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ORIGINAL PAPER

Comparative Study of the Fractional-Order Crime System as a Social Epidemic of the USA Scenario

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Abstract
Fractional derivatives are considered significant mathematical tools to design the fractional-order models of real phenomena. In this investigation, we are going to design and compare the non-integer models of the crime system by using three fractional-order operators called Atangana-Baleanu-Caputo, Caputo, and Caputo-Fabrizio derivatives for the first time. We use the real initial conditions for the subgroups of USA. To get the approximate solutions of the suggested models some numerical methods are derived. To see the performance of the numerical methods different values of the fractional orders are considered. The differences between the solutions under the used operators for each state variable are provided through some figures.

Keywords Numerical method · Fractional calculus · Fractional derivatives · Crime model · Lagrange interpolation

Introduction

Crime is considered a social problem, a problem as defined by society, such as homelessness, drug abuse, etc, and can be found across the globe. The cost to control crime worldwide was estimated as \$ 360 billion in 1997, of the total, 62 percent was spent on public policing, 3 percent on prosecutions, 18 percent on courts, and 17 percent on prisons [1]. The U.S. alone spends more on criminal justice than any other nation, investing approximately \$ 8 billion per year on state and federal prisons that currently hold 2.24 million Americans [2]. Study shows that state spending on prisons grew at six times the rate of state spending on higher education in the last two-decade [3]. Moreover, in the past decade, the rise in incarceration in the United States has been experienced disproportionately by minorities, particularly young African American men with low levels of education [4]. The minority population in the US has not only been facing disproportionately incarceration but recent studies show that the COVID-19 pandemic has a similar effect on minorities; COVID-19 cases and deaths are

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more in minority populations despite their population being low [5]. There are connections between poverty, crime, and diseases.

There are several studies with integer-order mathematical models of crime, based on ordinary differential equations (ODEs) [2, 6]. In [6] A nonlinear mathematical model was proposed to study the dynamics of extremism governed by different factors, and it was suggested that if an appropriate level of government efforts is applied to extremists, the spread of their ideology can be controlled in the general population. In the other study, [7] the effect of the police force in controlling crime in a society with variable population size was analyzed through the proposed model. Similarly, the models based on partial differential equations (PDEs) [8], have been proposed and analyzed in recent years. Where perturbative approach was applied to find amplitude equations that govern the development of crime hot spot patterns in the system in both the one-dimensional (1D) and two-dimensional (2D) cases. However, these models do not inherit non local property, which depicts behavior changes due to contact with criminals for a long period. To overcome this problem, very few the fractional order mathematical models of crime have also proposed [9, 10]. However, no any fractional order mathematical model has been yet studied considering certain important relationships and behavioral changes of the individuals. We have fractional order mathematical model of crime, extension of model studied by [2].

The main model assumption is that crime can spread by means of social epidemic transmission. In particular, it is assumed that crime spreads by social contact and this contact depends on the social environment that includes the criminal justice system. Social environment factors to consider are the penitentiary system, judges, police officers, media, and economical variables which have different degrees of influence over the probability of transmission of crime among individuals. Spatial effects are not considered explicitly in that model, however different classes for individuals with regard to the crime and justice system were taken into account. Using a population based approach of epidemiological type, the population was divided into six groups: susceptibles, free criminals, criminals arrested and in jail, convicted criminals in prison, judges and police officers. The resulting mathematical model of crime is a system of six nonlinear ordinary differential equations, which is analyzed to find the equilibrium and their stability, including the threshold parameter R0 for the extinction of criminality. Numerical simulations are also performed that support the established theoretical results. In addition, a sensitivity analysis is presented to determine the importance of specific model parameters in decreasing criminality.

Fractional order differential and integral operators are more useful, and many recent studies has shown their importance due to their effective use in the modelling of real-world phenoms with complicated dynamics [11–16]. It is found that non-integer order models show can define the dynamical behaviours more perfectly than integer order systems [17–19]. The is because the memory property of fractional-order operator can identify complex structure of numerous real-world problems [20–23]. Due to this reason, recently, fractional calculus has been recognized as a hot area for modelling and analyzing dynamical systems by disordered characteristics [24–30]. In this work, for the first attempt we compare the behaviour of the numerical solutions under three different fractional operators under the real initioal conditions. We consider the crime system with three fractional derivatives, Atangana-Baleanu-Caputo, Caputo and Caputo-Fabrizio considering the integer order of crime system is as follows [31]:

Table 1 The parameters description [27]

Parameter	Description
μ	The birth/death rate
β_1	The social crime transition rate from criminals to susceptible persons
β_2	The rate at which criminals are arrested by police and sent to jail
β_3	The rate at which the individuals in jail get convicted by an uncorrupted judge
β_4	The crime contagious rate from criminals to police officers
β_5	The rate at which criminals corrupt police officers
β_6	The rate at which criminals corrupt judges
a_1	The growth rates for judges
a_2	The growth rates for police officers
r_1	The rate at which individuals in jail leave to the susceptible class
r_2	The rate that accounts for the flow of individuals from the convicted class
to the susceptible one	

$$
\begin{cases}
\dfrac{dS(t)}{dt} = \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S, \\
\dfrac{dC(t)}{dt} = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\
\dfrac{dC_v(t)}{dt} = \beta 3 J J_a - r_2 C_v - \mu C_v, \\
\dfrac{dJ_a(t)}{dt} = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\
\dfrac{dJ(t)}{dt} = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\
\dfrac{dP(t)}{dt} = a_2 S - \beta_4 PC - \mu P, \\
S(0) = S_0, C(0) = C_0, C_v(0) = C_{v0}, J_a(0) = J_{a0}, J(0) = J_0, P(0) = P_0,
\end{cases}
\tag{1}
$$

In the above model, the population is divided into six main groups based on the criminal activity of the sub-populations: susceptible individuals (S), free criminals (C), criminals arrested in jail (J_a), convicted criminals (C_v), judges (J) and police officers (P) (Table 1).

We present the rest of the study as follows. In "Basic Definitions" section, essential definitions are provided. New fractional order models can be seen in "Fractional Order Models of the Crime System" section. Moreover, numerical algorithms to solve the suggested systems are provided in "Numerical Methods" section. Numerical results of the proposed models under practicing different values of fractional orders are supplied in "Numerical Results" section. Results and discussion can be found in "Results and Discussion" section. Finally, the conclusion of the current investigation can be observed in the last section.

Basic Definitions

Now, we present some needful definitions of the fractional derivatives. [32].

Definition 2.1 Let $\alpha \in \mathbb{R}^+$. The Mittag-Leffler function is a significant function that has a far-reaching application in the non-integer calculus. The Mittag-Leffler function arises naturally in the solution of fractional integral equations, and especially in the study of the fractional generalization of the kinetic equation, random walks, Lévy flights, and so-called superdiffusive transport. The ordinary and generalized Mittag-Leffler functions interpolate between a purely exponential law and power-like behavior of phenomena governed by ordinary kinetic equations and their fractional counterparts. The standard definition of Mittag-Leffler in order α is given as follows:

$$E_\alpha(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(j\alpha + 1)},$$

Definition 2.2 Some definitions regarding fractional-order derivatives have been introduced. One of the most famous and significant of them is the Caputo derivative suitable for the modeling of real-world phenomena. The following relation is showing its formulation [32]:

$$^{\mathrm{C}}\mathcal{D}_t^\alpha(\Upsilon(t)) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \Upsilon'(\tau)(t-\tau)^{-\alpha} \mathrm{d}\tau, \quad 0 < \alpha \leq 1. \tag{2}$$

Definition 2.3 The Caputo-Fabrizio derivative is stated by [32]

$$^{\mathrm{CF}}\mathcal{D}_t^\alpha(\Upsilon(t)) = \frac{(2-\alpha)M(\alpha)}{2(1-\alpha)} \int_0^t \Upsilon'(\tau) \exp\left[\frac{-\alpha(t-\tau)}{1-\alpha}\right] \mathrm{d}\tau, \quad 0 < \alpha \leq 1, \tag{3}$$

where $M(\alpha) = \dfrac{2}{(2-\alpha)}$.

This non-integer derivative has been classified as a nonlocal fractional operator carrying the exponential kernel. The Caputo-Fabrizio derivative enjoys the non-singularity property which has been regarded as a substantial advantage, especially from the approximation point of view.

Definition 2.4 [32] Atangana-Baleanu-Caputo derivative is stated as

$$^{\mathrm{ABC}}\mathcal{D}_t^\alpha(\Upsilon(t)) = \frac{AB(\alpha)}{1-\alpha} \int_0^t \Upsilon'(\tau) E_\alpha\left[-\alpha \frac{(t-\tau)^\alpha}{1-\alpha}\right] \mathrm{d}\tau, \quad 0 < \alpha \leq 1, \tag{4}$$

where $AB(\alpha) = 1 - \alpha + \dfrac{\alpha}{\gamma(\alpha)}$.

The Mittag-Leffler function plays an important role in the fractional. Atangana and Baleanu were motivated by this concern to change the exponential kernel in the Caputo-Fabrizio fractional kernel to the Mittag-Leffler kernel.

In the next section, we design the non-integer models of the system (1) using (2), (3), and (4) operators.

Fractional Order Models of the Crime System

By replacing the fractional derivatives defined in the previous section for the system (1) we have the following relations. In fact, using the Caputo derivative we can obtain

$$\begin{cases} {}^{\mathrm{C}}_{0}\mathcal{D}^{\alpha}_{t} S(t) = \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S, \\ {}^{\mathrm{C}}_{0}\mathcal{D}^{\alpha}_{t} C(t) = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\ {}^{\mathrm{C}}_{0}\mathcal{D}^{\alpha}_{t} C_v(t) = \beta 3 J J_a - r_2 C_v - \mu C_v, \\ {}^{\mathrm{C}}_{0}\mathcal{D}^{\alpha}_{t} J_a(t) = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\ {}^{\mathrm{C}}_{0}\mathcal{D}^{\alpha}_{t} J(t) = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\ {}^{\mathrm{C}}_{0}\mathcal{D}^{\alpha}_{t} P(t) = a_2 S - \beta_4 PC - \mu P, \end{cases} \tag{5}$$

Regarding the Caputo-Fabrizio sense the next system can be obtained

$$\begin{cases} {}^{\mathrm{CF}}_{0}\mathcal{D}^{\alpha}_{t} S(t) = \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S, \\ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\alpha}_{t} C(t) = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\alpha}_{t} C_v(t) = \beta 3 J J_a - r_2 C_v - \mu C_v, \\ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\alpha}_{t} J_a(t) = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\alpha}_{t} J(t) = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\ {}^{\mathrm{CF}}_{0}\mathcal{D}^{\alpha}_{t} P(t) = a_2 S - \beta_4 PC - \mu P, \end{cases} \tag{6}$$

Also, by using the Atangana-Baleanu-Caputo derivative we get:

$$\begin{cases} {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\alpha}_{t} S(t) = \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S, \\ {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\alpha}_{t} C(t) = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\ {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\alpha}_{t} C_v(t) = \beta 3 J J_a - r_2 C_v - \mu C_v, \\ {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\alpha}_{t} J_a(t) = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\ {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\alpha}_{t} J(t) = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\ {}^{\mathrm{ABC}}_{0}\mathcal{D}^{\alpha}_{t} P(t) = a_2 S - \beta_4 PC - \mu P, \end{cases} \tag{7}$$

Numerical Methods

This part of the study is dedicated to deriving some numerical approaches to solve the fractional orders (5), (6) and (7).

Numerical Technique in Caputo Frame

Suppose we have the following equation involving the Caputo derivative

$${}^{C}_{0}\mathscr{D}^{\alpha}_{t} y(t) = f(t, y(t)), \tag{8}$$

The relation (8) can be reformulated as a fractional integral problem:

$$y(t) - y(0) = \frac{1}{\Gamma(\alpha)} \sum_{m=0}^{n} \int_0^t f(\theta, y(\theta))(t-\theta)^{\alpha-1} \mathrm{d}\theta, \tag{9}$$

Eq. (9) can be written as

$$y(t_{n+1}) - y(0) = \frac{1}{\Gamma(\alpha)} \sum_{m=0}^{n} \int_{t_m}^{t_{m+1}} f(\theta, y(\theta))(t-\theta)^{\alpha(t_{n+1})-1} \mathrm{d}\theta \tag{10}$$

Using the two-step Lagrange polynomial interpolation, $f(\theta, y(\theta))$ in $[t_k, t_{k+1}]$ can be approximated as

$$P_k(\theta) \simeq \frac{f(t_m, y_m)}{h}(\theta - t_{m-1}) - \frac{f(t_{m-1}, y_{m-1})}{h}(\theta - t_m), \tag{11}$$

Using (10) and (11) we have

$$\begin{aligned} y_{n+1}(t) = y_0 + \frac{1}{\Gamma(\alpha)} \sum_{m=0}^{n} \Bigg(\frac{f(t_m, y_m)}{h} \int_{t_m}^{t_{m+1}} (t - t_{m-1}) \times (t_{m+1})^{\alpha-1} \mathrm{d}t - \frac{f(t_{m-1}, y_{m-1})}{h} \\ \times \int_{t_m}^{t_{m+1}} (t - t_m)(t_{m+1} - t)^{\alpha-1} \mathrm{d}t \Bigg). \end{aligned} \tag{12}$$

For simplicity, we write the next expressions

$$\begin{aligned} A_{\alpha,m,1} &= h^{\alpha+1} \frac{(n+1-m)^{\alpha}(n-m+2+\alpha) - (n-m)^{\alpha}(n-m+2+2\alpha)}{\alpha(\alpha+1)}, \\ A_{\alpha,m,2} &= h^{\alpha+1} \frac{(n+1-m)^{\alpha+1}(n-m+1+\alpha) - (n-m)^{\alpha}(n-m+1+\alpha)}{\alpha(\alpha+1)}, \end{aligned} \tag{13}$$

We obtain the approximate solution of (12) by using Eq. (13)

$$\begin{aligned} y_{n+1}(t) = y(0) + \frac{1}{\Gamma(\alpha)} \sum_{m=0}^{n} \Bigg(\frac{h^{\alpha} f(t_m, y_m)}{\alpha(\alpha+1)} \big((n+1-m)^{\alpha} \times (n-m+2+\alpha) \\ -(n-m)^{\alpha}(n-m+2+2\alpha) \big) \\ - \frac{h^{\alpha} f(t_{m-1}, y_{m-1})}{\alpha(\alpha+1)} \times \big((n+1-m)^{\alpha+1} - (n-m)^{\alpha}(n-m+1+\alpha) \big) \Bigg), \end{aligned} \tag{14}$$

In the next part of the manuscript, we will extract a numerical algorithm to solve the suggested models under the Caputo-Fabrizio derivative.

Numerical Technique in Caputo-Fabrizio Frame

Consider the following equation containing the Caputo-Fabrizio derivative of order α

$${}_0^C\mathscr{D}_t^{\alpha} y(t) = f(t, y(t)), \tag{15}$$

We can rewrite the above problem in the following form by employing the fundamental theorem of fractional calculus

$$y(t) - y(0) = \frac{1-\alpha}{M(\alpha)} f(t, y(t)) + \frac{\alpha}{M(\alpha)} \int_0^t f(\theta, y(\theta)) \mathrm{d}\theta, \tag{16}$$

where $M(\alpha) = \frac{2}{2-\alpha}$ is a normalization function such that $M(0) = M(1) = 1$. In this way

$$y(t_{n+1}) - y(0) = \frac{(2-\alpha)(1-\alpha)}{2} f(t_n, y(t_n)) + \frac{\alpha(2-\alpha)}{2} \int_0^{t_{n+1}} f(t, y(t)) \mathrm{d}t, \tag{17}$$

and

$$y(t_n) - y(0) = \frac{(2-\alpha)(1-\alpha)}{2} f(t_{n-1}, y(t_{n-1})) + \frac{\alpha(2-\alpha)}{2} \int_0^{t_n} f(t, y(t)) \mathrm{d}t, \tag{18}$$

Regarding (18) in (17) the following relation can be obtained

$$\begin{aligned} y(t_{n+1}) = y(t_n) + \frac{(2-\alpha)(1-\alpha)}{2} \times \left[f(t_n, y(t_n)) - f(t_{n-1}, y(t_{n-1})) \right] \\ + \frac{\alpha(2-\alpha)}{2} \int_{t_n}^{t_{n+1}} f(t, y(t)) \mathrm{d}t, \end{aligned} \tag{19}$$

where

$$\int_{t_n}^{t_{n+1}} f(t, y(t)) \mathrm{d}t = \frac{3h}{2} f(t_n, y_n) - \frac{h}{2} f(t_{n-1}, y_{n-1}), \tag{20}$$

Now, the following relation is displaying the approximate solution

$$\begin{aligned} y_{n+1} = y_n + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4} \alpha(t)(2-\alpha) \right] f(t_n, y_n) \\ - \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2} \alpha(2-\alpha) \right] f(t_{n-1}, y_{n-1}), \end{aligned} \tag{21}$$

The next section is supposed to provide a numerical method to get the approximate solutions of the suggested system involving the Atangana-Baleanu-Caputo derivative.

Numerical Method in Atangana-Baleanu-Caputo Frame

Consider following problem

$${}^{\mathrm{ABC}}_{0}\mathscr{D}^{\alpha}_{t} y(t) = f(t, y(t)), \tag{22}$$

Using the fundamental theorem of fractional calculus on (22), we have

$$y(t) - y(0) = \frac{1-\alpha}{B(\alpha)} f(t, y(t)) + \frac{\alpha}{\Gamma(\alpha) B(\alpha)} \int_0^t f(\theta, y(\theta))(t-\theta)^{\alpha-1} \mathrm{d}\theta, \tag{23}$$

where $B(\alpha) = 1 - \alpha + \frac{\alpha}{\Gamma(\alpha)}$ is a normalization function. Att_{n+1} , we own

$$y(t_{n+1}) - y(0) = \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha} f(t_n, y(t_n)) + \frac{\alpha}{\Gamma(\alpha)+\alpha(1-\Gamma(\alpha))} \sum_{m=0}^{n} \times \int_{t_m}^{t_{m+1}} f(\theta, y(\theta))(t_{n+1}-\theta)^{\alpha-1} \mathrm{d}\theta, \tag{24}$$

By considering the two-step Lagrange polynomial interpolation for $f(\tau, y(\tau))$ we get

$$P_k(\theta) \simeq \frac{f(t_m, y_m)}{h}(\theta - t_{m-1}) - \frac{t_{m-1}, y_{m-1}}{h}(\theta - t_m), \tag{25}$$

Eq. (25) is substituted in (24) to obtain

$$\begin{aligned} y_{n+1}(t) = y_0 &+ \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha+\alpha)} f(t_n, y(t_n)) + \frac{\alpha}{\Gamma(\alpha)+\alpha(1-\Gamma(\alpha))} \sum_{m=0}^{n} \\ &\times \left(\frac{f(t_m, y_m)}{h} \int_{t_m}^{t_{m+1}} (\theta - t_{m-1})(t_{n+1}-\theta)^{\alpha-1} \mathrm{d}\theta \right. \\ &\left. - \frac{f(t_{m-1}, y_{m-1})}{h} \int_{t_m}^{t_{m+1}} (\theta - t_m)(t_{n+1}-\theta)^{\alpha-1} \mathrm{d}\theta \right), \end{aligned} \tag{26}$$

that indicates

$$\begin{aligned} A_{\alpha,m,1} &= h^{\alpha+1} \frac{(n+1-m)^{\alpha}(n-m+2+\alpha) - (n-m)^{\alpha}(n-m+2+2\alpha)}{\alpha(\alpha+1)}, \\ A_{\alpha,m,2} &= h^{\alpha+1} \frac{(n+1-m)^{\alpha+1}(n-m+1+\alpha) - (n-m)^{\alpha}(n-m+1+\alpha)}{\alpha(\alpha+1)}, \end{aligned} \tag{27}$$

Integrating (27) and replaced in (26) , the following relations can be gained

$$\begin{aligned} y_{n+1}(t) = y_0 &+ \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha} f(t_n, y(t_n)) + \frac{1}{(\alpha + ((1-\alpha)) + \alpha)} \sum_{m=0}^{n} \\ &\times \left(h^{\alpha} f(t_m, y_m) \left((n+1-m)^{\alpha}(n-m+2+\alpha) - (n-m)^{\alpha}(n-m+2+2\alpha) \right) \right. \\ &\left. - h^{\alpha} f(t_{m-1}, y_{m-1}) \left((n+1-m)^{\alpha+1} - (n-m)^{\alpha} \times (n-m+1+\alpha) \right) \right), \end{aligned} \tag{28}$$

Numerical Results

Now, we provide the simulations of the crime models regarding the considered operators stated in the previous section.

Solutions in the Caputo Frame

Using (14), we have

$$\begin{cases} S_{n+1} = S_0 + \frac{1}{\Gamma(\alpha)} \sum_{b=0}^{n} \Bigg[\dfrac{h^\alpha \Lambda_1(t_b, S(t_b))}{\alpha(\alpha+1)} ((n+1-b)^\alpha (n-b+2+\alpha)) - (n-b)^\alpha \\ \times (n-b+2+2\alpha)) - \dfrac{h^\alpha \Lambda_1(t_{b-1}, S(t_{b-1}))}{\alpha(\alpha+1)} \times ((n+1-b)^{\alpha+1} - (n-b)^\alpha (n-b+1+\alpha)) \Bigg], \\ C_{n+1} = C_0 + \frac{1}{\Gamma(\alpha)} \sum_{b=0}^{n} \Bigg[\dfrac{h^\alpha \Lambda_2(t_b, C(t_b))}{\alpha(\alpha+1)} ((n+1-b)^\alpha (n-b+2+\alpha)) - (n-b)^\alpha \\ \times (n-b+2+2\alpha)) - \dfrac{h^\alpha \Lambda_2(t_{b-1}, C(t_{b-1}))}{\alpha(\alpha+1)} \times ((n+1-b)^{\alpha+1} - (n-b)^\alpha (n-b+1+\alpha)) \Bigg], \\ C_{v_{n+1}} = C_{v_0} + \frac{1}{\Gamma(\alpha)} \sum_{b=0}^{n} \Bigg[\dfrac{h^\alpha \Lambda_3(t_b, C_v(t_b))}{\alpha(\alpha+1)} ((n+1-b)^\alpha (n-b+2+\alpha)) - (n-b)^\alpha \\ \times (n-b+2+2\alpha)) - \dfrac{h^\alpha \Lambda_3(t_{b-1}, C_v(t_{b-1}))}{\alpha(\alpha+1)} \times ((n+1-b)^{\alpha+1} - (n-b)^\alpha (n-b+1+\alpha)) \Bigg], \\ J_{a_{n+1}} = J_{a_0} + \frac{1}{\Gamma(\alpha)} \sum_{b=0}^{n} \Bigg[\dfrac{h^\alpha \Lambda_4(t_b, J_a(t_b))}{\alpha(\alpha+1)} ((n+1-b)^\alpha (n-b+2+\alpha)) - (n-b)^\alpha \\ \times (n-b+2+2\alpha)) - \dfrac{h^\alpha \Lambda_4(t_{b-1}, J_a(t_{b-1}))}{\alpha(\alpha+1)} \times ((n+1-b)^{\alpha+1} - (n-b)^\alpha (n-b+1+\alpha)) \Bigg], \\ J_{n+1} = J_0 + \frac{1}{\Gamma(\alpha)} \sum_{b=0}^{n} \Bigg[\dfrac{h^\alpha \Lambda_5(t_b, J(t_b))}{\alpha(\alpha+1)} ((n+1-b)^\alpha (n-b+2+\alpha)) - (n-b)^\alpha \\ \times (n-b+2+2\alpha)) - \dfrac{h^\alpha \Lambda_5(t_{b-1}, J(t_{b-1}))}{\alpha(\alpha+1)} \times ((n+1-b)^{\alpha+1} - (n-b)^\alpha (n-b+1+\alpha)) \Bigg], \\ P_{n+1} = P_0 + \frac{1}{\Gamma(\alpha)} \sum_{b=0}^{n} \Bigg[\dfrac{h^\alpha \Lambda_6(t_b, P(t_b))}{\alpha(\alpha+1)} ((n+1-b)^\alpha (n-b+2+\alpha)) - (n-b)^\alpha \\ \times (n-b+2+2\alpha)) - \dfrac{h^\alpha \Lambda_6(t_{b-1}, P(t_{b-1}))}{\alpha(\alpha+1)} \times ((n+1-b)^{\alpha+1} - (n-b)^\alpha (n-b+1+\alpha)) \Bigg], \end{cases} \tag{29}$$

where

$$\begin{cases} \Lambda_1 = \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S, \\ \Lambda_2 = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\ \Lambda_3 = \beta 3 J J_a - r_2 C_v - \mu C_v, \\ \Lambda_4 = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\ \Lambda_5 = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\ \Lambda_6 = a_2 S - \beta_4 PC - \mu P, \end{cases} \tag{30}$$

Numerical Results in Atangana-Baleanu-Caputo Sense

Now, by utilizing the procedure produced by (28) we possess

$$\begin{cases} S_{n+1} = S_0 + \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha}\Lambda_1(t_n, S_n) \\ +\dfrac{1}{(\alpha+1)((1-\alpha)\Gamma(\alpha))+\alpha}\sum_{b=0}^{n}(h^\alpha\Lambda_1(t_b, S_b)((n+1-b)^\alpha \\ \times(n-b+2+\alpha)-(n-b)^\alpha(n-b+2+2\alpha))- \\ h^\alpha\Lambda_1(t_{b-1}, S_{b-1})((n+1-b)^{\alpha+1}-(n-b)^\alpha(n-b+1+\alpha))), \\ C_{n+1} = C_0 + \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha}\Lambda_2(t_n, C_n) \\ +\dfrac{1}{(\alpha+1)((1-\alpha)\Gamma(\alpha))+\alpha}\sum_{b=0}^{n}(h^\alpha\Lambda_2(t_b, C_b)((n+1-b)^\alpha \\ \times(n-b+2+\alpha)-(n-b)^\alpha(n-b+2+2\alpha))- \\ h^\alpha\Lambda_2(t_{b-1}, C_{b-1})((n+1-b)^{\alpha+1}-(n-b)^\alpha(n-b+1+\alpha))), \\ C_{v_{n+1}} = C_{v_0} + \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha}\Lambda_3(t_n, C_{v_n}) \\ +\dfrac{1}{(\alpha+1)((1-\alpha)\Gamma(\alpha))+\alpha}\sum_{b=0}^{n}(h^\alpha\Lambda_3(t_b, C_{v_b})((n+1-b)^\alpha \\ \times(n-b+2+\alpha)-(n-b)^\alpha(n-b+2+2\alpha))- \\ h^\alpha\Lambda_3(t_{b-1}, C_{v_{b+1}})((n+1-b)^{\alpha+1}-(n-b)^\alpha(n-b+1+\alpha))), \\ J_{a_{n+1}} = J_{a_0} + \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha}\Lambda_4(t_n, J_{a_n}) \\ +\dfrac{1}{(\alpha+1)((1-\alpha)\Gamma(\alpha))+\alpha}\sum_{b=0}^{n}(h^\alpha\Lambda_4(t_b, J_{a_b})((n+1-b)^\alpha \\ \times(n-b+2+\alpha)-(n-b)^\alpha(n-b+2+2\alpha))- \\ h^\alpha\Lambda_4(t_{b-1}, J_{a_{b-1}})((n+1-b)^{\alpha+1}-(n-b)^\alpha(n-b+1+\alpha))), \\ J_{n+1} = J_0 + \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha}\Lambda_5(t_n, J_n) \\ +\dfrac{1}{(\alpha+1)((1-\alpha)\Gamma(\alpha))+\alpha}\sum_{b=0}^{n}(h^\alpha\Lambda_5(t_b, J_b)((n+1-b)^\alpha \\ \times(n-b+2+\alpha)-(n-b)^\alpha(n-b+2+2\alpha))- \\ h^\alpha\Lambda_5(t_{b-1}, J_{b-1})((n+1-b)^{\alpha+1}-(n-b)^\alpha(n-b+1+\alpha))), \\ P_{n+1} = P_0 + \frac{\Gamma(\alpha)(1-\alpha)}{\Gamma(\alpha)(1-\alpha)+\alpha}\Lambda_6(t_n, P_n) \\ +\dfrac{1}{(\alpha+1)((1-\alpha)\Gamma(\alpha))+\alpha}\sum_{b=0}^{n}(h^\alpha\Lambda_6(t_b, P_b)((n+1-b)^\alpha \\ \times(n-b+2+\alpha)-(n-b)^\alpha(n-b+2+2\alpha))- \\ h^\alpha\Lambda_6(t_{b-1}, P_{b-1})((n+1-b)^{\alpha+1}-(n-b)^\alpha(n-b+1+\alpha))), \end{cases} \tag{31}$$

where

$$\begin{cases} \Lambda_1 = \mu N - \beta_1 SC - (a_1+a_2)S + r_1 J_a + r_2 C_v - \mu S, \\ \Lambda_2 = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\ \Lambda_3 = \beta 3 J J_a - r_2 C_v - \mu C_v, \\ \Lambda_4 = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\ \Lambda_5 = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\ \Lambda_6 = a_2 S - \beta_4 PC - \mu P, \end{cases} \tag{32}$$

Numerical Results in Caputo-Fabrizio Sense

Employing the numerical scheme offered by (21) for the crime model, we have

$$\begin{cases} S_{n+1} = S_n + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4}\alpha(2-\alpha)\right]\Lambda_1(t_n, S_n) - \\ \quad \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2}\alpha(2-\alpha)\right]\Lambda_1(t_{n-1}, S_{n-1}), \\ C_{n+1} = C_n + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4}\alpha(2-\alpha)\right]\Lambda_2(t_n, C_n) - \\ \quad \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2}\alpha(2-\alpha)\right]\Lambda_2(t_{n-1}, C_{n-1}), \\ C_{v_{n+1}} = C_{v_n} + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4}\alpha(2-\alpha)\right]\Lambda_3(t_n, C_{v_n}) - \\ \quad \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2}\alpha(2-\alpha)\right]\Lambda_3(t_{n-1}, C_{v_{n-1}}), \\ J_{a_{n+1}} = J_{a_n} + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4}\alpha(2-\alpha)\right]\Lambda_4(t_n, J_{a_n}) - \\ \quad \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2}\alpha(2-\alpha)\right]\Lambda_4(t_{n-1}, J_{a_{n-1}}), \\ J_{n+1} = J_n + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4}\alpha(2-\alpha)\right]\Lambda_5(t_n, J_n) - \\ \quad \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2}\alpha(2-\alpha)\right]\Lambda_5(t_{n-1}, J_{n-1}), \\ P_{n+1} = P_n + \left[\frac{(2-\alpha)(1-\alpha)}{2} + \frac{3h}{4}\alpha(2-\alpha)\right]\Lambda_6(t_n, P_n) - \\ \quad \left[\frac{(2-\alpha)(1-\alpha}{2} + \frac{h}{2}\alpha(2-\alpha)\right]\Lambda_6(t_{n-1}, P_{n-1}), \end{cases} \tag{33}$$

where

$$\begin{cases} \Lambda_1 = \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S, \\ \Lambda_2 = \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C, \\ \Lambda_3 = \beta 3 J J_a - r_2 C_v - \mu C_v, \\ \Lambda_4 = \beta_2 CP - \beta_3 J J_a - r_1 J_a - \mu J_a, \\ \Lambda_5 = a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J, \\ \Lambda_6 = a_2 S - \beta_4 PC - \mu P. \end{cases} \tag{34}$$

Results and Discussion

We used three different fractional-order derivatives in the frames of Atangana-Baleanu-Caputo, Caputo and Caputo derivatives to design the non-integer models of the crime system. To see the behavior of the numerical solutions of the suggested models, numerical schemes using the initial conditions are provided. In this work, we use the real initial conditions reported in [31] to see the performance of the suggested techniques for solving the considered systems different amounts of fractional orders are chosen. Fig. 1 is responsible to show the behaviour of solutions under three fractional operators with the initial condition $S(0) = 9.527 \times 10^(1)$, $C(0) = 3.823 \times 10^(0)$, $J_a(0) = 1.344 \times 10^(- 1)$, $C_v(0) = 5.097 \times 10^(- 1)$, $P(0) = 1.362 \times 10^(- 2)$ and $J(0) = 2.485 \times 10^(- 1)$ and fractional order $\alpha = 0.95$. It is clear that in spite of a decreasing trend for $C(t)$ and $C_v(t)$, number of susceptible group is experiencing an upward trend for the time period of 50 years. For the same period of time in fig. 2 the same trend(a decreasing trend) can be seen for $J(t)$, $J_a(t)$ and the number of police officers. The differences in the behaviour of the solutions under the considered operators are obvious.

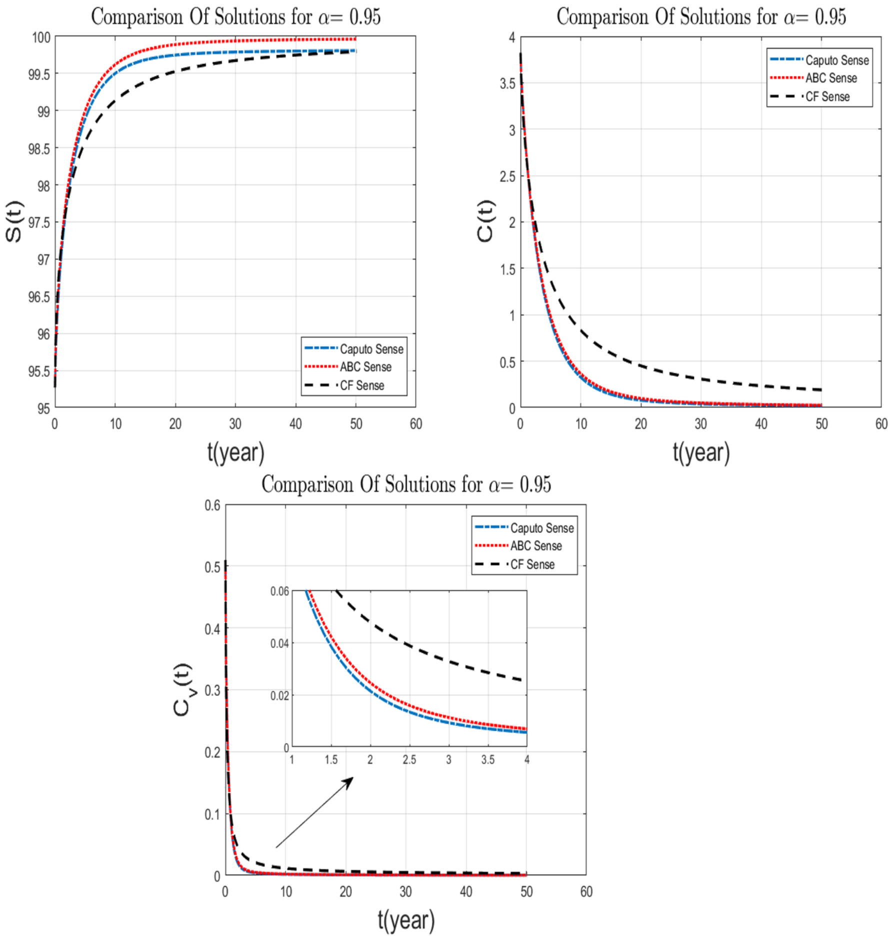

Fig. 1 Comparing the solutions for $\alpha = 0.95$

To see the efficacy and the applicability of the suggested numerical method we choose another fractional order as $\alpha = 0.99$. Similar to the first case, the time frame is considered for 50 years which the behaviour of the solutions for $S(t)$, $C(t)$ and $C_v(t)$ can be observed in the fig 3. Also, fig 4 show the approximate solutions of $J(t)$, $J_a(t)$ and $P(t)$. It is obvious that in the number of $C(t)$ and $C_v(t)$ start to fall in spite of the number of susceptible group which starts an increasing approach for the time period of 50 years. Also decreasing approach can be observed for the approximate behaviour of $J(t)$, $J_a(t)$ and the number of police officers. The differences in the behaviour of the solutions under the considered operators are obvious.

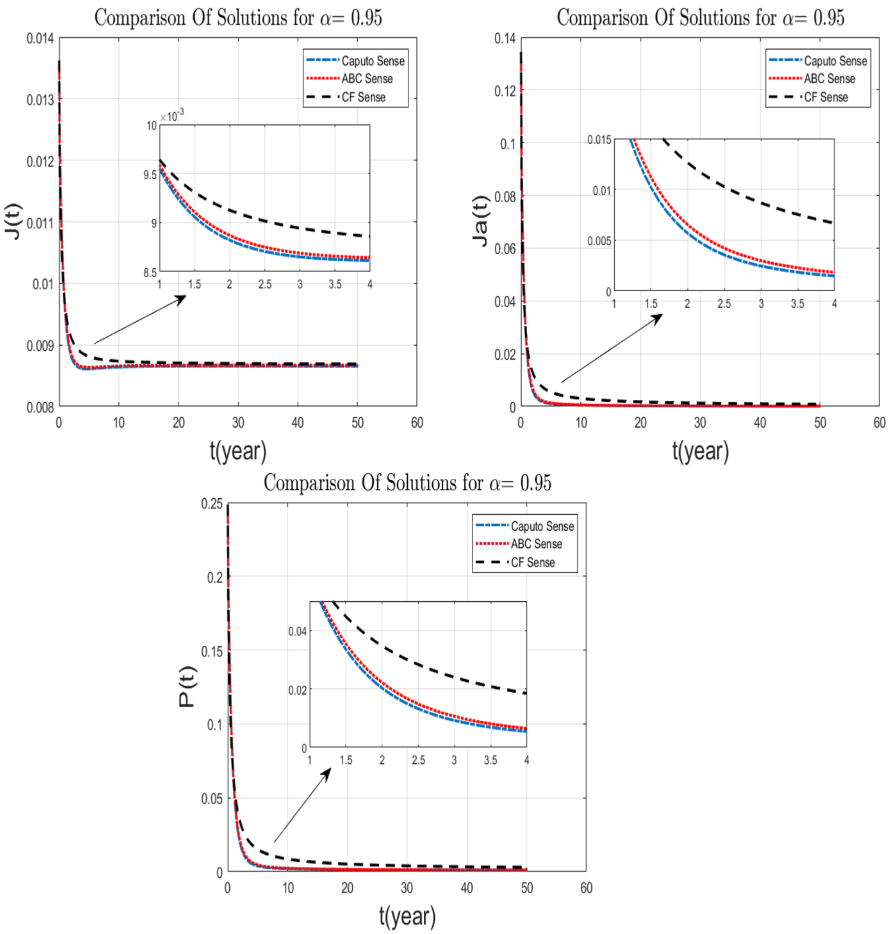

Fig. 2 Comparing the solutions for $\alpha = 0.95$

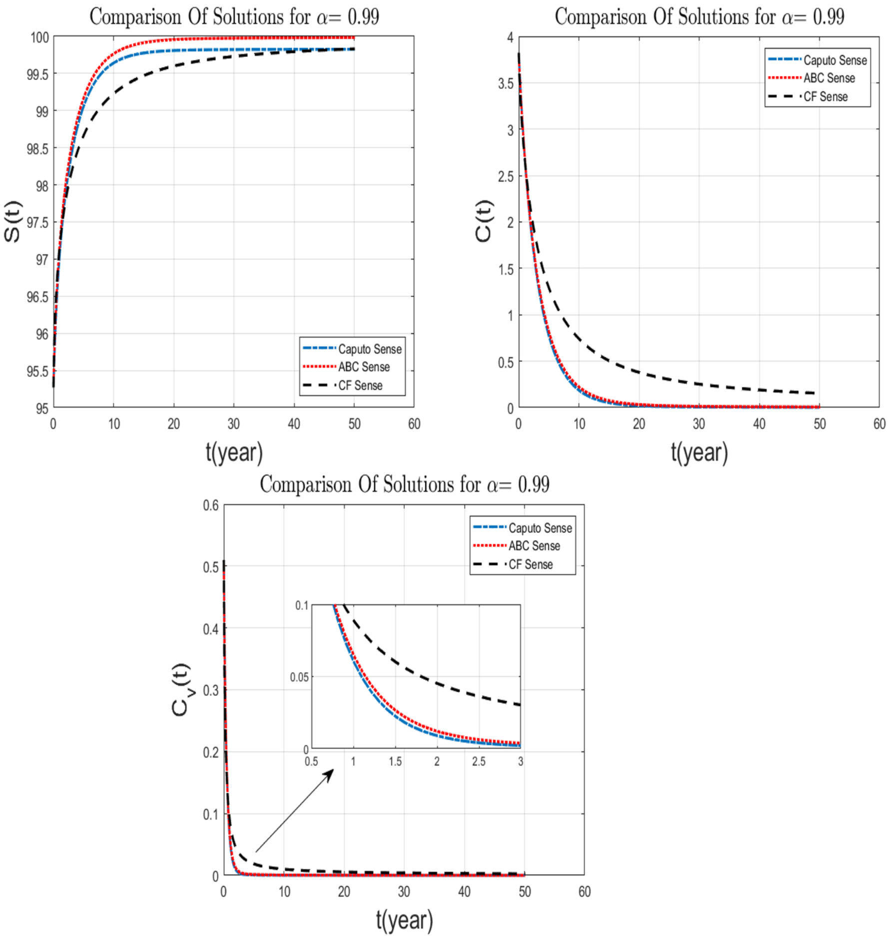

Fig. 3 Comparing the solutions for $\alpha = 0.99$

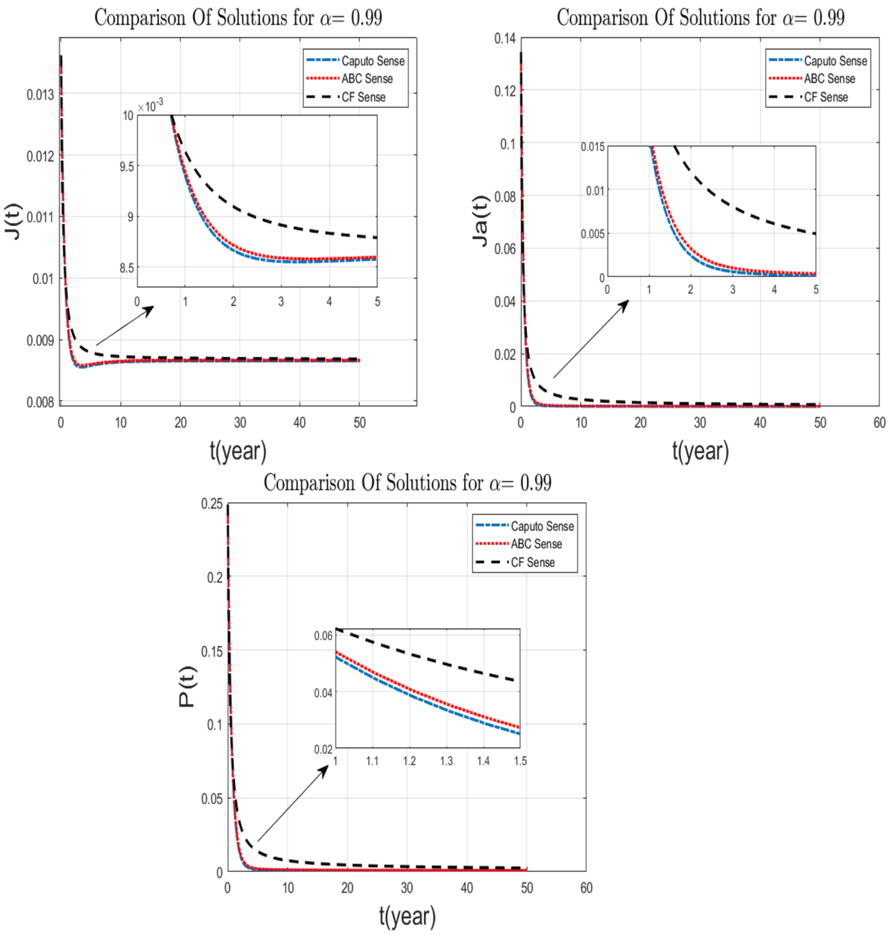

Fig. 4 Comparing the solutions for $\alpha = 0.99$

Conclusion

In this study, we applied three fractional-order derivatives in the frames of Caputo derivative, Caputo-Fabrizio derivative, and Atangana-Baleanu-Caputo derivative to make the fractional-order type model the crime system. Comparing the behavior of the solutions under the considered fractional operators was our main aim. After designing the fractional models of the crime system, effective numerical techniques were obtained to gain the approximate solutions to the problems. To see how the suggested models behave we selected different values of the fractional orders in a considerable time period. We plotted the solutions for each state variable in one figure to see the differences of the behaviors under the chosen initial conditions.

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Declarations

Competing interests The authors have not disclosed any competing interests.

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