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Fractional Model with Social Distancing Parameter for Early Estimation of COVID-19 Spread

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Abstract

COVID-19 disease has come up as a life-threatening outbreak at end of 2019. It has impacted almost all countries in the world. The major source of COVID-19 is a novel beta coronavirus. COVID-19 had a great impact on world throughout the year 2020. Now, the situation is becoming normal due to the invention of the vaccine. All major countries started large vaccination drives. Mathematical models are used to study the impact of different measures used to decrease pandemics. Mathematical models such as susceptible–infected–removed model and susceptible–exposed–infected–removed are used to predict the spread of diseases. But these models are not suitable to predict COVID-19 spread due to various preventive measures (social distancing and quarantine) applied to reduce spread. Hence, in the present manuscript, a novel fractional mathematical model with a social distancing parameter has been proposed to provide early COVID-19 spread estimation. Fractional calculus provides flexibility in choosing arbitrary order of derivative which controls data sensitivity. The model has been validated with real data set. It has been observed that the proposed model is highly accurate in spread estimation.

Keywords Coronavirus · COVID-19 · Mathematical modeling · Pandemic modeling

1 Introduction

COVID-19 disease is caused by a novel coronavirus [1]. COVID-19 came into existence as unexplained case of pneumonia in Wuhan City, Hubei Province in China in December 2019 [2,3]. It has been classified as a member of the severe acute respiratory syndrome (SARS) that outbroke also in South China in 2002–2003 [4]. Tyrell and Bynoe have introduced coronavirus in 1966 [5]. They divided the virus into alpha, beta, gamma, and delta. Alpha and beta virus is categorized as coronavirus and comes into the human body from bats, whereas pigs and birds are the major sources for gamma and delta category of the virus. Major symptoms include fever, tiredness, and dry cough, nasal congestion, runny nose, sore throat, or diarrhea [6]. The person who suffered from COVID-19 gets seriously ill and develops difficulty in breathing [7]. World Health Organization (WHO) has declared as a pandemic in January 2020 due to worldwide existence [6]. Peoples get infected with coronavirus from others (COVID patients) by coughs or sneezing in the incubation period [8]. On 30 January, India reported its first case of COVID-19 in Kerala. Novel coronavirus has on an average 10 days incubation period [9].

In the present manuscript, it is being assumed that the first patient infected with novel coronavirus already exists on January 20, 2020. These peoples are classified as latent peoples. When the latent peoples are diagnosed as infected people, they will be treated in isolation in the hospital. Due to the latent period, several infected peoples are more and they are counted into the exposed category. These exposed peoples are not isolated, they are free to infect other peoples who are in the close neighborhood. Social distancing is used as a preventive measure to reduce virus transmission. It includes cancellation of events, classes in schools and colleges, businesses have pushed work from home policies. All of these measures are adopted to slow the spread of the disease. Mathematical models are useful in the cases where disease dynamics are not unclear [10]. It estimates the number of cases in worst- and best-case scenarios. It is also helpful in estimating the effect of preventive measures adopted against novel viruses such as COVID-19. Susceptible–infected–removed (SIR) and susceptible–exposed–infected–removed

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(SEIR) models are the most popular models that are used to estimate disease spread in advance. The SIR model originated from the study of the plague almost one hundred years ago. Tremendous advance has been achieved in the dynamical epidemic model since the mid-twentieth century. In recent decades, some realistic factors influencing the epidemic transmission were included in the classic SIR model, such as the model considering the incubation stage. The SEIR model considering the population age, population exposed to epidemic, and birth and death of the susceptible peoples. Some dynamic models were also designed for specific epidemics. These models produce ordinary differential equations (ODEs) which are being solved using classical Euler's method. However, these models do not fit to predict the spread of novel coronavirus due to exclusion of preventive measures applied to prevent COVID-19 spread.

Many different mathematical and statistical models have been used to analyze virus behavior. Yadav et al. [12] have presented regression-based analysis models such as quadratic, cubic, exponential, polynomial of degree third, fourth, fifth, and sixth on Indian scenarios [11]. They have provided a functional analysis of data, but the prediction model is only feasible for 7 days. Yadav et al. (2020) have presented a regression-based method to carry out different analyses such as analyzing growth rate, predicting spread, the transmission of the virus, correlation to weather conditions, and predicting recovery rate [12]. Sharma et al. [13] have proposed the ARIMA model with Exponential Smoothing and Holt–Winters model for regression analysis of India's COVID-19 growth. The model underestimates the actual observations. They predicted the slowing of the cases in the upcoming days adopting proper guidelines. Owing to diversity, the difference in geography, a large population, the study would not be directly beneficial in predicting the growth rate for India. Chauhan et al. (2020) have presented linear and polynomial regression models on the datasets of various states of India [14]. They have highlighted a difference in national and state-level models, similar to the methodology proposed in this study. Gupta et al. (2020) and Bintihamzah et al. (2020) have performed analysis during the early onset of the corona [15,16]. Gupta (2020) has used an SEIR and regression model with the data following a linear pattern. It also predicted that community spread would increase cases exponentially.

In the present manuscript, a novel fractional mathematical model is being presented for estimating novel coronavirus patients. The proposed model has included a social distancing parameter which has proved to be one of the best preventive measures taken to reduce COVID-2019 spread [17,18]. Fractional Euler's method has been used to solve ODEs [19–21]. Fractional models incorporate historical data to calculate the gradient at any point due to which it approximates function with high precession. Fractional calculus has been used in mathematical modeling for cancer detection, segmentation, and classification [22,23]. The present work is organized as follows. The mathematical model for COVID-19 spread estimation is presented in Sect. 2. Section 3 discusses the results obtained by the proposed mathematical model.

2 Proposed Methodology with Social Distancing

The Classical SEIR model is defined as

$$\frac{dS}{dt} = -\beta \times S \times I \tag{1}$$

$$\frac{dE}{dt} = \beta \times S \times I - \alpha \times E \tag{2}$$

$$\frac{dI}{dt} = \alpha \times E - \gamma \times I \tag{3}$$

$$\frac{dR}{dt} = \gamma \times I. \tag{4}$$

Here, equations (1), (2), (3) and (4) represent four ordinary differential equations (ODE's). The three parameters are α, β and γ. Here, α is the inverse of the incubation period, β is the average contact rate in the population and γ is the inverse of the mean infectious period.

Equation (1) is the change in people susceptible to the disease and is moderated by the number of infected people and their contact with the infected. Equation 2 represents change of exposed peoples (those who have been infected but are not yet infectious) with time. The equation depends on contact rate and incubation period. If contact rate is high, the number of peoples in exposed category increases. After completion of incubation period, these peoples move to infected category and hence decrease. Equation (3) gives us the change in infected people based on the exposed population and the incubation period. It decreases based on the infectious period, so the higher γ is, the more quickly people die/recover and move on to the final stage in Equation (4). In the present manuscript, novel parameter ρ is being introduced for better estimation of COVID-19 spread on lockdown. The parameter ρ lies in the range 0 to 1, where 0 indicates everyone is locked down and quarantined while 1 is equivalent to our base case above. Equations (1) and (2) are being multiplied with parameter to have social distancing effect. The modified equations are defined as

$$\frac{dS}{dt} = -\rho \times \beta \times S \times I \tag{5}$$

$$\frac{dE}{dt} = \rho \times \beta \times S \times I - \alpha \times E. \tag{6}$$

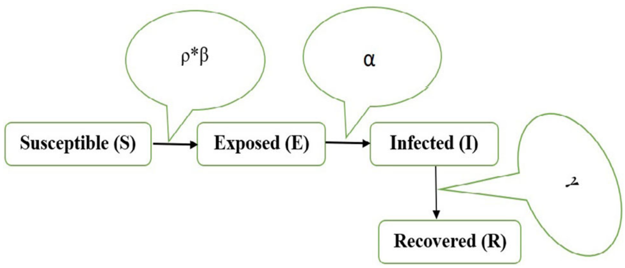


Fig. 1 Proposed mathematical model with social distancing

The proposed mathematical model is shown in Fig. 1

In the present manuscript, fractional ordinary differential equations (FODEs) are used to solve equations (1) to (6) [21]. In the literature, many definition exists for calculating fractional derivative of a function such as Riemann–Liouville, Grunwald–Letnikov, Miller and Ross and M. Caputo. It has been found that Caputo is most suitable definition for providing solution of initial value problem (IVP). The Caputo defines fractional derivative of a function as follow

$$\frac{\partial^{fo} f}{\partial (x-a)^{fo}} = \frac{1}{\Gamma(n-fo)} \int_{fo}^{t} \frac{f^n}{(t-z)^{\alpha+1-n}} \partial z \quad (n-1) \le fo < n. \tag{7}$$

Here, n is integer and fo is arbitrary order or fractional order. Caputo derivative has been used to derive generalized Euler's method for solving IVP [19,20].

Consider initial value problem (IVP)

$$D^{fo} y(t) = f(t, y(t)) \qquad y(0) = y_0 \qquad 0 < fo \le 1, t \ge 0. \tag{8}$$

In this method, a set points $(t_i, y(t_i))$ is generated and these points are used to approximate function $y(t)$. The interval $[0 \quad T]$ into k subintervals $[t_i \quad t_{i+1}]$ of equal width $h = T/k$ using the nodes ih for $i = 0, 1, 2, ...k$. Assume $y(t)$, $D^{fo}y(t)$, $D^{2fo}y(t)$ are continuous in the interval $[0 \quad T]$ and use the generalized Taylor formula to expand $y(t)$ about $t = t_0 = 0$. For each t, there is constant value C_1 so that

$$y(t_1) = y(t_0) + D^{fo}(y(t))(t_0) \frac{h^{fo}}{\Gamma(fo+1)} + D^{2fo}(y(t))(C_1) \frac{h^{2fo}}{\Gamma(2fo+1)} \cdots \tag{9}$$

Here, $D^{fo}(y(t))(t_0) = f(t_0, y(t_0, y_0))$ is substituted to equation 9, the resulting equation is

$$y(t_1) = y(t_0) + f(t_0, y(t_0, y_0)) \frac{h^{fo}}{\Gamma(fo+1)} + D^{2fo}(y(t))(C_1) \frac{h^{2fo}}{\Gamma(2fo+1)} \cdots \tag{10}$$

The step size h is chosen too small enough, then we may neglect the second order term involving h^{2fo} and get

$$y(t_1) = y(t_0) + f(t_0, y(t_0, y_0)) \frac{h^{fo}}{\Gamma(fo+1)}. \tag{11}$$

The above process is repeated to approximate function $y(t)$. Hence, the generalized Euler formula can be defined as

$$t_{i+1} = t_i + 1 \tag{12}$$

$$y(t_{i+1}) = y(t_i) + \frac{h^f o}{\Gamma(fo+1)} f(t_i, y(t_i, y_i)) \quad i = 0, 1, 2, ...k-1. \tag{13}$$

Equation (13) is used to solve equations (1) to (6) as

$$D^{fo} S(t) = -\rho \times \beta \times S \times I \tag{14}$$

$$D^{fo} E(t) = \rho \times \beta \times S \times I - \alpha \times E \tag{15}$$

$$D^{fo} I(t) = \alpha \times E - \gamma \times I \tag{16}$$

$$D^{fo} R(t) = \gamma \times I. \tag{17}$$

Flowchart of the proposed fractional model is shown in Fig. 2

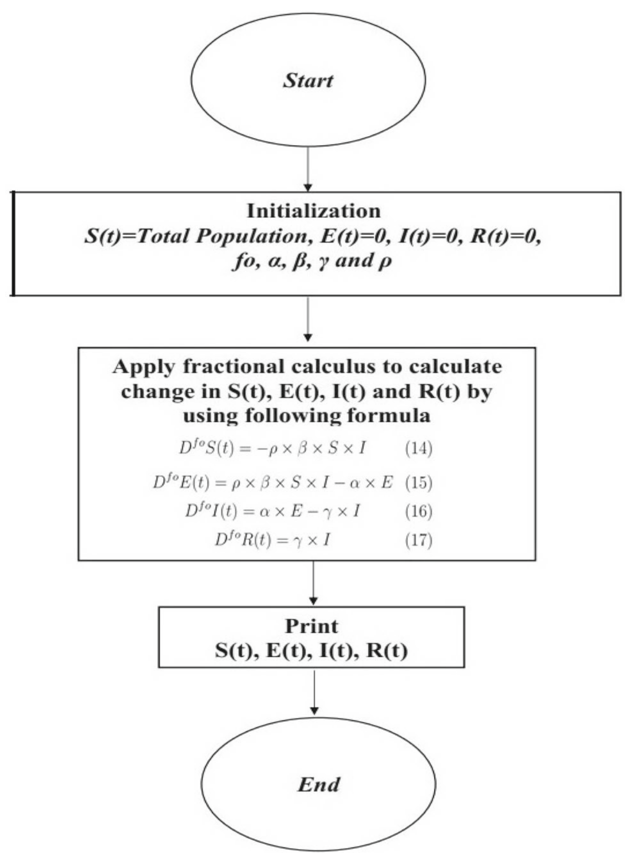


Fig. 2 Flowchart for proposed fractional model

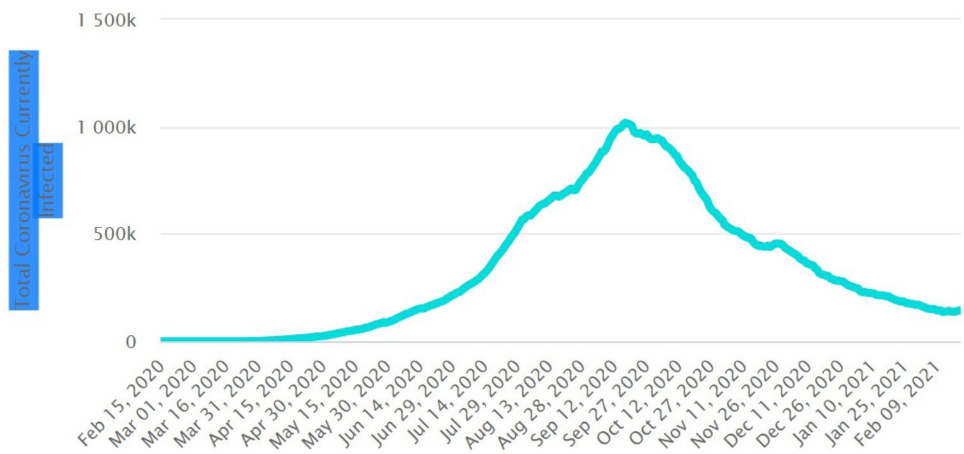


Fig. 3 Active COVID-19 cases in India

3 Results and Discussion

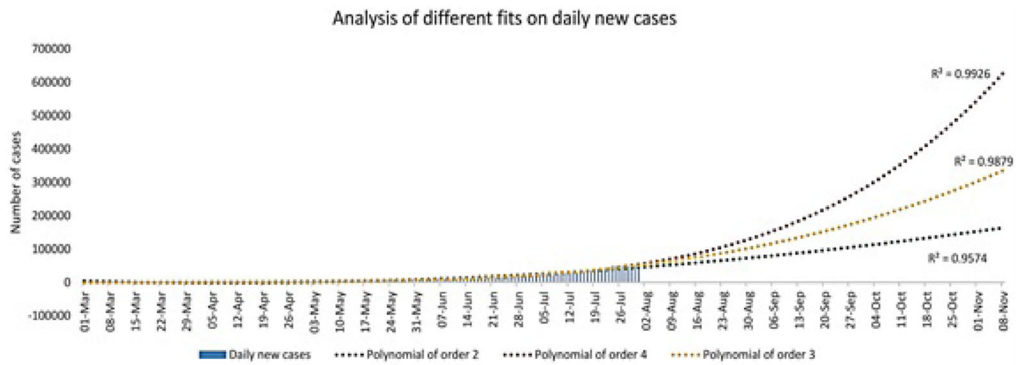
All the experimental works have been performed on PYTHON a machine with CPU clock speed 1.60 GHz, 8GB RAM, 256 KB L1 cache, 1.0 MB L2 cache, and 6.0 MB L3 cache. The proposed mathematical model has been validated on real COVID-19 active cases in India [24,25]. Total active cases in India till February 09, 2021, are shown in Fig. 3.

The COVID-19 spread behavior has been analyzed by curve fitting tools such as linear, logarithmic, exponential, and polynomial of order 2, polynomial of order 3, polynomial of order 4, and polynomial of order 5. Also the value of R^2 has been calculated for error analysis by different curve fitting tools. The results obtained are shown in Fig. 4. It can be easily analyzed from Fig. 4 that polynomial fit gives better approximation as compared to other fits. It is also found that by increasing polynomial order, curve fits better in analyzed data also reduce error. The quantitative analysis has been also performed with curve fitting and is tabulated in Table 1.

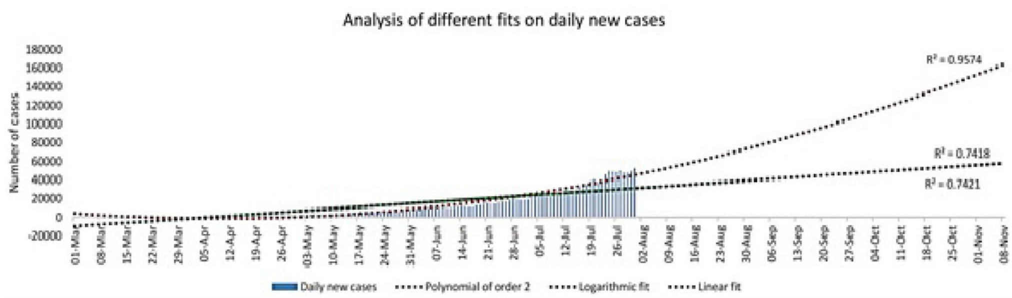
Also, spread behavior has been analyzed by using different parameters such as before and after applying social distancing and spread due to social gatherings. The behavior with different characteristics is shown in Fig. 5. It can be easily analyzed from Fig. 5 that spread behavior has been changed with time-based on different measurements adopted. In the analysis with Fig. 5a, polynomial spread with a degree 3 has been analyzed before applying social distancing. Linear behavior has been analyzed after applying social distancing as can be seen in Fig. 5b. Figure 5 c shows spread due to social gatherings. It can be analyzed from Fig. 5c that spread behavior has been changed from linear to a polynomial of degree 2. Hence, it can be said social distancing played a very important role in reducing spread behavior, and also social gathering has increased the spreading behavior. The overall spread scenario is shown in Fig. 5d. Overall all spread behavior is polynomial with degree 3. In the experimental work, 25000 population has been considered and these are considered as susceptible cases. The cases which do not belong to susceptible cases either belong to exposed, infected, or recovered. The peoples who have contacted recently to infected people move to exposed cases. The exposed cases move to infected after completion of the incubation period. The infected people's moves recover cases after successful completion of the coronavirus cycle, clinical process, or death.

The value of incubation period α has been fixed with 0.2 and an infection period of 0.5 days has been considered in all modes for validating proposed methodology [9]. The proposed fractional SEIR model with the social distancing parameter has been validated with the COVID-19 spread behavior in India. The proposed model has been tested with official data obtained. The experimental study has been performed with different values of α, β, γ and ρ. The spread behavior has been analyzed with different fractional derivative orders such as 0.5, 0.7 0.9, 1.0, $\beta = 1.75$ and $\rho = 1.0$ is shown in Fig. 6. In Fig. 6, the green, red, blue, and purple lines represent results obtained for fractional-order $fo =$ 0.5, 0.7, 0.9, and 1.0, respectively. The dotted line in the corresponding color represents the exposed category of peoples and the solid line in the corresponding color represents the infected people category. The COVID-19 spread is depicted in Fig. 6 based on social distancing parameters=1 which means when 100% of peoples follow social distancing which is not possible. Hence the proposed model has been extended to predict COVID-19 spread based on % of peoples follow social distancing. The results obtained with different % of social distancing are depicted in Figs. 7 and 8.

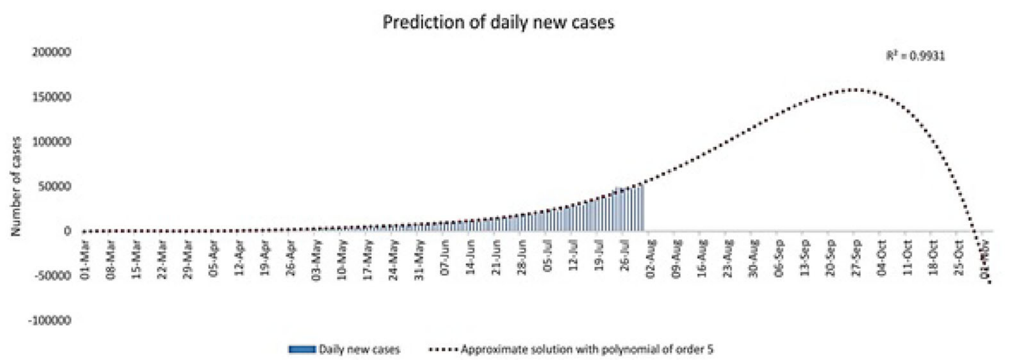
It can be analyzed that the number of an infected person closely matches with fractional near to 0.9. The spread analysis with different social distancing parameters has been also performed. The results obtained with different social distancing parameters $\rho = 0.8$ and $\rho = 0.5$ with $\beta = 1.75$ are shown in Figs. 7 and 8. The flattening effect can be seen in Figs. 7 and 8 which is due to the social distancing parameter. It can be analyzed from Figs. 7 and 8 that more social dis-



(a) Analysis of polynomial fits of order 2, 3 and 4 on daily new cases with R-squared value



(b) Analysis of polynomial fit of order 2, linear and logarithmic fit on daily new cases with R-squared value

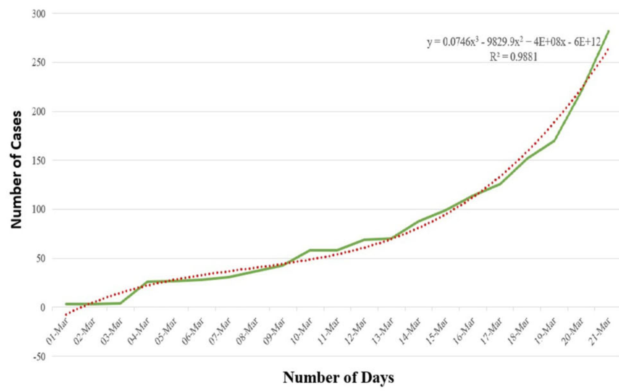


(c) Prediction of daily new cases for 120 days and approximate solution with polynomial fit of order 5

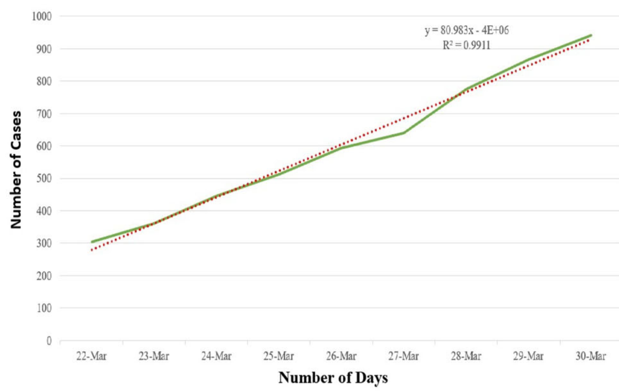
Fig. 4 COVID-19 spread analysis by different curve fitting tools

Table 1 Comparative study with existing methods

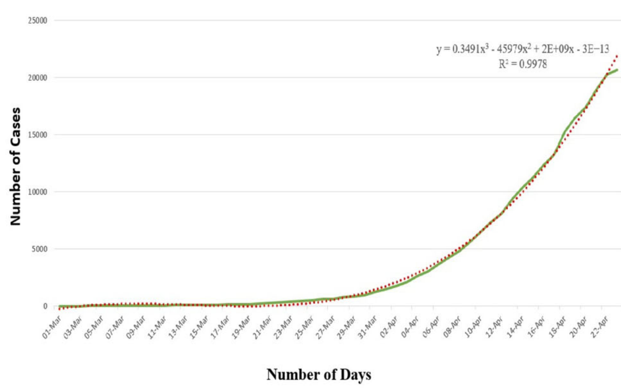
Types of functions	Equation	R-Squared value R^2
Exponential function	$y = 2946.3e^{0.0542x}$	0.7831
Linear function	$y = 10205x - 309462$	0.7831
Logarithmic function	$y = 299360ln(x) - 809412$	0.4252
Polynomial of order 2	$y = 153.25x2 - 9717.4x + 125501$	0.9792
Polynomial of order 3	$y = 1.46x3 - 131.45x2 + 5144.1x - 38599$	0.9974
Polynomial of order 4	$y = 0.0133x4 - 1.9892x3 + 157.55x2 - 3280.5x + 18062$	0.9986
Polynomial of order 5	$y = 0.0002x5 - 0.0395x4 + 4.1236x3 - 142.21x2 + 2373.4x - 7763.8$	0.9992



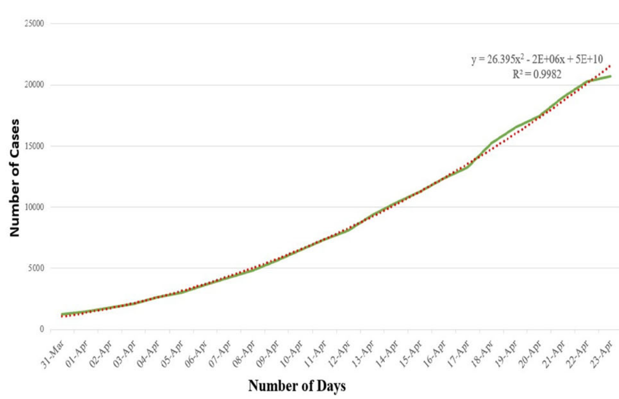
(a) Before applying social distancing



(b) After applying social distancing



(c) Cases due to breakage in social distancing



(d) Overall COVID-19 spread scenario

Fig. 5 COVID-19 trend in India with different scenarios

tancing means the slow spread of disease. It can be observed that with $\rho = 1.0$ that 7% peoples could have effected in 40 days whereas approximately 5% could have infected in 60 days with ρ =0.8. Also, less than 3% peoples could have infected in 112 days with $\rho = 0.5$.

The results obtained with fractional order $fo = 0.5$, 0.7 0.9, 1.0, $\beta = 2.1$ and $\rho = 1.0$ are shown in Fig. 9. It can be analyzed from Fig. 9 that about 12% people could have effected with COVID-19 in 20 days around which is almost the same as COVID-19 spread in India. The spreading behavior of in close matches with fractional order near to 0.9. The spread analysis with different social distancing parameters has been also performed. The results obtained with different social distancing parameters ρ0.8 and $\rho = 0.5$ with $\beta = 2.1$ are shown in Figs. 10 and 11. The flattening effect can be seen in Figs. 10 and 11 which is due to the social distancing parameter. It can be analyzed from Figs. 10 and 11 that more social distancing means the slow spread of disease. Also, it can be observed that with $\rho = 1.0$ that about 12% peoples could have effected in 20 days whereas approximately less than 10% could have infected in 20 days with $\rho = 0.8$ and 0.5.

Figure 12 shows results obtained by proposed for different values of social distancing parameter and wearing of face mask parameter. For $\rho = 1.0$ and wearing mask parameter 0.5, 0.8, and 1.0, the spread analysis will be same as presented as in Figs. 9, 10, and 11. Figure 12a represents result obtained by $\rho = 0.5$ and wearing mask=0.5. As it can be analyzed from Fig. 12a that if 50% peoples are following social distancing and wearing a mask then spread slightly lower than that of peoples not following social distancing and not wearing. Figure 12b represents result obtained by $\rho = 0.5$ and wearing mask=0.8. As it can be analyzed from Fig. 12b that if 50% peoples are following social distancing and 80% peoples are wearing a mask then spread decreases slightly. And the same result is obtained by interchanging the values of the social distancing parameter and wearing the mask parameter. Figure 12c represents result obtained by $\rho = 0.8$ and wearing mask=0.8. Spread reduction can be analyzed from Fig. 12c that if 80% peoples are following social distancing and wearing a mask.

Figure 13 shows results obtained by the SIR model on the same scenarios with the same parameters. From Fig. 13, it can be concluded that the model is not able to correctly analyze the spread scenario of Indian data. This is due to exclusion of some parameters such as the incubation period that plays a major role in COVID-19 spread analysis. Figure 14 shows results obtained by the SEIR model on the same scenarios with the same parameters. From Fig. 14, it can be said that the model does not fit to analyze Indian COVID-19 cases due to various countermeasures adopted to prevent the spread of COVID-19 cases.

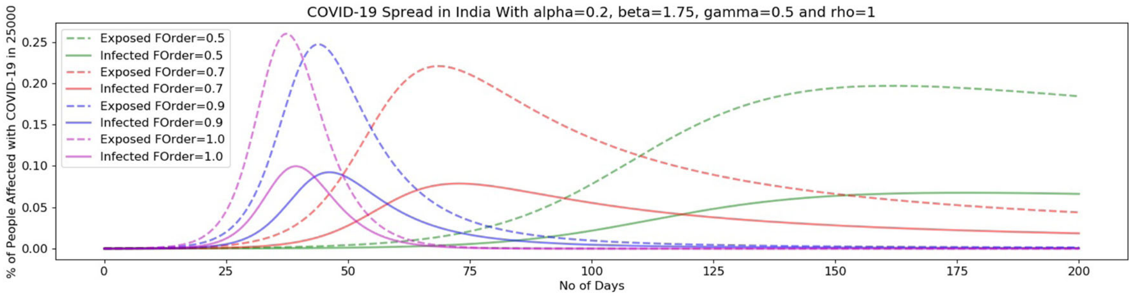


Fig. 6 COVID-19 spread before applying social distancing with $\rho = 1$

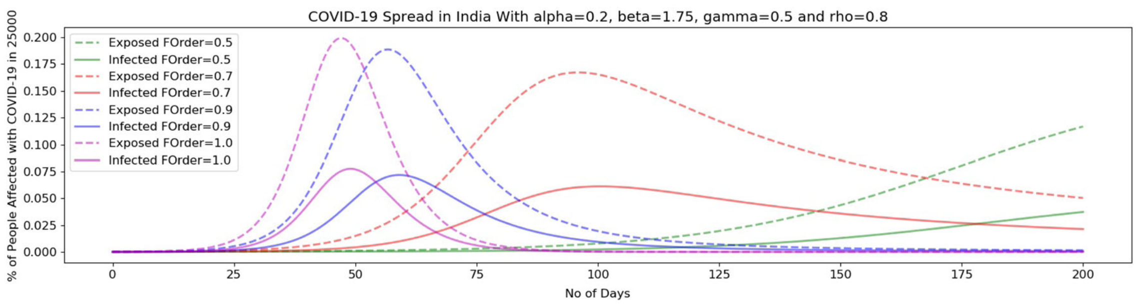


Fig. 7 COVID-19 spread due to breakage in social distancing with $\rho = 0.8$

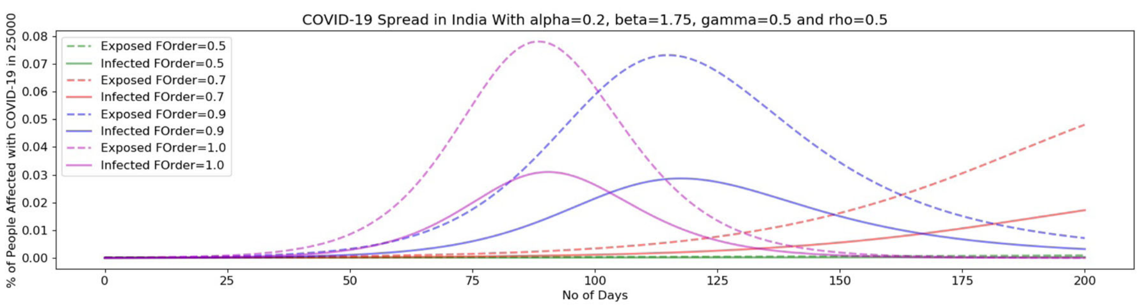


Fig. 8 COVID-19 spread due to breakage in social distancing with $\rho = 0.5$

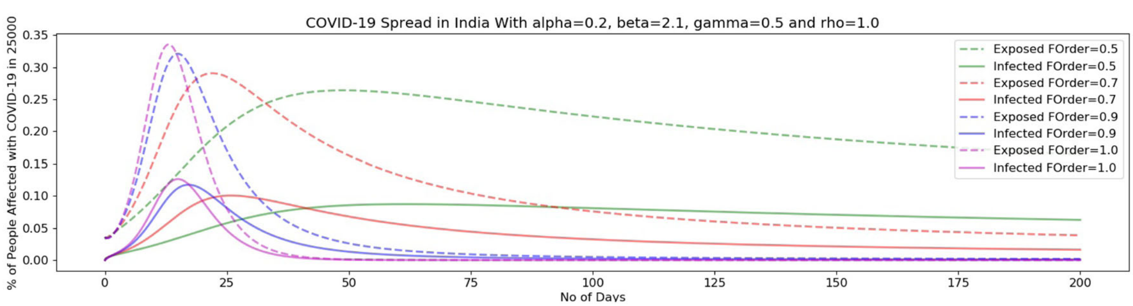


Fig. 9 COVID-19 spread after applying different social distancing with $\rho = 1.0$

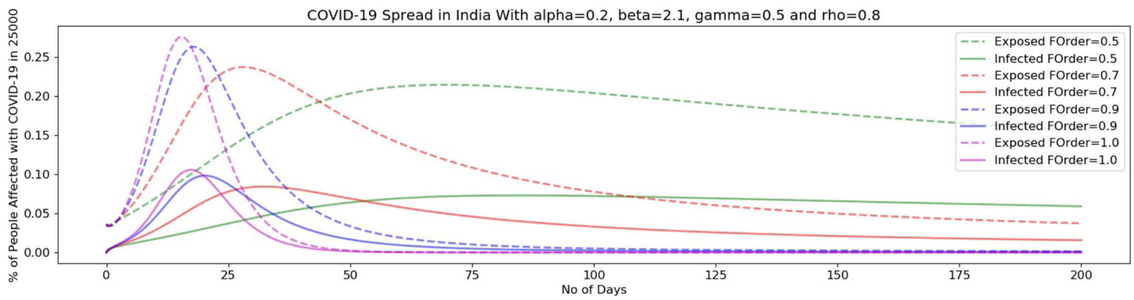


Fig. 10 COVID-19 spread due to breakage in social distancing with $\rho = 0.8$

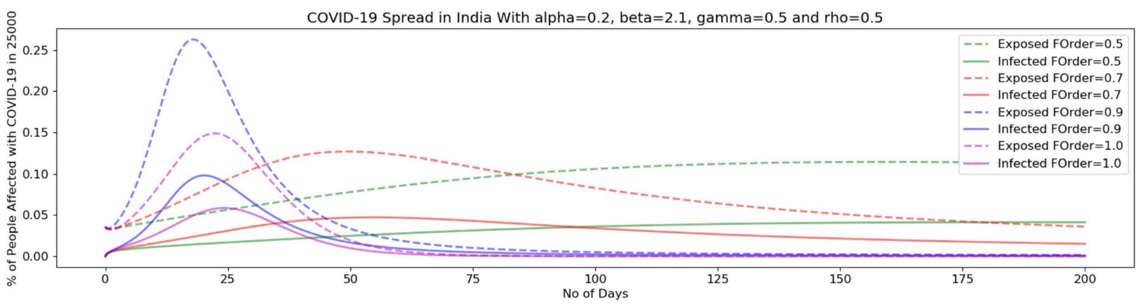


Fig. 11 COVID-19 spread due to breakage in social distancing with $\rho = 0.5$

The present manuscript has presented fractional SEIR with social distancing parameters. The proposed model has predicted COVID-19 spread in India. The model can be used to predict COVID-19 spread and other similar kind of diseases. The proposed model can be validated for any other dataset by selection appropriate fractional order and social distancing parameter.

4 Conclusion

The present work has investigated the COVID-19 spread scenario in India. A fractional mathematical model with social distancing has been established, which follows the actual data trend of COVID-19 spread in India. It has been proved from analytical (based on mathematical modeling) and simulation results that social distancing plays an important role in spread estimation. Different fractional-order derivatives with different social distancing rates and contact rates have been discussed. It has been found that social distancing can reduce spread from polynomial to linear. It has been also observed breakage in social distancing can rise spread from linear to a polynomial of high degree. It has been found that in the current spread scenario 60 days lock-down is required for complete recovery. The proposed model can be validated for any other dataset by selection appropriate fractional order, social distancing, and wearing of facial mask parameter. The proposed fractional model can also be used to design novel models to predict other similar kind of diseases.

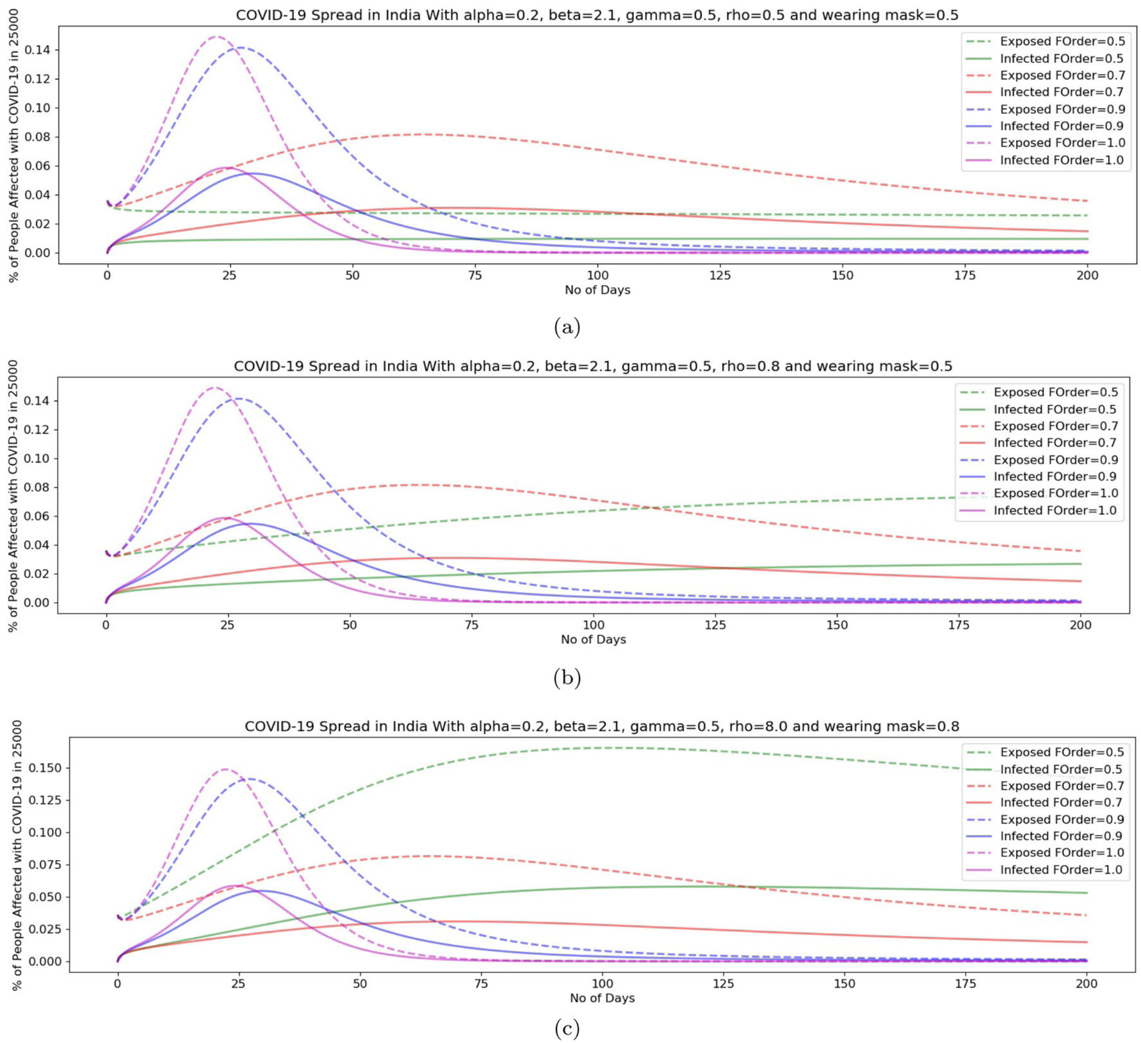


Fig. 12 COVID-19 spread after applying different social distancing and wearing of facial mask parameter

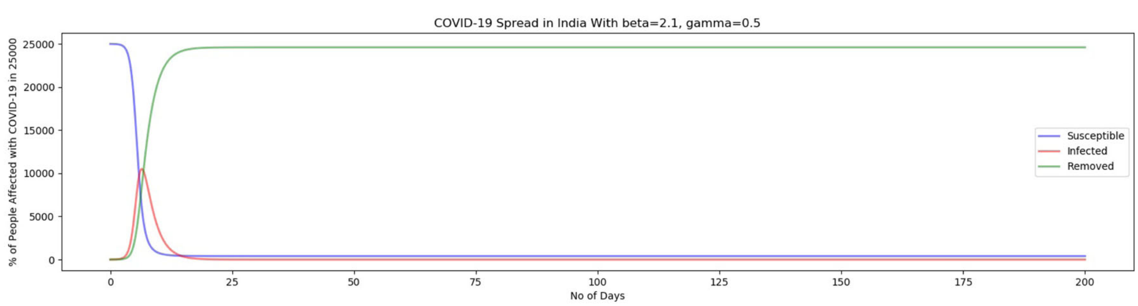


Fig. 13 COVID-19 spread analysis by SIR model

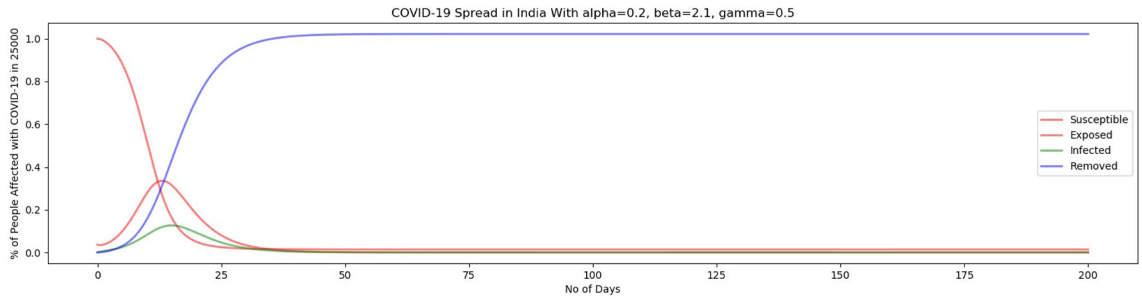


Fig. 14 COVID-19 spread analysis by SEIR model

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