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APPLICATION ARTICLE

An optimal control model of COVID-19 pandemic: a comparative study of five countries

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Abstract

This paper formulates an optimal control model of COVID-19 pandemic spreading. We discuss the health sector performance of Argentina, Hungary, Egypt, Malaysia, and Iraq. A mathematical model describes an actual case number of COVID-19. We investigate three strategies depend on recovery rate, death rate, and together (optimal). These strategies represent the percent of the health sector development. The explicit solution of the model using the Pontryagin maximum principle is derived. The results showed the ranking of countries based on the new percent of the recovery and death cases. A new percent as a result to the control variable value (health sector development). Also, the development percent of the health sector of each country, was determined. For example, 0.005 led to a significant reduce the death rates in Malaysia. Meanwhile, a half of death rates could reduce by this percent in Egypt.

Keywords COVID-19 pandemic · Optimal control · Argentina · Hungary · Egypt · Malaysia · Iraq

JEL Classification C44 · C61

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1 Introduction

Covid-19 pandemic threaten a life of millions of people around the world. Also, it led a huge loss in the world economy and trade exchange between countries. The pandemic has spread around the world during a few months. Most nations have isolated to face this virus and decrease the losses. Several studies have investigated COVID-19 virus around the world. The target understanding the composition of the virus and find the appropriate treatment.

To diagnose the symptoms of the disease, Wang et al. [1] clarified the long period of virus incubation without clinical symptoms earlier. It represents the most dangerous factor of a pandemic. Symptoms of COVID-19 and SARS-CoV-2-negative, such as fever, cough, levels of creatinine and Procalcitonin (PCT), were studied by Chen et al. [2]. For younger age patients, they discussed the difference in urea and creatinine, as well as parameters of routine blood work-up. Xu et al. [3] considered SARS-CoV-2 or liver injury may lead to COVID-19 injury. Also, they clarified the pathological features of COVID-19 are like to SARS and MERS. The main symptoms of the pregnant that injured by COVID-19 were indicated by X. Chen et al. [4]. Pregnant suffered from fever and cough, without vertical transmission of the virus in late pregnancy. World health organization (WHO) addressed the effect of COVID-19 pandemic on the mental health of children and elderly [5]. According to tests on 1014 patients in China, Ai et al. [6] clarified the chest CT is highly sensitive to virus diagnosis compared with reverse-transcription chain reaction.

As a suggested treatment, Lipsitch et al. [7] suggested using the treatment of influenza pandemic in 2009 to treat Covid-19 patients, with provide surveillance systems. Clinical trials in many hospitals in seven cities and more than 100 patients in China were conducted by Gao et al. [8]. They recommended using Chloroquine phosphate that used with malaria disease to treat COVID-19 patients. Zhou et al. [9] collected Viral RNA samples from survivors and deaths to explore the factors that cause death in hospital by using multivariable logistic regression.

As mathematical models, several researchers have formulated mathematical models to describe pandemics. Models have simulated the actual cases of diseases. Then clarify the effect of many factors on the spreading, such as vaccination, weather, isolation, and so on. The optimal control model is an example of these models. Lee et al. [10] determined the effect of treatment and isolation strategies, with a limited of antiviral on the spread of influenza. Three models of optimal control of the SIR epidemic were investigated by Hansen and Day [11] to reduce the outbreak size. Tuite et al. [12] reduced the potential effects of the cholera epidemic in Haiti by using the spread control strategies. The dengue vaccine as a control variable to paediatric and random mass, were discussed by Rodrigues et al. [13]. The total implementation costs and the number of infected cases of tuberculosis were reduced by P. Rodrigues et al. [14] by using different scenarios. Moualeu et al. [15] formulated a mathematical model of tuberculosis cases with lost-sight and latently infected. Pang et al. [16] simulated the actual cases of measles transmission, then suggest the optimal strategy of vaccine in US (1951–1962). Ebola virus spreading in West Africa was addressed by Rachah and Torres [17, 18]. The effect of vaccination on the virus spreading over time with different cases was investigated by the first paper. Meanwhile, the second paper addressed a control on the Ebola virus, with several strategies of exposed and vaccine. Gao and Huang [19] recommended a strategy with three controls to reduce the tuberculosis cases and the intervention cost.

Follows this introduction, information about COVID-19 pandemic spreading in Sect. 2. Section 3 describes the optimal control model and its explicit solution by using the Pontryagin maximum principle (PMP). The results and its explanation of several models are collected in Sect. 4. Section 5 describes conclusions and the future suggested works.

2 COViD-19 pandemic spreading

Wuhan, Hubei, China is the source of the first cases of COVID-19 that identified in December 2019 [2]. In January, the virus moved into three Asian countries; Thailand, Japan, and South Korea [20]. At the end of February, the cases have been registered in Europe, North America, Asia, African and Latin American [21]. At the begging of March, more than ten thousand patients have been dead out of China [1]. Until today, 1 July 2020, more than (10) million confirmed cases with more than (500) thousand deaths in the world [22].

The first case registered in Malaysia on 26 January [23], While in Egypt on 19 February [24]. 24 February, the first case registered in Iraq, while in Argentina and Hungary in 5 March [25]. Argentina reached to more than 100 cases by only (9) days, while Malaysia in (44) days. Other countries, Hungary, Iraq, and Egypt have reached to more than 100 cases in 17, 22, and 25 days, respectively.

3 Optimal control model

The objective function of optimal control model is defined by:

$$\text{Min} \cdot J = \sum_{t=0}^{T-1} \{x(t) + y(t)\}^2 \tag{1}$$

Subject to state equations of inpatient, recover and death:

$$\Delta I(t) = \beta(t)I(t) - \{\mu(t) + x(t)\}I(t) - \{N(t) + y(t)\}I(t) \tag{2}$$

$$\Delta R(t) = \{\mu(t) + x(t)\}I(t) \tag{3}$$

$$\Delta D(t) = \{N(t) + y(t)\}I(t) \tag{4}$$

where,

$x(t)$, $y(t)$: The control variables.
$I(t)$: The percent of inpatient cases.
$R(t)$: The percent of recovery cases.
$D(t)$: The percent of death cases.
$\beta(t)$: The percent of the new cases to the inpatient cases.
$\mu(t)$: The percent of the recovery cases to the inpatient cases.
$N(t)$: The percent of the death cases to the inpatient cases.
$\Delta I(t) = I(t+1) - I(t)$

The total number of inpatients increases by the new cases and decrease by the number of recovering and death every day (Eq. 2). Equations (3) and (4) represent the total number of recovering and death, respectively. Control variables represent the percentage of increase of the medical capabilities of the health sector.

We can find the solution of the optimal control model by using PMP. The Lagrangian function is as follows [26]:

$$\begin{aligned} L = \sum_{t=0}^{T-1} -\{x(t)+y(t)\}^2 + \sum_{t=0}^{T-1} \lambda_i(t+1)[\,\beta(t)I(t) - \{\mu(t)+x(t)\}I(t) \\ -\{N(t)+y(t)\}I(t) - I(t+1) + I(t)\big] + \sum\nolimits_{t=0}^{T-1} \lambda_r(t+1)[\{\mu(t)+x(t)\}I(t) \\ -R(t+1) + R(t)] + \sum\nolimits_{t=0}^{T-1} \lambda_d(t+1)\big[\{N(t)+y(t)\}I(t) - D(t+1) + D(t)\big] \end{aligned} \tag{5}$$

A Hamiltonian function is as follows:

$$\begin{aligned} H(t) = -\{x(t)+y(t)\}^2 + \lambda_i(t+1)\big[\,\beta(t)I(t) - \{\mu(t)+x(t)\}I(t) - \{N(t)+y(t)\}I(t)\big] \\ + \lambda_r(t+1)[\{\mu(t)+x(t)\}I(t)] + \lambda_d(t+1)\big[\{N(t)+y(t)\}I(t)\big] \end{aligned} \tag{6}$$

Substitute Eq. (6) into Eq. (5), yields:

$$\begin{aligned} L = \sum\nolimits_{t=0}^{T-1} \big[H(t) - \lambda_i(t+1)\{I(t+1) - I(t)\} - \lambda_r(t+1)\{R(t+1) - R(t)\} \\ -\lambda_d(t+1)\{D(t+1) - D(t)\}\big] \end{aligned} \tag{7}$$

Derive the Eq. (7) with respect to $I(t)$, $R(t)$, $D(t)$, separately, yields:

$$\begin{aligned} \lambda_i(t+1)\big[\,\beta(t) - \{\mu(t)+x(t)\} - \{N(t)+y(t)\}\big] + \lambda_r(t+1)\{\mu(t)+x(t)\} \\ + \lambda_d(t+1)\{N(t)+y(t)\} + \lambda_i(t+1) - \lambda_i(t) = 0, \end{aligned} \tag{8}$$

$$\lambda_r(t+1) - \lambda_r(t) = 0, \tag{9}$$

$$\lambda_d(t+1) - \lambda_d(t) = 0, \tag{10}$$

The adjoint equations can be found from Eqs. (8)-(10) is as follows:

$$\Delta\lambda_i(t) = -\beta(t)\lambda_i(t+1) + \{\mu(t) + x(t)\}\left[\lambda_i(t+1) - \lambda_r(t+1)\right] + \{N(t) + y(t)\}\left[\lambda_i(t+1) - \lambda_d(t+1)\right], \quad (11)$$

$$\Delta\lambda_r(t) = 0, \quad (12)$$

$$\Delta\lambda_d(t) = 0. \quad (13)$$

From Eq. (11), we get:

$$\lambda_i(t+1) = \frac{1}{\left[1 + \beta(t) - \{\mu(t) + x(t)\} - \{N(t) + y(t)\}\right]}\left[\lambda_i(t) - \{\mu(t) + x(t)\}\lambda_r(t+1) - \{N(t) + y(t)\}\lambda_d(t+1)\right] \quad (14)$$

Derive the Eq. (7) with respect to $x(t)$, $y(t)$, separately, yields:

$$-2\{x(t) + y(t)\} - \lambda_i(t+1)I(t) + \lambda_r(t+1)I(t) = 0, \quad (15)$$

$$-2\{x(t) + y(t)\} - \lambda_i(t+1)I(t) + \lambda_d(t+1)I(t) = 0. \quad (16)$$

From Eqs. (15) and (16), we get:

$$\lambda_r(t+1) = \lambda_d(t+1) \quad (17)$$

Rearrange Eq. (2), yields:

$$I(t+1) = \left[1 + \beta(t) - \{\mu(t) + x(t)\} - \{N(t) + y(t)\}\right]I(t) \quad (18)$$

From Eq. (14), we get:

$$\left[1 + \beta(t) - \{\mu(t) + x(t)\} - \{N(t) + y(t)\}\right] = \frac{1}{\lambda_i(t+1)}\left[\lambda_i(t) - \{\mu(t) + x(t)\}\lambda_r(t+1) - \{N(t) + y(t)\}\lambda_d(t+1)\right]. \quad (19)$$

By substituting Eq. (19) into Eq. (18), yields:

$$I(t+1) = \frac{I(t)}{\lambda_i(t+1)}\left[\lambda_i(t) - \{\mu(t) + x(t)\}\lambda_r(t+1) - \{N(t) + y(t)\}\lambda_d(t+1)\right] \quad (20)$$

Equation (20) represents the total number of inpatients over time. From Eqs. (3) and (20), we can find the total number of recovered cases over time is as follows:

$$R(t+1) = R(t) + \frac{\{\mu(t) + x(t)\}I(t+1)\lambda_i(t+1)}{\left[\lambda_i(t) - \{\mu(t) + x(t)\}\lambda_r(t+1) - \{N(t) + y(t)\}\lambda_d(t+1)\right]} \quad (21)$$

We can find the total number of death cases over time by substituting Eq. (20) into Eq. (4) is as follows:

$$D(t+1) = D(t) + \frac{\{N(t) + y(t)\}I(t+1)\lambda_i(t+1)}{\left[\lambda_i(t) - \{\mu(t) + x(t)\}\lambda_r(t+1) - \{N(t) + y(t)\}\lambda_d(t+1)\right]} \tag{22}$$

Now, the value of control variables is as follows:

$$y(t) = \begin{cases} -\gamma & \text{if } \gamma \le N \\ -N & O.W. \end{cases} \tag{23}$$

$$x(t) = \begin{cases} \delta & \text{if } y(t) = 0 \\ \gamma & O.W. \end{cases} \tag{24}$$

$$x(t) = y(t) = 0; \quad \textit{actual cases} \tag{25}$$

where γ and δ are deterministic values.

Equation (23) achieves the condition of a nonnegative value of the death rate. Initially the value of control variables is zero, which means the model represents the actual cases that confirmed with the real percentage of new cases, recovered, and death is as follows:

$$\beta_t = \frac{c_{t+1}}{I_t};\ \mu_t = \frac{r_{t+1}}{I_t};\ N_t = \frac{d_{t+1}}{I_t} \tag{26}$$

where *c, r,* and *d* represent the daily cases of new cases, recover, and death, respectively.

To fit the model with the real data of the COVID-19 cases, the initial values must be as follows:

$$I_1 = \frac{i_1}{\pi};\ R_1 = \frac{r_1}{\pi};\ D_1 = \frac{d_1}{\pi}; \lambda_i(1) = \frac{c_1}{\pi} \tag{27}$$

where *i* represents the inpatient cases, and π any fixed value, such as 10,000.

The results of the model will be percentages that must be multiplied by π to convert it to the integer numbers.

Then introduce the effect of the health sector capabilities by changing the values of control variables.

According to the values of control variables, there are three strategies are as follows:

- $x(t)$ represents increase the medical capabilities for health sector to increase the recovery cases. This means a decrease in the numbers of inpatient and death cases.

- $y(t)$ represents increase the medical capabilities to decrease the death cases. This means an increase in the rates of inpatient and recover cases. This strategy focuses on the critical cases.
- Increasing the medical capabilities with the values of the two control variables are equal regardless of the signal. This means the number of inpatient cases stay without change. This strategy focuses on the all cases, so it represents the optimal strategy.

Finally, the solution of the optimal control model depends on Eqs. (12–14, 17, 20–22).

4 Numerical results

The study deals with COVID-19 cases in five countries: Argentina, Hungary, Egypt, Malaysia and Iraq. Approximately 2750 confirmed cases in each country represent the study period. The study period begins from registering more than 100 cases in each country, so the length of the period is more than one month. Tables 1, 2, 3, 4 and 5 (see "Appendix") show the number of COVID-19 cases in countries and parameter values of the model (β, μ, N) by using Eq. (26).

Figure 1 shows the results of the three strategies and the actual case of injuries ($x = y = 0$) in Argentina. The length of the study period is 30 days from 128 to 2758 confirmed cases.

From Fig. 1, we can deduce that the number of inpatient of the third strategy (green line) is the same in the actual case (blue circle). Increase the

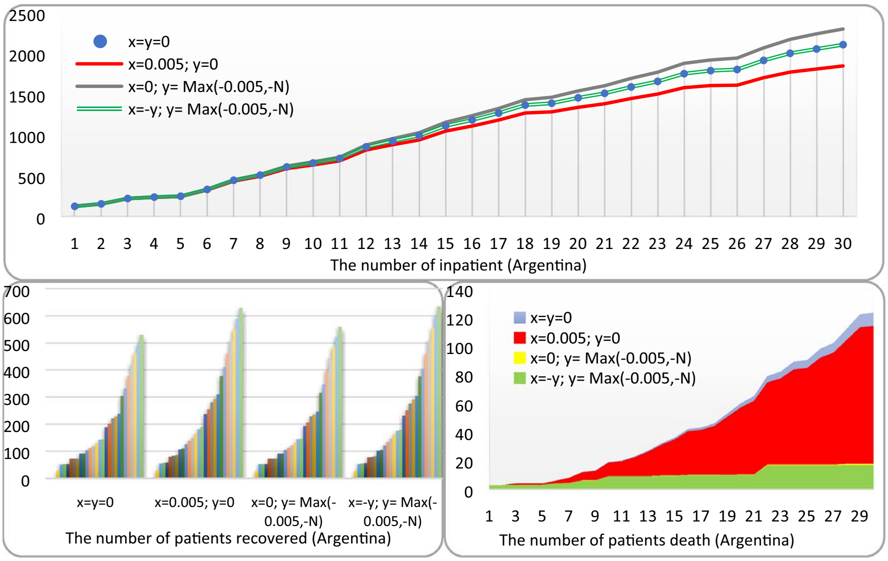

Fig. 1 The results of the optimal control model (Argentina)

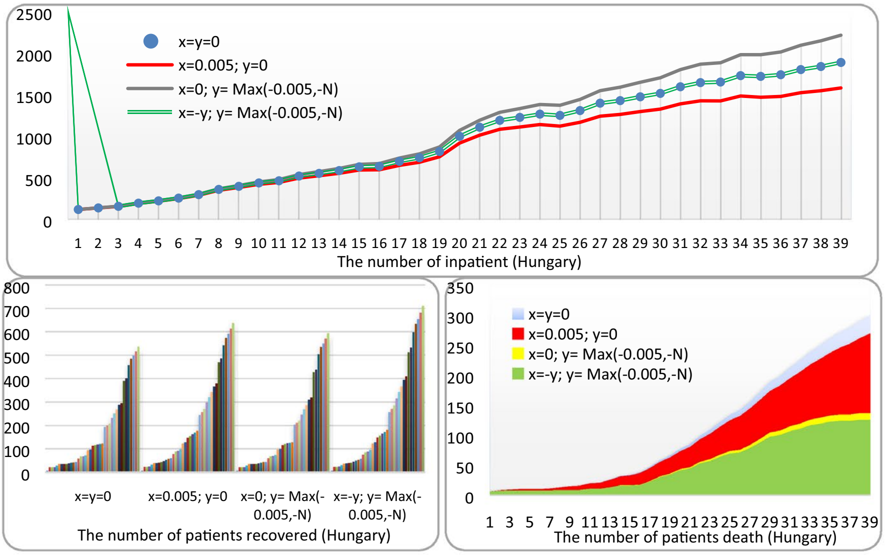

Fig. 2 The results of the optimal control model (Hungary)

medical capabilities affect only on the numbers of recovering and death with the same patients number. This means increased the recover cases by 0.2%, while a significant decrease the death cases (green area) by 0.861. A difference in percentages back to the big difference between the actual numbers of recovering and death cases.

The first strategy leads to decrease the number of inpatient (red line) by 0.123. At the same time a significant increase the recovers cases and decrease the death cases (red area) by 0.187 and 0.073, respectively.

The opposite case (the second strategy) leads to increase the number of inpatient (grey line) by 0.091 due to decrease the death cases. At the same time a slight increase the recover cases and a significant decrease the death cases (yellow area) by 0.057 and 0.856, respectively. Figure 2 shows the number of inpatient, recover and death cases in Hungary.

The length of the study period is 39 days from 131 to 2727 confirmed cases. This means COVID-19 cases spreads in Argentina is faster than Hungary. From Fig. 2, the number of cases a significant increase 17 days later of the beginning of the study period (April 7th). The third strategy leads to increase the recover cases and a significant decrease the death cases (green area) by 0.325 and 0.581, respectively. This means the most of the actual daily percentages of death are higher than 0.005 over the study period.

The first strategy leads to decrease the number of inpatient (red line) by 0.162. At the same time increase the recover cases and decrease the death cases (red area) by 0.191 and 0.102, respectively. The low change percentage of the recovery cases represent the best health sector. Thus, the health sector in Argentina is better than in Hungary, according to the change percentage of the recovery cases.

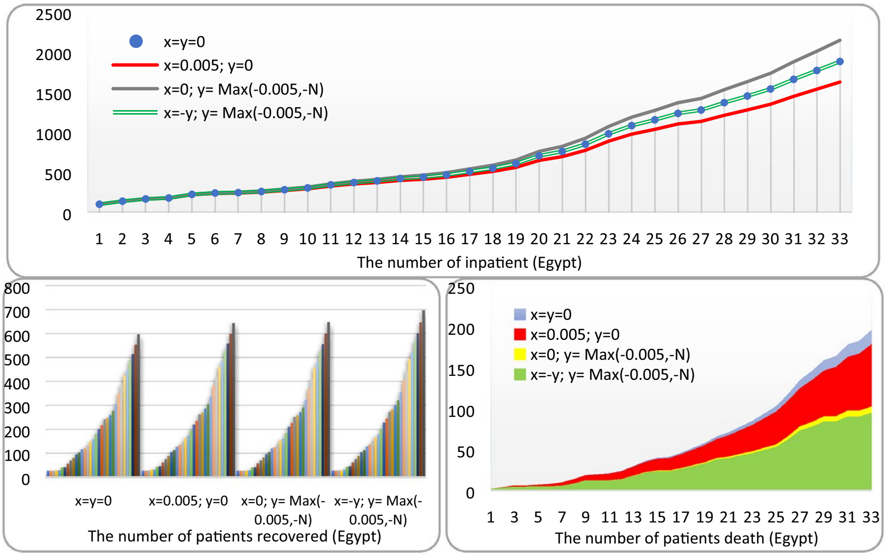

Fig. 3 The results of the optimal control model (Egypt)

Increase in the number of inpatient (grey line) by 0.173, according to the second strategy. At the same time a slight increase the recover cases and a significant decrease the death cases (yellow area) by 0.108 and 0.544, respectively. The high change percentage represents the best health sector in the case of death cases. Thus, the health sector in Argentina is better than in Hungary. Figure 3 shows the number of inpatient, recover and death cases in Egypt.

The length of the study period is 33 days from 126 to 2673 confirmed cases. This means the ranking of countries, according to the speed of spreading of COVID-19 cases is Argentina, Egypt and Hungary. From Fig. 3, the number of cases a significant increase 19 days later of the beginning of the study period (April 2th). Increase in the recover cases and a significant decrease the death cases (green area), according to the third strategy by 0.17 and 0.517, respectively. This means more than half of the actual daily percentages of death are higher than 0.005 over the study period.

The first strategy leads to decrease the number of inpatient (red line) by 0.136. At the same time a slight increase the recovered cases and decrease the death cases (red area) by 0.079 and 0.088, respectively. The ranking of the countries is Egypt, Argentina, and Hungary, according to the change percentage of the recovery cases.

Increase in the number of inpatient (grey line) by 0.141, according to the second strategy. At the same time a slight increase the recover cases and a significant decrease the death cases (yellow area) by 0.086 and 0.478, respectively. The ranking of the countries is Argentina, Hungary, and Egypt, according to the change percentage of the death cases. Figure 4 shows the number of inpatient, recover and death cases in Malaysia.

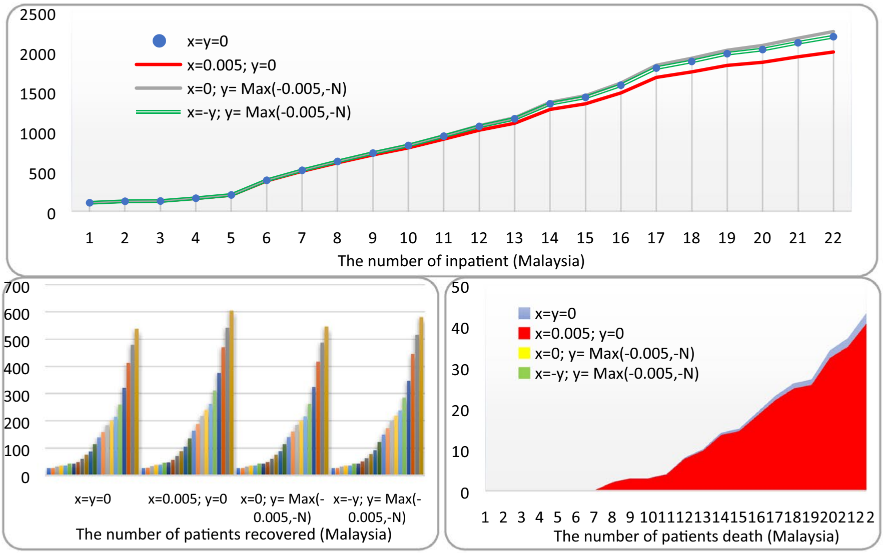

Fig. 4 The results of the optimal control model (Malaysia)

The length of the study period is 22 days from 129 to 2766 confirmed cases. This means the ranking of countries, according to the speed of spreading of COVID-19 cases is Malaysia, Argentina, Egypt and Hungary. From Fig. 4, the number of cases a significant increase 6 days later of the beginning of the study period (March 15th). The number of death cases is zero, according to the second and third strategies. This means the actual daily percent of death are less than 0.005, while the increase in the recover cases by 0.08.

The first strategy leads to decrease the number of inpatient (red line) by 0.087. At the same time a slight increase the recovered cases and decrease the death cases (red area) by 0.125 and 0.06, respectively. The ranking of the countries is Egypt, Malaysia, Argentina, and Hungary, according to the changing percent of the recovery cases.

Increase in the number of inpatient (grey line) by 0.029, according to the second strategy. At the same time a slight increase the recover cases by 0.016 percent. The ranking of the countries is Malaysia, Argentina, Hungary, and Egypt, according to the changing percent of the death cases. Figure 5 shows the number of inpatients, recover and death cases in Iraq.

The length of the study period is 55 days from 165 to 2767 confirmed cases. This means the ranking of countries is Malaysia, Argentina, Egypt, Hungary and Iraq, according to the speed of spreading. From Fig. 5, the number of cases decreases 27 days later of the beginning of the study period (April 12th) and increase again 43 days later (April 28th). The third strategy leads to increase in the recover cases by 0.037%, while a significant decrease the death cases (green area) by 0.601. This means less than half of the actual daily percentages of death are higher than 0.005 over the study period.

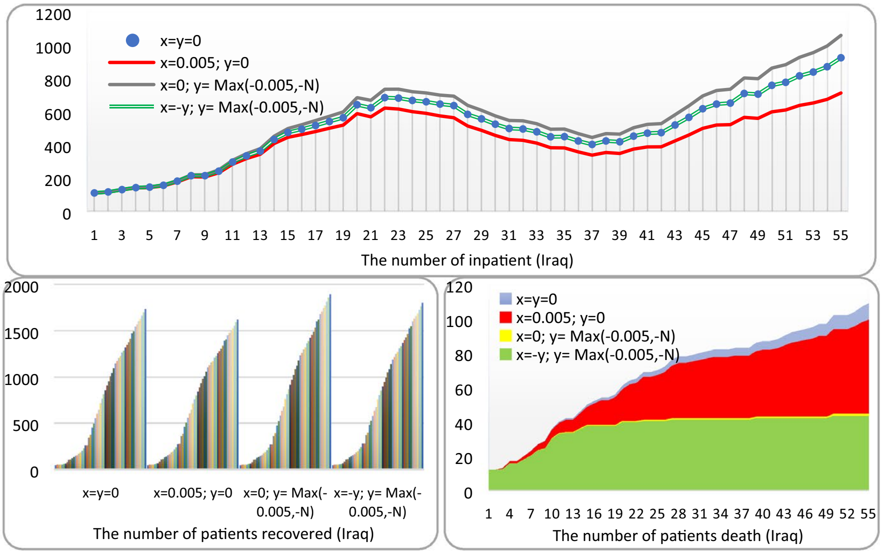

Fig. 5 The results of the optimal control model (Iraq)

The first strategy leads to decrease the number of inpatient (red line) by 0.229. At the same time a slight decrease in the recover and death cases (red area) by 0.066 and 0.087%, respectively.

Increase in the number of inpatient (grey line) by 0.146%, according to the second strategy. At the same time a slight increase the recover cases and a significant decrease the death cases (yellow area) by 0.092 and 0.59, respectively.

Overall, the speed of COVID-19 spread depends on many factors, such as health awareness, tourism, modern technology using. The ranking of countries is Malaysia, Argentina, Egypt, Hungary and Iraq. For example, one of the reasons of the speed of spreading of COVID-19 cases in Malaysia is a gathering of thousands of people in one place on March 14. The ranking of the countries is Iraq, Egypt, Malaysia, Argentina, and Hungary, according to the first strategy (see Fig. 6). Meanwhile, Malaysia, Argentina, Iraq, Hungary, and Egypt, represents the ranking, according to the second strategy (see Fig. 7).

The important point is to determine the effect of developing the medical capabilities on the number of the recover and death cases. Figures 6, 7 and 8 represent a change of percentages of inpatient (I), recovery (R) and death (D), according to the change in control variable's value.

From Fig. 6, a small change in the recovery percent (green line) means better health sector (Iraq), and vice versa (Hungary). Meanwhile, a big change in the death percent (red line) means better health sector (Malaysia), and vice versa (Egypt) (see Fig. 7. From Fig. 8, there is no change in the percent of inpatients as a result to the values of the two control variables are equal regardless of the signal.

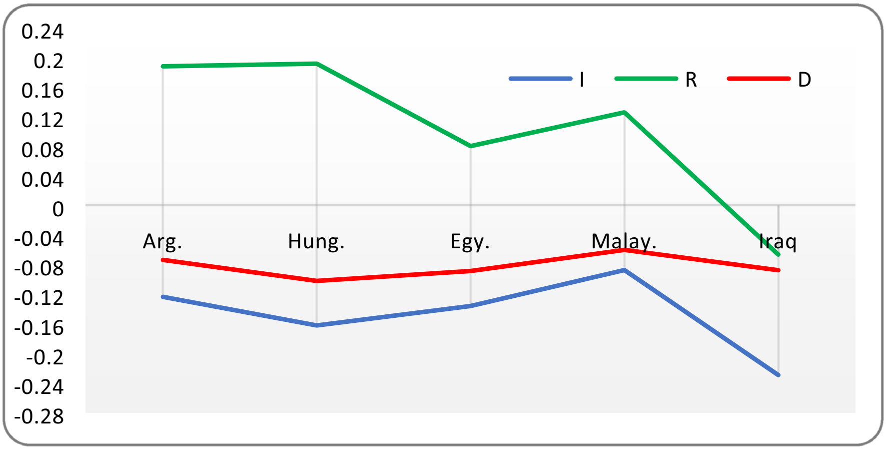

Fig. 6 Percent change, according to $x = 0.005$ and $y = 0$

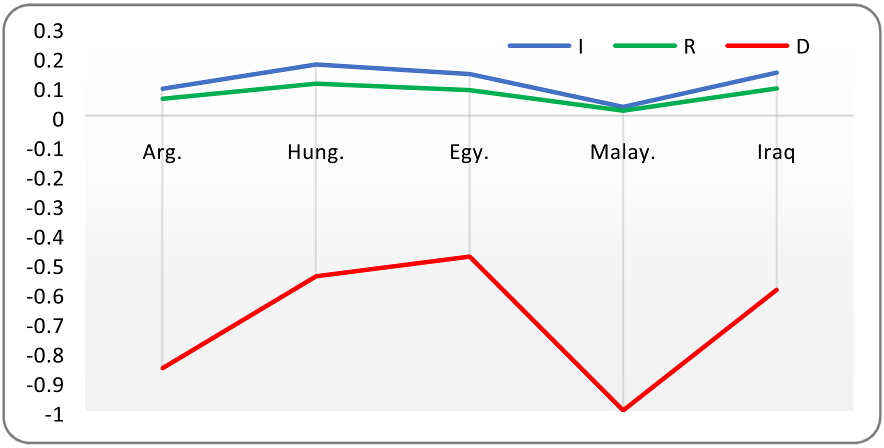

Fig. 7 Percent change, according to $x = 0$ and $y = Max(-0.005, -N)$

5 Conclusion

We developed an optimal control model of COVID-19 pandemic spreading. The health sector performance in Argentina, Hungary, Egypt, Malaysia, and Iraq, was discussed. The explicit solution of the model using the Pontryagin maximum principle was derived. At first, we formulated an optimal control model to fit the real case of COVID-19 in the countries. Control variables $x(t)$ and $y(t)$ representing the percentage of change in the rates of recovering and death, respectively. Zero value of the control variables representing the actual data of COVID-19 cases. Then, changed the value of the control variables, according to the three

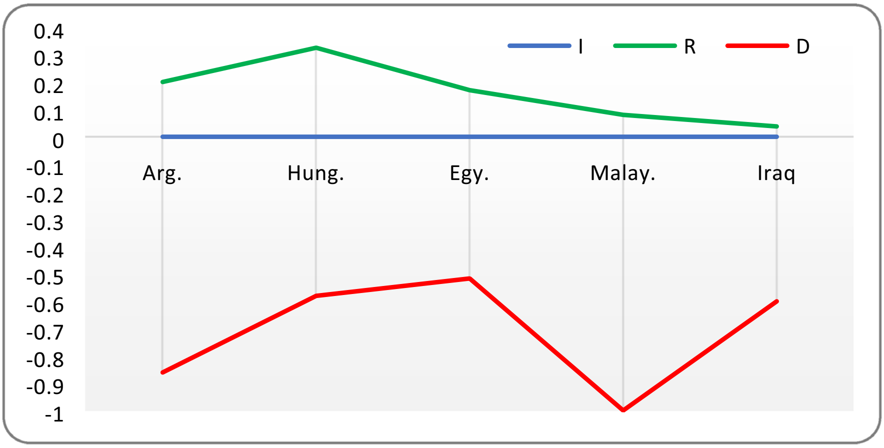

Fig. 8 Percent change, according to $x = 0.005$ and $y = Max(-0.005, -N)$

strategies of the health sector development. Finally, we determined the ranking of countries based on the change in the recovering and death percent.

It has been shown that speed of COVID-19 spreading (from 100 to 2750 cases) in Malaysia was the fastest with 22 days, and vice versa in Iraq with 55 days. The first strategy focused on increase the recovery cases by increase the medical capabilities by a specific percent. The ranking of the countries was Iraq, Egypt, Malaysia, Argentina, and Hungary. Meanwhile, the second strategy focused on reducing the death cases by increase the medical. The ranking was Malaysia, Argentina, Iraq, Hungary, and Egypt, according to the change percentage of the death cases. A change in the recovery and death cases without change in inpatient cases was the third strategy. Development the health sector by a specific percent (0.5%) led to reduce the death cases. Malaysia with the highest percent (100%) and Egypt with the lowest percent (48%). Meanwhile, 1.6% and 8.6% represented the increase in the recover cases in Malaysia and Egypt, respectively.

Our model can apply in other countries to determine the health sector performance and development percent that its need. Also, with other factors, such as isolation and communications, that can affect on pandemic spreading.

6 Availability of data and material

The datasets generated and/or analysed during the current study are available in the [https://www.worldometers.info/coronavirus/country/argentina/], [https://www.worldometers.info/coronavirus/country/hungary/], [https://www.coronatracker.com/country/hungary/], [https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports]

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Complaince with ethical standards

Conflict of interest The authors declare that they have no competing interests.

Appendix

See Tables 1, 2, 3, 4 and 5.

Table 1 The number of COVID-19 cases in Argentina[a]

Date	Cases	Recover	Death	N. cases	β	μ	N
14-Mar	128	3	3	31	0.245902	0	0
15-Mar	158	3	3	30	0.440789	0	0.006579
16-Mar	225	3	4	67	0.188073	0.110092	0
17-Mar	266	27	4	41	0.148936	0.102128	0
18-Mar	301	51	4	35	0.349593	0.004065	0.00813
19-Mar	387	52	6	86	0.349544	0	0.006079
20-Mar	502	52	8	115	0.196833	0.045249	0.00905
21-Mar	589	72	12	87	0.2	0	0.00198
22-Mar	690	72	13	101	0.090909	0	0.009917
23-Mar	745	72	19	55	0.114679	0.029052	0.001529
24-Mar	820	91	20	75	0.205924	0	0.004231
25-Mar	966	91	23	146	0.103286	0.014085	0.004695
26-Mar	1054	103	27	88	0.085498	0.008658	0.005411
27-Mar	1133	111	32	79	0.133333	0.008081	0.00404
28-Mar	1265	119	36	132	0.079279	0.009009	0.005405
29-Mar	1353	129	42	88	0.08291	0.010998	0.000846
30-Mar	1451	142	43	98	0.081359	0.00079	0.00237
31-Mar	1554	143	46	103	0.054212	0.032967	0.005128
1-Apr	1628	188	53	74	0.062725	0.009373	0.005047
2-Apr	1715	201	60	87	0.055021	0.013755	0.003439
3-Apr	1795	221	65	80	0.065606	0.004639	0.009278
4-Apr	1894	228	79	99	0.05104	0.006301	0.00189
5-Apr	1975	238	82	81	0.100906	0.039275	0.00423
6-Apr	2142	303	89	167	0.037714	0.016	0.000571
7-Apr	2208	331	90	66	0.038612	0.026301	0.004477
8-Apr	2277	378	98	69	0.08884	0.024431	0.002221
9-Apr	2437	422	102	160	0.070047	0.019341	0.005227
10-Apr	2571	459	112	134	0.049	0.0175	0.005
11-Apr	2669	494	122	98	0.043351	0.017048	0.000487
12-Apr	2758	529	123	89			

[a]https://www.worldometers.info/coronavirus/country/argentina/

Table 2 The number of COVID-19 cases in Hungary*

Date	Cases	Recover	Death	N. cases	β	μ	N
22-Mar	131	6	6	28	0.302521	0.12605	0.016807
23-Mar	167	21	8	36	0.144928	0	0.007246
24-Mar	187	21	9	20	0.248408	0	0.006369
25-Mar	226	21	10	39	0.179487	0.035897	0
26-Mar	261	28	10	35	0.174888	0.026906	0
27-Mar	300	34	10	39	0.167969	0	0.003906
28-Mar	343	34	11	43	0.218121	0	0.006711
29-Mar	408	34	13	65	0.108033	0	0.00554
30-Mar	447	34	15	39	0.113065	0.007538	0.002513
31-Mar	492	37	16	45	0.075171	0.006834	0.009112
1-Apr	525	40	20	33	0.129032	0.004301	0.002151
2-Apr	585	42	21	60	0.072797	0.001916	0.009579
3-Apr	623	43	26	38	0.099278	0.027076	0.01083
4-Apr	678	58	32	55	0.093537	0.013605	0.003401
5-Apr	733	66	34	55	0.017378	0.00158	0.006319
6-Apr	744	67	38	11	0.114241	0.00626	0.014085
7-Apr	817	71	47	73	0.111588	0.032904	0.015737
8-Apr	895	94	58	78	0.114401	0.002692	0.010767
9-Apr	980	96	66	85	0.256724	0.01956	0.013447
10-Apr	1190	112	77	210	0.11988	0.002997	0.007992
11-Apr	1310	115	85	120	0.09009	0.002703	0.012613
12-Apr	1410	118	99	100	0.040235	0.001676	0.008382
13-Apr	1458	120	109	48	0.043938	0.001627	0.010578
14-Apr	1512	122	122	54	0.052839	0.055205	0.009464
15-Apr	1579	192	134	67	0.05826	0.005587	0.006385
16-Apr	1652	199	142	73	0.084668	0.006102	0.010679
17-Apr	1763	207	156	111	0.050714	0.017143	0.011429
18-Apr	1834	231	172	71	0.057303	0.013277	0.01188
19-Apr	1916	250	189	82	0.046039	0.01151	0.00677
20-Apr	1984	267	199	68	0.075099	0.013175	0.009223
21-Apr	2098	287	213	114	0.043805	0.005006	0.007509
22-Apr	2168	295	225	70	0.070388	0.057646	0.008495
23-Apr	2284	390	239	116	0.059819	0.006647	0.006647
24-Apr	2383	401	250	99	0.034642	0.03291	0.006928
25-Apr	2443	458	262	60	0.033082	0.01567	0.005804
26-Apr	2500	485	272	57	0.047619	0.007458	0.00459
27-Apr	2583	498	280	83	0.036565	0.009972	0.006094
28-Apr	2649	516	291	66	0.042345	0.010858	0.004886
29-Apr	2727	536	300	78			

*https://www.worldometers.info/coronavirus/country/hungary/

*https://www.coronatracker.com/country/hungary/

Table 3 The number of COVID-19 cases in Egypt[a]

Date	Cases	Recover	Death	N. Cases	β	μ	N
15-Mar	126	26	2	16	0.408163	0	0.020408
16-Mar	166	26	4	40	0.220588	0	0.014706
17-Mar	196	26	6	30	0.085366	0.012195	0
18-Mar	210	28	6	14	0.261364	0	0.005682
19-Mar	256	28	7	46	0.131222	0.049774	0.004525
20-Mar	285	39	8	29	0.037815	0.008403	0.008403
21-Mar	294	41	10	9	0.135802	0.061728	0.016461
22-Mar	327	56	14	33	0.151751	0.046693	0.019455
23-Mar	366	68	19	39	0.129032	0.043011	0.003584
24-Mar	402	80	20	36	0.178808	0.049669	0.003311
25-Mar	456	95	21	54	0.114706	0.020588	0.008824
26-Mar	495	102	24	39	0.111111	0.03794	0.01626
27-Mar	536	116	30	41	0.102564	0.012821	0.015385
28-Mar	576	121	36	40	0.078759	0.026253	0.009547
29-Mar	609	132	40	33	0.107551	0.04119	0.002288
30-Mar	656	150	41	47	0.116129	0.015054	0.010753
31-Mar	710	157	46	54	0.136095	0.043393	0.011834
1-Apr	779	179	52	69	0.156934	0.040146	0.010949
2-Apr	865	201	58	86	0.19802	0.024752	0.013201
3-Apr	985	216	66	120	0.12091	0.035562	0.007112
4-Apr	1070	241	71	85	0.135884	0.007916	0.009235
5-Apr	1173	247	78	103	0.175708	0.014151	0.008255
6-Apr	1322	259	85	149	0.130879	0.017382	0.009202
7-Apr	1450	276	94	128	0.101852	0.026852	0.008333
8-Apr	1560	305	103	110	0.12066	0.037326	0.013021
9-Apr	1699	348	118	139	0.077048	0.029197	0.013788
10-Apr	1794	384	135	95	0.113725	0.032941	0.008627
11-Apr	1939	426	146	145	0.085589	0.015362	0.00951
12-Apr	2056	447	159	117	0.092414	0.028276	0.003448
13-Apr	2190	488	164	134	0.104031	0.016905	0.009103
14-Apr	2350	514	178	160	0.093486	0.023522	0.003016
15-Apr	2505	553	183	155	0.094969	0.024308	0.007349
16-Apr	2673	596	196	168			

[a]The ministry of health/Egypt

Table 4 The number of COVID-19 cases in Malaysia[a]

Date	Cases	Recover	Death	N. Cases	β	μ	N
10-Mar	129	25	0	12	0.192308	0.009615	0
11-Mar	149	26	0	20	0.073171	0.04878	0
12-Mar	158	32	0	9	0.309524	0.031746	0
13-Mar	197	36	0	39	0.254658	0	0
14-Mar	238	36	0	41	0.940594	0.029703	0
15-Mar	428	42	0	190	0.323834	0	0
16-Mar	553	42	0	125	0.234834	0.013699	0.003914
17-Mar	673	49	2	120	0.188103	0.017685	0.001608
18-Mar	790	60	3	117	0.151307	0.020633	0
19-Mar	900	75	3	110	0.158151	0.014599	0.001217
20-Mar	1030	87	4	130	0.162939	0.028754	0.00426
21-Mar	1183	114	8	153	0.115928	0.023563	0.001885
22-Mar	1306	139	10	123	0.183232	0.017286	0.003457
23-Mar	1518	159	14	212	0.07881	0.017844	0.000743
24-Mar	1624	183	15	106	0.120617	0.01122	0.002805
25-Mar	1796	199	19	172	0.148923	0.010139	0.002535
26-Mar	2031	215	23	235	0.072504	0.02454	0.001673
27-Mar	2161	259	26	130	0.084755	0.032516	0.000533
28-Mar	2320	320	27	159	0.076026	0.046123	0.003548
29-Mar	2470	411	34	150	0.077037	0.03358	0.001481
30-Mar	2626	479	37	156	0.066351	0.027488	0.002844
31-Mar	2766	537	43	140			

[a]The ministry of health/Malaysia

Table 5 The number of COVID-19 cases in Iraq[a]

Date	Cases	Recover	Death	N. Cases	β	μ	N
17-Mar	165	43	12	11	0.10909	0.054545	0
18-Mar	177	49	12	12	0.12931	0	0.008621
19-Mar	192	49	13	15	0.12308	0	0.030769
20-Mar	208	49	17	16	0.04225	0.021127	0
21-Mar	214	52	17	6	0.13103	0.034483	0.02069
22-Mar	233	57	20	19	0.21154	0.032051	0.019231
23-Mar	266	62	23	33	0.27624	0.071823	0.022099
24-Mar	316	75	27	50	0.14019	0.130841	0.009346
25-Mar	346	103	29	30	0.16822	0.009346	0.03271
26-Mar	382	105	36	36	0.31535	0.070539	0.016598
27-Mar	458	122	40	76	0.16216	0.030405	0.006757
28-Mar	506	131	42	48	0.12312	0.036036	0
29-Mar	547	143	42	41	0.22928	0.024862	0.01105
30-Mar	630	152	46	83	0.14815	0.041667	0.009259
31-Mar	694	170	50	64	0.07173	0.025316	0.004219
1-Apr	728	182	52	34	0.08907	0.040486	0.004049
2-Apr	772	202	54	44	0.09302	0.046512	0
3-Apr	820	226	54	48	0.10741	0.061111	0.003704
4-Apr	878	259	56	58	0.14742	0	0.008881
5-Apr	961	259	61	83	0.1092	0.132605	0.00468
6-Apr	1031	344	64	70	0.14607	0.046549	0.001605
7-Apr	1122	373	65	91	0.11696	0.115497	0.005848
8-Apr	1202	452	69	80	0.04405	0.064611	0
9-Apr	1232	496	69	30	0.07046	0.08096	0.001499
10-Apr	1279	550	70	47	0.05918	0.07739	0.003035
11-Apr	1318	601	72	39	0.05271	0.060465	0.006202
12-Apr	1352	640	76	34	0.04088	0.121069	0.003145
13-Apr	1378	717	78	26	0.03774	0.084048	0
14-Apr	1400	766	78	22	0.02698	0.082734	0.001799
15-Apr	1415	812	79	15	0.03626	0.083969	0.001908
16-Apr	1434	856	80	19	0.09639	0.100402	0.002008
17-Apr	1482	906	81	48	0.06263	0.094949	0.00202
18-Apr	1513	953	82	31	0.05439	0.117155	0
19-Apr	1539	1009	82	26	0.07813	0.075893	0
20-Apr	1574	1043	82	35	0.06236	0.11804	0.002227
21-Apr	1602	1096	83	28	0.06856	0.118203	0
22-Apr	1631	1146	83	29	0.11443	0.062189	0
23-Apr	1677	1171	83	46	0.07329	0.078014	0.007092
24-Apr	1708	1204	86	31	0.13158	0.047847	0.002392
25-Apr	1763	1224	87	55	0.12611	0.086283	0
26-Apr	1820	1263	87	57	0.05745	0.048936	0.002128
27-Apr	1847	1286	88	27	0.17125	0.069767	0.004228

Table 5 (continued)

Date	Cases	Recover	Death	N. Cases	β	μ	N
28-Apr	1928	1319	90	81	0.14451	0.052023	0.003854
29-Apr	2003	1346	92	75	0.14513	0.051327	0.00177
30-Apr	2085	1375	93	82	0.11021	0.063209	0.001621
1-May	2153	1414	94	68	0.10233	0.091473	0.00155
2-May	2219	1473	95	66	0.11828	0.026114	0.003072
3-May	2296	1490	97	77	0.07052	0.076164	0
4-May	2346	1544	97	50	0.12057	0.038298	0.007092
5-May	2431	1571	102	85	0.06464	0.040897	0
6-May	2480	1602	102	49	0.08119	0.030928	0
7-May	2543	1626	102	63	0.07362	0.042945	0.002454
8-May	2603	1661	104	60	0.09069	0.048926	0.00358
9-May	2679	1702	107	76	0.10115	0.036782	0.002299
10-May	2767	1734	109	88			

[a]The ministry of health/Iraq

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