

# Excited States with Selected Configuration Interaction-Quantum Monte Carlo: Chemically Accurate Excitation Energies and Geometries

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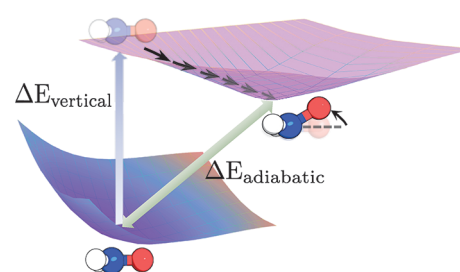
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## Supporting Information

**ABSTRACT:** We employ quantum Monte Carlo to obtain chemically accurate vertical and adiabatic excitation energies, and equilibrium excited-state structures for the small, yet challenging, formaldehyde and thioformaldehyde molecules. A key ingredient is a robust protocol to obtain balanced ground- and excited-state Jastrow–Slater wave functions at a given geometry, and to maintain such a balanced description as we relax the structure in the excited state. We use determinantal components generated via a selected configuration interaction scheme which targets the same second-order perturbation energy correction for all states of interest at different geometries, and fully optimize all variational parameters in the resultant Jastrow–Slater wave functions. Importantly, the excitation energies as well as the structural parameters in the ground and excited states are converged with very compact wave functions comprising few thousand determinants in a minimally augmented double- $\zeta$  basis set. These results are obtained already at the variational Monte Carlo level, the more accurate diffusion Monte Carlo method yielding only a small improvement in the adiabatic excitation energies. We find that matching Jastrow–Slater wave functions with similar variances can yield excitation energies compatible with our best estimates; however, the variance-matching procedure requires somewhat larger determinantal expansions to achieve the same accuracy, and it is less straightforward to adapt during structural optimization in the excited state.

QMC-CIPSI : Excited states with chemical accuracy



## 1. INTRODUCTION

Quantum Monte Carlo (QMC) methods are first-principle approaches that approximately solve the Schrödinger equation in a stochastic manner. The two most commonly used variants, namely, variational (VMC) and diffusion Monte Carlo (DMC), typically employ so-called Jastrow–Slater wave functions where a determinant expansion is multiplied by a Jastrow factor which explicitly depends on the interparticle distances and accounts for a significant portion of electronic correlation. Because of the presence of the Jastrow factor, much shorter expansions are often needed to describe the Slater component and obtain accurate VMC results, which can then be further improved with the use of DMC. Thanks to the favorable scaling with system size, these methods have been routinely employed to compute the electronic properties, particularly total energies, of relatively large molecules and solids.<sup>1–5</sup>

Recent methodological advances have reduced the cost per Monte Carlo step of computing energy derivatives to the one of the energy itself, also for large multideterminant wave functions.<sup>6,7</sup> This has enabled us not only to simultaneously

optimize geometry and wave function with as many as 200 000 determinants but also to explore the dependence of the results on different lengths and types of Slater expansions, foraging into QMC wave function choices beyond conventional small-active-space definitions. Capitalizing on these developments, our recent thorough investigation<sup>8</sup> of constructing the Slater component by an automated determinant selection through a selected configuration interaction (CIPSI) approach<sup>9–21</sup> has led to accurate predictions of the ground-state energies and structural parameters of butadiene with relatively short Slater expansions. Relevant determinants could be systematically introduced into the wave function that would not have been chosen otherwise based on a manual, intuitive selection.

These developments also open very interesting prospects for the application of QMC to geometry relaxation in the excited state, where most electronic structure methods either lack the required accuracy or are computationally quite expensive due to their scaling with system size. To date, there are very few

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studies to assess the ability of QMC to predict excited-state geometries,<sup>22–25</sup> while most of the relatively limited literature on excited-state QMC calculations is primarily concerned with vertical excitation energies.<sup>26–42</sup> Importantly, all these studies are characterized by the use of very different wave functions ranging from the simple ansatz of a CI singles wave function to complete active space (CAS) expansions, sometimes truncated or only partially optimized in the presence of the Jastrow factor due to the limitations previously faced in sampling and optimizing large numbers of determinants. In this work, we want to overcome this empiricism in the application of QMC to excited states and (a) identify the most efficient protocol to obtain a balanced and robust description of the ground and excited states in VMC at a given geometry, (b) extend this protocol to the optimization of the geometry as well as the computation of energy differences between different potential energy surfaces at distinct geometries, and (c) demonstrate the competence of VMC in determining accurate vertical excitation energies, optimal excited-state structures, and adiabatic excitations.

To this aim, we focus on the low-lying singlet  $n \rightarrow \pi^*$  excitation of formaldehyde and thioformaldehyde, and compute these excited-state properties employing relatively compact wave functions containing between a few thousand to about 45 000 determinants obtained through different CIPSI selection schemes. These molecules are small but theoretically challenging: a recent VMC study reported an error as large as 0.2 eV on the vertical excitation energy of thioformaldehyde,<sup>39</sup> and DMC calculations without a Jastrow factor required wave functions with as many as 100 000 determinants to achieve high accuracy for formaldehyde.<sup>38</sup> Here, we obtain VMC vertical excitation energies within chemical accuracy ( $\sim 0.04$  eV) of the extrapolated full CI (FCI) estimates for both molecules with expansions as small as a few thousand determinants in combination with a minimally augmented double- $\zeta$  basis set. These excellent results are obtained using CIPSI expansions constructed to yield a comparable second-order perturbation (PT2) correction in the ground and excited states, and fully reoptimized in the presence of the Jastrow factor. Matching the energy variance of the Jastrow–Slater wave functions of the two states<sup>39,43</sup> to estimate the excitation energy appears to be a more delicate procedure which, in the case of formaldehyde, recovers VMC excitation energies within 0.05 eV of our best estimates for determinantal expansions comprising at least 7000 determinants.

When optimizing the structure, we follow two different selection routes to maintain a balanced treatment of the wave function while changing the geometry: we construct the determinantal component targeting a roughly constant value of either the perturbation correction or the variance of the CIPSI expansions. We find that both schemes are viable to obtain robust VMC geometries, also in the more demanding case of formaldehyde where different correlated methods give a range of variations in the prediction of the CO bond and the out-of-plane angle in the excited state as large as 80 mÅ and 20°, respectively.<sup>44</sup> With just a minimal basis, we obtain optimal VMC structures converged with fewer than 1000 determinants and in excellent agreement with the coupled cluster estimates, namely, with deviations smaller than a couple of degrees in the angles and 3 and 10 mÅ in the ground- and excited-state bond lengths, respectively. Finally, we compute the difference between the variational minima of the VMC ground- and excited-state potential energy surfaces to evaluate the adiabatic

excitation energies. For both molecules, irrespective of the determinant selection mode, we obtain VMC and DMC estimates within better than 0.05 eV of the corresponding coupled cluster values.

The paper is organized as follows. We describe the CIPSI selection scheme employed to obtain a balanced description of multiple states in section 2 and present the computational details in section 3. The VMC and DMC vertical excitation energies of formaldehyde and thioformaldehyde are given in section 4, and the results of the VMC structural relaxation in the ground and excited states are given in section 5. We conclude in section 6 by summarizing the most important outcomes of our investigation and the future prospects of the applicability of our approach.

## 2. METHODS

The wave functions used in the QMC calculations are of the Jastrow–Slater form, namely, the product of a determinantal component and a positive Jastrow correlation function

$$\Psi = \mathcal{J} \sum_k^{N_{\text{det}}} c_k D_k \quad (1)$$

where  $N_{\text{det}}$  is the total number of determinants and the Jastrow factor  $\mathcal{J}$  depends here on the electron–nucleus and electron–electron distances, explicitly ensuring that the electronic cusp conditions are satisfied.

To construct the determinantal part of the wave function, we employ the CIPSI selected CI algorithm that iteratively allows us to identify energetically important determinants from the FCI space. When one is interested in multiple electronic states, it is important to obtain a description of the CI subspace which leads to a uniform and balanced treatment of all states of interest. To ensure a consistent quality of the wave functions, the selection of the determinants for the multiple states is done in a single run even if the states belong to different symmetry classes. In practice, starting from an initial reference subspace  $\mathcal{S}$  typically given by the CI singles wave functions of all states, at every iteration, we expand the space by selecting among all singly and doubly excited determinants those which contribute the most to a state-average PT2 energy contribution.

If we represent a newly selected determinant with  $|\alpha\rangle$ , we then compute its second-order energy contribution using Epstein–Nesbet perturbation theory<sup>45,46</sup> as

$$\delta E_n^{(2)} = \frac{|\langle \alpha | \hat{H} | \Psi_n \rangle|^2}{\langle \Psi_n | \hat{H} | \Psi_n \rangle - \langle \alpha | \hat{H} | \alpha \rangle} \quad (2)$$

and the corresponding CI coefficients as

$$c_{\alpha,n} = \frac{\langle \Psi_n | \hat{H} | \alpha \rangle}{\langle \Psi_n | \hat{H} | \Psi_n \rangle - \langle \alpha | \hat{H} | \alpha \rangle} \quad (3)$$

where  $n$  denotes a state and  $\Psi_n$  is the current CI wave function in the space  $\mathcal{S}$ . As suggested by Angeli et al.,<sup>47</sup> the selection criterion for a determinant  $|\alpha\rangle$  in the external space is based on the energy contribution

$$e_\alpha = \sum_n^{N_{\text{states}}} w_n \delta E_n^{(2)} \quad (4)$$

where  $w_n$  is the inverse of the maximum  $c_{k,n}^2$  with the index  $k$  running over the determinants in the current subspace  $\mathcal{S}$ . The determinant  $|\alpha\rangle$  is added to  $\mathcal{S}$  if its energy contribution  $e_\alpha$  is

higher than a threshold, which is automatically adjusted in such a way that the number of determinants in  $\mathcal{S}$  is doubled at each iteration. In the applications presented in this work, the ground state is a closed-shell singlet and the excited state an open-shell singlet, both having single-reference character with the dominant configuration state function (CSF) comprising a single determinant in the ground state and two determinants in the excited state. Since the expansion is performed in the basis of the determinants and not of the CSFs, reweighting the energy contributions by  $w_n$  recovers the balance for the different states. As a consequence, we find that multiple states resulting from the expansion have approximately equivalent net PT2 corrections at every CIPSI iteration. For sufficiently large expansions, the PT2 correction can be viewed as an estimate of the current error in the variational energy with respect to the FCI value and, therefore, wave functions of different states with similar PT2 corrections will have comparable quality and yield a reasonable estimate of the excitation energies already at the CI level.

The many-body Hamiltonian,  $\hat{H}$ , is diagonalized in the subspace  $\mathcal{S}$  at every iteration to determine the current CI coefficients. One keeps expanding until the wave function has a required number of determinants or until another chosen criterion is met, as for instance a desired total PT2 correction or variance of the CI wave functions as further discussed below in the results sections.<sup>8,48–51</sup> Additionally, we impose that the selected wave functions are spin-adapted, namely, eigenfunctions of the  $\hat{S}^2$  operator: all determinants corresponding to the spatial occupation patterns currently present in  $\mathcal{S}$  are added to the reference space before the Hamiltonian is diagonalized.<sup>52,53</sup> Although wave functions for all the states are obtained through this common selection process, determinants of one symmetry character do not contribute, of course, to a state of a different character. Therefore, we separate these determinants based on space symmetry prior to proceeding with the VMC optimization.

In what we name the “expansion” scheme, we simply use subsequent wave functions of increasing length generated according to the CIPSI algorithm we just described. While the “expansion” scheme is more easily transferable to larger systems where one could experience difficulty in obtaining a large expansion to then truncate, we will also present some tests with wave functions generated from a “truncation” scheme. After having introduced a large number of determinants in the reference space and obtained the wave functions for both states by diagonalizing the Hamiltonian, we project the wave functions in a common subspace of determinants as follows: for each group of determinants,  $\mathcal{P}$ , of the internal space sharing the same spatial occupation pattern, we compute the quantity

$$d(\mathcal{P}) = \sum_{k \in \mathcal{P}} \sum_n^{N_{\text{states}}} c_{k,n}^2 \quad (5)$$

All the individual spatial occupation patterns of the internal space are sorted in decreasing order of  $d(\mathcal{P})$  values, and the truncated determinant space is built by taking the union of the first patterns of the list. This guarantees that the truncated wave functions are spin-adapted and that the final determinants kept after truncation are the most important for all states of interest.

### 3. COMPUTATIONAL DETAILS

All QMC computations are carried out with the program package CHAMP.<sup>54</sup> We employ scalar-relativistic energy-consistent Hartree–Fock pseudopotentials and the correlation-consistent Gaussian basis sets specifically constructed for these pseudopotentials.<sup>55,56</sup> For the majority of our calculations, we use a minimally augmented double- $\zeta$  basis set denoted here as maug-cc-pVDZ and constructed by augmenting the cc-pVDZ basis with  $s$  and  $p$  diffuse functions on the heavy atoms. Basis-set convergence tests are performed with the fully augmented aug-cc-pVTZ pseudopotential basis. In both cases, the diffuse functions are taken from the corresponding all-electron Dunning’s correlation-consistent basis sets.<sup>57</sup> The Jastrow factor includes two-body electron–electron and electron–nucleus correlation terms.<sup>58</sup>

We optimize all wave function parameters (Jastrow, orbital, and CI coefficients) by energy minimization in VMC using the stochastic reconfiguration (SR) method<sup>59</sup> in a conjugate gradient implementation.<sup>60</sup> We optimize the ground and excited states separately since the two states have different symmetries at both the ground- and excited-state optimal structures. We relax the geometry in Z-matrix coordinates and simply follow the direction of steepest descent, appropriately rescaling the interatomic forces and using an approximate constant diagonal Hessian. After convergence, we perform additional optimization steps and average the last 40 structures to estimate the structural parameters presented below. To remove occasional spikes in the forces, we use an improved estimator of the forces obtained by sampling the square of a modified wave function close to the nodes.<sup>61</sup> In the DMC calculations, we treat the pseudopotentials beyond the locality approximation using the T-move algorithm<sup>62</sup> and employ an imaginary time step of 0.02 au. As shown in the [Supporting Information](#), this time step yields DMC excitation energies converged to better than 0.01 eV for the smallest wave function in formaldehyde and is therefore appropriate for all wave functions of higher quality considered in this work.

The CIPSI calculations are performed with Quantum Package<sup>53</sup> using orbitals obtained from small complete active space self-consistent field (CASSCF) calculations in the program GAMESS(US).<sup>63,64</sup> As explained above, the CIPSI expansions are constructed to be eigenstates of  $\hat{S}^2$  and the selected determinants are subsequently mapped into the basis of configuration state functions, thereby effectively reducing the number of optimization parameters for QMC.

### 4. VERTICAL EXCITATION ENERGIES

We begin our investigation by computing the lowest singlet vertical excitation energies of  $n \rightarrow \pi^*$  character of formaldehyde ( $\text{CH}_2\text{O}$ ) and thioformaldehyde ( $\text{CH}_2\text{S}$ ). In the ground state, both molecules possess  $C_{2v}$  symmetry and the relevant ground ( $S_0$ ) and excited ( $S_1$ ) states belong to the  $A_1$  and the  $A_2$  irreducible representation, respectively. The QMC vertical excitation energies are computed on the ground-state structures optimized at the iterative approximate coupled cluster singles, doubles, and triples (CC3) level with an aug-cc-pVTZ basis and without the frozen-core approximation.<sup>38,65</sup> Importantly, as we show below, the CC3 geometries are identical to the optimal ground-state structures obtained at the VMC level.

**4.1. Formaldehyde.** The lowest  $n \rightarrow \pi^*$  excited state of formaldehyde has been the subject of a recent DMC

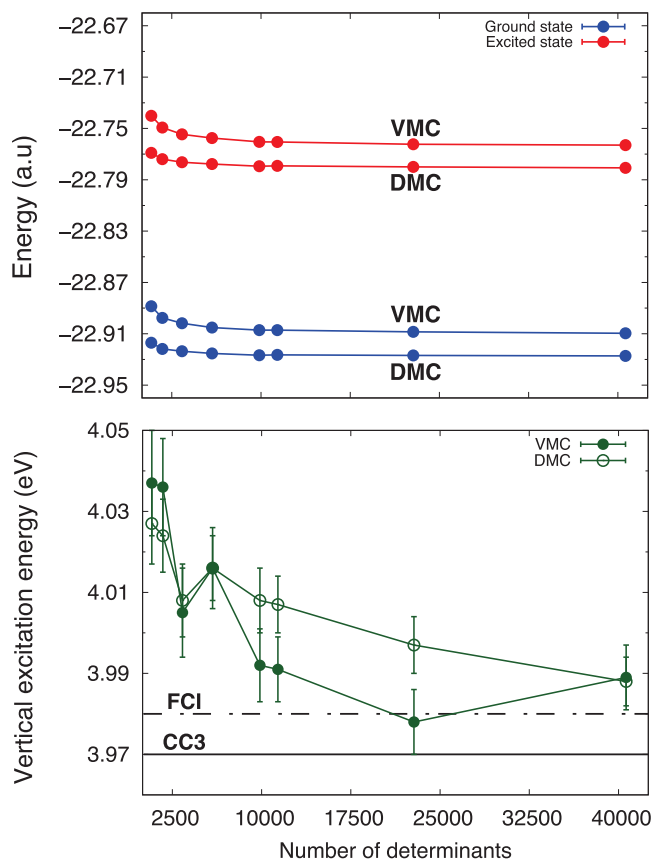
**Table 1.** VMC and DMC Ground- and Excited-State Energies (au), and Vertical Excitation Energies (eV) of Formaldehyde Obtained with Fully Optimized Jastrow-CIPSI Wave Functions, Where a Series of Increasing Determinantal Expansions Are Generated with the “Expansion” Scheme

basis	no. det		no. param		VMC			DMC		
	S0	S1	S0	S1	E(S0)	E(S1)	$\Delta E$	E(S0)	E(S1)	$\Delta E$
maug-cc-pVDZ	343	436	848	614	-22.88852(32)	-22.74015(33)	4.037(13)	-22.91702(26)	-22.76903(26)	4.027(10)
	580	1124	946	961	-22.89762(32)	-22.74929(31)	4.036(12)	-22.92182(25)	-22.77393(24)	4.024(09)
	994	2360	1104	1357	-22.90177(30)	-22.75459(27)	4.005(11)	-22.92362(23)	-22.77635(23)	4.008(09)
	1703	4182	1375	1937	-22.90512(25)	-22.75753(25)	4.016(10)	-22.92530(22)	-22.77772(22)	4.016(08)
	2747	7110	1762	2805	-22.90717(25)	-22.76046(24)	3.992(09)	-22.92675(22)	-22.77947(22)	4.008(08)
	3050	8320	1874	3141	-22.90719(20)	-22.76051(20)	3.991(08)	-22.92642(18)	-22.77916(17)	4.007(07)
	5932	16871	2915	5550	-22.90852(20)	-22.76233(20)	3.978(08)	-22.92687(18)	-22.77997(17)	3.997(07)
	10854	29786	4681	9177	-22.90961(20)	-22.76303(20)	3.989(08)	-22.92728(14)	-22.78072(16)	3.988(06)
aug-cc-pVTZ	675	912	3237	2256	-22.90151(19)	-22.75361(19)	4.024(07)	-22.92260(16)	-22.77499(16)	4.017(06)
	1488	3214	3592	3474	-22.91364(17)	-22.76638(17)	4.007(06)	-22.92778(14)	-22.78068(13)	4.015(05)
	3058	7672	4210	5050	-22.92021(15)	-22.77242(15)	4.021(06)	-22.93112(12)	-22.78359(12)	4.014(05)
	5849	15338	5222	7401	-22.92050(16)	-22.77333(14)	4.005(06)	-22.93136(11)	-22.78360(10)	4.021(04)
	12987	31710	7731	12040	-22.92188(15)	-22.77466(14)	4.006(06)	-22.93145(11)	-22.78393(13)	4.014(05)

investigation,<sup>38</sup> where Jastrow-free CIPSI wave functions with as many as 300 000 determinants were employed to recover a vertical excitation energy within 0.08(3) eV of the best theoretical estimate of 3.97 eV.<sup>65</sup> In fact, calculations involving about 9000 and 75 000 determinants yielded excitation energies about 0.2 and 0.1 eV higher, respectively, than the reference value. Given the size of the molecule and the single-reference character of the excitation, the large number of determinants required to recover a satisfactory result is rather surprising and warrants an investigation where we generate optimal wave functions in the presence of the Jastrow factor prior to the DMC step.

Here, we revisit the VMC and DMC computation of this vertical excitation energy with compact CIPSI expansions in the Jastrow–Slater wave functions, containing between 1000 and 40 000 determinants. These determinant components are the result of subsequent expansions at the CIPSI level constructed to achieve a balanced description of the two states of interest at each step by selecting determinants that yield comparable PT2 contributions for both states as discussed above. The determinants corresponding to  $A_1$  and  $A_2$  symmetry are then isolated and the Jastrow–Slater wave functions are separately fully optimized by energy minimization in VMC. The resulting VMC and DMC total and vertical excitation energies are listed in Table 1 and plotted in Figure 1.

The VMC vertical excitation energies computed with the maug-cc-pVDZ basis are already within 0.05 eV of the CC3/aug-cc-pVTZ value with about 1000 and 2400 determinants in the ground state and the excited state, respectively. Further increasing the size of the wave functions by a factor of 10, we observe a rapid convergence to within  $\sim 0.02$  eV of the CC3 value. The DMC calculations with the VMC optimized wave functions gain about 17–30 mhartrees for the range of expansions studied, so the gain is uniform for both states. Consequently, the gap is quite stable also at the DMC level and consistently compatible within statistical error (to better than 0.02 eV) with the corresponding VMC values. A similar behavior is observed when employing the fully augmented triple- $\zeta$  basis set with the VMC excitation energy being rather stable as a function of the expansion size and in agreement within statistical error with the corresponding DMC value. From these calculations, we can therefore make three important observations: (a) the weighted CIPSI algorithm



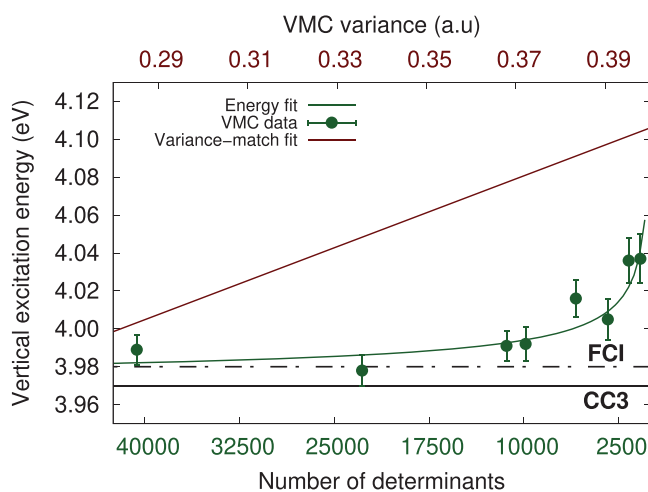
**Figure 1.** Convergence of the VMC and DMC energies of the ground and excited states (top) and of the excitation energy (bottom) of the  $n \rightarrow \pi^*$  excitation of formaldehyde with the combined number of determinants in the CIPSI expansions. The maug-cc-pVDZ basis is used. We also show the extrapolated FCI and CC3 values obtained with the aug-cc-pVTZ basis set.<sup>65</sup>

used here yields an automated, balanced determinant selection of the two states; (b) in combination with VMC optimization, we obtain reliable estimates for the vertical excitation energy with rather small and compact Jastrow–Slater wave functions; (c) to estimate the excitation energy, there is no need to perform a DMC calculation as the main correlation effects on

the energy difference have already been captured at the VMC level.

The extrapolated FCI estimate of the excitation energy computed with the same pseudopotential maug-cc-pVDZ basis set is 3.99 eV (see Figure S1 in the Supporting Information), in perfect agreement with the all-electron value of 3.99 eV obtained with the corresponding aug-cc-pVDZ basis set. In the all-electron calculations, the use of the larger aug-cc-pVTZ only reduces the FCI value to 3.98 eV, which can be further corrected for basis set and frozen-core errors to yield the best theoretical estimate of 3.97 eV.<sup>65</sup> Our results of 3.99 and 4.02 eV obtained with the maug-cc-pVDZ and aug-cc-pVTZ, respectively, are in excellent agreement with the reference value.

So far, we have obtained a balanced description of both states by selecting determinants in the CIPSI expansion that yield comparable net PT2 contribution to both states at every step. In Figure 2, we plot the resulting excitation energies



**Figure 2.** VMC vertical excitation energies (circles) of formaldehyde obtained with the “expansion” scheme. The excitation energy is also estimated as the difference of the fits of the energies against the total number of determinants (bottom *x*-axis) and against the variance of the two states (top *x*-axis). We also show the extrapolated FCI and CC3 values obtained with the aug-cc-pVTZ basis set.<sup>65</sup>

together with the estimates obtained as difference of the fits of the ground- and excited-state VMC energies against the number of total determinants as  $E_{\text{EX}}^{\text{fit}}(N_{\text{det}}) - E_{\text{GS}}^{\text{fit}}(N_{\text{det}})$  (see section S3 in the Supporting Information). Although we

optimize our wave functions separately for the two states in VMC, we use the total number of determinants as a common index since the determinants were obtained from the same CIPSI selection.<sup>66</sup> Furthermore, since we now have at hand multiple wave functions for the ground and excited states, we can alternatively follow another physically appropriate approach to compute the excitation energy by matching the variances of the states of interest.<sup>39,43</sup> To do so, we fit the VMC energies separately for the ground and the excited states against the corresponding variances (see Figure S3) and then subtract the two fits as  $E_{\text{EX}}^{\text{fit}}(\sigma^2) - E_{\text{GS}}^{\text{fit}}(\sigma^2)$ . The resulting variance-matched excitation energies are plotted in Figure 2, where smaller variances correspond to larger expansions.

For our fully optimized Jastrow-CIPSI wave functions, both schemes to estimate the excitation energy yield reasonable values, which become of course more compatible for the larger expansions. The excitation energies computed by fitting the energies against the number of determinants display a clear and fast convergence, and are closer to the extrapolated FCI estimate over the whole range of expansions explored here. For the variance-matching scheme, the fit at high variances deviate from the FCI value by 0.08–0.12 eV and, only beyond about 7000 determinants and a variance of about 0.31 au, the fit starts giving reasonable estimates within 0.05 eV of our best estimate. Therefore, to make a reliable variance-matched prediction, one needs to employ larger wave functions containing several thousand determinants. We note that we are not in a regime<sup>67</sup> where a reliable estimate of the exact energy can be obtained by linear extrapolation to zero variance.

Finally, while the “expansion” scheme represents the most practical route to generate the determinantal component in a Jastrow-CIPSI wave function for larger systems, given the small size of formaldehyde, we can also investigate the effect of using the “truncation” scheme where a large CIPSI expansion is truncated either to yield wave functions with a similar norm for all states (Table S1, truncation from 40 000 total determinants) or to include the most relevant patterns (Table S2, truncation from 3 million total determinants) as discussed in section 2. We find that both truncated wave functions yield energetically equivalent estimates of the vertical excitation energies at the VMC and DMC levels as those obtained with comparable expansion sizes with the “expansion” scheme of Table 1. We note that the “truncation” scheme should be applied to a relatively large expansion since, during the CIPSI iterations, the largest coefficients vary much at the beginning but tend to stabilize as the size of the expansion grows. If the size of the initial expansion is such that the determinants kept

**Table 2.** VMC and DMC Ground- and Excited-State Energies (au), and Vertical Excitation Energies (eV) of Thioformaldehyde Obtained with Fully Optimized Jastrow-CIPSI Wave Functions Generated with the “Expansion” Scheme<sup>a</sup>

no. det		no. param		VMC			DMC		
S0	S1	S0	S1	$E(S0)$	$E(S1)$	$\Delta E$	$E(S0)$	$E(S1)$	$\Delta E$
353	475	854	607	-17.04424(26)	-16.96212(25)	2.234(10)	-17.06968(25)	-16.98671(24)	2.258(9)
702	1488	995	1044	-17.04998(25)	-16.96814(24)	2.227(09)	-17.07246(23)	-16.99023(23)	2.237(9)
1165	2702	1168	1461	-17.05239(24)	-16.97019(24)	2.237(09)	-17.07284(22)	-16.99086(22)	2.231(9)
1834	4692	1419	2034	-17.05388(23)	-16.97100(23)	2.255(09)	-17.07394(22)	-16.99171(22)	2.237(9)
2500	6316	1662	2514	-17.05494(23)	-16.97260(23)	2.241(09)	-17.07434(21)	-16.99211(22)	2.238(8)
3432	8338	1997	3112	-17.05622(23)	-16.97313(23)	2.261(09)	-17.07482(22)	-16.99219(22)	2.248(8)
5712	14562	2810	4848	-17.05698(22)	-16.97497(22)	2.232(08)	-17.07542(22)	-16.99314(26)	2.239(9)
14218	30142	5650	9162	-17.05780(15)	-16.97590(19)	2.236(07)	-17.07566(18)	-16.99345(26)	2.237(8)

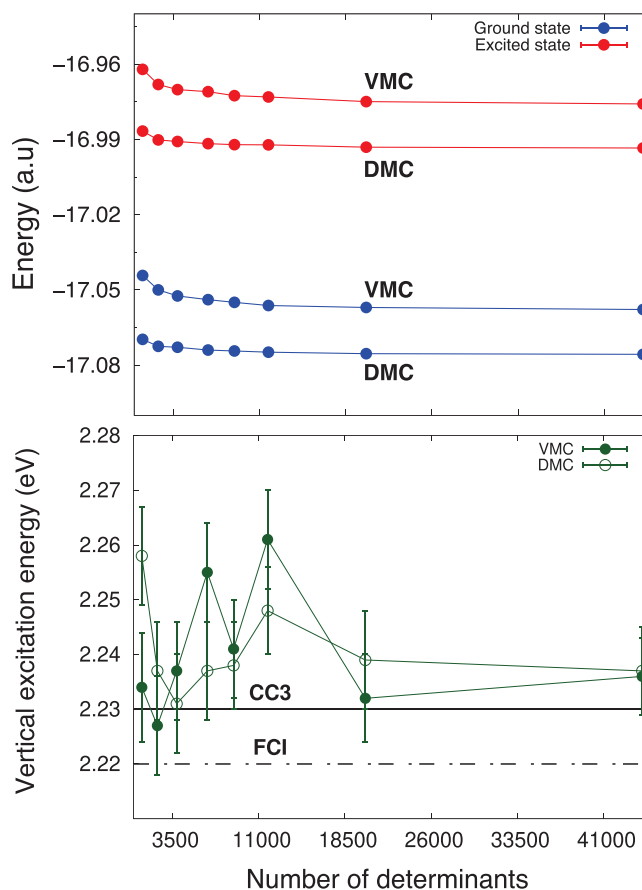
<sup>a</sup>The maug-cc-pVDZ basis set is used.

after truncation have converged coefficients relative to the largest coefficient, then truncating the wave function will be independent of the size of the starting expansion and, therefore, will be equivalent to truncating the FCI wave function. In general, fully reoptimizing the trial wave functions appears to yield accurate and robust estimates of the excitation energies of formaldehyde for both the “expansion” scheme and the “truncation” scheme.

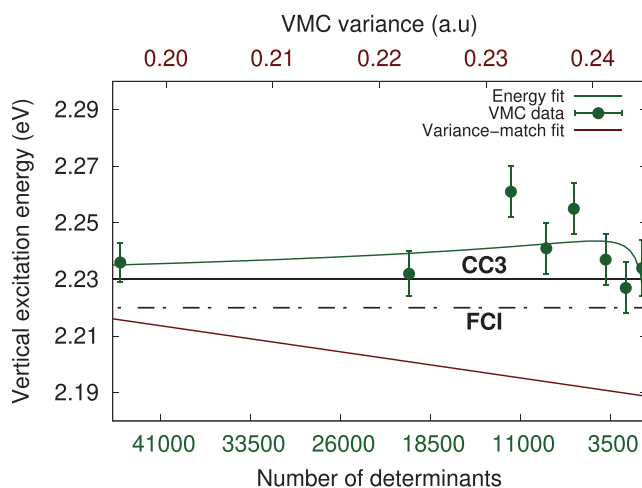
**4.2. Thioformaldehyde.** This small molecule has been the subject of many recent investigations<sup>39,68–70</sup> and is a variant of formaldehyde, wherein the O atom is replaced by a S atom. Interestingly, the very recent study by Pineda Flores et al.<sup>39</sup> employed Jastrow-CIPSI wave functions to compute the excitation energy of the  $n \rightarrow \pi^*$  transition but obtained a VMC excitation energy of 2.07(2) eV, that is, about 0.2 eV lower than the reference extrapolated stochastic-heat-bath CI value of 2.31(1) eV on their geometry. They optimized all wave function parameters, albeit in variance minimization. In view of our success with the computation of the vertical excitation energy of formaldehyde at the VMC level, it is somewhat puzzling that the simple substitution of oxygen with sulfur would worsen so much the performance of the method. Therefore, we revisit here the same molecule using fully optimized Jastrow-CIPSI wave functions and both the CIPSI selection procedure of multiple states with similar PT2 energy correction and the variance-matching scheme.

We summarize our QMC results obtained from the CIPSI “expansion” scheme and the maug-cc-pVDZ basis set in Table 2 and also present them in Figure 3. As in the case of formaldehyde, we find that we are able to obtain a stable estimate of the vertical excitation energy at both the VMC and DMC levels with relatively little effort. The excitation energy is essentially the same within statistical error at both the VMC and the DMC levels when increasing the total number of determinants from 800 to 44 000. The use of larger expansions yields of course a gain in the total energies, which is however less than 1 and 0.5 mhartree in VMC and DMC, respectively, when doubling the expansion size between the last two entries of Table 2. Computation of the DMC energies on top of the final VMC optimized wave functions uniformly lowers the energies of both states by about 18–20 mhartrees, thereby unaltering the energy separation between them. Therefore, the CIPSI selection scheme combined with wave function optimization ensures a balanced description of the two states and yields a VMC excitation energy of 2.23 eV in excellent agreement with the CC3 value of 2.23 eV and the extrapolated FCI estimate of 2.22 eV obtained with a larger aug-cc-pVTZ basis.<sup>68</sup>

In Figure 4, we plot the excitation energies obtained for the different CIPSI expansions together with the two estimates one obtains by computing the difference of the fits of the ground- and excited-state energies against the total number of determinants and against the variances of the two states as done for formaldehyde. We find that both estimates are compatible well within 0.05 eV over the whole range of expansion sizes/variances. We therefore do not reproduce the large error of 0.2 eV reported in ref 39, which cannot therefore be due to the use of the variance-matching recipe with reoptimized Jastrow-CIPSI wave functions.



**Figure 3.** Convergence of the VMC and DMC energies with the number of CIPSI determinants of the ground and excited states (top) and of the excitation energy (bottom) of the  $n \rightarrow \pi^*$  excitation of thioformaldehyde. The maug-cc-pVDZ basis is used. We also show the extrapolated FCI and CC3 values obtained with the aug-cc-pVTZ basis set.<sup>65</sup>



**Figure 4.** VMC vertical excitation energies (circles) of thioformaldehyde obtained with the “expansion” scheme. The excitation energy is also estimated as the difference of the fits of the energies against the total number of determinants (bottom  $x$ -axis) and against the variance of the two states (top  $x$ -axis). We also show the extrapolated FCI and CC3 values obtained with the aug-cc-pVTZ basis set.<sup>65</sup>

**Table 3. Optimal VMC Ground-State Bond Lengths (Å) and Bond Angles (deg) of Formaldehyde Obtained Using Jastrow-CIPSI Wave Functions with the maug-cc-pVDZ Basis<sup>a</sup>**

no. det	no. param	CO	CH	H–C–H	$\langle E_{\text{VMC}} \rangle$
580	946	1.20754(08)	1.09871(9)	117.09(6)	–22.89824(4)
2747	1762	1.20791(13)	1.09952(6)	116.85(3)	–22.90716(4)
5932	2915	1.20819(08)	1.09952(3)	117.05(3)	–22.90867(3)
CC3 <sup>44</sup>		1.208	1.100	116.4	
expt <sup>71</sup>		1.208	1.116	116.3	

<sup>a</sup>The VMC energy (au) is averaged over the last 40 iterations of geometry optimization. We also list the CC3/aug-cc-pVTZ and the experimental values.

**Table 4. Optimal VMC Ground- and Excited-State Bond Lengths (Å) and Bond Angles (deg), and VMC and DMC Adiabatic Excitation Energies (eV) of Formaldehyde Obtained Using Jastrow-CIPSI Wave Functions with Targeted PT2 Correction (Iso-PT2)<sup>a</sup>**

state	$\delta E_{\text{PT2}}$	no. det	no. param	CO	CH	H–C–H	o.o.p.	$\langle E_{\text{VMC}} \rangle$	$\Delta E_{\text{adia}}^{\text{VMC}}$	$\Delta E_{\text{adia}}^{\text{DMC}}$
S0	0.20	515	1183	1.20663(05)	1.09851(03)	116.84(4)	–0.098(25)	–22.89892(5)		
	0.15	1184	1496	1.20795(08)	1.09832(05)	116.96(6)	0.075(15)	–22.90411(6)		
	0.10	2784	2208	1.20752(19)	1.09947(05)	117.00(5)	0.073(54)	–22.90723(5)		
	0.07	4799	2946	1.20710(14)	1.09959(10)	117.01(8)	–0.030(29)	–22.90805(4)		
CC3 <sup>44</sup>				1.208	1.100	116.4	0.000			
expt <sup>71</sup>				1.208	1.116	116.3	0.000			
S1	0.20	1088	1363	1.33948(07)	1.08468(02)	119.83(03)	32.916(055)	–22.76787(5)	3.566(2)	3.612(9)
	0.15	3106	2037	1.33971(36)	1.08630(06)	119.03(20)	34.778(051)	–22.77342(5)	3.556(2)	3.614(8)
	0.10	8058	3520	1.33606(21)	1.08647(12)	119.34(17)	34.027(233)	–22.77626(3)	3.564(2)	3.597(8)
	0.07	15278	5589	1.33597(15)	1.08645(08)	119.56(09)	33.294(070)	–22.77746(3)	3.553(1)	3.592(8)
CC3 <sup>44,68</sup>				1.326	1.089	118.3	36.8		3.602	
expt <sup>72–74</sup>				1.321–1.323	1.092–1.103	118.1–121.5	20.5–34			

<sup>a</sup>The VMC energy (au) is averaged over the last 40 iterations of geometry optimization. o.o.p denotes the out-of-plane angle of the CO bond.

**Table 5. Same as Table 4 Using Jastrow-CIPSI Wave Functions with Targeted CI Variance (Iso-Variance)**

state	$\sigma_{\text{CI}}^2$	no. det.	no. param	CO	CH	H–C–H	o.o.p.	$\langle E_{\text{VMC}} \rangle$	$\Delta E_{\text{adia}}^{\text{VMC}}$	$\Delta E_{\text{adia}}^{\text{DMC}}$
S0	0.80	515	1183	1.20663(05)	1.09851(03)	116.84(4)	–0.098(025)	–22.89892(5)		
	0.60	1459	1605	1.20761(14)	1.09886(04)	116.98(5)	–0.091(023)	–22.90489(6)		
	0.40	3755	2566	1.20732(23)	1.09886(05)	116.99(4)	–0.057(235)	–22.90761(6)		
S1	0.80	782	1167	1.33888(11)	1.08562(06)	119.40(07)	34.061(104)	–22.76474(6)	3.651(2)	3.652(8)
	0.60	3106	2037	1.33971(36)	1.08630(06)	119.03(20)	34.778(051)	–22.77342(5)	3.577(2)	3.624(8)
	0.40	9820	4031	1.33713(31)	1.08641(06)	119.23(11)	33.522(079)	–22.77682(6)	3.563(2)	3.611(8)

## 5. OPTIMAL EXCITED-STATE STRUCTURES AND ADIABATIC EXCITATION ENERGIES

Upon relaxation of formaldehyde in the first singlet excited state, the oxygen atom moves out of the plane. The optimal excited-state structure therefore possesses  $C_s$  symmetry with the symmetry plane being perpendicular to the initial molecular plane and passing through the CO bond. The excited-state optimization therefore follows the  $A''$  state as the symmetry of the molecule is lowered from  $C_{2v}$  to  $C_s$ . In the first singlet excited state, thioformaldehyde remains instead planar and preserves  $C_{2v}$  symmetry so that the excited state maintains  $A_2$  character.

**5.1. Formaldehyde.** We first optimize the structure of formaldehyde in the ground state for some of the Jastrow–Slater CIPSI wave functions that we have used to compute the vertical excitation energies of Table 1. Similar to our previous findings for butadiene,<sup>8</sup> we obtain an accurate geometry in the ground state with relatively short expansions. As shown in Table 3, the optimal bond lengths and bond angles computed with the compact aug-cc-pVDZ basis are within 1 mÅ and 1° of the corresponding CC3/aug-cc-pVTZ values already with the smallest wave function considered. Energetically, we gain

about 1 mhartree upon structure optimization with the smallest wave function of 580 determinants and the average VMC energies of the two larger cases are identical to the initial energies on top of the starting CC3 geometry.

To relax the geometry in the excited state, we can simply start from the ground-state structure, with the oxygen slightly displaced out of the plane, and use the same excited-state expansion as in the calculation of the vertical excitation. Even though such a wave function misses all determinants of  $B_2$  character which will acquire nonzero weight as the molecule moves out of the plane, this quick procedure gives us already a good estimate of the excited-state geometry with a CO bond of about 1.334 Å and an out-of-plane angle of 31.5° (see Table S4). These structural parameters compare very favorably with the corresponding values obtained in CC3 and a full valence CASPT2 calculation.<sup>44</sup> We stress that the excited-state geometry of formaldehyde has been found to depend significantly on the level of theory, with highly correlated methods spanning differences of as much as 0.08 Å in the CO bond length and 20° in the out-of-plane angle which measures how much this bond is out of the HCH plane.<sup>44</sup>

**Table 6.** Optimal VMC Ground- and Excited-State Bond Lengths (Å) and Bond Angles (deg), and VMC and DMC Adiabatic Excitation Energies (eV) of Thioformaldehyde Obtained Using Jastrow-CIPSI Wave Functions with Targeted PT2 Correction (Iso-PT2)<sup>a</sup>

state	$\delta E_{PT2}$	no. det	no. param	CS	CH	H–C–H	o.o.p.	$\langle E_{VMC} \rangle$	$\Delta E_{adia}^{VMC}$	$\Delta E_{adia}^{DMC}$
S0	0.17	702	995	1.62218(14)	1.08427(04)	116.869(53)	−0.042(50)	−17.05044(4)		
	0.10	2500	1662	1.62250(16)	1.08469(05)	116.862(32)	0.070(36)	−17.05570(4)		
	0.07	5712	2810	1.62200(11)	1.08464(06)	116.786(34)	−0.058(46)	−17.05715(6)		
	0.045	14218	5650	1.62207(10)	1.08480(07)	116.756(26)	−0.041(53)	−17.05817(3)		
CC3 <sup>44</sup>				1.619	1.083	116.1	0.000			
expt <sup>71,75,76</sup>				1.611–1.614	1.093–1.096	116.2–116.9	0.000			
S1	0.17	858	764	1.72393(27)	1.07879(07)	121.309(163)	−0.024(46)	−16.97123(4)	2.155(2)	2.152(6)
	0.10	3982	1820	1.72514(04)	1.07895(04)	121.388(27)	−0.106(79)	−16.97809(5)	2.112(2)	2.131(6)
	0.07	8418	3110	1.72309(07)	1.07909(05)	121.149(26)	0.115(79)	−16.97998(4)	2.100(2)	2.119(6)
	0.045	16866	5475	1.72182(09)	1.07921(03)	121.144(28)	0.041(85)	−16.98090(3)	2.103(1)	2.112(6)
CC3 <sup>44,68</sup>				1.709	1.078	120.2	0.0		2.112	
expt <sup>71,75,77,78</sup>				1.682–1.708	1.077–1.093	116.8–121.6	0.0–8.9			

<sup>a</sup>The VMC energy (au) is averaged over the last 40 iterations of geometry optimization. o.o.p. denotes the out-of-plane angle of the SO bond.

Starting from such an out-of-plane excited-state geometry, we generate a series of CIPSI expansions of symmetry  $A''$  and continue the structural relaxation in the excited state to further investigate the dependence of the resultant geometry on the complexity of the wave function. Since we also want to compute the adiabatic excitation energy, we have to evaluate differences of the ground-/excited-state energies on the ground-/excited-state optimal geometries. To obtain a balanced description, we therefore generate the CIPSI wave functions at different geometries targeting a given value either of the PT2 correction or of the CI variance for all four states. For convenience, we shall hereafter refer to them as the iso-PT2 and the iso-variance procedure, respectively. The results of these calculations are shown in Tables 4 and 5.

We perform four iso-PT2 optimization tests. Starting from a VMC geometry of Table 3, we generate four ground-state wave functions targeting different values of PT2 correction and further optimize the structure with these wave functions. We follow a similar procedure for the excited state starting from an out-of-plane VMC excited-state geometry of Table S4. For the ground state, the resulting bond lengths and angles remain within 1 mÅ and 1°, respectively, of the CC3 values for all four wave functions. In the excited state, the CO bond undergoes a marginal lengthening and the out-of-plane angle slightly increases by 2–3°, becoming closer to the CC3 angle. The adiabatic excitation energies are computed as differences between iso-PT2 ground and excited-state energies on the corresponding equilibrium structures, and are highly stable irrespective of the size of the wave function: the VMC values lie between 3.55 and 3.56 eV and the DMC ones are somewhat higher and compatible within statistical error with the CC3 estimate. The iso-variance tests yield results very similar to those of the iso-PT2 ones, and the procedure is therefore also a viable route to compute excited-state structures and adiabatic energies.

**5.2. Thioformaldehyde.** Also for thioformaldehyde, we investigate the VMC convergence of the ground-state optimization with three ground-state wave functions used in the computation of the vertical excitation energies. As shown in Table 6, we are able to obtain converged bond lengths and bond angles within less than 3 mÅ and 1° of the CC3 values already with the smallest set of 702 determinants and a maug-cc-pVDZ basis. Energetically, we gain much less than 1

mhartee in all three cases upon optimization starting from the CC3 structure.

In the first singlet excited state, unlike its oxygen counterpart, thioformaldehyde remains planar and the excited state does not change character. We therefore start by simply relaxing the structure with one of the excited-state wave functions employed in the estimation of the vertical excitation energy. To proceed with the structural optimization, we then generate new CIPSI expansions following the iso-PT2 scheme and matching the PT2 corrections used in the corresponding ground-state calculations. As shown in Table 6, despite testing a large range of expansions, there is no significant variation in the length of the CS bond, which is 10 mÅ longer than the CC3 value. The adiabatic excitation energy is again rather stable across all cases and within 0.01–0.04 eV of the CC3 estimate.

## 6. DISCUSSION

We have demonstrated the excellent performance of QMC in the accurate computation of the vertical excitation energies of two small but theoretically challenging systems, formaldehyde and thioformaldehyde. Using fully optimized Jastrow-CIPSI wave functions where the determinantal components are constructed to yield a similar PT2 correction for both states, we are able to obtain VMC excitation energies compatible within less than 0.02 eV of the extrapolated FCI values using relatively compact expansions of a few thousand determinants and a minimally augmented double- $\zeta$  basis. Performing DMC calculations on top of the fully optimized VMC wave functions leads to a uniform gain across both states, thereby not affecting the VMC estimate of the excitation. If we compute the excitation energy by matching instead wave functions with similar variances, we obtain a less robust procedure in the case of formaldehyde, where relatively large expansions of about 7000 determinants are required to recover results within 0.05 eV of our best estimates.

Next, we have investigated the ability of QMC to obtain accurate ground- and excited-state structures. In the ground state, we easily obtain geometries in excellent agreement with those produced by other high-level correlated approaches such as CC3 and full valence CASPT2, using quite compact Jastrow-CIPSI wave functions. The maximum deviation is a meager 3 mÅ in the CS bond of thioformaldehyde. During the relaxation of the excited-state structure, we regenerate the



determinantal component in our Jastrow-CIPSI wave functions following two different determinant selection schemes: (i) keeping a roughly constant PT2 energy correction during the optimization and (ii) targeting a fixed value of the CI energy variance. Like the vertical excitation energies, the excited-state structural parameters show relatively low sensitivity to the size of the wave function and both selection schemes yield similar structures with no clear distinction in the convergence properties. The largest deviation with respect to the CC3 bond lengths is of about 10 mÅ when using the double- $\zeta$  basis. Geometrical relaxation of the excited state additionally allows us to estimate the adiabatic excitation energies, which we find to be compatible within less than 0.05 eV of the CC3 values for all wave function sizes.

To summarize, the use of a CIPSI selection scheme targeting similar PT2 corrections for the states of interest in combination with the full optimization of the Jastrow–Slater wave function enables us to obtain extremely stable and accurate estimates of the vertical excitation energies already at the VMC level, namely, without the need for DMC. This can be achieved with relatively compact wave functions, demonstrating the accuracy of the “expansion” scheme, which is viable also for larger systems. Furthermore, our iso-PT2 protocol to regenerate the wave function during relaxation along a potential energy surface leads to consistent and high-quality structures for both ground and excited states and to accurate adiabatic excitation energies. Our robust estimates of the structural parameters and the vertical and adiabatic excitation energies with moderate Slater expansions open important prospects for the use of VMC structural optimization with Jastrow-CIPSI wave functions as an efficient and reliable approach to characterize excited-state potential energy surfaces.

## ■ ASSOCIATED CONTENT

### ■ Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: [10.1021/acs.jctc.9b00476](https://doi.org/10.1021/acs.jctc.9b00476).

CIPSI energies and FCI/maug-cc-pVDZ extrapolation for formaldehyde and thioformaldehyde; performance of “truncation” scheme in VMC; fits of ground- and excited-state VMC energies as a function of the total number of determinants and as a function of the energy variance; DMC time-step extrapolation; intermediate excited-state geometry optimizations (PDF)

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