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Research article

Selection of appropriate location based on signed distance based ranking approach with interval type-2 trapezoidal fuzzy preference relations

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ABSTRACT

In real life situation, it is often difficult to judge the relative importance of different parameters being considered for evaluating some alternatives. In the context of fuzzy sets, it is a situation where it is difficult to define precise membership grades for attribute values. Here we require more generalized type of fuzzy sets which have a greater representational power than ordinary fuzzy sets. For this purpose we use “interval type-2 trapezoidal fuzzy preference relations (IT2TrFPRs)” in this article as a generalization of fuzzy preference relations and consider the environment discussed above, where there is no information on priority weights. A collective decision matrix will be constructed on the basis of hybrid averages using weighted averaging and signed distance based OWA operation. Then a least deviation model will be employed in order to determine the priority weight vectors. Finally, the alternatives will be ranked on the basis of weighted normalized signed distance of each alternative from the ideal solution. Moreover, a real life example of location selection is illustrated to elaborate the effectiveness of the proposed scheme.

1. Introduction

In real life, the decision making often becomes difficult due to the uncertainty and vagueness associated with certain situations. The typical two valued logic often fails to tackle such situations due to improper modelling or lack of appropriate representational power to model the situation. Particularly, it is the case when a decision varies from person to person, like out of a set of houses one wants to select a house which is usually dependent on the person’s choice as per his/her compatibility. The concept of fuzzy sets proposed by Zadeh [1] in 1965 is considered as a paradigmatic change in dealing with uncertain and ambiguous decision making problems. Fuzzy set is considered as the generalization of the classical sets with members and non-members as the special cases of totally compatible or non-compatible with the concept represented by the fuzzy set. Zadeh [2] further extended this concept to linguistic variables thereby dealing the real life situations in a more effective way of qualitative reasoning instead of quantifying the objects which is often not possible.

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The idea of fuzzy sets has further been extended to interval-valued fuzzy sets by [3–5]. Different authors in [6,17,7,18] have employed these extended fuzzy sets in uncertain situations where it is not possible to precisely identify the membership degree of an element of the universe of discourse. The idea of fuzzy sets is still under-developing stage and further generalizations are in process, like one of the three dimensional extensions of fuzzy sets is the type-2 fuzzy sets and its restriction to interval type-2 fuzzy sets introduced by Medel [8], which is currently in extensive use for many real life decision making problems as can be seen in [9,10,12,13,11,14]. An extension of fuzzy sets involves the idea of intuitionistic fuzzy sets by Atannasov [15], which considers not only the compatibility of an element of universe of discourse to the concept represented by the fuzzy set but also the deviation from the concept. Xu [9,16] defined the aggregation operations Choquet integrals for intuitionistic fuzzy sets.

Nowadays, the process of group decision making is extensively making use of different type of fuzzy preference relations where the alternatives are assessed on the basis on their preference degrees on the other alternatives. In [19–24], authors have employed different types of fuzzy preference relations for group decision making. Here we present a group decision making approach based on “interval type-2 trapezoidal preference relations” IT2TrFPRs which has a better representational power than the existing fuzzy preference relations. Also the hybrid averaging operator used in the proposed scheme considers not only the individual weight-age of the decision makers but also the ordered positions based on closeness to the ideal solution, which makes this approach more authentic than the existing techniques for combining the individual opinions into a collective opinion. Moreover, the technique has the capability of dealing with situations where there is no information on the priority weights. The following sections describe how the article is structured. The ideas of IVFSs and IT2TrFNs are briefly reviewed in Section 2. The signed distance between IT2TrFNs is defined in Section 3, and numerous properties are discussed. Section 4 presents a GDM problem using IT2TrFN data. In addition, this section develops a deviation model for group decision analysis based on signed distances. Section 5 describes the algorithm. Section 6 uses a selection problem to demonstrate and discuss the suggested strategy. In addition, this section compares the method’s applicability to other methods. Lastly, the whole study is concluded in Section 7.

2. Preliminaries

Some related definitions are briefly reviewed in the following section.

Definition 2.1. [3] A non-empty set X with the mapping Z : X $\longrightarrow \{[x,y]|x \le y, x, y \in [0,1]\}$; is known as IVFS in the universal set X. Each IVFS on X is represented as IVFS(X).

Definition 2.2. [3] If $Z \in IT2FS(X)$, assume $Z(x) = [Z^{\bar{L}}(x), Z^{\bar{U}}(x)]$, where $0 \le Z^{\bar{L}} \le Z^{\bar{U}} \le 1$ where $x \in X$ then the ordinary FS $Z^{\bar{L}}$ where $Z^{\bar{L}}$: X $\longrightarrow$ [0,1] is known as ‘lower FS of Z’ and ordinary FS $Z^{\bar{U}}$ where $Z^{\bar{U}}$: X $\longrightarrow$ [0,1] is known as ‘upper FS of Z’

Definition 2.3. [10] Assume that $Z^{\bar{U}}$ and $Z^{\bar{L}}$ are both generalized trapezoidal fuzzy numbers, the relative heights of $Z^{\bar{U}}$ and $Z^{\bar{L}}$ are $h^Z_{\bar{U}}$ and $h^Z_{\bar{L}}$, and $h^Z_{\bar{U}}, h^Z_{\bar{L}} \in [0,1]$, an IT2TrFN Z is represented as below:

$$Z = [Z^{\bar{L}}, Z^{\bar{U}}] = [(z^{\bar{L}}_1, z^{\bar{L}}_2, z^{\bar{L}}_3, z^{\bar{L}}_4; h^Z_{\bar{L}}), (z^{\bar{U}}_1, z^{\bar{U}}_2, z^{\bar{U}}_3, z^{\bar{U}}_4; h^Z_{\bar{U}})]$$

where $0 \le z^{\bar{L}}_1 \le z^{\bar{L}}_2 \le z^{\bar{L}}_3 \le z^{\bar{L}}_4 \le 1$, $0 \le z^{\bar{U}}_1 \le z^{\bar{U}}_2 \le z^{\bar{U}}_3 \le z^{\bar{U}}_4 \le 1$, $z^{\bar{U}}_1 \le z^{\bar{L}}_1$, $z^{\bar{L}}_4 \le z^{\bar{U}}_4$, $0 \le h^Z_{\bar{L}} \le h^Z_{\bar{U}} \le 1$, and ‘lower membership function (LMF)’ $Z^{\bar{L}}$ and the ‘upper membership function (UMF)’ $Z^{\bar{U}}$ of Z are represented as:

$$Z^{\bar{L}}(x) = \begin{cases} \frac{(x - z^{\bar{L}}_1) h^A_{\bar{L}}}{z^{\bar{L}}_2 - z^{\bar{L}}_1}, & z^{\bar{L}}_1 \le x \le z^{\bar{L}}_2 \\ h^Z_{\bar{L}}, & z^{\bar{L}}_2 \le x \le z^{\bar{L}}_3 \\ \frac{(z^{\bar{L}}_4 - x) h^Z_{\bar{L}}}{z^{\bar{L}}_4 - z^{\bar{L}}_3}, & z^{\bar{L}}_3 \le x \le z^{\bar{L}}_4 \\ 0 & otherwise \end{cases}$$

$$Z^{\bar{U}}(x) = \begin{cases} \frac{(x - z^{\bar{U}}_1) h^Z_{\bar{U}}}{z^{\bar{U}}_2 - z^{\bar{U}}_1}, & z^{\bar{U}}_1 \le x \le z^{\bar{U}}_2 \\ h^Z_{\bar{U}}, & z^{\bar{U}}_2 \le x \le z^{\bar{U}}_3 \\ \frac{(z^{\bar{U}}_4 - x) h^Z_{\bar{U}}}{z^{\bar{U}}_4 - z^{\bar{U}}_3}, & z^{\bar{U}}_3 \le x \le z^{\bar{U}}_4 \\ 0 & otherwise \end{cases}$$

The operations described on IT2TrFNs [25,26] are defined as below:

1. Addition:
$\bar{Z} \oplus \bar{B} = [(\bar{z}^{\bar{L}}_1 + \bar{b}^{\bar{L}}_1, \bar{z}^{\bar{L}}_2 + \bar{b}^{\bar{L}}_2, \bar{z}^{\bar{L}}_3 + \bar{b}^{\bar{L}}_3, \bar{z}^{\bar{L}}_4 + \bar{b}^{\bar{L}}_4; min(h^{\bar{L}}_Z, h^{\bar{L}}_B)), (\bar{z}^{\bar{U}}_1 + \bar{b}^{\bar{U}}_1, \bar{z}^{\bar{U}}_2 + \bar{b}^{\bar{U}}_2, \bar{z}^{\bar{U}}_3 + \bar{b}^{\bar{U}}_3, \bar{z}^{\bar{U}}_4 + \bar{b}^{\bar{U}}_4; min(h^{\bar{U}}_Z, h^{\bar{U}}_B)]$
2. Subtraction:
$\bar{Z} \ominus \bar{B} = [(\bar{z}^{\bar{L}}_1 - \bar{b}^{\bar{L}}_1, \bar{z}^{\bar{L}}_2 - \bar{b}^{\bar{L}}_2, \bar{z}^{\bar{L}}_3 - \bar{b}^{\bar{L}}_3, \bar{z}^{\bar{L}}_4 - \bar{b}^{\bar{L}}_4; min(h^{\bar{L}}_Z, h^{\bar{L}}_B)), (\bar{z}^{\bar{U}}_1 - \bar{b}^{\bar{U}}_1, \bar{z}^{\bar{U}}_2 - \bar{b}^{\bar{U}}_2, \bar{z}^{\bar{U}}_3 - \bar{b}^{\bar{U}}_3, \bar{z}^{\bar{U}}_4 - \bar{b}^{\bar{U}}_4; min(h^{\bar{U}}_Z, h^{\bar{U}}_B)]$

3. Multiplication:
$\bar{Z} \otimes \bar{B} = [(\bar{z}_1^{\bar{L}} \times \bar{b}_1^{\bar{L}}, \bar{z}_2^{\bar{L}} \times \bar{b}_2^{\bar{L}}, \bar{z}_3^{\bar{L}} \times \bar{b}_3^{\bar{L}}, \bar{z}_4^{\bar{L}} \times \bar{b}_4^{\bar{L}}; min(h_Z^{\bar{L}}, h_B^{\bar{L}})), (\bar{z}_1^{\bar{U}} \times \bar{b}_1^{\bar{U}}, \bar{z}_2^{\bar{U}} \times \bar{b}_2^{\bar{U}}, \bar{z}_3^{\bar{U}} \times \bar{b}_3^{\bar{U}}, \bar{z}_4^{\bar{U}} \times \bar{b}_4^{\bar{U}}; min(h_Z^{\bar{U}}, h_B^{\bar{U}})]$
4. Division:
$\bar{Z} \oslash \bar{B} = [(\frac{\bar{z}_1^{\bar{L}}}{\bar{b}_4^{\bar{L}}}, \frac{\bar{z}_2^{\bar{L}}}{\bar{b}_3^{\bar{L}}}, \frac{\bar{z}_3^{\bar{L}}}{\bar{b}_2^{\bar{L}}}, \frac{\bar{z}_4^{\bar{L}}}{\bar{b}_1^{\bar{L}}}; min(h_Z^{\bar{L}}, h_B^{\bar{L}})), (\frac{\bar{z}_1^{\bar{U}}}{\bar{b}_4^{\bar{U}}}, \frac{\bar{z}_2^{\bar{U}}}{\bar{b}_3^{\bar{U}}}, \frac{\bar{z}_3^{\bar{U}}}{\bar{b}_2^{\bar{U}}}, \frac{\bar{z}_4^{\bar{U}}}{\bar{b}_1^{\bar{U}}}; min(h_Z^{\bar{U}}, h_B^{\bar{U}})]$
5. Multiplication by a number (r≠0):
$r.Z = Z.r = [(r \times \bar{z}_1^{\bar{L}}, r \times \bar{z}_2^{\bar{L}}, r \times \bar{z}_3^{\bar{L}}, r \times \bar{z}_4^{\bar{L}}, h_Z^{\bar{L}}), (r \times \bar{z}_1^{\bar{U}}, r \times \bar{z}_2^{\bar{U}}, r \times \bar{z}_3^{\bar{U}}, r \times \bar{z}_4^{\bar{U}}, h_Z^{\bar{U}})]$ if $r > 0$
$[(r \times \bar{z}_4^{\bar{L}}, r \times \bar{z}_3^{\bar{L}}, r \times \bar{z}_2^{\bar{L}}, r \times \bar{z}_1^{\bar{L}}, h_Z^{\bar{L}}), (r \times \bar{z}_4^{\bar{U}}, r \times \bar{z}_3^{\bar{U}}, r \times \bar{z}_2^{\bar{U}}, r \times \bar{z}_1^{\bar{U}}, h_Z^{\bar{U}})]$ if $r < 0$
6. Division by a number (r≠0):
$\frac{Z}{r} = [(\frac{\bar{z}_1^{\bar{L}}}{r}, \frac{\bar{z}_2^{\bar{L}}}{r}, \frac{\bar{z}_3^{\bar{L}}}{r}, \frac{\bar{z}_4^{\bar{L}}}{r}, h_Z^{\bar{L}}), (\frac{\bar{z}_1^{\bar{U}}}{r}, \frac{\bar{z}_2^{\bar{U}}}{r}, \frac{\bar{z}_3^{\bar{U}}}{r}, \frac{\bar{z}_4^{\bar{U}}}{r}, h_Z^{\bar{U}})]$ if $r > 0$
$[(\frac{\bar{z}_4^{\bar{L}}}{r}, \frac{\bar{z}_3^{\bar{L}}}{r}, \frac{\bar{z}_2^{\bar{L}}}{r}, \frac{\bar{z}_1^{\bar{L}}}{r}, h_Z^{\bar{L}}), (\frac{\bar{z}_4^{\bar{U}}}{r}, \frac{\bar{z}_3^{\bar{U}}}{r}, \frac{\bar{z}_2^{\bar{U}}}{r}, \frac{\bar{z}_1^{\bar{U}}}{r}, h_Z^{\bar{U}})]$ if $r < 0$

Definition 2.4. [27] Let $Z = (z_{ij})_{n\times n}$ be the real valued matrix if $z_{ij} \in [0,1]$, and

$$z_{ij} + z_{ji} = 1, z_{ii} = 0.5, \forall i, j \in N$$

then Z is FPR, where z_{ij} denotes the strength of preference for alternative z_i over alternative z_j.

We propose the idea of IT2TrFPRs in the following section, which is inspired by FPRs, to accurately express basic uncertain decision making information.

Definition 2.5. [30] let a finite collection X of the alternatives where $X = \{x_1, x_2, ..., x_n\}$, an IT2TrFPR $\hat{Z}$ on X is characterized by one matrix $\hat{Z} = (\hat{Z}_{ij})_{n\times n} \subset X \times X$, where $\hat{Z}_{ij} = [(z_{ij(1)}^{\bar{L}}, z_{ij(2)}^{\bar{L}}, z_{ij(3)}^{\bar{L}}, z_{ij(4)}^{\bar{L}}; h_{ij}^{\bar{L}}), (z_{ij(1)}^{\bar{U}}, z_{ij(2)}^{\bar{U}}, z_{ij(3)}^{\bar{U}}, z_{ij(4)}^{\bar{U}}; h_{ij}^{\bar{U}})]$ is an interval type two trapezoidal fuzzy numbers (IT2TrFNs) which represent the strength of preference of alternative x_i over x_j. The following conditions must be satisfied:

$$\begin{aligned} &z_{ii(t)}^{\bar{L}} = z_{ii(t)}^{\bar{U}} = h_{ii}^{\bar{U}} = h_{ii}^{\bar{U}} = 0.5 \\ &z_{ij(t)}^{\bar{L}} + z_{ji(5-t)}^{\bar{L}} = 1, z_{ij(t)}^{\bar{U}} + z_{ji(5-t)}^{\bar{U}} = 1, h_{ij}^{\bar{L}} + h_{ji}^{\bar{U}} = 1, \end{aligned} \tag{1}$$

where $0 \le z_{ij(1)}^{\bar{L}} \le z_{ij(2)}^{\bar{L}} \le z_{ij(3)}^{\bar{L}} \le z_{ij(4)}^{\bar{L}} \le 1, 0 \le z_{ij(1)}^{\bar{U}} \le z_{ij(2)}^{\bar{U}} \le z_{ij(3)}^{\bar{U}} \le z_{ij(4)}^{\bar{U}} \le 1, z_{ij(1)}^{\bar{U}} \le z_{ij(1)}^{\bar{L}}, a_{ij(4)}^{\bar{L}} \le z_{ij(4)}^{\bar{U}}, 0 \le h_{ij}^{\bar{L}} \le h_{ij}^{\bar{U}} \le 1.$

3. A signed based distance approach

The scheme proposed here involves the signed distance of each alternative from the ideal solution which is considered to be ordinary value of $x = 1$ as the attribute values considered here all lie in the unit interval $[0,1]$. Signed distance is used to calculate the preference order of the values for aggregation based on OWA operator and also the final ranking of alternatives is also done based on their signed distances. The motivating idea to determine the SBD of an IT2TrFN from at x = 1 on the y-axis, which is proposed by Chen [10], is as follows;

Proposition 1. *Suppose Z be an IT2TrFN defined on X and* $Z = [Z^{\bar{L}}, Z^{\bar{U}}] = [(z_1^{\bar{L}}, z_2^{\bar{L}}, z_3^{\bar{L}}, z_4^{\bar{L}}; h_{\bar{L}}^{Z}), (z_1^{\bar{U}}, z_2^{\bar{U}}, z_3^{\bar{U}}, z_4^{\bar{U}}; h_{\bar{U}}^{Z})]$. *The SBD of* $Z^{\bar{L}}$ *and* $Z^{\bar{L}}$ *from* 1_1 *is given below;*
$d(Z^{\bar{L}}, 1_1) = \frac{1}{4}((z_1^{\bar{L}} + z_2^{\bar{L}} + z_3^{\bar{L}} + z_4^{\bar{L}} - 4)$
$d(Z^{\bar{U}}, 1_1) = \frac{1}{4}((z_1^{\bar{U}} + z_2^{\bar{U}} + z_3^{\bar{U}} + z_4^{\bar{U}} - 4).$

Property 1. *Suppose Z be an IT2TrFN defined on X and* $Z = [Z^{\bar{L}}, Z^{\bar{U}}] = [(z_1^{\bar{L}}, z_2^{\bar{L}}, z_3^{\bar{L}}, z_4^{\bar{L}}; h_{\bar{L}}^{Z}), (z_1^{\bar{U}}, z_2^{\bar{U}}, z_3^{\bar{U}}, z_4^{\bar{U}}; h_{\bar{U}}^{Z})]$. *If both* $Z^{\bar{L}}$ *and* $Z^{\bar{U}}$ *are interval type 2 ordinary numbers then numerical value of the SBD in Proposition 1 is similar to hamming distance in the middle of comparable ordinary number (*$z^{\bar{L}}$ *and* $z^{\bar{U}}$*) and* 1_1.

Property 2. *Suppose Z be an IT2TrFN defined on X and* $Z = [Z^{\bar{L}}, Z^{\bar{U}}]$. $Z^{\bar{L}}$ *is placed at* 1_1 *iff* $d(Z^{\bar{L}}, 1_1) = 0$. $Z^{\bar{L}}$ *and* $Z^{\bar{U}}$ *both are placed at* 1_1 *iff* $d(Z^{\bar{U}}, 1_1) = 0$.

Proposition 2. *Suppose Z be an IT2TrFN defined on X and* $Z = [Z^{\bar{L}}, Z^{\bar{U}}] = [(z_1^{\bar{L}}, z_2^{\bar{L}}, z_3^{\bar{L}}, z_4^{\bar{L}}; h_{\bar{L}}^{Z}), (z_1^{\bar{U}}, z_2^{\bar{U}}, z_3^{\bar{U}}, z_4^{\bar{U}}; h_{\bar{U}}^{Z})]$, *where* $0 < h_{\bar{L}}^{Z} \le h_{\bar{U}}^{Z} \le 1$. *The SBD from* 1_1 *is given below;*
$d(Z, 1_1) = \frac{1}{8}\left[z_1^{\bar{L}} + z_2^{\bar{L}} + z_3^{\bar{L}} + z_4^{\bar{L}} + 4z_1^{\bar{U}} + 2z_2^{\bar{U}} + 2z_3^{\bar{U}} + 4z_4^{\bar{U}} + 3(z_2^{\bar{U}} + z_3^{\bar{U}} - z_1^{\bar{U}} - z_4^{\bar{U}})\frac{h_Z^{\bar{L}}}{h_Z^{\bar{U}}} - 16\right]$, *and when* $0 < h_{\bar{L}}^{Z} = h_{\bar{U}}^{Z} \le 1$, *then*
$d(Z, 1_1) = \frac{1}{8}\left[z_1^{\bar{L}} + z_2^{\bar{L}} + z_3^{\bar{L}} + z_4^{\bar{L}} + z_1^{\bar{U}} + 5z_2^{\bar{U}} + 5z_3^{\bar{U}} + z_4^{\bar{U}} - 16\right].$

Property 3. *Suppose Z be an IT2TrFN defined on X and* $Z = [(z_1^{\bar{L}}, z_2^{\bar{L}}, z_3^{\bar{L}}, z_4^{\bar{L}}; h_{\bar{L}}^{Z}), (z_1^{\bar{U}}, z_2^{\bar{U}}, z_3^{\bar{U}}, z_4^{\bar{U}}; h_{\bar{U}}^{Z})]$. *Z is placed at* 1_1 *iff* $d(Z, 1_1) = 0$, *where* $z_1^{\bar{L}} = z_2^{\bar{L}} = z_3^{\bar{L}} = z_4^{\bar{L}} = z_1^{\bar{U}} = z_2^{\bar{U}} = z_3^{\bar{U}} = z_4^{\bar{U}} = 1$.

Property 4. *Suppose M, N and O are three IT2TrFNs defined on X, where* $M = [(m_1^{\bar{L}}, m_2^{\bar{L}}, m_3^{\bar{L}}, m_4^{\bar{L}}; h_{\bar{L}}^{M}), (m_1^{\bar{U}}, m_2^{\bar{U}}, m_3^{\bar{U}}, m_4^{\bar{U}}; h_{\bar{U}}^{M})]$, $N = [(n_1^{\bar{L}}, n_2^{\bar{L}}, n_3^{\bar{L}}, n_4^{\bar{L}}; h_{\bar{L}}^{N}), (n_1^{\bar{U}}, n_2^{\bar{U}}, n_3^{\bar{U}}, n_4^{\bar{U}}; h_{\bar{U}}^{N})]$ *and* $O = [(1, 1, 1, 1; 1)(1, 1, 1, 1; 1)]$. *The IT2TrFNs M is near to IT2TrFNs O then N iff* $d(M, 1_1) > d(N, 1_1)$.

Definition 3.1. [10] Let Z_1 and Z_2 be two IT2TrFNs on X. By concept of "greater is superior", ranking of Z_1 and Z_2 according to signed based distance $d(Z_1, 1_1)$ and $d(Z_2, 1_1)$ is defined as below:
(1) $d(Z_1, 1_1) > d(Z_2, 1_1)$ iff $Z_1 > Z_2$.
(2) $d(Z_1, 1_1) < d(Z_2, 1_1)$ iff $Z_1 < Z_2$.
(3) $d(Z_1, 1_1) = d(Z_1, 1_1)$ iff $Z_1 = Z_2$.

4. GDM technique based on IT2TrFNs and incomplete weights

Firstly, this part defines a decision setting for GDM issues based on IT2TrFNs. This part then introduces a unified method for constructing a collective decision matrix with aggregating evaluative opinions of various decision-makers. For combining IT2TrFN data, an average hybrid technique is also suggested as an accumulating technique addressing the comparative understanding of degrees and the significance of decision-makers.

Next, this segment describes a valuable technique using an incorporated programming model to access the weights. In the end, a ranking procedure is offered in this segment, centered on a signed based distance approach and established for GDM; in addition, a proper algorithm is settled to solve a GDM problem within the perspective of IT2TrFNs.

4.1. Collective decision environment based on IT2TrFNs

Let $Z = \{z_1, z_2, ..., z_t\}$ be the alternative set which contain of t non-inferior decision alternatives. Every alternative is estimated on each alternative, and the evaluation is stated as an IT2TrFNs. To attain a ranking of every alternative found on the full evaluation of alternatives, GDM can also pick out the most favored alternative from Z.

Let $S = \{S_1, S_2, ..., S_q\}$ be the decision maker's set engaged in the decision-taking process. As said above, the ratings of alternative assessment on different alternatives are indicated as IT2TrFNs. For ease, it is proposed that the DMs use the IT2TrFPRs (Z_1, Z_2, Z_3, Z_4) to define the rating of alternatives over different alternatives.

By applying Z_1, Z_2, Z_3 and Z_4, every DM constructs positive IT2TrFNs to estimate alternatives for every alternative according to their judgments and experience. Assume an alternative value $Z_{ij}^{q} = [Z_{ij}^{q\bar{L}}, Z_{ij}^{q\bar{U}}]$ where q = 1, 2,..., Q is given by the mth DMs. The alternative value as IT2TrFNs estimates the alternatives Z_i w.r.t the alternative Z_j. $Z_{ij}^{q\bar{L}}$ represents the lower extreme and $Z_{ij}^{q\bar{U}}$ represents the upper extreme of IT2TrFNs Z_{ij}^{q}. Observe that,

$Z_{ij}^{q\bar{L}} = (z_{1ij}^{q\bar{L}}, z_{2ij}^{q\bar{L}}, z_{3ij}^{q\bar{L}}, z_{4ij}^{q\bar{L}}; h_{Z_{ij}}^{q\bar{L}}), Z_{ij}^{q\bar{U}} = (z_{1ij}^{q\bar{U}}, z_{2ij}^{q\bar{U}}, z_{3ij}^{q\bar{U}}, z_{4ij}^{q\bar{U}}; h_{Z_{ij}}^{q\bar{U}})$ and $Z_{ij}^{q\bar{L}} \subset Z_{ij}^{q\bar{U}}$

4.2. To collect decision matrix through hybrid average

Building an aggregation procedure to collect all the separate decision thinking to denote the general opinion is the main problem for evaluating the decision. To attain the evaluation of alternatives w.r.t. every alternative, by the motivation of XU [28], this research established a newly 'Hybrid Average' (HA) operation. This HA operation is used to accumulate IT2TrFNs data and make a collective decision matrix on signed-based distance ordered weighted average (OWA) operation.

4.2.1. Signed based distance OWA operation for aggregation technique

As different DMs have their own different opinions or individual expected ratings to every alternative for every alternative, it's vital to attain a group consent work that accumulates those ratings to develop an ordinary opinion. This research determines a technique to collect the different suggestions using signed-based distance OWA operation. It's a reordering procedure that reorders the whole opinions in descending order and weighting them. Corresponding to Proposition 2, the signed based distance of IT2TrFNs $Z_{ij}^{q} = [(z_{1ij}^{q\bar{L}}, z_{2ij}^{q\bar{L}}, z_{3ij}^{q\bar{L}}, z_{4ij}^{q\bar{L}}; h_{Z_{ij}}^{q\bar{L}}), (z_{1ij}^{q\bar{U}}, z_{2ij}^{q\bar{U}}, z_{3ij}^{q\bar{U}}, z_{4ij}^{q\bar{U}}; h_{Z_{ij}}^{q\bar{U}})]$ from 1_1 is calculated below:

$$d(Z_{ij}^{q}, 1_1) = \frac{1}{8}\left[z_{1ij}^{q\bar{L}} + z_{2ij}^{q\bar{L}} + z_{3ij}^{q\bar{L}} + z_{4ij}^{q\bar{L}} + 4z_{1ij}^{q\bar{U}} + 2z_{2ij}^{q\bar{U}} + 2z_{3ij}^{q\bar{U}} + 4z_{4ij}^{q\bar{U}} + 3(z_{2ij}^{q\bar{U}} + z_{3ij}^{q\bar{U}} - z_{1ij}^{q\bar{U}} - z_{4ij}^{q\bar{U}})\frac{h_{Z_{ij}}^{q\bar{L}}}{h_{Z_{ij}}^{q\bar{U}}} - 16\right] \tag{2}$$

Assume a more excellent value of $d(Z_{ij}^{q}, 1_1)$, because Z_{ij}^{q} and 1_1 are nearer in values. Hence the SBD signifies a valuable means of evaluating the IT2TrFN values.

One significant problem in OWA operation is establishing its related weights. A process established by Xu [29] was implemented to arise a weighting vector, that process is known as the normal distribution process. This process can minimalize the effect of partial arguments on averaging outcomes by allocating the small weights and then form the outcomes more realistic. Assume that the mean is μ_q of the set of 1,2,3,..., Q and the standard deviation is v_q of the set 1,2,3,..., Q., i.e.:

$$\mu_Q = \frac{1}{Q} \cdot \frac{Q(1+Q)}{2} = \frac{1+Q}{2} \tag{3}$$

$$v_Q = \sqrt{\frac{1}{Q}\sum_{q=1}^{Q}(q-\mu_Q)^2} \tag{4}$$

the OWA weight is represented as below according to normal distribution based on the equations (3) and (4):

$$\varsigma_q = \frac{e^{-\frac{(q-\mu_Q)^2}{2\cdot v_Q{}^2}}}{\sum_{q=1}^{Q} e^{-\frac{(q-\mu_Q)^2}{2\cdot v_Q{}^2}}},\ q = 1,2,...,Q, \tag{5}$$

where $\sum_{q=1}^{Q} \varsigma_q = 1$ and $\varsigma_q \in [0,1]$

Definition 4.1. Assume the set of decision makers $S = \{S_1, S_2, ..., S_q\}$. Consider an IT2TrFN $Z_{ij}^q = [(z_{1ij}^{q\bar{L}}, z_{2ij}^{q\bar{L}}, z_{3ij}^{q\bar{L}}, z_{4ij}^{q\bar{L}}; h_{Z_{ij}}^{q\bar{L}}), (z_{1ij}^{q\bar{U}}, z_{2ij}^{q\bar{U}}, z_{3ij}^{q\bar{U}}, z_{4ij}^{q\bar{U}}; h_{Z_{ij}}^{q\bar{U}})]$ represent the ranking of alternative $Z_i \in$ Z w.r.t alternatives Z_j offered by the DMs S_q. The signed based distance OWA operator on the Q IT2TrFNs with the related weight vector is described by:

$$\begin{aligned}\dot{Z}_{ij} &= \text{HA}(\tilde{Z}_{ij}^1, \tilde{Z}_{ij}^2, ..., \tilde{Z}_{ij}^Q) = (\varsigma_1 \cdot \tilde{Z}_{ij}^{\varrho(1)}) \oplus (\varsigma_2 \cdot \tilde{Z}_{ij}^{\varrho(2)}) \oplus ... \oplus (\varsigma_Q \cdot \tilde{Z}_{ij}^{\varrho(Q)}) \\ &= \Bigg[\Bigg(\sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{1ij}^{\varrho(q)\bar{L}}), \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{2ij}^{\varrho(q)\bar{L}}), \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{3ij}^{\varrho(q)\bar{L}}), \sum_{q=1}^{M}(\varsigma_q \times \tilde{z}_{4ij}^{\varrho(q)\bar{L}}); \min_q(h_{\tilde{Z}_{ij}}^{\varrho(q)\bar{L}}\Bigg), \\ &\Bigg(\sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{1ij}^{\varrho(q)\bar{U}}), \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{2ij}^{\varrho(q)\bar{U}}), \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{3ij}^{\varrho(q)\bar{U}}), \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{4ij}^{\varrho(q)\bar{U}}); \min_q(h_{\tilde{Z}_{ij}}^{\varrho(q)\bar{U}})\Bigg)\Bigg]\end{aligned}$$

where $(\varrho(1), \varrho(2), ..., \varrho(Q))$ describes the permutation of $(1,2,...Q)$ such that $\mathrm{d}(Z_{ij}^{\varrho(q-1)}, 1_1) \geq \mathrm{d}(Z_{ij}^{\varrho(q)}, 1_1)$ ∀q.

Indicate $\dot{z}_{1ij}^{\bar{L}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{1ij}^{\varrho(q)\bar{L}})$, $\dot{z}_{2ij}^{\bar{L}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{2ij}^{\varrho(q)\bar{L}})$, $\dot{z}_{3ij}^{\bar{L}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{3ij}^{\varrho(q)\bar{L}})$, $\dot{z}_{4ij}^{\bar{L}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{4ij}^{\varrho(q)\bar{L}})$, $\dot{z}_{1ij}^{\bar{U}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{1ij}^{\varrho(q)\bar{U}})$, $\dot{z}_{2ij}^{\bar{U}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{2ij}^{\varrho(q)\bar{U}})$, $\dot{z}_{3ij}^{\bar{U}} = \sum_{q=1}^{Q}(\varsigma_q \times \tilde{z}_{3ij}^{\varrho(q)\bar{U}})$, $\dot{z}_{4ij}^{\bar{U}} = \sum_{q=1}^{Q}(\varsigma_m \times \tilde{z}_{4ij}^{\varrho(q)\bar{U}})$, $h_{\dot{Z}_{ij}}^{\bar{L}} = h_{\tilde{Z}_{ij}}^{\varrho(q)\bar{L}}$, $h_{\dot{Z}_{ij}}^{\bar{U}} = h_{\tilde{Z}_{ij}}^{\varrho(q)\bar{U}}$.

The OWA estimation based on signed distance of alternative Z_i over alternative Z_j is as follows:

$$\dot{Z}_{ij} = [\dot{Z}_{ij}^{\bar{L}}, \dot{Z}_{ij}^{\bar{U}}] = [(\dot{z}_{1ij}^{\bar{L}}, \dot{z}_{2ij}^{\bar{L}}, \dot{z}_{3ij}^{\bar{L}}, \dot{z}_{4ij}^{\bar{L}}; h_{\dot{Z}_{ij}}^{\bar{L}}), (\dot{z}_{1ij}^{\bar{U}}, \dot{z}_{2ij}^{\bar{U}}, \dot{z}_{3ij}^{\bar{U}}, \dot{z}_{4ij}^{\bar{U}}; h_{\dot{Z}_{ij}}^{\bar{U}})]$$

where

$0 \leq \dot{z}_{1ij}^{\bar{L}} \leq \dot{z}_{2ij}^{\bar{L}} \leq \dot{z}_{3ij}^{\bar{L}} \leq \dot{z}_{4ij}^{\bar{L}} \leq 1$,
$0 \leq \dot{z}_{1ij}^{\bar{U}} \leq \dot{z}_{2ij}^{\bar{U}} \leq \dot{z}_{3ij}^{\bar{U}} \leq \dot{z}_{4ij}^{\bar{U}} \leq 1$,
$0 \leq h_{\dot{Z}_{ij}}^{\bar{U}} \leq h_{\dot{Z}_{ij}}^{\bar{U}} \leq 1$,
$\dot{z}_{1ij}^{\bar{U}} \leq \dot{z}_{1ij}^{\bar{L}}$, $\dot{z}_{4ij}^{\bar{L}} \leq \dot{z}_{4ij}^{\bar{U}}$, and $\dot{Z}_{ij}^{\bar{L}} \subset \dot{Z}_{ij}^{\bar{U}}$.

The collected decision matrix $\dot{\mathrm{F}}$ is presented as follows:

$$\dot{\mathrm{F}} = \begin{bmatrix} [\dot{Z}_{11}^{\bar{U}}, \dot{Z}_{11}^{\bar{L}}] & [\dot{Z}_{12}^{\bar{U}}, \dot{Z}_{12}^{\bar{L}}] & \cdots & [\dot{Z}_{1n}^{\bar{U}}, \dot{Z}_{1n}^{\bar{L}}] \\ [\dot{Z}_{21}^{\bar{U}}, \dot{Z}_{21}^{\bar{L}}] & [\dot{Z}_{22}^{\bar{U}}, \dot{Z}_{22}^{\bar{L}}] & \cdots & [\dot{Z}_{2n}^{\bar{U}}, \dot{Z}_{2n}^{\bar{L}}] \\ \vdots & \vdots & \ddots & \vdots \\ [\dot{Z}_{m1}^{\bar{U}}, \dot{Z}_{m1}^{\bar{L}}] & [\dot{Z}_{m2}^{\bar{U}}, \dot{Z}_{m2}^{\bar{L}}] & \cdots & [\dot{Z}_{mn}^{\bar{U}}, \dot{Z}_{mn}^{\bar{L}}] \end{bmatrix} \tag{6}$$

4.3. Collective decision matrix normalization

We can normalize the collective information about the values of alternatives. The suggested normalization procedure is given below:

Let $\dot{Z}_j^* = \max_i\ \dot{Z}_{1ij}^{\bar{U}}$ and $\dot{Z}_j^- = \min_i\ \dot{Z}_{4ij}^{\bar{L}}$, the converted conclusion of $\dot{Z}_{ij}$ is attained as:

$$\ddot{Z}_{ij} = [\ddot{Z}_{ij}^{\bar{L}}, \ddot{Z}_{ij}^{\bar{U}}] = [(\ddot{z}_{1ij}^{\bar{L}}, \ddot{z}_{2ij}^{\bar{L}}, \ddot{z}_{3ij}^{\bar{L}}, \ddot{z}_{4ij}^{\bar{L}}; h_{\ddot{Z}_{ij}}^{\bar{L}}), (\ddot{z}_{1ij}^{\bar{U}}, \ddot{z}_{2ij}^{\bar{U}}, \ddot{z}_{3ij}^{\bar{U}}, \ddot{z}_{4ij}^{\bar{U}}; h_{\ddot{Z}_{ij}}^{\bar{U}})]$$

$$= \begin{cases} \left[\left(\frac{\dot{z}_{1ij}^{\bar{L}}}{\dot{z}_j^+}, \frac{\dot{z}_{2ij}^{\bar{L}}}{\dot{z}_j^+}, \frac{\dot{z}_{3ij}^{\bar{L}}}{\dot{z}_j^+}, \frac{\dot{z}_{4ij}^{\bar{L}}}{\dot{z}_j^+}; h_{\dot{Z}_{ij}}^{\bar{L}}\right), \left(\frac{\dot{z}_{1ij}^{\bar{U}}}{\dot{z}_j^+}, \frac{\dot{z}_{2ij}^{\bar{U}}}{\dot{z}_j^+}, \frac{\dot{z}_{3ij}^{\bar{U}}}{\dot{z}_j^+}, \frac{\dot{z}_{4ij}^{\bar{U}}}{\dot{z}_j^+}; h_{\dot{Z}_{ij}}^{\bar{U}}\right)\right] & \text{if Z is admissible} \\ \left[\left(\frac{\dot{z}_j^-}{\dot{z}_{4ij}^{\bar{L}}}, \frac{\dot{z}_j^-}{\dot{z}_{3ij}^{\bar{L}}}, \frac{\dot{z}_j^-}{\dot{z}_{2ij}^{\bar{L}}}, \frac{\dot{z}_j^-}{\dot{z}_{1ij}^{\bar{L}}}; h_{\dot{Z}_{ij}}^{\bar{L}}\right), \left(\frac{\dot{z}_j^-}{\dot{z}_{4ij}^{\bar{U}}}, \frac{\dot{z}_j^-}{\dot{z}_{3ij}^{\bar{U}}}, \frac{\dot{z}_j^-}{\dot{z}_{2ij}^{\bar{U}}}, \frac{\dot{z}_j^-}{\dot{z}_{1ij}^{\bar{U}}}; h_{\dot{Z}_{ij}}^{\bar{U}}\right)\right] & \text{if Z is inadmissible} \end{cases} \tag{7}$$

collective decision matrix $\ddot{F}$ after normalization is presented as below:

$$\ddot{F} = \begin{bmatrix} [\ddot{Z}_{11}^{\bar{L}}, \ddot{Z}_{11}^{\bar{U}}] & [\ddot{Z}_{12}^{\bar{L}}, \ddot{Z}_{12}^{\bar{U}}] & \cdots & [\ddot{Z}_{1n}^{\bar{L}}, \ddot{Z}_{1n}^{\bar{U}}] \\ [\ddot{Z}_{21}^{\bar{L}}, \ddot{Z}_{21}^{\bar{U}}] & [\ddot{Z}_{22}^{\bar{L}}, \ddot{Z}_{22}^{\bar{U}}] & \cdots & [\ddot{Z}_{2n}^{\bar{L}}, \ddot{Z}_{2n}^{\bar{U}}] \\ \vdots & \vdots & \ddots & \vdots \\ [\ddot{Z}_{m1}^{\bar{L}}, \ddot{Z}_{m1}^{\bar{U}}] & [\ddot{Z}_{m2}^{\bar{L}}, \ddot{Z}_{m2}^{\bar{U}}] & \cdots & [\ddot{Z}_{mn}^{\bar{L}}, \ddot{Z}_{mn}^{\bar{U}}] \end{bmatrix} \tag{8}$$

4.4. An optimization model

For finding the weights of normalized matrix we will use a least deviation model [19] in this section. The suggested model is presented as below:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4}(a_{ijk} - 0.5 - log_{(81)}\pi_{ik} + log_{(81)}\pi_{j(5-k)})^2$$

$$[\text{mdl1}]\ s.t \begin{cases} 0 \le \sum_{i=1}^{n} \pi_{i1} \le 1, \\ \sum_{i=1}^{n} \pi_{i4} \ge 1, \\ 0 \le \pi_{i1} \le \pi_{i2} \le \pi_{i3} \le \pi_{i4} \le 1, \\ i = 1, 2, 3, \ldots.n \end{cases} \tag{9}$$

$\pi = (\pi_1, \pi_2, \ldots., \pi_n)$ be the set of weights.

4.5. Signed distances based unified programming model

Assuming the comparative significance values of different alternatives, the weighted normalization value of Z_{ij} is calculated as below:

$$\bar{Z}_{ij} = \pi_{ij}.\ddot{Z}_{ij} = [(\pi_{ij}^{\bar{L}}.\ddot{z}_{1ij}^{\bar{L}}, \pi_{ij}^{\bar{L}}.\ddot{z}_{2ij}^{\bar{L}}, \pi_{ij}^{\bar{L}}.\ddot{z}_{3ij}^{\bar{L}}, \pi_{ij}^{\bar{L}}.\ddot{z}_{4ij}^{\bar{L}}; h_{\ddot{Z}_{ij}}^{\bar{L}}), (\pi_{ij}^{\bar{U}}.\ddot{z}_{1ij}^{\bar{U}}, \pi_{ij}^{\bar{U}}.\ddot{z}_{2ij}^{\bar{U}}, \pi_{ij}^{\bar{U}}.\ddot{z}_{3ij}^{\bar{U}}, \pi_{ij}^{\bar{U}}.\ddot{z}_{4ij}^{\bar{U}}; h_{\ddot{A}_{ij}}^{\bar{U}})] \tag{10}$$

we can also denote as:

$$\bar{Z}_{ij} = [\bar{Z}_{ij}^{\bar{L}}, \bar{Z}_{ij}^{\bar{U}}] = [(\bar{z}_{1ij}^{\bar{L}}, \bar{z}_{2ij}^{\bar{L}}, \bar{z}_{3ij}^{\bar{L}}, \bar{z}_{4ij}^{\bar{L}}; h_{\bar{Z}_{ij}}^{\bar{L}}), (\bar{z}_{1ij}^{\bar{U}}, \bar{z}_{2ij}^{\bar{U}}, \bar{z}_{3ij}^{\bar{U}}, \bar{z}_{4ij}^{\bar{U}}; h_{\bar{Z}_{ij}}^{\bar{U}})]$$

further, the normalized weighted matrix can be defined as:

$$\ddot{F}_{\pi} = \begin{bmatrix} [\bar{A}_{11}^{\bar{L}}, \bar{A}_{11}^{\bar{U}}] & [\bar{A}_{12}^{\bar{L}}, \bar{A}_{12}^{\bar{U}}] & \cdots & [\bar{A}_{1n}^{\bar{L}}, \bar{A}_{1n}^{\bar{U}}] \\ [\bar{A}_{21}^{\bar{L}}, \bar{A}_{21}^{\bar{U}}] & [\bar{A}_{22}^{\bar{L}}, \bar{A}_{22}^{\bar{U}}] & \cdots & [\bar{A}_{2n}^{\bar{L}}, \bar{A}_{2n}^{\bar{U}}] \\ \vdots & \vdots & \ddots & \vdots \\ [\bar{A}_{m1}^{\bar{L}}, \bar{A}_{m1}^{\bar{U}}] & [\bar{A}_{m2}^{\bar{L}}, \bar{A}_{m2}^{\bar{U}}] & \cdots & [\bar{A}_{mn}^{\bar{L}}, \bar{A}_{mn}^{\bar{U}}] \end{bmatrix} \tag{11}$$

the SBD of every alternative from A can be computed by Proposition 2 as following;

$$d_{*i} = \sum_{j=1}^{n} d_{*i}(\bar{Z}_{ij}, 1_1)$$

$$= \sum_{j=1}^{n} \frac{1}{8}\left[\bar{z}_{1ij}^{\bar{L}} + \bar{z}_{2ij}^{\bar{L}} + \bar{z}_{3ij}^{\bar{L}} + \bar{z}_{4ij}^{\bar{L}} + 4\bar{z}_{1ij}^{\bar{U}} + 2\bar{z}_{2ij}^{\bar{U}} + 2\bar{z}_{3ij}^{\bar{U}} + 4\bar{z}_{4ij}^{\bar{U}} + 3(\bar{z}_{2ij}^{\bar{U}} + \bar{z}_{3ij}^{\bar{U}} - \bar{z}_{1ij}^{\bar{U}} - \bar{z}_{4ij}^{\bar{U}})\frac{h_{\bar{Z}_{ij}}^{\bar{L}}}{h_{\bar{Z}_{ij}}^{\bar{U}}} - 16\right]$$

here i = 1,2,...,q and normalized signed based distance is calculated as:

$$\bar{d}_{*i} = \frac{1}{2n}\sum_{j=1}^{n} d_{*i}(\bar{Z}_{ij}, 1_1)$$

$$= \frac{1}{16n} \sum_{j=1}^{n} \left[\bar{z}_{1ij}^{\bar{L}} + \bar{z}_{2ij}^{\bar{L}} + \bar{z}_{3ij}^{\bar{L}} + \bar{z}_{4ij}^{\bar{L}} + 4\bar{z}_{1ij}^{\bar{U}} + 2\bar{z}_{2ij}^{\bar{U}} + 2\bar{z}_{3ij}^{\bar{U}} + 4\bar{z}_{4ij}^{\bar{U}} + 3(\bar{z}_{2ij}^{\bar{U}} + \bar{z}_{3ij}^{\bar{U}} - \bar{z}_{1ij}^{\bar{U}} - \bar{z}_{4ij}^{\bar{U}}) \frac{h_{\bar{Z}_{ij}}^{\bar{L}}}{h_{\bar{Z}_{ij}}^{\bar{U}}} - 16 \right] \tag{12}$$

here i = 1,2,...,q.

Signed-distance $\bar{d}_{*i}$ denotes the proximity of the alternative Z_i to the ideal solution for i = 1,2,...,q (by Property 4). Using (2), alternatives may be ranked via their equivalent signed-based distance from the ideal solution. It shows that the alternative's preference order is ranked with the aid of ascending order of $\bar{d}_{*i}$, and also, the best choice is the minimum value of $\bar{d}_{*i}$.

5. Algorithm

The proposed group decision making approach based on IT2TrFPRs is an extension of simple fuzzy preference relations based approaches. Therefore it has a better representational power and has the ability to cope with the situations where it is difficult to identify the preference grades precisely. Also the hybrid averaging operator used in the proposed scheme considers not only the individual weight-age of the decision makers but also the ordered positions based on closeness to the ideal solution, which makes this approach more authentic than the existing techniques for combining the individual opinions into a collective opinion. Moreover, the technique has the capability of dealing with situations where there is no information on the priority weights. It is amongst the only few of the existing techniques where the aggregated opinion results in the IT2TrFPRs since it is often the case the aggregated opinion isn't is the fuzzy preference relation and therefore one has to use defuzzified aggregated opinion which is obviously not authentic because of its restrictions. Moreover, the proposed scheme involves the signed distance based ranking approach which is more effective than complex existing approaches of TOPSIS, VIKOR, Grey relation projection method all of which require an extensive amount of computational activity for ranking the alternatives. Also the technique of hybrid averaging employed here values not only the individual preference of the decision maker but also the ordered position based on closeness to the ideal solution, which makes the results more authentic, thereby adding to the superiority of this aggregation approach on the existing aggregation techniques.

Following is a precise set of steps for the proposed technique.

Step 1: Ask the DMs to give their preference information in the form IT2TrFPRs (using equation (1)).
Step 2: Calculate the signed-based distance of the IT2TrFPRs matrix using equation (2) and reorder the signed distances in descending order.
2.1: Find weight vector $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_m)$ by using OWA operation by standard distribution technique applying equation (5).
2.2: Construct a collective decision matrix $\dot{F}$ by applying the HA approach. (using equation (6)).
Step 3: Normalize the constructed matrix of step 2.2 by equation (7). After that, construct a new matrix $\ddot{F}$ as discussed in equation (8).
Step 4: Find the weights of the normalized matrix by using model provided in equation (9).
Step 5: Calculate the weighted normalization criterion value from equation (10) and construct F_{π} using equation (11).
Step 6: Establish normalized signed based distance of $\bar{d}_{*i}$ of every alternative from the ideal solution as in equation (12).
Step 7: Rank the order of each alternative by way of increasing $\bar{d}_{*i}$.

6. Numerical example

Consider a decision-making situation where it is needed to install a radar to monitor the air security. There are four locations $Z = \{Z_1, Z_2, Z_3, Z_4\}$ under consideration out of three are the hilltops $\{Z_1, Z_3, Z_4\}$ considered as an acceptable alternative, and one $\{Z_2\}$ is within the populated region which will be considered as an inadmissible alternative. Moreover, there are three decision-makers $S_m (m = 1, 2, 3)$ who will consider the preferences of locations in terms of interval type-2 trapezoidal fuzzy preference relations.

Step 1: Matrices based on IT2TrFPRs are constructed by the DMs.

$$Z^{(1)} = \begin{bmatrix} [(0.5, 0.5, 0.5, 0.5; 0.5), (0.5, 0.5, 0.5, 0.5; 0.5)] \\ [(0.4, 0.5, 0.7, 0.8; 0.4), (0.3, 0.4, 0.6, 0.9; 0.7)] \\ [(0.2, 0.3, 0.4, 0.6; 0.3), (0.1, 0.4, 0.5, 0.7; 0.4)] \\ [(0.3, 0.4, 0.6, 0.7; 0.6), (0.2, 0.3, 0.5, 0.8; 0.8)] \\ \\ [(0.2, 0.3, 0.5, 0.6; 0.3), (0.1, 0.4, 0.6, 0.7; 0.6)] \\ (0.5, 0.5, 0.5, 0.5; 0.5), (0.5, 0.5, 0.5, 0.5; 0.5) \\ [(0.4, 0.6, 0.7, 0.8; 0.7), (0.3, 0.5, 0.6, 0.9; 0.8)] \\ [(0.5, 0.6, 0.7, 0.8; 0.4), (0.4, 0.5, 0.8, 0.9; 0.7)] \\ \\ [(0.4, 0.6, 0.7, 0.8; 0.6), (0.3, 0.5, 0.6, 0.9; 0.7)] \\ [(0.2, 0.3, 0.4, 0.6; 0.2), (0.1, 0.4, 0.5, 0.7; 0.3)] \\ [(0.5, 0.5, 0.5, 0.5; 0.5), (0.5, 0.5, 0.5, 0.5; 0.5)] \\ [(0.3, 0.5, 0.6, 0.8; 0.2), (0.2, 0.4, 0.7, 0.9; 0.3)] \\ \\ [(0.3, 0.4, 0.6, 0.7; 0.2), (0.2, 0.5, 0.7, 0.8; 0.4)] \\ [(0.2, 0.3, 0.4, 0.5; 0.3), (0.1, 0.2, 0.5, 0.6; 0.6)] \\ [(0.2, 0.4, 0.5, 0.7; 0.7), (0.1, 0.3, 0.6, 0.8; 0.8)] \\ [(0.5, 0.5, 0.5, 0.5; 0.5), (0.5, 0.5, 0.5, 0.5; 0.5)] \end{bmatrix}$$

$$Z^{(2)}=\begin{bmatrix} [(0.5,0.5,0.5,0.5;0.5),(0.5,0.5,0.5,0.5;0.5)] \\ [(0.3,0.4,0.5,0.6;0.6),(0.2,0.3,0.4,0.7;0.8)] \\ [(0.2,0.5,0.6,0.7;0.2),(0.2,0.4,0.6,0.8;0.3)] \\ [(0.1,0.3,0.6,0.8;0.3),(0.1,0.3,0.5,0.9;0.6)] \\ \\ [(0.4,0.5,0.6,0.7;0.2),(0.3,0.6,0.7,0.8;0.4)] \\ [(0.5,0.5,0.5,0.5;0.5),(0.5,0.5,0.5,0.5;0.5)] \\ [(0.2,0.3,0.6,0.8;0.6),(0.1,0.3,0.5,0.9;0.7)] \\ [(0.3,0.4,0.6,0.7;0.5),(0.1,0.3,0.4,0.8;0.8)] \\ \\ [(0.3,0.4,0.5,0.8;0.7),(0.2,0.4,0.6,0.8;0.8)] \\ [(0.2,0.4,0.7,0.8;0.3),(0.1,0.5,0.7,0.9;0.4)] \\ [(0.5,0.5,0.5,0.5;0.5),(0.5,0.5,0.5,0.5;0.5)] \\ [(0.3,0.5,0.6,0.7;0.3),(0.2,0.4,0.5,0.8;0.4)] \\ \\ [(0.2,0.4,0.7,0.9;0.4),(0.1,0.5,0.7,0.9;0.7)] \\ [(0.3,0.4,0.6,0.7;0.2),(0.2,0.6,0.7,0.9;0.5)] \\ [(0.3,0.4,0.5,0.7;0.6),(0.2,0.5,0.6,0.8;0.7)] \\ [(0.5,0.5,0.5,0.5;0.5),(0.5,0.5,0.5,0.5;0.5)] \end{bmatrix}$$

$$Z^{(3)}=\begin{bmatrix} [(0.5,0.5,0.5,0.5;0.5)(0.5,0.5,0.5,0.5;0.5)] \\ [(0.5,0.6,0.7,0.8;0.1)(0.2,0.4,0.5,0.9;0.3) \\ [(0.2,0.3,0.5,0.7;0.8)(0.1,0.4,0.7,0.9;0.9)] \\ [(0.3,0.4,0.6,0.7;0.4)(0.2,0.5,0.6,0.8;0.7)] \\ \\ [(0.2,0.3,0.4,0.5;0.7)(0.1,0.5,0.6,0.8;0.9)] \\ [(0.5,0.5,0.5,0.5;0.5)(0.5,0.5,0.5,0.5;0.5)] \\ [(0.4,0.5,0.6,0.8;0.6)(0.2,0.5,0.6,0.9;0.8)] \\ [(0.2,0.4,0.6,0.7;0.2)(0.1,0.2,0.3,0.8;0.4)] \\ \\ [(0.3,0.5,0.7,0.8;0.1)(0.1,0.3,0.6,0.9;0.2)] \\ [(0.2,0.4,0.5,0.6;0.2)(0.1,0.4,0.5,0.8;0.4)] \\ [(0.5,0.5,0.5,0.5;0.5)(0.5,0.5,0.5,0.5;0.5)] \\ [(0.3,0.5,0.7,0.9;0.1)(0.1,0.2,0.4,0.9;0.2)] \\ \\ [(0.3,0.4,0.6,0.7;0.3)(0.2,0.4,0.5,0.8;0.6)] \\ [(0.3,0.4,0.6,0.8;0.6)(0.2,0.7,0.8,0.9;0.8)] \\ [(0.1,0.3,0.5,0.7;0.8)(0.1,0.6,0.8,0.9;0.9)] \\ [(0.5,0.5,0.5,0.5;0.5)(0.5,0.5,0.5,0.5;0.5)] \end{bmatrix}$$

Step 2: The signed distance of above matrices using equation (2) are calculated as:

$$d(Z^1_{13},1_1)=\frac{1}{8}\left[z^{1\bar{L}}_{113}+z^{1\bar{L}}_{213}+z^{1\bar{L}}_{313}+z^{1\bar{L}}_{413}+4z^{1\bar{U}}_{113}+2z^{1\bar{U}}_{213}+2z^{1\bar{U}}_{313}+4z^{1\bar{U}}_{413}+3(z^{1\bar{U}}_{213}+z^{1\bar{U}}_{313}-z^{1\bar{U}}_{113}-z^{1\bar{U}}_{413})\frac{h^{1\bar{L}}_{Z_{13}}}{h^{1\bar{U}}_{Z_{13}}}-16\right]$$

$$=\frac{1}{8}\left[0.2+0.3+0.4+0.6+4\times 0.1+2\times 0.4+2\times 0.5+4\times 0.7+3(0.4+0.5-0.1-0.7)\frac{0.3}{0.4}-16\right]$$

$= -1.159375$

same as above, we use $d(\mathrm{Z}^2_{13},1_1))=-1$, $d(\mathrm{Z}^3_{13},1_1)=-0.9792$ As $d(\mathrm{Z}^3_{13},1_1)>d(\mathrm{Z}^2_{13},1_1)>d(\mathrm{Z}^1_{13},1_1)$ so $\varrho(1)=3$, $\varrho(2)=2$, $\varrho(3)=1$. Hence, $\mathrm{Z}^{\varrho(1)}_{13}=\mathrm{Z}^3_{13}$, $\mathrm{Z}^{\varrho(2)}_{13}=\mathrm{Z}^2_{13}$, $\mathrm{Z}^{\varrho(3)}_{13}=\mathrm{Z}^1_{13}$.

Step 2.1: The OWA weight vector is $\varsigma=(\varsigma_1,\varsigma_2,\varsigma_3)=(0.2429,0.5142,0.2429)$.

Step 2.2: By using equation (6), collective decision matrix $\dot{F}$ is constructed as:

$$\dot{\mathrm{Z}}_{13}=\mathrm{HA}(\tilde{\mathrm{Z}}^1_{13},\tilde{\mathrm{Z}}^2_{13},\tilde{\mathrm{Z}}^3_{13})=(\varsigma_1\cdot\tilde{\mathrm{Z}}^{\varrho(1)}_{13})\oplus(\varsigma_2\cdot\tilde{\mathrm{Z}}^{\varrho(2)}_{13})\oplus(\varsigma_3\cdot\tilde{\mathrm{Z}}^{\varrho(3)}_{13})=\Bigg[\Bigg(\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{L}}_{113}),\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{L}}_{213}),\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{L}}_{313}),$$
$$\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{L}}_{413});\min_m(h^{\varrho(m)\bar{L}}_{\tilde{Z}_{13}})\Bigg),\Bigg(\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{U}}_{113}),\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{U}}_{213}),\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{U}}_{313}),\sum_{m=1}^{M}(\varsigma_m\times\tilde{z}^{\varrho(m)\bar{U}}_{413});\min_m(h^{\varrho(m)\bar{U}}_{\tilde{Z}_{13}})\Bigg)\Bigg]$$

$=[(0.2429\times 0.2+0.5142\times 0.2+0.2429\times 0.2, 0.2429\times 0.3+0.5142\times 0.5+0.2429\times 0.3, 0.2429\times 0.5+0.5142\times 0.6+0.2429\times 0.4, 0.2429\times 0.7+0.5142\times 0.7+0.2429\times 0.6; \min(0.8,0.2,0.3))(0.2429\times 0.1+0.5142\times 0.2+0.2429\times 0.1, 0.2429\times 0.4+0.5142\times 0.4+0.2429\times 0.4, 0.2429\times 0.7+0.5142\times 0.6+0.2429\times 0.5, 0.2429\times 0.9+0.5142\times 0.8+0.2429\times 0.7; \min(0.9,0.3,0.4)]$

$=[(0.2,0.40284,0.52713,0.67571;0.2)(0.15142,0.4,0.6,0.8;0.3)]$

Step 3: From Table 1, we know that $\dot{\mathrm{Z}}^+_1=0.84858, \dot{\mathrm{Z}}^-_2=0.1, \dot{\mathrm{Z}}^+_3=0.9, \dot{\mathrm{Z}}^+_4=0.84858$. By using Table 1 and (7) construct a normalized matrix. Here $\ddot{\mathrm{Z}}_{11}$ and $\ddot{\mathrm{Z}}_{12}$ are computed as:

Table 1
Aggregated rating of alternatives values in $\dot{F}$.

	$\dot{Z}^{\bar{L}}_{ij}$					$\dot{Z}^{\bar{U}}_{ij}$				
	$\dot{Z}^{\bar{L}}_{1ij}$	$\dot{Z}^{\bar{L}}_{2ij}$	$\dot{Z}^{\bar{L}}_{3ij}$	$\dot{Z}^{\bar{L}}_{4ij}$	$h^{\bar{L}}_{\dot{Z}_{ij}}$	$\dot{Z}^{\bar{U}}_{1ij}$	$\dot{Z}^{\bar{U}}_{2ij}$	$\dot{Z}^{\bar{U}}_{3ij}$	$\dot{Z}^{\bar{U}}_{4ij}$	$h^{\bar{U}}_{\dot{Z}_{ij}}$
$\dot{Z}_{11}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\dot{Z}_{12}$	0.42713	0.52713	0.65142	0.75142	0.1	0.22429	0.37571	0.5	0.85142	0.3
$\dot{Z}_{13}$	0.2	0.40284	0.52713	0.67571	0.2	0.15142	0.4	0.6	0.8	0.3
$\dot{Z}_{14}$	0.25142	0.37571	0.6	0.72429	0.3	0.17571	0.34858	0.52429	0.82429	0.6
$\dot{Z}_{21}$	0.24858	0.34858	0.47287	0.57287	0.2	0.14858	0.5	0.62429	0.77571	0.4
$\dot{Z}_{22}$	0.5	0.5	0.5	0.5	0.5	0.5	0.	0.5	0.5	0.5
$\dot{Z}_{23}$	0.35142	0.47571	0.62429	0.8	0.6	0.2	0.45142	0.57571	0.9	0.7
$\dot{Z}_{24}$	0.32429	0.44858	0.62429	0.72429	0.2	0.17287	0.32429	0.47287	0.82429	0.4
$\dot{Z}_{31}$	0.32429	0.47287	0.59716	0.8	0.1	0.2	0.4	0.6	0.84858	0.2
$\dot{Z}_{32}$	0.2	0.34858	0.49716	0.64858	0.2	0.1	0.42429	0.54858	0.77287	0.3
$\dot{Z}_{33}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\dot{Z}_{34}$	0.3	0.5	0.62429	0.77287	0.1	0.17571	0.35142	0.52429	0.84858	0.2
$\dot{Z}_{41}$	0.27571	0.4	0.62429	0.74858	0.2	0.17571	0.47571	0.65412	0.82429	0.4
$\dot{Z}_{42}$	0.27571	0.37571	0.55142	0.67571	0.2	0.17571	0.52713	0.67571	0.82713	0.5
$\dot{Z}_{43}$	0.22713	0.37571	0.5	0.7	0.6	0.15142	0.47571	0.64858	0.82429	0.7
$\dot{Z}_{44}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 2
Normalized decision matrix $\ddot{F}$.

	$\ddot{Z}^{\bar{L}}_{ij}$					$\ddot{Z}^{\bar{U}}_{ij}$				
	$\ddot{Z}^{\bar{L}}_{1ij}$	$\ddot{Z}^{\bar{L}}_{2ij}$	$\ddot{Z}^{\bar{L}}_{3ij}$	$\ddot{Z}^{\bar{L}}_{4ij}$	$h^{\bar{L}}_{\ddot{Z}_{ij}}$	$\ddot{Z}^{\bar{U}}_{1ij}$	$\ddot{Z}^{\bar{U}}_{2ij}$	$\ddot{Z}^{\bar{U}}_{3ij}$	$\ddot{Z}^{\bar{U}}_{4ij}$	$h^{\bar{U}}_{\ddot{Z}_{ij}}$
$\ddot{Z}_{11}$	0.5892	0.5892	0.5892	0.5892	0.5	0.5892	0.5892	0.5892	0.5892	0.5
$\ddot{Z}_{12}$	0.1331	0.1535	0.1897	0.2341	0.1	0.1171	0.2	0.2662	0.4459	0.3
$\ddot{Z}_{13}$	0.2222	0.4476	0.5857	0.7508	0.2	0.1682	0.4444	0.6667	0.8889	0.3
$\ddot{Z}_{14}$	0.2962	0.44275	0.7071	0.8535	0.3	0.2071	0.4108	0.6178	0.9714	0.6
$\ddot{Z}_{21}$	0.2929	0.4108	0.5572	0.6751	0.2	0.1751	0.5892	0.7357	0.9141	0.4
$\ddot{Z}_{22}$	0.2	0.2	0.2	0.2	0.5	0.2	0.2	0.2	0.2	0.5
$\ddot{Z}_{23}$	0.3905	0.5286	0.69365	0.8889	0.6	0.2222	0.5016	0.6397	1	0.7
$\ddot{Z}_{24}$	0.38126	0.5286	0.7357	0.8535	0.2	0.2037	0.38216	0.5572	0.9714	0.4
$\ddot{Z}_{31}$	0.38215	0.5572	0.7073	94275	0.1	0.2357	0.4714	0.7071	1	0.2
$\ddot{Z}_{32}$	0.1542	0.2011	0.2869	0.5	0.2	0.1294	0.1823	0.2357	1	0.3
$\ddot{Z}_{33}$	0.5556	0.5556	0.5556	0.5556	0.5	0.5556	0.5556	0.5556	0.5556	0.5
$\ddot{Z}_{34}$	0.3535	0.5892	0.7357	0.9108	0.1	0.2071	0.4141	0.6178	1	0.2
$\ddot{Z}_{41}$	0.3249	0.4714	0.7357	0.88215	0.2	0.2071	0.5606	0.7708	0.9714	0.4
$\ddot{Z}_{42}$	0.14799	0.1813	0.2662	0.3627	0.2	0.1209	0.14799	0.1897	0.5691	0.5
$\ddot{Z}_{43}$	0.2524	0.41745	0.5556	0.7778	0.6	0.1682	0.5286	0.7206	0.9159	0.7
$\ddot{Z}_{44}$	0.5892	0.5892	0.5892	0.5892	0.5	0.5892	0.5892	0.5892	0.5892	0.5

$$\ddot{Z}_{11} = \left[\left(\frac{\dot{Z}^{\bar{L}}_{111}}{\dot{Z}^{+}_{1}}, \frac{\dot{Z}^{\bar{L}}_{211}}{\dot{Z}^{+}_{1}}, \frac{\dot{Z}^{\bar{L}}_{311}}{\dot{Z}^{+}_{1}}, \frac{\dot{Z}^{\bar{L}}_{411}}{\dot{Z}^{+}_{1}}; h^{\bar{L}}_{\dot{Z}_{11}}\right), \left(\frac{\dot{Z}^{\bar{U}}_{111}}{\dot{Z}^{+}_{1}}, \frac{\dot{Z}^{\bar{U}}_{211}}{\dot{Z}^{+}_{1}}, \frac{\dot{Z}^{\bar{U}}_{311}}{\dot{Z}^{+}_{1}}, \frac{\dot{Z}^{\bar{U}}_{411}}{\dot{Z}^{+}_{1}}; 0.5\right)\right]$$
$$= \left[\left(\frac{0.5}{0.84858}, \frac{0.5}{0.84858}, \frac{0.5}{0.84858}, \frac{0.5}{0.84858}; 0.5\right), \left(\frac{0.5}{0.84858}, \frac{0.5}{0.84858}, \frac{0.5}{0.84858}, \frac{0.5}{0.84858}; 0.5\right)\right]$$
$$= [(0.5892, 0.5892, 0.5892, 0.5892; 0.5)(0.5892, 0.5892, 0.5892, 0.5892; 0.5)]$$
$$\ddot{Z}_{12} = \left[\left(\frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{L}}_{412}}, \frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{L}}_{312}}, \frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{L}}_{212}}, \frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{L}}_{112}}; h^{\bar{L}}_{\dot{Z}_{ij}}\right), \left(\frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{U}}_{412}}, \frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{U}}_{312}}, \frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{U}}_{212}}, \frac{\dot{Z}^{-}_{2}}{\dot{Z}^{\bar{U}}_{112}}; h^{\bar{U}}_{\dot{Z}_{ij}}\right)\right]$$
$$= \left[\left(\frac{0.1}{0.75142}, \frac{0.1}{0.65142}, \frac{0.1}{0.52713}, \frac{0.1}{0.42713}; 0.1\right), \left(\frac{0.1}{0.85142}, \frac{0.1}{0.5}, \frac{0.1}{0.37571}, \frac{0.1}{0.22429}; 0.3\right)\right]$$
$$= [(0.1331, 0.1535, 0.1897, 0.2341; 0.1)(0.1171, 0.2, 0.2662, 0.4459; 0.3)]$$

The normalized collective decision matrix denoted by $\ddot{F}$ is constructed in Table 2.

Step 4: Weights are constructed from mdl1 using LINGO.

$$\pi^{(\bar{L})}_{ij} = \begin{bmatrix} 0.09778976 & 0.1370183 & 0.2949429 & 0.4942435 \\ 0.6263346 & 0.6263346 & 0.7045276 & 0.8268057 \\ 0.1908924 & 0.2734641 & 0.3802469 & 0.6831259 \\ 0.08498323 & 0.09278762 & 0.2815136 & 0.5385731 \end{bmatrix}$$

$$\pi^{(\bar{U})}_{ij} = \begin{bmatrix} 0.1953395 & 0.1953395 & 0.3190868 & 0.5898702 \\ 0.3924413 & 0.5856498 & 0.6155769 & 0.7569166 \\ 0.2276497 & 0.2354403 & 0.3505917 & 0.7414761 \\ 0.1845695 & 0.3166867 & 0.4288888 & 0.6201069 \end{bmatrix}$$

Table 3
Normalization weighted matrix F_π by using programming model.

	$\bar{Z}^{\bar{L}}_{ij}$					$\bar{Z}^{\bar{U}}_{ij}$				
	$\bar{Z}^{\bar{L}}_{1ij}$	$\bar{Z}^{\bar{L}}_{2ij}$	$\bar{Z}^{\bar{L}}_{3ij}$	$\bar{Z}^{\bar{L}}_{4ij}$	$h^{\bar{L}}_{\bar{Z}_{ij}}$	$\bar{Z}^{\bar{U}}_{1ij}$	$\bar{Z}^{\bar{U}}_{2ij}$	$\bar{Z}^{\bar{U}}_{3ij}$	$\bar{Z}^{\bar{U}}_{4ij}$	$h^{\bar{U}}_{\bar{Z}_{ij}}$
$\bar{Z}_{11}$	0.0576	0.0576	0.0576	0.0576	0.5	0.1151	0.1151	0.1151	0.1151	0.5
$\bar{Z}_{12}$	0.0182	0.0210	0.02599	0.0321	0.1	0.0229	0.0391	0.05199	0.0871	0.3
$\bar{Z}_{13}$	0.0655	0.1320	0.1727	0.2214	0.2	0.0537	0.1418	0.2127	0.2836	0.3
$\bar{Z}_{14}$	0.1464	0.2188	0.3495	0.4218	0.3	0.1222	0.2434	0.3644	0.57299	0.6
$\bar{Z}_{21}$	0.18345	0.2573	0.34899	0.4228	0.2	0.0687	0.2312	0.2887	0.3587	0.4
$\bar{Z}_{22}$	0.1253	0.1253	0.1253	0.1253	0.5	0.1171	0.1171	0.1171	0.1171	0.5
$\bar{Z}_{23}$	0.2751	0.3724	0.4887	0.62625	0.6	0.1368	0.3087	0.3938	0.61557	0.7
$\bar{Z}_{24}$	0.3152	0.4370	0.6083	0.7057	0.2	0.1542	0.2892	0.4217	0.7353	0.4
$\bar{Z}_{31}$	0.0729	0.1064	0.1350	0.17996	0.1	0.0536	0.1073	0.16097	0.2276	0.2
$\bar{Z}_{32}$	0.0422	0.05499	0.0784	0.1367	0.2	0.0305	0.0429	0.0555	0.2354	0.3
$\bar{Z}_{33}$	0.2113	0.2113	0.2113	0.2113	0.5	0.1948	0.1948	0.1948	0.1948	0.5
$\bar{Z}_{34}$	0.2415	0.4025	0.5026	0.6222	0.1	0.1535	0.3070	0.4581	0.7415	0.2
$\bar{Z}_{41}$	0.0276	0.0401	0.0625	0.07496	0.2	0.0382	0.1035	0.1423	0.1793	0.4
$\bar{Z}_{42}$	0.0137	0.0168	0.0247	0.0336	0.2	0.0383	0.0469	0.0601	0.1802	0.5
$\bar{Z}_{43}$	0.07105	0.1175	0.1564	0.21896	0.6	0.0721	0.2267	0.3090	0.3928	0.7
$\bar{Z}_{44}$	0.3173	0.3173	0.3173	0.3173	0.5	0.3654	0.3654	0.3654	0.3654	0.5

Table 4
Signed based distances corresponding to every alternative.

Alternatives	$\bar{d}_{*i}(\bar{Z}_{ij}, 1_1)$
$\bar{Z}_1$	-0.8437791406
$\bar{Z}_2$	-0.7057604241
$\bar{Z}_3$	-0.7924367969
$\bar{Z}_4$	-0.8127628572

Step 5: The weighted normalized value of Z_{ij} can be achieved with (9), hence the weighted normalized matrix $\ddot{F}_\pi$ is constructed by (10). Take $\ddot{\bar{Z}}_{13} = [\bar{Z}^{\bar{L}}_{13}, \bar{Z}^{\bar{U}}_{13}]$ as an example:

$\bar{Z}_{13} = \pi_{13}.\ddot{Z}_{13}$

$= [(\pi^{\bar{L}}_{13} \times \ddot{z}^{\bar{L}}_{113}, \pi^{\bar{L}}_{13} \times \ddot{z}^{\bar{L}}_{213}, \pi^{\bar{L}}_{13} \times \ddot{z}^{\bar{L}}_{313}, \pi^{\bar{L}}_{13} \times \ddot{z}^{\bar{L}}_{413}; h^{\bar{L}}_{\ddot{A}_{13}}), (\pi^{\bar{U}}_{13} \times \ddot{z}^{\bar{U}}_{113}, \pi^{\bar{U}}_{13} \times \ddot{z}^{\bar{U}}_{213}, \pi^{\bar{U}}_{13} \times \ddot{z}^{\bar{U}}_{313}, \pi^{\bar{U}}_{13} \times \ddot{z}^{\bar{U}}_{413}; h^{\bar{U}}_{\ddot{Z}_{13}})]$

$= [(0.2949429 \times 0.2222, 0.2949429 \times 0.4476, 0.2949429 \times 0.5857, 0.2949429 \times 0.7508; 0.2), (0.3190868 \times 0.1682, 0.3190868 \times 0.4444, 0.3190868 \times 0.6667, 0.3190868 \times 0.8889; 0.3)]$

$= [(0.0655, 0.1320, 0.1727, 0.2214; 0.2)(0.0537, 0.1418, 0.2127, 0.2836; 0.3)]$

Step 6: In this step, normalized signed based distances are computed using Table 3 and (12), results are shown in Table 4.

Step 7: Hence, ranking of the alternatives is: $Z_2 \succ Z_3 \succ Z_4 \succ Z_1$. Thus Z_2 is the best choice.

6.1. Comparative analysis

A comparative study is carried out to observe the consequences of the proposed technique in conjunction with those from the other methods. In this comparative examination, we suppose the same information as used in the above problem and use the WA approach that is defined as follows;

$$\dot{Z}_{ij} = \mathrm{WA}(\tilde{Z}^1_{ij}, \tilde{Z}^2_{ij}, ..., \tilde{Z}^Q_{ij}) = (\upsilon_1 \cdot \tilde{Z}^1_{ij}) \oplus (\upsilon_2 \cdot \tilde{Z}^2_{ij}) \oplus ... \oplus (\upsilon_Q \cdot \tilde{Z}^Q_{ij})$$
$$= \Bigg[\Bigg(\sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{L}}_{1ij}), \sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{L}}_{2ij}), \sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{L}}_{3ij}), \sum_{q=1}^{M} (\upsilon_q \times \tilde{z}^{q\bar{L}}_{4ij}); \min_q (h^{q\bar{L}}_{\tilde{Z}_{ij}} \Bigg),$$
$$\Bigg(\sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{U}}_{1ij}), \sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{U}}_{2ij}), \sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{U}}_{3ij}), \sum_{q=1}^{Q} (\upsilon_q \times \tilde{z}^{q\bar{U}}_{4ij}); \min_q (h^{q\bar{U}}_{\tilde{Z}_{ij}}) \Bigg) \Bigg] \tag{13}$$

where $\upsilon = \{\upsilon_1, \upsilon_2, ..., \upsilon_Q\}$ be the weighted vector with $\upsilon_Q \in [0,1]$ and $\sum_{q=1}^{Q} \upsilon_q = 1$, here consider $\upsilon = \{\upsilon_1, \upsilon_2, \upsilon_3\} = \{0.22, 0.65, 0.13\}$, then by solving equation (13), we get Table 5.

After that normalize the values of Table 5 by equation (7) and then get Table 6, and then find the weights of normalized matrix and then by using equation (9) we obtain a new Table 7, apply equation (12) and find the distances of alternatives as shown in Table 8 and rank them. Hence, the ranking of the 4 alternatives is $Z_2 \succ Z_3 \succ Z_4 \succ Z_1$ with the best choice Z_2.

The proposed group decision making approach based on IT2TrFPRs is an extension of simple fuzzy preference relations based approaches. Therefore it has a better representational power and has the ability to cope with the situations where it is difficult to

Table 5
Aggregated rating of alternatives values in $\dot{F}$.

	$\dot{Z}^{\bar{L}}_{ij}$					$\dot{Z}^{\bar{U}}_{ij}$				
	$\dot{Z}^{\bar{L}}_{1ij}$	$\dot{Z}^{\bar{L}}_{2ij}$	$\dot{Z}^{\bar{L}}_{3ij}$	$\dot{Z}^{\bar{L}}_{4ij}$	$h^{\bar{L}}_{\dot{Z}_{ij}}$	$\dot{Z}^{\bar{U}}_{1ij}$	$\dot{Z}^{\bar{U}}_{2ij}$	$\dot{Z}^{\bar{U}}_{3ij}$	$\dot{Z}^{\bar{U}}_{4ij}$	$h^{\bar{U}}_{\dot{Z}_{ij}}$
$\dot{Z}_{11}$	0.5	0.5	0.5	0.5	0.5	O.5	0.5	0.5	0.5	0.5
$\dot{Z}_{12}$	0.384	0.448	0.57	0.67	0.1	0.222	0.335	0.457	0.77	0.3
$\dot{Z}_{13}$	0.2	0.43	0.543	0.678	0.2	0.165	0.4	0.591	0.791	0.3
$\dot{Z}_{14}$	0.17	0.335	0.6	0.765	0.3	0.135	0.326	0.513	0.865	0.6
$\dot{Z}_{21}$	0.33	0.43	0.552	0.652	0.2	0.23	0.543	0.665	0.778	0.4
$\dot{Z}_{22}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\dot{Z}_{23}$	0.27	0.392	0.622	0.8	0.6	0.157	0.37	0.535	0.9	0.7
$\dot{Z}_{24}$	0.331	0.444	0.622	0.722	0.2	0.166	0.331	0.475	0.822	0.4
$\dot{Z}_{31}$	0.322	0.457	0.57	0.8	0.1	0.209	0.409	0.6	0.835	0.2
$\dot{Z}_{32}$	0.2	0.378	0.608	0.73	0.2	0.1	0.465	0.63	0.843	0.3
$\dot{Z}_{33}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\dot{Z}_{34}$	0.3	0.5	0.613	0.748	0.1	0.187	0.374	0.531	0.835	0.2
$\dot{Z}_{41}$	0.235	0.4	0.655	0.83	0.2	0.135	0.487	0.674	0.865	0.4
$\dot{Z}_{42}$	0.278	0.378	0.556	0.669	0.2	0.178	0.525	0.669	0.834	0.5
$\dot{Z}_{43}$	0.252	0.387	0.5	0.7	0.6	0.165	0.469	0.626	0.813	0.7
$\dot{Z}_{44}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 6
Normalized decision matrix $\ddot{F}$.

	$\ddot{Z}^{\bar{L}}_{ij}$					$\ddot{Z}^{\bar{U}}_{ij}$				
	$\ddot{Z}^{\bar{L}}_{1ij}$	$\ddot{Z}^{\bar{L}}_{2ij}$	$\ddot{Z}^{\bar{L}}_{3ij}$	$\ddot{Z}^{\bar{L}}_{4ij}$	$h^{\bar{L}}_{\ddot{Z}_{ij}}$	$\ddot{Z}^{\bar{U}}_{1ij}$	$\ddot{Z}^{\bar{U}}_{2ij}$	$\ddot{Z}^{\bar{U}}_{3ij}$	$\ddot{Z}^{\bar{U}}_{4ij}$	$h^{\bar{U}}_{\ddot{Z}_{ij}}$
$\ddot{Z}_{11}$	0.5780	0.5780	0.5780	0.5780	0.5	0.5780	0.5780	0.5780	0.5780	0.5
$\ddot{Z}_{12}$	0.1492	0.1754	0.2232	0.2604	0.1	0.1299	0.2188	0.2985	0.4504	0.3
$\ddot{Z}_{13}$	0.222	0.4778	0.6033	0.7533	0.2	0.1833	0.444	0.6567	0.8789	0.3
$\ddot{Z}_{14}$	0.1965	0.3873	0.6936	0.8843	0.3	0.1561	0.3769	0.5931	1	0.6
$\ddot{Z}_{21}$	0.3815	0.4971	0.6381	0.7537	0.2	0.2659	0.6277	0.7688	0.8994	0.4
$\ddot{Z}_{22}$	0.2	0.2	0.2	0.2	0.5	0.2	0.2	0.2	0.2	0.5
$\ddot{Z}_{23}$	0.3	0.435	0.6911	0.8889	0.6	0.1744	0.4111	0.5944	1	0.7
$\ddot{Z}_{24}$	0.3826	0.5133	0.7191	0.8347	0.2	0.1919	0.3826	0.5491	0.9503	0.4
$\ddot{Z}_{31}$	0.391	0.5283	0.6589	0.9248	0.1	0.2416	0.4728	0.6936	0.9653	0.2
$\ddot{Z}_{32}$	0.1369	0.1645	0.2645	0.5	0.2	0.1186	0.1587	0.2150	1	0.3
$\ddot{Z}_{33}$	0.555	0.555	0.555	0.555	0.5	0.555	0.555	0.555	0.555	0.5
$\ddot{Z}_{34}$	0.3468	0.5780	0.7086	0.8647	0.1	0.2612	0.4324	0.6139	0.9653	0.2
$\ddot{Z}_{41}$	0.2717	0.4624	0.7572	0.9595	0.2	0.1561	0.5630	0.7791	1	0.4
$\ddot{Z}_{42}$	0.1495	0.1799	0.2645	0.3597	0.2	0.1199	0.1495	0.1905	0.5618	0.5
$\ddot{Z}_{43}$	0.2913	0.4474	0.5780	0.8092	0.6	0.19075	0.5421	0.7237	0.9398	0.7
$\ddot{Z}_{44}$	0.5780	0.5780	0.5780	0.5780	0.5	0.5780	0.5780	0.5780	0.5780	0.5

Table 7
Normalization weighted matrix F_{π} by using programming model.

	$Z^{\bar{L}}_{ij}$					$\bar{Z}^{\bar{U}}_{ij}$				
	$\bar{Z}^{\bar{L}}_{1ij}$	$\bar{Z}^{\bar{L}}_{2ij}$	$\bar{Z}^{\bar{L}}_{3ij}$	$\bar{Z}^{\bar{L}}_{4ij}$	$h^{\bar{L}}_{\bar{Z}_{ij}}$	$\bar{Z}^{\bar{U}}_{1ij}$	$\bar{Z}^{\bar{U}}_{2ij}$	$\bar{Z}^{\bar{U}}_{3ij}$	$\bar{Z}^{\bar{U}}_{4ij}$	$h^{\bar{U}}_{\bar{Z}_{ij}}$
$\bar{Z}_{11}$	0.0473	0.0473	0.0473	0.0473	0.5	0.1107	0.1107	0.1107	0.1107	0.5
$\bar{Z}_{12}$	0.0236	0.0277	0.0353	0.0412	0.1	0.0249	0.0419	0.0571	0.0862	0.3
$\bar{Z}_{13}$	0.0659	0.1418	0.1790	0.2235	0.2	0.0577	0.1399	0.2069	0.2769	0.3
$\bar{Z}_{14}$	0.0908	0.1789	0.3205	0.4086	0.3	0.0915	0.2209	0.3477	0.5862	0.6
$\bar{Z}_{21}$	0.2221	0.2894	0.3715	0.4388	0.2	0.1054	0.2488	0.3048	0.3565	0.4
$\bar{Z}_{22}$	0.1164	0.1164	0.1164	0.1164	0.5	0.1158	0.1158	0.1158	0.1158	0.5
$\bar{Z}_{23}$	0.2005	0.2907	0.4619	0.5941	0.6	0.1078	0.2540	0.3673	0.6179	0.7
$\bar{Z}_{24}$	0.2896	0.3886	0.5444	0.6319	0.2	0.1441	0.2873	0.4123	0.7136	0.7
$\bar{Z}_{31}$	0.0799	0.1080	0.1347	0.1891	0.1	0.0559	0.1094	0.1605	0.2234	0.2
$\bar{Z}_{32}$	0.0357	0.0429	0.0689	0.1303	0.2	0.0297	0.0397	0.0538	0.2502	0.3
$\bar{Z}_{33}$	0.1977	0.1977	0.1977	0.1977	0.5	0.2018	0.2018	0.2018	0.2018	0.5
$\bar{Z}_{34}$	0.2138	0.3564	0.4369	0.5332	0.1	0.1909	0.3160	0.4486	0.7055	0.2
$\bar{Z}_{41}$	0.0357	0.0607	0.0995	0.1260	0.2	0.0282	0.1017	0.1407	0.1806	0.4
$\bar{Z}_{42}$	0.0229	0.0276	0.0407	0.0553	0.2	0.0399	0.0497	0.0634	0.1869	0.5
$\bar{Z}_{43}$	0.0929	0.1426	0.1843	0.2579	0.6	0.0838	0.2383	0.3197	0.4131	0.7
$\bar{Z}_{44}$	0.3171	0.3171	0.3171	0.3171	0.5	0.3653	0.3653	0.3653	0.3653	0.5

Table 8
Signed based distances corresponding to every alternative.

Alternatives	$\bar{d}_{*i}(\bar{Z}_{ij}, 1_1)$
$\bar{Z}_1$	-0.849890625
$\bar{Z}_2$	-0.7149188594
$\bar{Z}_3$	-0.7954242188
$\bar{Z}_4$	-0.80088625

identify the preference grades precisely. Also the hybrid averaging operator used in the proposed scheme considers not only the individual weight-age of the decision makers but also the ordered positions based on closeness to the ideal solution, which makes this approach more authentic than the existing techniques for combining the individual opinions into a collective opinion. Moreover, the technique has the capability of dealing with situations where there is no information on the priority weights. It is amongst the only few of the existing techniques where the aggregated opinion results in the IT2TrFPRs since it is often the case the aggregated opinion isn't is the fuzzy preference relation and therefore one has to use defuzzified aggregated opinion which is obviously not authentic because of its restrictions. Moreover, the proposed scheme involves the signed distance based ranking approach which is more effective than complex existing approach of weighted aggregation which requires an extensive amount of computational activity for ranking the alternatives. Also the technique of hybrid averaging employed here values not only the individual preference of the decision maker but also the ordered position based on closeness to the ideal solution, which makes the results more authentic, thereby adding to the superiority of this aggregation approach on the existing weighted aggregation approach.

7. Conclusion

This research suggested an algorithmic procedure for assessing alternatives in GDM issues by which weights are unknown. This technique presents rating values as FPR, which can be extracted in IT2TrFNs. The HA approach can collect all individual rankings to set up the collective decision matrix. However, the aggregation technique utilized signed-based distance OWA operations. The precept of the suggested GDM approach is that the selected alternative would have to most extensive SBD from 1_1. Consequently, the SBD of a IT2TrFN from 1_1 turned into used as the idea of determining the concern of alternatives. A least-deviation model is suggested to evaluate the top-rated weights of the alternatives. Lastly, an executive technique of the suggested approach was explained by its request for the problem of alternatives. In precise, this research has recorded the background of IT2TrFNs in multiple groups of DMs environment. More specifically, a technique becomes advanced for producing an SBD method for weight assessment and decision evaluation.

CRediT authorship contribution statement

Muhammad Touqeer: Writing – review & editing, Supervision, Methodology, Formal analysis. **Rabia Irfan:** Writing – review & editing, Writing – original draft, Software, Methodology. **Hajrah Khatoon:** Methodology, Formal analysis, Data curation. **Shalan Alkarni:** Writing – original draft, Visualization, Validation, Funding acquisition. **Awais Ahmed:** Writing – original draft, Software. **Abdullah Mohamed:** Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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