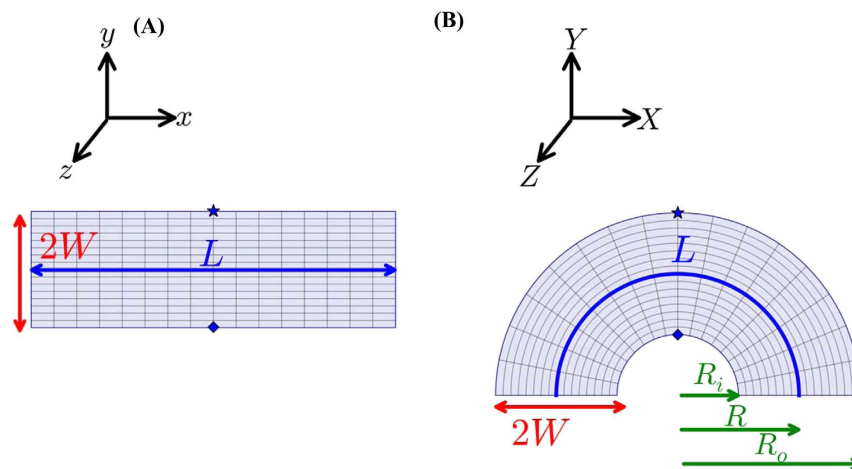


Figure 2. Isotropic heat transfer concomitant with uniform temperature distribution is obtained through the conformal mapping of the thermal conductivity of a (A) rectangular block of length L and width $2W$ on to a (B) semi-circle, with inner radius R_i , outer radius R_o and width $2W$. Both the geometries in (A,B) have the same out of plane thickness. Two representative correspondent points (indicated, for example, by the \blacklozenge and \blacksquare) are indicated.

the thermal conductivity is denoted through κ^{rect}) could be related to another (say, the circular system with thermal conductivity: κ^c) through a transformation^{6–8} described through: $\kappa^c = (J\kappa^{rect}J^T)/\det(J)$, where J is the Jacobian for the considered transformation, J^T is the transpose, and $\det(J)$ is the determinant of the Jacobian. It can then be derived (see Section I of the Supplementary Materials for details) that the thermal conductivity of the semi-circular diffuser in Fig. 2(B), would be a function of the distance from the heat flux source (r), through:

$$\kappa^c(r, \theta, z) = \begin{pmatrix} \kappa_r^c & 0 & 0 \\ 0 & \kappa_\theta^c & 0 \\ 0 & 0 & \kappa_z^c \end{pmatrix} = \begin{pmatrix} R/r & 0 & 0 \\ 0 & r/R & 0 \\ 0 & 0 & R/r \end{pmatrix} \kappa; (R - W) \leq r \leq (R + W), 0 \leq \theta \leq 2\phi \quad (2)$$

The physical implication, of Eqn. (2), is that the regularization of the heat transport, to obtain a uniform temperature distribution and isotropic heat transfer, requires the modification of matter following the path of heat transfer. Consider, for instance, the thermal conductivity in the radial direction κ_r^c . At $r < R$ ($r > R$), $\kappa_r^c (= [R/r]\kappa)$ would be larger (/smaller) than the nominal isotropic thermal conductivity κ , implying that the resultant distance between isotherms in the anisotropic diffuser: Fig. 1(B), would be farther (/closer) than its isotropic counterpart: Fig. 1(A). Such imposed variation, yielding a respective expansion (/contraction) of the isotherm spacing close to

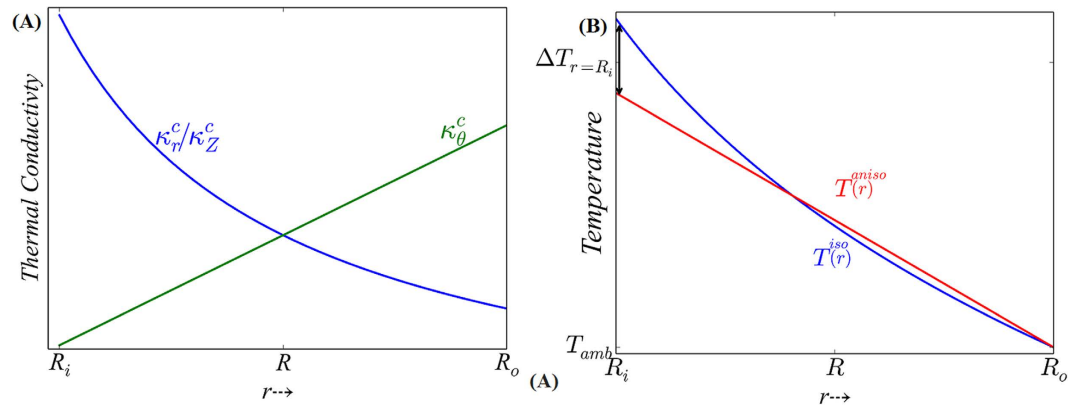


Figure 3. (A) The variation of the *in plane* radial (κ_r^c), and azimuthal (κ_θ^c) thermal conductivity, along with the *cross-plane* thermal conductivity (κ_z^c) with radial distance (r) from the heat source. (B) While the radial variation of the temperature, *i.e.*, $T^{iso}(r)$, is nonlinear for a diffuser constituted from isotropic material, a linear temperature gradient together with a resultant lower source temperature may be obtained through the use of an anisotropic material (the temperature variation of which is depicted through $T^{aniso}(r)$).

(/further away) from the heat source in the spherical geometry, would help achieve a linear temperature profile along the heat flux direction. The spatial change of the $\kappa^c(r)$, indicated in Fig. 3(A), would indicate that the thermal conductivity of the semi-circular heat diffuser would be identical to that of the isotropic thermal conductivity only at $r = R$, and $\det(J) = 1$ for the semi-circular geometry with respect to the rectangular geometry. While the through thickness variation in the diffuser, through the κ_z^c may be similarly interpreted, the thermal conductivity change in the azimuthal direction $\kappa_\theta^c(r)$, is proportional to the variation of r . As such unusual anisotropy cannot be found in nature, at any particular r , the radial/perpendicular and azimuthal variation of the thermal conductivity must be considered through suitable and specific material placement^{7,9} for obtaining uniform temperature distribution.

In this regard, we have experimentally demonstrated in previous work⁶ that a layered composite of just two disparate materials (say, of thermal conductivity: κ_1 and κ_2) and of thickness (l) would be sufficient to obtain any desired anisotropic thermal conductivity at a given r (see Section II of the Supplementary Materials). Moreover, the influence of any particular value of l on the propagation of heat flux can be eliminated through the creation of an *effective thermal medium* (ETM)¹⁰. An ETM based theory essentially assumes a homogeneous composite and the lack of interfaces. While such an idealization is not readily apparent in the case of heat diffusion due to the lack of a definable length scale, possible ways to alleviate the problem include choosing the contacting layers with a high thermal conductivity contrast, or the use of a layer/contacting adhesive with a small thermal conductivity contrast¹¹. Following Fig. 3(A), at $r < R$, the thermal conductivity in the radial (/perpendicular) direction: κ_r^c ($/\kappa_z^c$) would be larger than the κ_θ^c , implying an orientation of elements parallel to the heat flux, while for $r > R$, *i.e.*, when κ_r^c ($/\kappa_z^c$) is smaller compared to the κ_θ^c , the orientation of the elements would be perpendicular to the heat flux. Essentially, the size of a composite element (yielding the desired κ^c) should be such that a constant temperature gradient across the element could be assumed, in accordance with the tenets of an ETM. We will show later, through computational simulations, that the related arrangement of materials indeed ensures uniform heat flow and a linear temperature gradient. The facilitating methodology involves a metamaterial architecture^{12–15}, constituted from individual *thermal meta-atoms* at a given r , and with spatially varying values of the thermal conductivity (*i.e.*, through κ_r^c , κ_z^c , and κ_θ^c). Such r dependent variation yields an *anisotropic*¹⁰ character to the diffuser.

We discuss next the temperature variation in the thermal diffuser employing such spatial conductivity modulation. Through consideration of the heat flux continuity in (a) an *isotropic* cylindrical diffuser: Fig. 1(A), *vis-à-vis*, (b) an *anisotropic* cylindrical diffuser: Fig. 1(B), and a convective heat transfer coefficient h between the outer radial surface $r = R_o$ and the ambient (at T_{amb}), it can be derived (see Section III of the Supplementary Materials) that the source temperature (at $r = R_i$), in the *isotropic* and the *anisotropic* case, would be respectively:

$$T_{r=R_i}^{iso} = q_{in} \left[\frac{R_i}{\kappa} \ln \left(\frac{R_o}{R_i} \right) + \frac{1}{h} \left(\frac{R_i}{R_o} \right) \right] + T_{amb} \tag{3a}$$

$$T_{r=R_i}^{aniso} = q_{in} \left[\frac{R_i}{\kappa R} (R_o - R_i) + \frac{1}{h} \left(\frac{R_i}{R_o} \right) \right] + T_{amb} \tag{3b}$$

While Eqn. 3(a) predicts a logarithmic temperature variation, it is apparent from Eqn. 3(b) that a linear temperature profile is obtained through the use of an anisotropic materials architecture: Fig. 3(B). The latter attribute now yields a lower temperature at the source ($r = R_i$) by an amount, $\Delta T_{r=R_i}$, obtained by subtracting (3b) from (3a), and is:

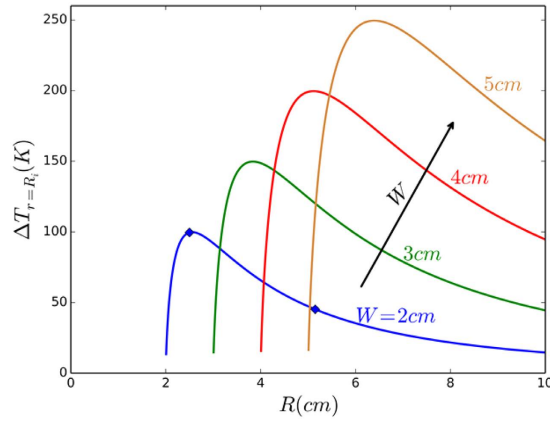


Figure 4. The variation of the difference in temperature, at the source ($r = R_i$): $\Delta T_{r=R_i}$, between using a thermal diffuser constituted from a single material with *isotropic* thermal conductivity and the desired *anisotropically* architected material, with the mean radius ($R = \frac{1}{2} [R_i + R_o]$) and the width of the diffuser ($2W = [R_i - R_o]$), per Eqn. (4). The marked points (\blacklozenge) indicate results from the computational simulations.

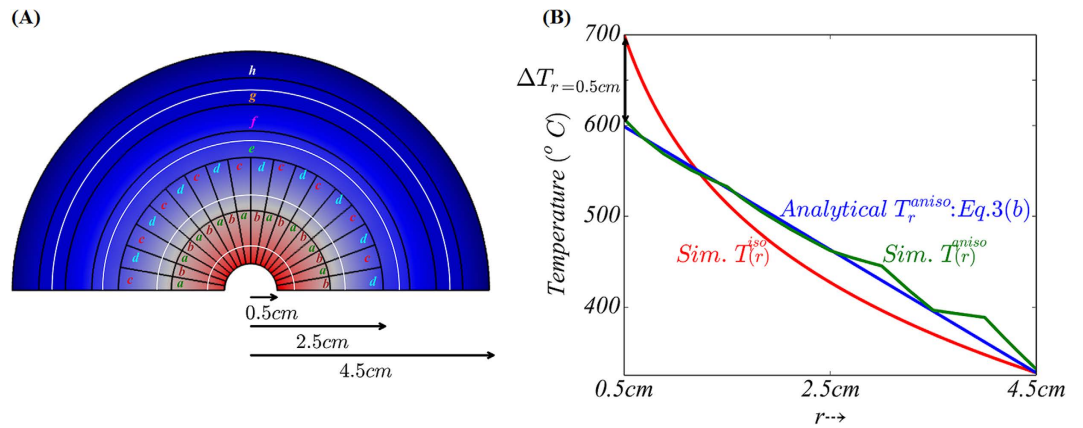


Figure 5. (A) Computational simulations indicating the notion of a perfect thermal diffuser, for an input heat flux $q_{in} = 2 \times 10^6 \text{ W/m}^2$. A uniform temperature distribution in the composite was obtained through the use of an anisotropic arrangement (see Table S1 of the Supplementary Materials for the details on the arrangement scheme), composed of materials of constant and isotropic thermal conductivity. The symbols refer to the nominal thermal conductivity of the used isotropic materials, i.e., a ($=282 \text{ W/mK}$), b ($=12 \text{ W/mK}$), c ($=118 \text{ W/mK}$), d ($=29 \text{ W/mK}$), e ($=110 \text{ W/mK}$), f ($=31 \text{ W/mK}$), g ($=169 \text{ W/mK}$), and h ($=20 \text{ W/mK}$). (B) The resulting temperature variation, for the isotropic case: $T^{iso}(r)$, and the anisotropic case: $T^{aniso}(r)$, determined from the simulations (*Sim.*), was found to be in excellent accord with the analytical (*Analytical*) relation derived in Eqn. 3(b). The ambient temperature (T_{amb}) was chosen to be 293 K. The respective temperatures at the center ($r = R_i$) are 587 K and 687 K, for the anisotropic and the isotropic cases, respectively.

$$\Delta T_{r=R_i} = \frac{q_{in} R_i}{\kappa} \left[\ln \left(\frac{R_o}{R_i} \right) - 2 \left(\frac{R_o - R_i}{R_o + R_i} \right) \right] \quad (4)$$

One way to understand Eqn. (4), from a physical perspective, involves considering the first term: $\frac{q_{in} R_i}{\kappa}$, as the thermal energy input, while the multiplying bracketed term represents the reduced temperature due to induced anisotropy. The resulting plots of $\Delta T_{r=R_i}$ as a function of the mean radius ($R = \frac{1}{2} [R_i + R_o]$) and the width of the diffuser ($W = \frac{1}{2} [R_i - R_o]$) are shown in Fig. 4. The constraint of $R_i > 0$, mandates that $R > W$ and marks the beginning of the curve. At large R (small W), the anisotropic composite would degenerate to an isotropic case, implying a very small $\Delta T_{r=R_i}$. We also observe that at increasingly larger R , with a fixed W , the logarithm in Eqn. (4) could be linearized yielding a correspondingly diminished temperature difference. The peak in the $\Delta T_{r=R_i}$ corresponds to an r where the enhanced thermal diffusion due to the induced anisotropy would be balanced by the increasing heat input. A very significant reduction of the source temperature can be obtained, in principle to infinitesimally above the ambient temperature, and lays the basis for a *perfect* thermal diffuser with maximal heat dissipation.

Computational simulations were performed to validate our analytical derivations and the principles underlying the perfect thermal diffuser design: Fig. 5(A). We consider a diffuser needing to dissipate a q_{in} of 2×10^6 W/m², as may occur in heat assisted magnetic recording (HAMR)^{16,17}, with $R = 2.5$ cm and $W = 2$ cm (related to the point at the *maximum* of the curve in Fig. 4). The corresponding anisotropic thermal conductivity of the diffuser and placement of the material, per Eqn. (2), was derived (also see Section IV of the Supplementary Materials) assuming a nominal isotropic $\kappa = 60$ W/mK. The corresponding simulations and agreement with the theoretical predictions for a point *away from the maximum* of the curve (with $R = 5$ cm and $W = 2$ cm) –as indicated by the marked point in Fig. 4 has also been shown in Section IV of the Supplementary Information (Figure S3). The suggested thermal conductivity values are very practicable and may easily be experimentally implemented through the use of existing and readily available materials. A comparison of the analytically predicted temperature variation from Eqn. 3b, with the computational results is illustrated in Fig. 5(B). The temperature variation was found to be linear in excellent accord with the predictions, and has been achieved through the use of relatively few layers. As predicted, a remarkable reduction in the source temperature (following Eqn. 4) by ~ 100 K in the engineered composite, is a highlight of our design and indicates the validity of our approach as a proposal for a new type of a thermal management technique.

In summary, we have implemented a simultaneous variation of the radial and the azimuthal thermal conductivity at a given point in an anisotropic material and have hence demonstrated the path towards creating a perfect thermal diffuser. The significant lowering of the source temperature in such a diffuser, compared to that obtained through the use of a single material with isotropic thermal conductivity, would find many applications. Our work also exemplifies the utility of anisotropic architectures. As heat transfer is fundamentally anisotropic, as (a) in thermal conduction, *e.g.*, due to phonon dispersion^{18,19}, (b) in convection, *e.g.*, due to buoyancy effects²⁰, and (c) in radiative heat transfer, *e.g.*, due to view factor related effects²¹, the regularization of diffusive heat transfer through the use of anisotropic composites may indeed be appropriate.

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Author Contributions

K.P.V. and P.R.B. contributed to the conceptualization, design, and the development of the project and to the writing of the paper. K.P.V. performed the simulations.

Additional Information

Supplementary information accompanies this paper at <http://www.nature.com/srep>

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