# Design and performance analysis of a 3-RRR spherical parallel manipulator for hip exoskeleton applications 

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#### Abstract

This paper presents the design and performance analysis and experimental study of a 3-RRR spherical parallel manipulator in the context of hip exoskeleton applications. First, the mechanism's inverse kinematics analysis and Jacobian matrix development are revisited. Manipulability, dexterity, and rotational sensitivity indices are then evaluated for two different methods of attachment to the human body. The superior attachment method in terms of these performance measures is indicated, and an experimental study based on the selected method is conducted; the experiment involves testing the capability of a $3-R R R$ manipulator's end-effector in tracking the motions experienced by a human hip joint during normal gait cycles. Finally, the results of the experimental study indicate that the manipulator represents a feasible hip exoskeleton solution providing total kinematic compliance with the human hip joint's 3-degree-of-freedom motion capabilities.


## Keywords

Hip exoskeleton, spherical parallel manipulator, inverse kinematics, Jacobian, dexterity, manipulability, rotational sensitivity, experimental data

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## Introduction

An exoskeleton is a wearable robot designed to supplement one or more abilities of the human body part to which it is connected. Exoskeleton usage is often motivated by energy conservation for functional bodily limbs or strength assistance for limbs that have weakened or total loss of functionality. These capabilities stand to improve the quality of life for people suffering from mobility disablements, which have been reported to affect approximately $20,639,200$ noninstitutionalized individuals in the United States (7.1\% of the total US population) in $2013^{1}$ and $2,512,800$ Canadians ( $7.2 \%$ of the total Canadian population) in 2012 . $^{2}$

The presence of one or more joints with multiple active degrees-of-freedom (DOFs) in the pertinent limb complicates the design of an exoskeleton with complete kinematic compliance. One method to address this challenge is to restrict the motions that the exoskeleton supports about the multi-DOF joint, instead of providing total kinematic compliance.

This is the common design method for current exoskeleton research and technologies. ${ }^{3-6}$ Consequently, most present-day exoskeletons are composed of kinematic open chains: serially connected single-DOF rotary or prismatic joints between rigid linkages. However, Kizir and Bingul conclude that closed-chain parallel manipulators (PMs) have better performance than their serial manipulator counterparts with regard to positioning accuracy, speed, force application, and pay-load-to-weight ratio. ${ }^{7}$ Thus, in order to improve the robotic performance and kinematic functionality of exoskeletons, we propose the use of parallel robots paired with a mechanical structure that transmits motions to the targeted body part in a comfortable,

[^0]non-restrictive way. One parallel robotic structure that has potential for use in such an application is the 3-RRR spherical parallel manipulator. Among previous works in the literature, Gosselin and Angeles present an inverse kinematics analysis, along with discussions of design optimization and singularities, for this manipulator. ${ }^{8}$ Gosselin and Lavoie further discuss the kinematic design and Jacobian derivation for the mechanism. ${ }^{9}$ Gosselin and Hamel have gone on to present a specific embodiment of the manipulator, the Agile Eye; ${ }^{10}$ Gosselin and St-Pierre have further developed its kinematic description and experimentation. ${ }^{11}$ More recently, Bai et al. have revisited the forward displacement analysis of the $3-\underline{R} R R$ manipulator and introduced a new embodiment, called the Agile Wrist. ${ }^{12}$ Wang et al. have conceptualized, analyzed (i.e. kinematic performance), and completed experiments (i.e. torque study) on the use of a redundantly actuated 3-RUS/RRR manipulator for 3-DOF ankle rehabilitation. ${ }^{13}$ Most recently, Niyetkaliyev and Shintemirov detail one method of obtaining forward and inverse kinematics solutions for the Agile Wrist design, including simulation results and numerical examples for verification. ${ }^{14}$

This paper investigates the performance of the 3-RRR in the context of exoskeleton applications. Specifically, manipulability, dexterity, and rotational sensitivity performance indices are evaluated for two different body-interfacing schemes of the manipulator when it is applied as a hip exoskeleton device; here it is assumed that the manipulator supports 3-DOF rotational motions of the upper leg with respect to the pelvis. Our findings suggest that a $3-R R R$ manipulator can be employed as the hip actuator in an exoskeleton system; this represents an original contribution to the field of exoskeleton research.

## Kinematic considerations for the 3-RRR manipulator

## Kinematic architecture

Figure 1 shows a geometrical schematic of a generalized 3-RRR manipulator. This device is considered a 3- $\overline{\mathrm{D}} \mathrm{OF}$ spherical mechanism because all of its moving linkages perform spherical motions about a common point, $O$, which is stationary with respect to its base structure. ${ }^{15,16}$ That is, all particles' motions within the system can be unambiguously described by radial projections on the surface of a unit sphere centered at the aforementioned stationary point. Consequently, the only permissible lower-pair joint within a spherical mechanism's limbs is a revolute joint; furthermore, all joint axes must intersect at the common stationary point mentioned above. In Figure 1, linkages are


Figure I. Schematic illustration of a generalized 3-RRR manipulator. ${ }^{16}$
labeled $0-7$, where 0 indicates the fixed base structure and 7 corresponds to the manipulator's end-effector (i.e. the moving platform). $A_{i}, B_{i}$, and $C_{i}$ denote the three revolute joints of each limb $i$, where $i=1,2,3$ and only $A_{i}$ joints are active.

Note that two notable embodiments of the 3-RRR manipulator are the Agile Eye and Agile Wrist, as mentioned in the previous section and shown in Figure 2(a) and (b), respectively. Although mechanically distinct, these two embodiments have the same inverse kinematics procedure, which is reviewed in the subsection that follows.

## Inverse kinematics derivation

Inverse kinematics analysis for the 3-RRR manipulator has been examined extensively. ${ }^{8,9,11,12}$ One approach is briefly revisited here for the sake of completeness and to acclimatize the reader to the notations and naming conventions used subsequently in this paper.

To start, direction vectors $\mathbf{u}_{1}, \mathbf{u}_{2}$, and $\mathbf{u}_{3}$ specify the rotational axes of the system's three active $A_{i}$ joints, as shown in Figure 3. These vectors have constant values with respect to the global frame (with origin $O$ ) because they correspond to fixed joints. Next, input scalar variables $\theta_{1}, \theta_{2}$, and $\theta_{3}$ define the angular states of the respective active joints. Direction vectors $\mathbf{w}_{\mathbf{1}}, \mathbf{w}_{\mathbf{2}}$, and $\mathbf{w}_{\mathbf{3}}$ in turn specify the rotational axes of the joints between the three proximal-distal link pairs (i.e. the $C_{i}$ joints). These vectors vary in element values with respect to the global frame because they correspond to free joints. The final set of direction vectors, $\mathbf{v}_{\mathbf{1}}, \mathbf{v}_{\mathbf{2}}$, and $\mathbf{v}_{3}$, specify the rotational axes of the joints between the three connection points of the distal links to the end effector (i.e. the $B_{i}$ joints). Again, these vectors vary with respect to the global frame because they correspond to free joints.

Scalar constant $\alpha_{1}$ specifies the angle between each actuated $A_{i}$ joint and the corresponding proximal $C_{i}$ joint within the plane containing both of these joints as well as the global origin, $O$. The value of $\alpha_{1}$ used for the $3-R R R$ design analyzed here is $90^{\circ}$. The second


Figure 2. (a) Agile Eye and (b) Agile Wrist embodiments of the 3-RRR manipulator.


Figure 3. 3-RRR schematics with parameters and direction vectors labelled.
scalar constant, $\alpha_{2}$, specifies the angle between each proximal $C_{i}$ joint and the corresponding distal $B_{i}$ joint within the plane containing both of these joints as well as the global origin. The value of $\alpha_{2}$ used for the 3 - RRR design considered here is also $90^{\circ}$. Third, scalar constant $\beta$ indicates the angle between the $\mathbf{v}_{\boldsymbol{i}}$ direction vectors and the global $z$-axis when the device is in its 'home' position (i.e. when the plane created by $A_{i}$ joint positions is parallel to that defined by the $B_{i}$ points). The value of $\beta$ used here is $54.75^{\circ}$. Fourth, scalar constant $\gamma$ indicates the angle between the $\mathbf{u}_{i}$ direction vectors and the vertical axis (i.e. the global $z$-axis). Unlike $\beta$, this value is constant for all mechanism states because the joints corresponding to the $\mathbf{u}_{i}$ direction vectors are fixed relative to the global frame. The value of $\gamma$ used in this analysis is also $54.75^{\circ}$.

Finally, scalar constants $\eta_{1}, \eta_{2}$, and $\eta_{3}$ are used to specify the locations of the active joints associated with direction vectors $\mathbf{u}_{1}, \mathbf{u}_{2}$, and $\mathbf{u}_{3}$ and 'homepositioned' distal passive joints associated with $\mathbf{v}_{\mathbf{1}}, \mathbf{v}_{\mathbf{2}}$, and $\mathbf{v}_{3}$ within the global $x-y$ plane. Measured with respect to the positive $y$-axis, the values of $\eta_{1}, \eta_{2}$, and $\eta_{3}$ are $0^{\circ}, 120^{\circ}$, and $240^{\circ}$, respectively. Using this convention, $\eta_{i}$ directly specifies the directions $\mathbf{u}_{i}$ in the global $x-y$ plane and specifies the directions $\mathbf{v}_{i}$ in the global $x-y$ plane when added to $60^{\circ}$ and the mechanism is in its 'home' position. Note that the above parameter values are not independent, as they are related through geometry.

Equations for the $\mathbf{u}_{i}$ direction vectors can be derived in terms of the $\eta_{i}$ and $\gamma$ parameters discussed above. This derivation involves the following fixed-frame rotation process: rotation of a local frame $F_{1}$ (i.e. originally identical to the global frame) by $\left(90^{\circ}-\eta\right)$ about the global ${ }^{0} y$-axis and then rotation of $F_{1}$ by $\eta_{i}$ about the ${ }^{0} z$-axis. This overall transformation is represented mathematically in Kucuk and Bingul. ${ }^{17}$ Note that a superscript ' 0 ' indicates an axis or vector expressed with respect to the global frame.

$$
\begin{align*}
R_{01} & =R_{z}\left(\eta_{i}+90^{\circ}\right) R_{y}\left(90^{\circ}-\gamma\right) \\
& =\left[\begin{array}{ccc}
-S \eta_{i} S \gamma & -C \eta_{i} & -S \eta_{i} C \gamma \\
C \eta_{i} S \gamma & -S \eta_{i} & C \eta_{i} C \gamma \\
-C \gamma & 0 & S \gamma
\end{array}\right] \tag{1}
\end{align*}
$$

It follows that the $x$-axis of the resulting $R_{01}$ orientation frame is equal to the direction vector $\mathbf{u}_{i}$.

$$
\mathbf{u}_{i}=\left[\begin{array}{c}
-S \eta_{i} S \gamma  \tag{2}\\
C \eta_{i} S \gamma \\
-C \gamma
\end{array}\right]^{T}
$$

Direction vectors $\mathbf{w}_{i}$ are in turn related to the corresponding $\mathbf{u}_{i}$ vectors through a fixed rotation by $\alpha_{1}$ within the plane containing $O, A_{i}$, and $C_{i}$, along with a variable rotation dependent on actuator angle $\theta_{i}$.

The parameterization of this transformation can be considered as a set of current frame rotations: first a rotation of $\theta_{i}$ about the local ${ }^{1} x$-axis and then a rotation of $\alpha_{1}$ about the updated local $z$-axis. In matrix format, an expression for this is as follows.

$$
R_{x}\left(\theta_{i}\right) R_{z}\left(\alpha_{1}\right)=\left[\begin{array}{ccc}
C \alpha_{1} & -S \alpha_{1} & 0  \tag{3}\\
-S \theta_{i} S \alpha_{1} & -S \theta_{i} C \alpha_{1} & -C \theta_{i} \\
C \theta_{i} S \alpha_{1} & C \theta_{i} C \alpha_{1} & -S \theta_{i}
\end{array}\right]
$$

Now, to obtain expression in terms of the global coordinate system, the set of rotations described above must be pre-multiplied by $R_{01}$. Finally, the set of direction vectors $\mathbf{w}_{\boldsymbol{i}}$ is obtained from the resulting matrix set as the $x$-axes for each $i$, as shown below.

$$
\mathbf{w}_{i}=\left[\begin{array}{c}
\left(-S \eta_{i} C \gamma C \theta_{i}+C \eta_{i} S \theta_{i}\right) S \alpha_{1}-S \eta_{i} S \gamma C \alpha_{1}  \tag{4}\\
\left(C \eta_{i} C \gamma C \theta_{i}+S \eta_{i} S \theta_{i}\right) S \alpha_{1}+C \eta_{i} S \gamma C \alpha_{1} \\
-C \gamma C \alpha_{1}+S \gamma C \theta_{i} S \alpha_{1}
\end{array}\right]^{T}
$$

Similarly to the derivation for $\mathbf{u}_{i}$ vectors summarized in equations (1) and (2), the $\mathbf{v}_{\boldsymbol{i}}$ vectors can be established via two spatial rotations as follows when the device is in its 'home' position.

$$
\begin{equation*}
R_{03}=R_{z}\left(\eta_{i}\right) R_{y}(\beta) \tag{5}
\end{equation*}
$$

Again, $\mathbf{v}_{\boldsymbol{i}}$ is given as the $x$-axis component of the orientation matrix shown in equation (5). To determine the $\mathbf{v}_{\boldsymbol{i}}$ directions after the mechanism's end-effector has undergone roll, pitch, and/or yaw rotations, $R_{03}$ must be pre-multiplied by another transformation.

$$
\begin{equation*}
R_{04}=R_{r p y} R_{03} \tag{6}
\end{equation*}
$$

where $R_{r p y}$ represents the orientation of the end-effector with respect to the global frame. If it is assumed that $R_{r p y}$ is expressed as fixed-frame rotations about the global $x$-axis by $\theta, y$-axis by $\phi$, and $z$-axis by $\psi$, respectively, then the $\mathbf{v}_{\boldsymbol{i}}$ vector can be explicitly derived as follows.

$$
\left[\begin{array}{l}
v_{i x}  \tag{7}\\
v_{i y} \\
v_{i z}
\end{array}\right]=\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right] \cdot R_{z}(\psi) R_{y}(\phi) R_{x}(\theta) R_{03}
$$

Given that all direction vectors $\mathbf{w}_{\boldsymbol{i}}$ and $\mathbf{v}_{\boldsymbol{i}}$ are of unit length, the angle between corresponding $\mathbf{w}_{i}$ and $\mathbf{v}_{i}$ vectors is $\alpha_{2}$ (by the parameter's definition), and the geometric definition of the vector dot product, the following equation relates the two sets of direction vectors.

$$
\begin{equation*}
\mathbf{w}_{i} \cdot \mathbf{v}_{\boldsymbol{i}}=\cos \alpha_{2} \quad i=1,2,3 \tag{8}
\end{equation*}
$$

Now, through substitution of equations (4) and (7) into equation (8), a set of relationships between the system inputs and outputs is obtained. Upon performing this substitution and simplifying the result, the following equation is produced.

$$
\begin{equation*}
A \times \tan ^{2}\left(\theta_{i} / 2\right)+B \times \tan \left(\theta_{i} / 2\right)+C=0 \quad i=1,2,3 \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
A= & \left(-C \eta_{i} C \gamma S \alpha_{1}+C \eta_{i} S \gamma C \alpha_{1}\right) v_{i y} \\
& +\ldots\left(S \eta_{i} C \gamma S \alpha_{1}-S \eta_{i} S \gamma C \alpha_{1}\right) v_{i x}  \tag{10}\\
& +\ldots\left(-C \gamma C \alpha_{1}-S \alpha_{1} S \gamma\right) v_{i z}-c \alpha_{2} \\
C= & B=2 S \eta_{i} S \alpha_{1} v_{i y}+2 C \eta_{i} S \alpha_{1} v_{i x}  \tag{11}\\
C= & \left(-S \eta_{i} C \gamma S \alpha_{1}-S \eta_{i} S \gamma C \alpha_{1}\right) v_{i x} \\
+ & \left(-C \gamma C \alpha_{1}+S \alpha_{1} S \gamma\right) v_{i z}  \tag{12}\\
+ & \left(C \eta_{i} C \gamma S \alpha_{1}+C \eta_{i} S \gamma C \alpha_{1}\right) v_{i y}-c \alpha_{2}
\end{align*}
$$

It follows that the input angle required to achieve a desired end-effector positional output can be found with the following equation.

$$
\begin{equation*}
\theta_{i}=2 \operatorname{atan} 2\left(\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}\right) \quad i=1,2,3 \tag{13}
\end{equation*}
$$

Equations (10)-(13) represent the solution to the inverse kinematics problem for the 3-RRR manipulator because they provide the required active joint states, $\theta_{i}$, necessary to achieve a desired orientation of the endeffector. That is, once end-effector rotations $\theta, \phi$, and $\psi$ are established, the associated angular states of the active revolute joints can be identified.

## Jacobian analysis

A number of generally accepted performance indices for parallel manipulators are often published as a method for comparing various robotic manipulators. ${ }^{16}$ The values of these indices usually have physical significance and applications for design optimization. ${ }^{17}$ The three indices considered in this paper, which are manipulability, dexterity, and rotational sensitivity, all derive from the Jacobian matrix of a manipulator. Thus, the $3-\underline{R} R R$ device's Jacobian development is discussed in this section, before the performance indices are examined in the next section.

To start, a vector $\mathbf{q}$ is assigned to represent active joint variables while $\mathbf{x}$ is used to characterize the endeffector's position. The kinematic constraints associated
with the device's limbs can be expressed as follows.

$$
\begin{equation*}
f(\mathbf{x}, \mathbf{q})=\mathbf{0} \tag{14}
\end{equation*}
$$

where $f$ is an $n$-dimensional implicit function of $\mathbf{q}$ and $\mathbf{x}$, and $n$ is the active joint count within the mechanism. Now, time-differentiating equation (14) yields the following relationship between input joint rates and endeffector velocity.

$$
\begin{equation*}
\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \dot{\mathbf{X}}+\frac{\partial \mathbf{f}}{\partial \mathbf{q}} \dot{\mathbf{q}}=0 \rightarrow \mathrm{~J}_{x} \dot{\mathbf{X}}=\mathrm{J}_{q} \dot{\mathbf{q}} \tag{15}
\end{equation*}
$$

As shown above, two components of the Jacobian are produced: $\mathrm{J}_{x}$ and $\mathrm{J}_{q}$. The combination of these components yields the complete Jacobian matrix.

$$
\begin{equation*}
\dot{\mathbf{q}}=\mathbf{J}_{q}^{-1} \mathbf{J}_{x} \dot{\mathbf{X}}=\mathbf{J} \dot{\mathbf{X}} \tag{16}
\end{equation*}
$$

It is important to note that the Jacobian associated with a parallel manipulator, as in equation (16), is derived as the inverse of a serial manipulator's Jacobian. ${ }^{15}$
when equation (15) is written once for each of $i=1$, 2 , and 3 , three scalar equations are produced. These can be arranged in matrix form as follows.

$$
\left[\begin{array}{c}
\left(\mathbf{w}_{\mathbf{1}} \times \mathbf{v}_{\mathbf{1}}\right)^{T}  \tag{17}\\
\left(\mathbf{w}_{\mathbf{2}} \times \mathbf{v}_{\mathbf{2}}\right)^{T} \\
\left(\mathbf{w}_{\mathbf{3}} \times \mathbf{v}_{3}\right)^{T}
\end{array}\right] \omega_{b}=-\left[\begin{array}{ccc}
\mathbf{w}_{\mathbf{1}} \times \mathbf{u}_{\mathbf{1}} \cdot \mathbf{v}_{\mathbf{1}} & 0 & 0 \\
0 & \mathbf{w}_{\mathbf{2}} \times \mathbf{u}_{\mathbf{2}} \cdot \mathbf{v}_{\mathbf{2}} & 0 \\
0 & 0 & \mathbf{w}_{\mathbf{3}} \times \mathbf{u}_{\mathbf{3}} \cdot \mathbf{v}_{\mathbf{3}}
\end{array}\right] \dot{\mathbf{q}}
$$

Combining equations (16) and (17) yields a complete form of the 3-RRR manipulator's Jacobian matrix.

$$
\begin{align*}
\mathbf{J}= & \mathbf{J}_{q}^{-1} \mathbf{J}_{x}=-\left[\begin{array}{ccc}
\mathbf{w}_{\mathbf{1}} \times \mathbf{u}_{\mathbf{1}} \cdot \mathbf{v}_{\mathbf{1}} & 0 & 0 \\
0 & \mathbf{w}_{\mathbf{2}} \times \mathbf{u}_{\mathbf{2}} \cdot \mathbf{v}_{\mathbf{2}} & 0 \\
0 & 0 & \mathbf{w}_{\mathbf{3}} \times \mathbf{u}_{3} \cdot \mathbf{v}_{\mathbf{3}}
\end{array}\right]^{-1} \\
& \times\left[\begin{array}{c}
\left(\mathbf{w}_{\mathbf{1}} \times \mathbf{v}_{\mathbf{1}}\right)^{T} \\
\left(\mathbf{w}_{\mathbf{2}} \times \mathbf{v}_{\mathbf{2}}\right)^{T} \\
\left(\mathbf{w}_{\mathbf{3}} \times \mathbf{v}_{\mathbf{3}}\right)^{T}
\end{array}\right] \tag{18}
\end{align*}
$$

Recall that vectors $\mathbf{u}_{\boldsymbol{i}}, \mathbf{w}_{\boldsymbol{i}}$, and $\mathbf{v}_{\boldsymbol{i}}$ can be computed from equations (2), (4), and (7), respectively.

## Hip exoskeleton design based on performance indices

With the $3-\underline{R} R R$ manipulator's Jacobian matrix derived, it is now possible to evaluate several of the device's performance indices. In doing so, two methods for attaching the device to the human body are considered, as shown in Figure 4. Furthermore, only flex-ion-extension and abduction-adduction motions are considered; the final major DOF of the hip joint (i.e. internal/external rotation) is assumed to be constant and oriented such that the knee's axis of rotation is perpendicular to the sagittal plane of the body. As can be deduced from Figure 3, the device's $\psi$ angle corresponds to flexion/extension motions for Attachment Method 1, while $\phi$ is associated with those motions in Attachment Method 2; for both cases, $\theta$ corresponds to abduction/ adduction motions. Additionally, a workspace range of [-0.2 0.2] radians for both flexion-extension and abduc-tion-adduction motions was considered for all local performance studies. Finally, the results below are only applicable when the parameter values (i.e. for $\alpha_{1}, \alpha_{2}, \beta$, $\gamma, \eta_{1}, \eta_{2}$, and $\eta_{3}$ ) are selected as per the discussion in Kinematic architecture section.

## Manipulability

Forces experienced by joints within parallel manipulators tend to become large when such a device nears singular configurations. ${ }^{16}$ Thus, the ability to quantify a manipulator's nearness to singular configurations is useful. Manipulability is a common performance index used to accomplish this quantification. It is defined as the absolute value of the Jacobian's determinant, ${ }^{18}$ as given in equation (19). Alternatively, this index can be interpreted as the Jacobian matrix's minimum-magnitude eigenvalue.

$$
\begin{equation*}
\mu=\sqrt{\left|\mathbf{J}^{T} \mathbf{J}\right|} \tag{19}
\end{equation*}
$$



Figure 4. Considered 3-RRR attachment methods as a hip exoskeleton. (a) Interfacing scheme I; (b) interfacing scheme 2.

In mechanical terms, manipulability represents a manipulator's ability to successfully create a desired velocity at its end-effector. ${ }^{19}$ Alternatively, this index can be understood as the ellipsoid volume resulting when a unit sphere is mapped from the manipulator's $n$-dimensional joint space into Cartesian space through its Jacobian matrix and a constant proportionality factor, ${ }^{20}$ recall that $n$ represents the active joint count for the manipulator. It follows that a manipulator achieves greater manipulability performance if its ellipsoid has a greater uniformity, or isotropy, characteristic. ${ }^{21}$ Such an isotropy index for manipulability can be quantified as follows.

$$
\begin{equation*}
\mu_{i s o}=\sigma_{\min } / \sigma_{\max } \tag{20}
\end{equation*}
$$

where $\sigma_{\min }$ and $\sigma_{\max }$ are the minimum and maximum singular values of the Jacobian matrix, respectively. The $\mu_{\text {iso }}$ value in equation (20) is limited to the range [ 0,1 ], where 0 indicates inability to transmit velocity to the end-effector (i.e. a singular configuration) and 1 indicates ability to transmit velocity to the end-effector uniformly in all directions. Figure 5 shows the 3 -RRR device's manipulability deviation and statistical distribution within the considered workspace for the two attachment methods depicted in Figure 4.

According to the surface plots, the manipulability of the $3-\underline{R} R R$ is greatest when operating near its 'home' configuration and least near the boundaries of the considered workspace for both attachment methods. Comparatively, Attachment Method 1 achieves a greater average value for manipulability than Attachment Method 2. Furthermore, Method 1 achieves less variance in performance within the workspace considered.

Therefore, Method 1 is superior to Method 2 in terms of manipulability.

## Dexterity (condition number)

Because a manipulator's control scheme generally relies on its joint position coordinates, any errors between the expected and actual joint coordinates cause errors in the end-effector's position and orientation. ${ }^{16}$ This end-effector error can be determined through multiplication of the errors in the joint coordinates by a scaling factor: the condition number, $k .^{22}$ A manipulator's condition number is obtained from the Jacobian matrix as follows. ${ }^{22-25}$

$$
\begin{equation*}
k(\mathbf{J})=\|\mathbf{J}\|\left\|\mathbf{J}^{-1}\right\| \tag{21}
\end{equation*}
$$

where $\mathbf{J}$ is the Jacobian matrix. Here, $\|\mathbf{J}\|$ denotes the Jacobian's Euclidean norm.

$$
\begin{equation*}
\|\mathbf{J}\|=\sqrt{\operatorname{tr}\left(\frac{1}{n} \mathbf{J J}^{T}\right)} \tag{22}
\end{equation*}
$$

Gosselin proposes that the condition number's inverse be used to quantify a manipulator's kinematic accuracy; ${ }^{24}$ this criterion is called the local dexterity index, denoted by $\nu$.

$$
\begin{equation*}
v=\frac{1}{\|\mathbf{J}\|\left\|\mathbf{J}^{-1}\right\|} \tag{23}
\end{equation*}
$$

Again, allowable values for $v$ are constrained between 0 and 1 ; zero indicates a singularity, and





Figure 5. 3-RRR manipulability for (a) Attachment Method I and (b) Attachment Method 2.


Figure 6. 3-RRR dexterity for (a) Attachment Method I and (b) Attachment Method 2.


Figure 7. 3-RRR rotational sensitivity for (a) Attachment Method I and (b) Attachment Method 2.


Figure 8. (a) Experimental prototype of the $3-\underline{R} R R$ manipulator and (b) $3-\underline{R} R R$ manipulator mounted on the Hip mannequin.
greater values correspond to increasingly accurate motion generation at the end-effector.

Figure 6 depicts dexterity index surface plots and statistical box plots for both body-attachment arrangements of the $3-\underline{R} R R$ manipulator across its considered workspace. Similarly to manipulability, these plots suggest that the mechanism's dexterity is greatest when configured in close proximity to its 'home' orientation and that it decreases as the device moves towards the boundaries of its considered workspace. Additionally, greater average dexterity and less dexterity variation are achieved when the $3-\mathrm{RR}$ R robot is interfaced with the human body according to Attachment Method 1 as opposed to Method 2, which makes the former preferable.

## Rotational sensitivity

The rotational sensitivity index of a manipulator indicates how reactive its end-effector is to changes in active joint states. More specifically, it is the maximum-magnitude rotation of the end-effector under a unit-norm actuator displacement; ${ }^{20}$ it is given by either the 2 -norm or the $\infty$-norm of the Jacobian matrix as follows.

$$
\begin{equation*}
\tau_{r}=\|\mathbf{J}\| \tag{24}
\end{equation*}
$$

Figure 7 shows the sensitivity results for the $3-R R R$ manipulator when subject to the body-interfacing schemes of Figure 4 and constrained to the $\left[\begin{array}{ll}-0.2 & 0.2\end{array}\right]$ radian workspace range in both flexion-extension and abduction-adduction motions. Again, Attachment Method 1 demonstrates preferable performance to that of Method 2 because the former possesses the smaller-magnitude average and variance range in sensitivity index value. Furthermore, sensitivity performance is optimal for both arrangements near the device's 'home' orientation and degrades as the workspace limits are approached.

## Experimental study on the 3-RRR manipulator

## Mechanism fabrication details

In preparation for experimental tests on the $3-R R R$ manipulator design proposed in this paper, a prototype system was fabricated. As shown in Figure 8, all linkage components of the device are 3D-printed, including the base structure, proximal and distal links, and end-effector platform. The prototype's passive revolute joints are composed of off-the-shelf shoulder screws, rotary ball bearings, and thrust bearings. Meanwhile, the active revolute joints are prototyped with Maxon RE-max 29 brushed DC motors. Lastly, a VectorNAV

VN-100 Rugged inertial measurement unit (IMU) is attached to the end-effector platform for capturing orientation data during system operation.

## Experimental results

The purpose of our experimental study on the $3-\underline{R} R R$ prototype is to confirm its end-effector's ability to perform the 3-DOF motions experienced by the human hip joint during normal gait cycles. In order to complete this test, the prototype's motors are controlled with a simple proportional-integral (PI) scheme; angular feedback is provided by the actuator's attached encoders.


Figure 9. Comparison of reference signal and encoder feedback from each actuator during gait motion tracking.

In terms of test execution, reference signals for the endeffector to track are provided by Stanford University's OpenSim software. ${ }^{26,27}$ Subsequently, motor reference signals are obtained by applying the inverse kinematics algorithm discussed in Kinematic architecture section to the OpenSim angular motion signals. Because Attachment Method 1 is expected to provide manipulability, dexterity, and sensitivity performance that are superior to those of Method 2, the motion strategy associated with Method 1 is utilized. That is, the
prototype's $\psi$ motions are matched to hip flexionextension motions, $\theta$ to abduction-adduction, and $\phi$ to internal/external rotations.

The experimental results of Figure 9 depict the reference and response signals associated with the individual system motors. These are the motions required by the selected design and body-attachment scheme to achieve the hip motions associated with normal gait cycles at the end-effector, as determined by the inverse kinematics algorithm. In turn, Figure 10 presents an


Figure 10. Comparison of desired end-effector orientation and IMU-measured value during gait motion tracking.


Figure II. Absolute error between desired end-effector orientation and IMU-measured value during gait motion tracking.
overlay of the resulting end-effector orientation angles, as captured by the system IMU, and the desired angles, as provided by the OpenSim software.

The results shown in Figures 9 and 10 indicate that the $3-\underline{R} R \mathrm{R}$ manipulator can achieve the same motion ranges as the human hip during normal gait cycles. Furthermore, the plots suggest that the mechanism can complete these motions with a similar rate to that of the human hip. The maximum absolute error between a single desired end-effector angle and the measured angle is $7.6^{\circ}$, and it applies to $\psi$ (i.e. flexion-extension motions); the root mean squared error values for $\theta, \phi$, and $\psi$ are $1.2^{\circ}, 0.7^{\circ}$, and $3.1^{\circ}$, respectively.

As shown in the absolute error plots of Figure 11, the error in $\psi$ rises periodically during a rapid extension motion of the hip joint. This systematic error can be primarily attributed to the experiment's non-optimal control method, which does not account for inherent nonlinearities of the device's dynamics and inhibits the device from adequately tracking its reference signal. Therefore, the development of a more effective control algorithm would likely reduce the end-effector's orientation errors. Given this solution and the otherwise small magnitudes of error, it is feasible that the 3-RRR manipulator could be used within a hip exoskeleton system.

## Conclusion and future work

This paper proposes the use of the well-established 3-RRR manipulator as a robotic component within a hip exoskeleton system. Before investigating the mechanism's performance for two different body-attachment methods and presenting the results of a motion-tracking experiment, the device's inverse kinematics and Jacobian matrix development procedures were revisited.

The performance study results indicate that the body-interfacing arrangement that orients the manipulator's $x-y$ plane parallel to the body's sagittal plane is superior in terms of average value and variability for manipulability, dexterity, and rotational sensitivity indices. As can be expected, the manipulator's performance is optimal when configured at its initial 'home' orientation and degrades as the end-effector moves away from this state.

For the experimental study, a prototype manipulator's end-effector was controlled to track the motions experienced by a human hip joint during normal gait cycles. In summary, the general agreement between input and output signals depicted in the resulting figures suggests that application of this $3-\underline{R} R$ d design as a hip exoskeleton is feasible. Furthermore, this application poses a motion assistance solution with total kinematic compliance for multi-DOF body joints. Future work includes singularity, dynamic, and workspace
analyses, design and analysis of the complete exoskeleton system with bodily interfacing details considered, and development of an effective closed-loop control algorithm for the $3-\underline{R} R$ manipulator.

## Declaration of Conflicting Interests

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