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Experimental determination of dynamic pseudo-equilibrium moisture content: A practical limit for the drying process[☆]



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Determination of dynamic pseudo-equilibrium moisture contents in an experimental drying kinetics study, and calculation of the practical moisture ratio for the modeling of said experimental kinetics, for use in rigorous design and simulation of industrial dryers

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ABSTRACT

Simulation and rigorous design of industrial dryers combine a large number of models, which feed three fundamental balances: (1) mass; (2) energy; and (3) quantity of movement of the material through the dryer. Many of these models represent physical phenomena affecting the three balances at the same time, which makes these calculations extremely complex, hence, accurate models are essential. The hypothesis that the kinetic stage of drying of any material culminates in the thermodynamic moisture equilibrium between solid and drying gas has been in effect for many years. However, recent findings show that there is a transition stage between the kinetic stage and the thermodynamic equilibrium, which, experimentally, looks like an equilibrium. The beginning of this transition stage or dynamic pseudo-equilibrium stage would mark the end of the drying kinetics models, which has been named as the dynamic pseudo-equilibrium moisture contents (X_{dpe}). The non-observance of this phenomenon presupposes a model limited in its prediction capacity, especially in the last stages of drying and even more so at low drying temperatures. As a consequence, sizes of industrial dryers could be underestimated during the simulation and rigorous design process, or underestimate drying times, in batch dryers. On the other hand, the optimal conditions may never be found, during the optimization of existing industrial drying processes. The objective of this work is to present the procedure to determine X_{dpe} , during the experimental determination of drying curves of any material. Likewise, to propose the practical moisture ratio, which uses X_{dpe} , instead of the equilibrium moisture, to be used in the modeling of the drying kinetics.

- The drying process is divided into three stages: kinetic, transition, and equilibrium.
- The dynamic pseudo-equilibrium moisture content divides the kinetic and the transition stages.
- The practical moisture ratio should be used in rigorous industrial dryer design calculations.

[☆] **Related research article:** A. Faneite, A. Rincón, A. Ferrer, I. Angós, G. Arguello, G., Mathematical modeling of thin-layer drying of green plantain (*Musa paradisiaca* L.) peel, *Int. Food Res. J.* 23(5) (2016) 2088–2095. URL: [http://www.ifrj.upm.edu.my/23%20\(05\)%202016/\(33\).pdf](http://www.ifrj.upm.edu.my/23%20(05)%202016/(33).pdf)

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Specification table

Subject area	Chemical Engineering
More specific subject area	Drying kinetics
Name of your method	Determination of dynamic pseudo-equilibrium moisture contents in an experimental drying kinetics study, and calculation of the practical moisture ratio for the modeling of said experimental kinetics, for use in rigorous design and simulation of industrial dryers
Name and reference of original method	Mathematical modeling of thin-layer drying of green plantain (<i>Musa paradisiaca</i> L.) peel
Resource availability	No specific resources are required.

Method details

Background

Smith [1], in 1918, mentioned the term moisture ratio for the first time to refer to moisture on a dry basis (db), in units of kg/kg (water/dry matter). However, Lewis [2], three years later, used it to refer to the moisture ratio (dimensionless). In his famous work, where he published the first known thin-layer drying model, being the Lewis' concept of moisture ratio, which remains valid to date. The traditional theoretical moisture ratio (Mr), expresses the amount of water evaporated in time t , in relation to the total amount of water that can be evaporated by drying, and whose obvious limit is the equilibrium (Eq. (1)).

$$Mr = \frac{X_t - X_\infty}{X_0 - X_\infty} \quad (1)$$

where Mr is de traditional theoretical moisture ratio, X is the moisture content on a dry basis (db) (kg/kg) (water/dry matter), and the subscripts 0, t , and ∞ are the initial state, the state at time t , and the equilibrium state, respectively.

Mr was used to express the moisture content, in studies of drying kinetics, for 80 years. The drying kinetics data, in terms of Mr , obtained from said studies of drying kinetics are correlated as a function of time to obtain the kinetic models. In 2001, Freire et al. [3], had already required to correct Mr to consider an external resistance (initial period of thermal stabilization) that occurs, at the beginning of the olive bagasse drying process. This modification to Mr calculations, however, was not generalized. In 2010, Li et al. [4], found a pseudo-equilibrium period, baptized as practical equilibrium, which divided the dynamic period, and the real equilibrium, in the osmotic dehydration processes of fruits. Since real equilibrium is extraordinarily hard to achieve, Li et al. [4], instead recommend the use of practical equilibrium.

In 2016, Faneite et al. [5], found a region of the drying curves of green plantain (*Musa paradisiaca* L.) peel, at a moisture contents below 10% on a wet basis, (wb), in which the initial monotonic decreasing slows down appreciably. In said region of the drying curves, all the thin-layer drying models, lost their predictive capacity to a greater or lesser degree. The moisture content where the change of the initial monotonic decrease as a function of time (or kinetic stage, which has a characteristic kinetic behavior) is visualized, was named as dynamic pseudo-equilibrium (dpe) moisture content. In a subsequent study on thin-layer drying kinetics of peeled cassava root slices (year 2020), Faneite et al. [6], also found the existence of this stage for this other material. More recently, in 2022, Kappaun et al. [7], reported the drying kinetics of the green banana peel, including a part of the dynamic pseudo-equilibrium stage, but erroneously, they classified it as the real equilibrium.

The dynamic pseudo-equilibrium moisture content is then a thermophysical property of drying, whose numerical value is normally below the storage moisture content, and above of the real equilibrium moisture content. And the pseudo-dynamic equilibrium stage, likewise, is a transition stage between the initial kinetic stage and the final stage of real thermodynamic equilibrium and can also be called the kinetic/equilibrium transition stage. Its behavior, in terms of moisture content as a function of time, is approximately an average between the mentioned stages. These findings imply that Eq. (1) can no longer be used, as an expression of the moisture content, in the modeling of experimental drying kinetics. This is because the ultimate goal of modeling the thin-film drying kinetics of any material of interest is to make the resulting model useful for rigorous design and simulation. As will be explained later, the calculation of the output moisture of the solid from the dryer will be erroneously estimated by the simulator when it is below X_{dpe} .

Thus, for simulation and design purposes, Eq. (2) should be used [5,6], being the experimental determination methodology of X_{dpe} , detailed in this method.

$$pMr = \frac{X_t - X_{dpe}}{X_0 - X_{dpe}} \quad (2)$$

where pMr is the practical moisture ratio (term proposed in this method), X_{dpe} is the moisture content (db) at the beginning of the dynamic pseudo-equilibrium stage (kg/kg) (water/dry matter). X_t and X_0 were already previously defined, in the definition of terms of Eq. (1).

The prefix "pseudo" (false) comes from the fact that pseudo-equilibrium, seen in a small period of time, looks like a true equilibrium state. However, when viewed in its entirety, it can be noticed that It is a transitory process of very slow moisture change over time. This stage begins at the end of the typical kinetic process and ends when true thermodynamic equilibrium has been reached. It is dynamic because it occurs in studies where there is air circulation, and not in traditional gravimetric studies of macroscopic static equilibrium, where air does not flow over the material [8]. But in addition, it is dynamic because it preserves a small fraction of the

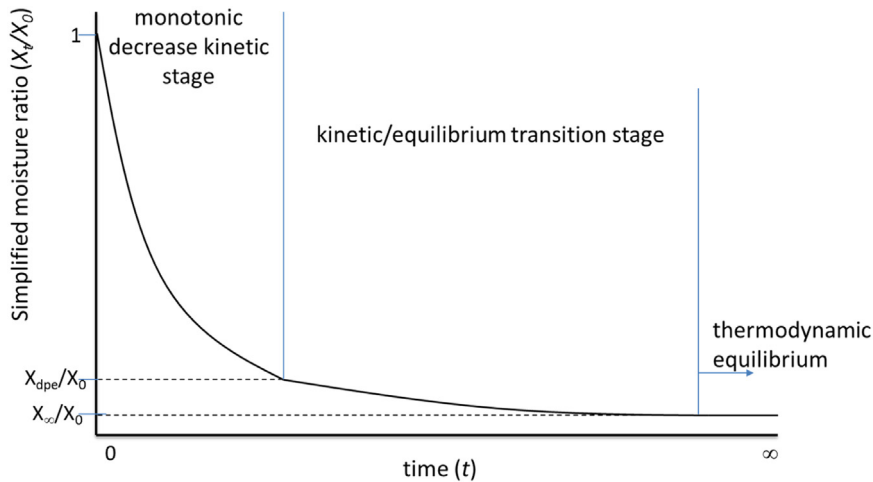


Fig. 1. Representation of the entire drying process, up to equilibrium moisture.

dynamic process of material transport, through the solid, typical of the initial kinetic stage [4]. The dpe moisture content is lower than the storage moisture content (except at very low drying temperatures), but higher than the equilibrium moisture content.

The complete drying process, in terms of simplified moisture ratio (Eq. (3)) vs time (t) can then be visualized in Fig. 1, where the process starts in a X_t/X_0 of 1 at t_0 . Next, a first stage of monotonic decrease in moisture content, widely described in the literature, is established. Eventually, this curve reaches the dynamic pseudo-equilibrium simplified moisture ratio (X_{dpe}/X_0) , where the behavior of the curve changes. At this point, the drying process enters in a transition stage between the kinetics and the equilibrium, which culminates in the simplified equilibrium moisture ratio (X_∞/X_0) . Although the entire kinetic/equilibrium transition stage curve has not been measured, it is estimated to be much longer in time than the kinetic stage, due to the slope found between X_{dpe}/X_0 and X_∞/X_0 [6].

$$sMr = \frac{X_t}{X_0} \tag{3}$$

where sMr represents the simplified moisture ratio. X_t and X_0 were already previously defined, in the definition of terms of Eq. (1).

For simulation purposes, the importance of recognizing the kinetic/equilibrium transition stage, can be appreciated in Fig. 2, where a drying curve is simulated. A retention time inside the dryer is estimated equal to t_{sim} . The simulation would erroneously

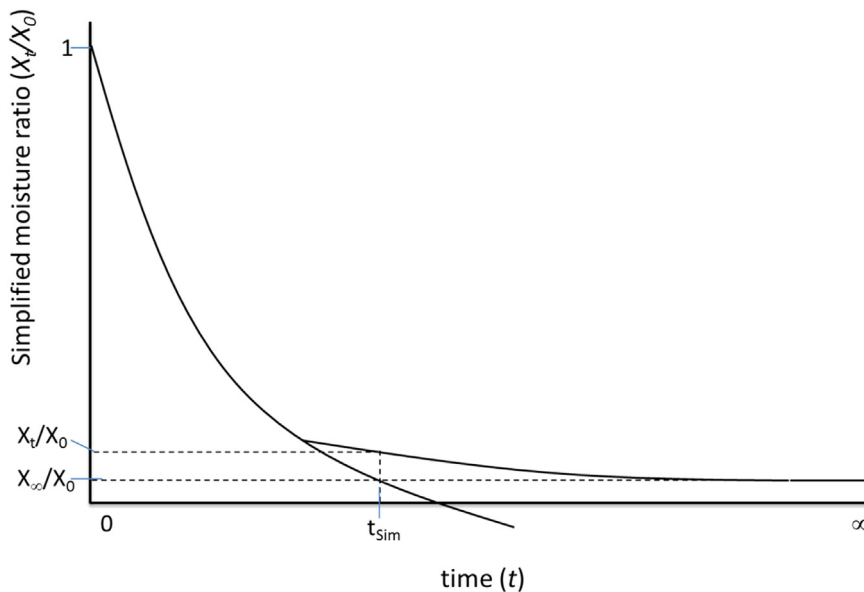


Fig. 2. Simulation of a drying curve up to a time t_{sim} . This graphic representation is only for explanatory purposes, since inside a real continuous dryer, the gas temperature normally changes.

assume that the moisture at time t_{sim} is X_{∞}/X_0 , because the real moisture content reached in that time is X_t/X_0 , since as it falls on the kinetic/equilibrium transition stage curve. Negative values of the outlet moisture content in a simulation, using Eq. (2), would imply that said moisture values would be located on the kinetic/equilibrium transition curve. Adjustments in the drying conditions must be made, in order to visualize a positive value. On the other hand, if a positive value resulted, that is, above the dynamic pseudo-equilibrium moisture content (but inside the kinetic curve), it would by no means be miscalculated. An ideal simulation for industrial dryers would require a model taking into account the entire drying process, where the first stage should be modeled with Eq. (2). This, however, would be impractical, and could rather be rigorously simulated, using pMr instead of Mr , together with a model for X_{dpe} .

The identification of the trend of the dynamic pseudo-equilibrium moisture content, with respect to temperature, is of vital importance in the simulation of co-current dryers, where the temperature of the exit gas decreases dramatically with respect to its value at the entrance [9,10]. This is because the dynamic pseudo-equilibrium moisture content decreases with decreasing drying gas [5,6], as will be seen later in the validation section.

In the pseudo-dynamic equilibrium stage, both kinetic *phenomena* [11] and surface thermodynamic phenomena coexist. As is well known, in real equilibrium, the rate of condensation and evaporation of steam and liquid water, respectively, at the molecular level, it is the same. Furthermore, this phenomenon is mainly superficial. The effects related to the kinetic phenomena decrease to the extent that the drying process advances to the limit, where aforementioned equilibrium thermodynamic *phenomena* prevail. For this reason, the dynamic pseudo-equilibrium moisture content cannot be predicted by traditional thermodynamic equations.

The numerical value of the dynamic pseudo-equilibrium moisture content (X_{dpe}) decreases dramatically with increasing temperature, being predicted by exponential and potential functions [5,6]. This was to be expected since the drying curves get closer to each other as the drying temperature increases. This separation between curves is not proportional.

Just like any other method of analysis, the conditions for determining X_{dpe} (a thermophysical property of drying) have been refined in successive published experiments [5,6], and in unpublished experiments. At this point it is good to clarify that this property was not being sought to be determined, but rather that it emerged as a scientific finding.

Goal

The objective of this method is to describe the procedure to determine the dynamic pseudo-equilibrium moisture contents in an experimental drying kinetics study, and the calculation of the practical moisture ratio for the modeling of said experimental kinetics, for use in rigorous design and simulation of industrial dryers.

Method

The method consists, first of all, in explaining the experimental determination of the dynamic pseudo-equilibrium moisture content for each drying curve, determined for a material of interest. Secondly, explain how to incorporate this property in the thin layer drying kinetics model that best fits the aforementioned curve. To convert the experimental data to data suitable for modeling, it is enough to have a spreadsheet such as Excel®, as a computational tool. Below is the step-by-step explanation of the method.

The method is divided into four main steps. The first step is to identify that the drying process is within the dynamic pseudo-equilibrium stage, during the measurement of an experimental drying curve. The second step is to process the experimental data in terms of simplified moisture content vs time and identify the dynamic pseudo-equilibrium moisture content for each curve. As a third step, the dynamic pseudo-equilibrium moisture content trend as a function of temperature must be identified and the correlation generated. As a fourth and last step, the kinetic model is reported in terms of moisture content on a dry basis, for use in simulation. These steps are presented schematically in Fig. 3.

In the first step, the moment in which the mass of the samples begins to vary in an order of 10 times the precision of the balance used must be identified, during the experimental measurement of its drying curve. For example, if the precision of the balance is 0.1 mg, it should be identified when the mass of the material is changing in the order of milligrams. At this moment, the drying process is considered to have reached the kinetic/equilibrium transition stage. Identify the mass where this kinetic/equilibrium transition stage begins (m_t), and write down the time at which it was measured (t_b). Keep measuring the mass of the material as a function of time, for a time between 10% and 15% additional to the time of the kinetic stage ($t_b \times 1.25$), to verify the permanence in the kinetic/equilibrium transition stage. Stop the measurements and measure the mass of dry solid (m_{DS}), as usual (105 °C, for 24 h, in a laboratory oven).

In the second step, calculate the moisture content on a dry basis (X_t) for each mass measurement or point on the kinetic curve (m_t), as usual, using Eq. (4). Divide all moistures by X_0 , to obtain sMr (Eq. (3)), and plot as a function of time (t). Using the sMr vs t graph, identify, in each drying curve, the point where the kinetic behavior culminates, whose time (t_{dpe}) must be close to the time identified in the experimental phase (t_b), and write down this simplified moisture ratio as X_{dpe}/X_0 , one for each experimental temperature (T_C). Validate that the equilibrium moisture contents, at the experimental temperatures, have numerical values below the dynamic pseudo-equilibrium moisture contents.

$$X_t = \frac{m_t - m_{DS}}{m_{DS}} \quad (4)$$

where X_t is the moisture content (db) of a point on the drying curve at time t (kg/kg) (water/dry matter), m_t is the mass of a point on the drying curve at time t (g), and m_{DS} is the dry mass of the sample subjected to the measurement of its drying curve (g). The initial masses and moistures are denoted as m_0 and X_0 , replacing m_t and X_t in Eq. (4), respectively, when $t = 0$.

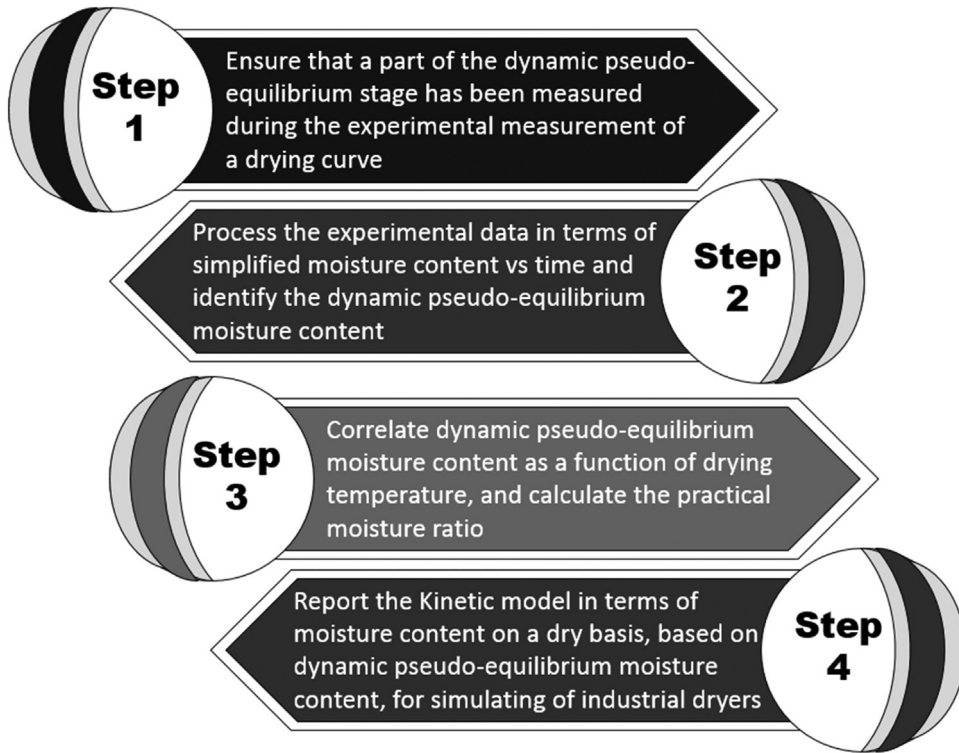


Fig. 3. Scheme of the dynamic pseudo-equilibrium moisture content determination method in an experimental drying kinetics study, and calculation of the practical moisture ratio for the modeling of said experimental kinetics, for use in rigorous design and simulation of industrial dryers.

In the third step, graph X_{dpe}/X_0 , vs T_G . Model the X_{dpe}/X_0 vs T_G data, with the most efficient algebraic expression (X_{dpe} Function) and measure its performance with the preferred statistics. We recommend the use of some of the statistics proposed by Faneite et al. [5,6].

In the fourth step, calculate the drying curves in terms of pMr vs t , calculating pMr with Eq. (2). Model the pMr vs t curves, with the most efficient thin-layer drying kinetics model, for example, as Faneite et al. [5,6]. (*Kinetic model* in Eq. (5)) (The modeling of the kinetic stage of the drying curves is beyond the scope of this method).

$$pMr = \text{Kinetic model} \tag{5}$$

Equating Eqs. (2) and (5), Eq. (6) is obtained.

$$\frac{X_t - X_{dpe}}{X_0 - X_{dpe}} = \text{Kinetic model} \tag{6}$$

Dividing the left side of Eq. (6) by X_0 , Eq. (7) is obtained.

$$\frac{X_t/X_0 - X_{dpe}/X_0}{X_0/X_0 - X_{dpe}/X_0} = \text{Kinetic model} \tag{7}$$

Simplifying $X_0/X_0 = 1$ and substituting X_{dpe}/X_0 for X_{dpe} function in Eq. (7), Eq. (8) is obtained.

$$\frac{X_t/X_0 - X_{dpe} \text{ function}}{1 - X_{dpe} \text{ function}} = \text{Kinetic model} \tag{8}$$

Rearranging Eq. (8) step by step, Eqs. (9), (10), and (11) are obtained successively.

$$X_t/X_0 - X_{dpe} \text{ function} = (1 - X_{dpe} \text{ function}) \cdot (\text{Kinetic model}) \tag{9}$$

$$X_t/X_0 = X_{dpe} \text{ function} + (1 - X_{dpe} \text{ function}) \cdot (\text{Kinetic model}) \tag{10}$$

$$X_t = X_0 \cdot [X_{dpe} \text{ function} + (1 - X_{dpe} \text{ function}) \cdot (\text{Kinetic model})] \tag{11}$$

where X_{dpe} function is the correlation of the dynamic pseudo-equilibrium moisture content as a function of the temperature of the drying gas in terms of the simplified moisture ratio, *Kinetic model* is the thin layer drying model that best fits the experimental drying

data in terms of the practical moisture ratio. X_t , X_0 , X_{dpe} , and pMr were already previously defined, in the definition of terms of Eqs. (1) and (2).

Eq. (11) is the Kinetic model in terms of moisture content on a dry basis, based on dynamic pseudo-equilibrium moisture content.

Applications

Eq. (11) applies to the first finite volume, where the input moisture content corresponds to the initial moisture of the material to be dried. In the simulation of industrial dryers, the equipment is divided, in a virtual way, into volumes of finite size (hence the name of the technique), for its rigorous design. In each of these volumes a balance of mass, energy and movement of matter through the dryer is established [9]. The output of matter and energy from a previous finite volume is the input of matter and energy from the next finite volume (this does not apply, of course, to lost heat) [9]. The material and energy output of the last finite volume is the output of the dryer [9]. For the subsequent finite volumes, Eq. (11) must be adjusted. The adjustment of Eq. (11), for the rest of the finite volumes, is beyond the scope of this method. Eq. (11) also applies to batch dryers that work with a fixed gas temperature.

Validation of the method

Two cases of experimental thin-layer drying kinetics are presented below, where the method described in the previous section was applied, and where its validity is demonstrated. These two cases correspond to the first two published studies, of a group of more than ten studies, each related to a material of biological origin of interest, for the areas of biomass, biorefining, postharvest processing, and animal feed. As previously mentioned, the methodology established from the first of these studies, whose objective was to generate inputs for the simulation and rigorous design of industrial dryers, allowed us to visualize this finding that was not being sought. That same stated objective allowed us to discover the importance of dynamic pseudo-equilibrium moisture for the ultimate use of drying kinetics experimental data. Finally, the strength of the research group in the areas of phase equilibrium thermodynamics, transport phenomena and surface phenomena, typical areas of Chemical Engineering, finished giving theoretical foundation to the finding, and differentiating real equilibrium from pseudo-equilibrium. The drying kinetics data, presented in the following validation cases, was processed in Excel®.

Case 1. Green plantain (*Musa paradisiaca* L.) peel

The thin-layer drying kinetics of green plantain (*Musa paradisiaca* L.) peel was published by Faneite et al. [5], in the temperature range from 60 °C to 110 °C. This was the first work published, where the method was applied. Table 1 shows the final experimental points of the drying curves, where the difference in masses is included, point by point, with respect to the previous point. Mass differences in the order of milligrams, 10 times the precision of the balance (of 0.1 mg), indicate the end of the experiment, as established by this method.

As a next step, the complete experimental data is processed in Excel® to obtain the drying curves and identify the dynamic pseudo-equilibrium moisture content. The drying curves for the green plantain peel, from 60 °C to 110 °C, in terms of the simplified moisture ratio (X_t/X_0), within the time range of interest, can be seen in Fig. 4. Kinetic and pseudo-dynamic equilibrium stages, as well as pseudo-dynamic equilibrium moisture content were indicated with markers of different colors. It is possible to clearly appreciate the change in the behavior of the curve, towards a more horizontal one.

As a last step, the trend of the experimental points of the dynamic pseudo-equilibrium moisture content is identified and the corresponding mathematical model is established. This model will be included in the Kinetic model in terms of moisture content on a dry basis. The X_{dpe}/X_0 trend is potential, both in terms of time and as a function of temperature (Figs. 4 and 5, respectively), coinciding with the hypothesis expressed in the Background. Eq. (12) was the model that best fits the behavior of X_{dpe}/X_0 as a function

Table 1

Period of the drying curve, expressed in average mass of the replicates, where the change from kinetic behavior to the kinetic/equilibrium transition stage occurs for the green plantain (*Musa paradisiaca* L.) peel at different temperatures.

60 °C			70 °C			80 °C			90 °C			100 °C			110 °C		
<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)
320	2.7589	-15.07	265	2.7157	-17.87	185	2.4417	-14.80	150	2.6548	-18.73	138	2.7246	-21.23	117	2.8026	-14.73
325	2.7443	-14.60	270	2.7074	-8.30	190	2.4294	-12.37	153	2.6368	-17.93	141	2.7022	-22.40	120	2.7856	-17.00
330	2.7338	-10.47	275	2.6944	-13.07	195	2.4192	-10.13	156	2.6248	-12.07	144	2.6879	-14.30	123	2.7739	-11.73
335	2.7323	-1.50	280	2.6870	-7.33	200	2.4111	-8.17	159	2.6175	-7.30	147	2.6821	-5.80	126	2.7654	-8.47
340	2.7288	-3.57	285	2.6776	-9.47	205	2.4046	-6.47	162	2.6137	-3.80	150	2.6748	-7.33	129	2.7640	-1.40
345	2.7225	-6.30	290	2.6715	-6.07	210	2.3997	-4.87	165	2.6125	-1.20	153	2.6765	1.77	132	2.7613	-2.67
350	2.7187	-3.80	295	2.6688	-2.67	215	2.3940	-5.73	168	2.6068	-5.63	156	2.6706	-5.97	135	2.7572	-4.17

t: time; *m*: mass (measured with analytical balance with a precision of ± 0.1 mg); *DIFF*: difference in masses, previous point minus the current one. Bold: mass where the change in mass began to have an order of 10 times the precision of the balance. Source: Faneite et al. [5].

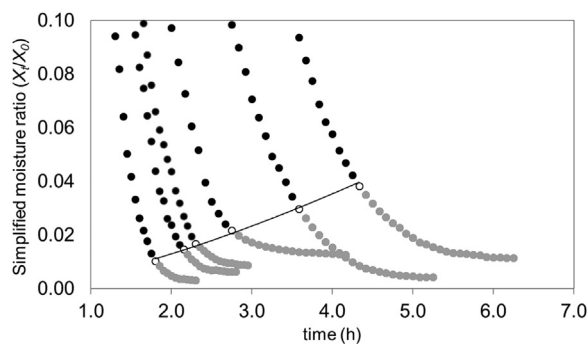


Fig. 4. End of the experimental drying curves of green plantain (*Musa paradisiaca* L.) peel at temperatures from 60 °C to 110 °C [5], from 10 to 10 °Celsius, from right to left. Experimental points: kinetic stage (●), dynamic pseudo-equilibrium moisture content (○), dynamic pseudo-equilibrium stage (●). X_{dpe}/X_0 tendency (—).

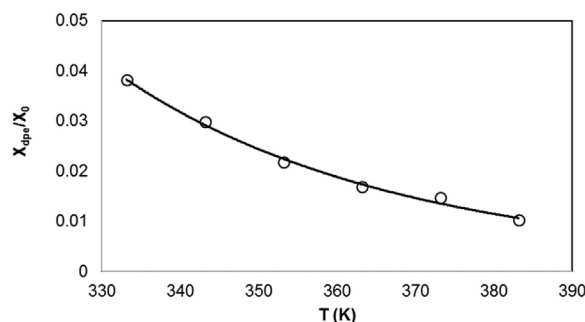


Fig. 5. Simplified moisture ratio of dpe moisture content (X_{dpe}/X_0) as a function of absolute temperature (K) for green plantain (*Musa paradisiaca* L.) peel. The markers ○ represent the experimental points. The continuous line represents Eq. (12).

of temperature, for this material.

$$X_{dpe}/X_0 = 5.12656418274477E21 \cdot (T + 273.15)^{-9.16792923} \quad (12)$$

where X_{dpe}/X_0 is the simplified moisture ratio at the beginning of the dynamic pseudo-equilibrium stage at the drying gas temperature T , and T is the temperature of the drying gas (°C).

The performance of Eq. (12) was excellent. A very high Coefficient of Determination (R^2), of 99.48%, indicates how closely the model fits experimental behavior, where 100 % is a perfect fit. A very low Mean Bias Error (MBE:10,000), of 0.14, indicates how evenly the model errors are distributed around 0, where 0 is a perfect distribution. A very low Root Mean Square Error (RMSE:10,000), of 6.8, indicates the bias with which the model makes a prediction, where 0 indicates no bias at all. A Standard Error of Estimation (SEE:10,000) of 8.4, of the same order as RMSE, indicates that the number of parameters of the model is not exaggerated.

Dynamic pseudo-equilibrium moisture contents could not be compared with equilibrium moisture contents for green plantain peels. It only exists in the literature, a data reported by Kappaun et al. [7], as equilibrium moisture content. They erroneously established a value of 6.0 kg/kg \pm 0.3 kg/kg (water/dry matter) for 40 °C, 60 °C, and 80 °C, but what they really got were average values for a portion of the kinetic/equilibrium transition stage at the temperatures mentioned. Finally, the modeling of the thin-layer drying kinetics of the green plantain peels can be reviewed in the work of Faneite et al. [5].

Case 2. Peeled cassava root slices

The kinetics of thin-layer drying of cassava root, peeled and cut into 5 mm and 6 mm slices, was published by Faneite et al. [6], in the temperature range from 55 °C to 105 °C. This was the second work published, where the method was applied. They found the times in which the kinetic behavior culminated for each drying curve, as was done in the previous case, presenting the results in Tables 2 and 3, respectively (bold numbers). Unlike the case of the drying data of green plantain (*Musa paradisiaca* L.) peel, reported in Table 1, the average of the masses of Tables 2 and 3, does not generate four endpoints with a difference in milligrams in all cases. However, in the replicas, this requirement was visualized in full compliance. This is due to slight differences in the initial masses of the replicas. Likewise, the identification of these points in tables 2 and 3 is purely referential since the final decision is made by looking at the drying curves on a graph.

Drying curves for cassava root, peeled and cut into 5 mm and 6 mm slices, at the temperatures studied and within the time range of interest (in terms of the simplified moisture ratio) are shown in Figs. 6(A) and (B), respectively. The behavior or shape of the drying curve at the end of it coincides with that of the drying of green banana peels. As in Fig. 3, kinetic and pseudo-dynamic equilibrium

Table 2

Period of the drying curve, expressed in average mass of the replicates, where the change from kinetic behavior to the kinetic/equilibrium transition stage occurs for peeled cassava root, sliced 5 mm thick at different temperatures.

55 °C			65 °C			75 °C			85 °C			95 °C			105 °C		
<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)
340	14.7395	72.03	290	14.2269	57.20	285	11.2697	15.77	235	12.0896	23.30	177	10.5685	18.90	132	12.0013	29.53
345	14.6467	92.83	295	14.1833	43.57	290	11.2555	14.17	240	12.0700	19.60	180	10.5531	15.43	135	11.9822	19.17
350	14.6223	24.37	300	14.1425	40.80	295	11.2350	20.47	245	12.0549	15.10	183	10.5399	13.17	138	11.9626	19.57
355	14.6084	13.90	305	14.1037	38.80	300	11.2211	13.93	250	12.0472	7.63	186	10.5302	9.77	141	11.9561	6.47
360	14.5972	11.23	310	14.1035	0.27	305	11.2083	12.83	255	12.0404	6.83	189	10.5188	11.33	144	11.9390	17.10
365	14.5937	3.50	315	14.1032	0.23	310	11.1976	10.63	260	12.0347	5.67	192	10.5108	8.03	147	11.9124	26.67
370	14.5904	3.33	320	14.1030	0.27	315	11.1953	2.30	265	12.0293	5.40	195	10.5067	4.13	150	11.8904	22.00

t: time; *m*: mass (measures with analytical balance with a precision of 0.1 mg); *DIFF*: difference in masses, previous point minus the current one. Bold: mass where the change in mass began to have an order of 10 times the precision of the balance. Source: Faneite et al. [6].

Table 3

Period of the drying curve, expressed in average mass of the replicates, where the change from kinetic behavior to the kinetic/equilibrium transition stage occurs for peeled cassava root, sliced 6 mm thick at different temperatures.

55 °C			65 °C			75 °C			85 °C			95 °C			105 °C		
<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)	<i>t</i> (min)	<i>m</i> (g)	<i>DIFF</i> (mg)
370	14.3818	103.63	330	14.5985	19.13	295	11.8951	43.57	260	11.3468	44.03	219	12.4343	16.66	189	12.1828	25.33
375	14.3062	75.57	335	14.5892	9.27	300	11.8543	40.80	265	11.3023	44.43	222	12.4004	33.86	192	12.1491	33.77
380	14.2953	10.97	340	14.5172	72.03	305	11.8155	38.80	270	11.2758	26.57	225	12.3667	33.69	195	12.1204	28.71
385	14.2713	23.97	345	14.4243	92.83	310	11.8153	0.27	275	11.2539	21.90	228	12.3577	9.05	198	12.1153	5.07
390	14.2643	7.00	350	14.4000	24.37	315	11.8150	0.23	280	11.2434	10.43	231	12.3486	9.05	201	12.1119	3.38
395	14.2624	1.87	355	14.3861	13.90	320	11.8148	0.27	285	11.2360	7.40	234	12.3396	9.05	204	12.1102	1.69
400	14.2612	1.27	360	14.3748	11.23	325	11.8146	0.20	290	11.2306	5.40	237	12.3305	9.05	207	12.1085	1.69

t: time; *m*: mass (measures with analytical balance with a precision of 0.1 mg); *DIFF*: difference in masses, previous point minus the current one. Bold: mass where the change in mass began to have an order of 10 times the precision of the balance. Source: Faneite et al. [6].

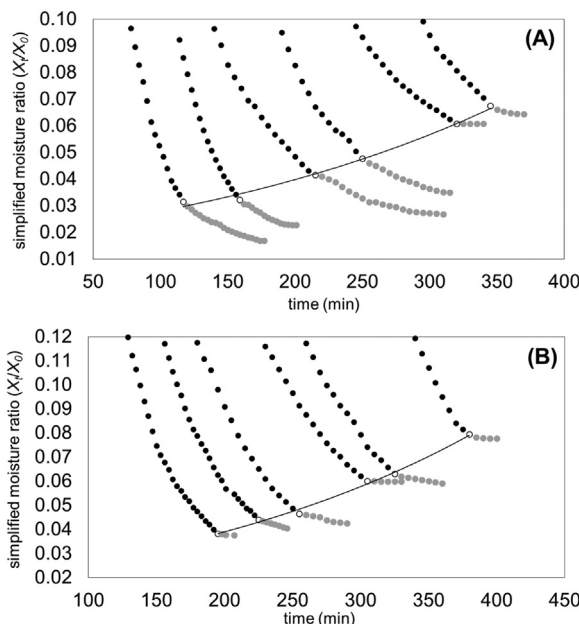


Fig. 6. End of the experimental drying curves of peeled cassava roots (●), (A) sliced 5 mm thick, and (B) sliced 6 mm thick, at temperatures from 55 °C to 105 °C [6], from 10 to 10 °Celsius, from right to left. Experimental points: kinetic stage (●), dynamic pseudo-equilibrium moisture content (○), dynamic pseudo-equilibrium stage (●). X_{dpe}/X_0 tendency (—).

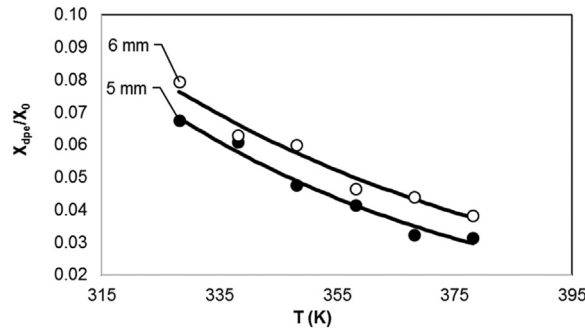


Fig. 7. Simplified moisture ratio of dpe moisture content as a function of absolute temperature for peeled cassava root, (●) sliced 5 mm thick, and (○) sliced 6 mm thick. The solid line represents Eq. (13).

stages, as well as pseudo-dynamic equilibrium moisture content were indicated with markers of different colors. The trend of X_{dpe}/X_0 is exponential in both cases.

The behavior of the X_{dpe}/X_0 as a function of the absolute temperature (Fig. 7) responds to an exponential behavior, and the prediction model is represented by Eq. (13).

$$X_{dpe}/X_0 = (-8.356210 \cdot L + 58.268958) \cdot e^{(0.002498 \cdot L - 0.029209) \cdot (T+273.15)} \tag{13}$$

where L is the layer height (mm). X_{dpe}/X_0 and T were already previously defined, in the definition of terms of Eq. (12).

The capacity of the Eq. (13) to reproduce the trend of the experimental data was excellent, being the results R^2 for 5 mm and 6 mm, of 97.98% and 96.76%, respectively. The capacity of the model to distribute the errors around 0 was excellent too, obtaining 0.658 and 0.813, respectively, in terms of $MBE \cdot 10,000$. The values of $RMSE \cdot 10,000$ indicated a low bias between the model and the experimental data of 19.0 and 23.8, respectively. The values of $SEE \cdot 10,000$ were in the same order and with numerical values quite close to the $RMSE \cdot 10,000$, obtaining 21.9 and 27.5, respectively. This last result indicate that the number of parameters of this model is not exaggerated, since it does not significantly affect the $RMSE$ bias.

The fact that the layer height influences the numerical value of X_{dpe} implies that transport phenomena prevail over surface phenomena. Likewise, despite the fact that in this transition stage, both transport phenomena (typical of kinetics) and those typical of thermodynamic equilibrium coexist, in the X_{dpe} limit, the first prevail, due to its proximity to this first stage.

Once the X_{dpe}/X_0 model has been obtained, as a function of temperature, and its performance has been measured, it is verified whether, indeed, the equilibrium moisture content is below the dynamic pseudo-equilibrium moisture content. For this, the X_{dpe}/X_0 values are multiplied by the initial moisture content on an average dry basis, of each curve, and the X_{dpe} values are obtained, which can be seen in Table 4. Likewise, using the proposed model by Moreno et al. [12], with parameters calculated from the data of Best [13], and using the environmental and drying chamber conditions from the experiments of Faneite et al. [6], the equilibrium moisture contents were obtained, which are also shown in Table 4. It can be seen that the dynamic pseudo-equilibrium moisture content is higher in all temperature and layer height conditions than the true equilibrium moisture content.

The relative difference (RD) of the equilibrium moisture content with respect to the dynamic pseudo-equilibrium moisture content can be calculated with Eq. (14).

$$RD = (X_{dpe} - X_{\infty}) \cdot 100\% / X_{\infty} \tag{14}$$

Table 4

Comparison of equilibrium and dynamic pseudo-equilibrium moisture contents of peeled cassava root in 5 mm and 6 mm slices at conditions reported by Faneite et al. [6].

Drying kinetic experiments at 5 mm layer height					Drying kinetic experiments at 6 mm layer height				
T_{DC} (°C)	RH_A (%)	RH_{DC} (%)	X_{∞} (kg/kg)	X_{dpe} (kg/kg)	T_{DC} (°C)	RH_A (%)	RH_{DC} (%)	X_{∞} (kg/kg)	X_{dpe} (kg/kg)
55	71.61	10.65	0.0620	0.0918	55	62.28	15.25	0.0654	0.1179
65	68.50	9.89	0.0575	0.0870	65	68.50	9.89	0.0575	0.1047
75	69.33	4.96	0.0505	0.0810	75	68.50	6.43	0.0515	0.0985
85	73.14	3.38	0.0463	0.0667	85	64.89	4.04	0.0468	0.0783
95	72.38	2.13	0.0427	0.0563	95	65.30	2.88	0.0432	0.0688
105	72.38	0.00	0.0388	0.0522	105	68.80	0.00	0.0388	0.0564

T_{DC} : Drying chamber temperature. RH_A : Ambient relative humidity. RH_{DC} : Relative humidity inside the drying chamber. X_{∞} : Equilibrium moisture content (kg/kg) (water/dry solid) calculated with the procedure suggested by Moreno et al. [12], using the parameters obtained by correlating the data from Best [13], and the conditions of the drying chamber reported by Faneite et al. [6]. X_{dpe} : Dynamic pseudo-equilibrium moisture content (kg/kg) (water/dry solid) obtained from Faneite et al. [6].

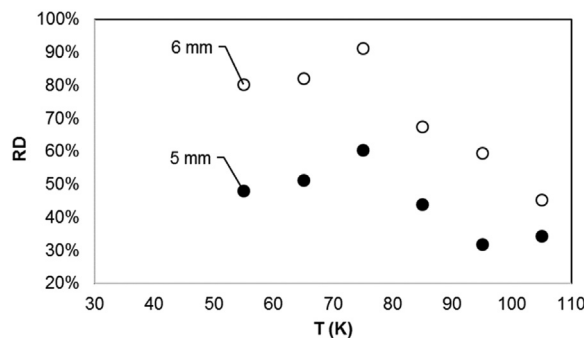


Fig. 8. The relative difference of the equilibrium moisture (RD) with respect to the dynamic pseudo-equilibrium moisture as a function of temperature at 5 mm and 6 mm layer height.

where RD is the relative difference (%) between the equilibrium moisture content (X_{∞} in kg/kg (water/dry matter)), and the dynamic pseudo-equilibrium moisture content X_{dpe} in kg/kg (water/dry matter), measured or estimated at the same drying conditions (temperature and relative humidity of the drying gas).

Fig. 8 is obtained by plotting DR as a function of temperature for each layer height. It can be seen in both cases, and with an almost identical behavior, that when the drying temperature rises from 55 °C to 105 °C, RD increases up to 75 °C, and then decrease. An in-depth analysis on RD behavior is beyond the scope of this validation. Finally, the modeling of the drying kinetics in a thin-layer of the cassava root, peeled and cut into 5 mm and 6 mm slices, at a temperature range between 55 °C and 105 °C, can be reviewed in the work of Faneite et al. [6].

Conclusion

Both for the drying of the green banana peel and for the cassava root, peeled and cut into 5 mm and 6 mm slices, a limit was found in the moisture content, where the initial kinetic behavior of monotonic decrease in moisture content, changes to a more horizontal trend. Said limit, baptized as the dynamic pseudo-equilibrium moisture content, is below the typical storage moisture content (around 10%, on a wet basis), but above the real equilibrium moisture content, so this latter moisture should not be used to calculate the practical moisture ratio. The practical moisture ratio (calculated with the dynamic pseudo-equilibrium moisture content instead of the real equilibrium moisture content) is the correct expression for the moisture content used as a dependent variable in the kinetic models for simulation and rigorous design of industrial dryers.

There are no data published in the literature about the equilibrium moisture content of the green plantain peels, for comparative effects with the dynamic pseudo-equilibrium moisture content. However, dynamic pseudo-equilibrium moisture contents are higher than the equilibrium moisture contents, for all temperatures, of the drying of the cassava root, peeled and cut into 5 mm and 6 mm slices. The dynamic pseudo-equilibrium moisture content of cassava root drying at 6 mm layer height was above that at 5 mm layer height. These results confirm the existence of a transition stage between the initial kinetic stage (governed by transport phenomena) and the final state of thermodynamic equilibrium (governed by surface phenomena), where both phenomena are conjugated.

CRedit author statement

Alexis Manuel Faneite Noguera: Finding, Conceptualization, Methodology, and Validation. **Ignacio Angós Iturgaiz:** Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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