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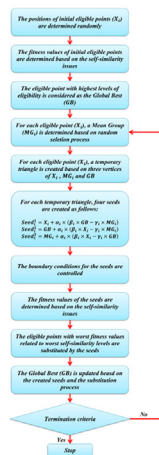
Shape and size optimization of truss structures by Chaos game optimization considering frequency constraints

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HIGHLIGHTS

- Shape and size Optimization of truss Structures is considered.
- Chaos Game Optimization (CGO) is utilized for optimization purposes.
- Benchmark 10-bar, 37-bar, 52-bar, 72-bar and 120-bar truss structures are utilized.

GRAPHICAL ABSTRACT



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ABSTRACT

Introduction: An engineering system consists of properly established activities and put together to achieve a predefined goal. These activities include analysis, design, construction, research, and development. Designing and constructing structural systems, including buildings, bridges, highways, and other complex systems, have been developed over the centuries. However, the evolution of these systems has been prolonged because the overall process is very costly and time-consuming, requiring primary human and material resources to be utilized. One of the options for overcoming these shortcomings is the utilization of metaheuristic algorithms as recently developed intelligent techniques. These algorithms can be utilized as upper-level search techniques for optimization procedures to achieve better results. **Objectives:** Shape and size optimization of truss structures are considered in this paper utilizing the Chaos Game Optimization (CGO) as one of the recently developed metaheuristic algorithms. The principles of chaos theory and fractal configuration are considered inspirational concepts. **Methods:** For the numerical purpose, the 10-bar, 37-bar, 52-bar, 72-bar, and 120-bar truss structures as four of the benchmark problems in this field are considered as design examples in which the frequency constraints are considered as limits that have to be dealt with during the optimization procedure. Multiple optimization runs are also conducted for having a comprehensive statistical analysis, while a comparative investigation is also conducted with other algorithms in the literature.

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Results: Based on the results of the CGO and other approaches from the literature, the CGO can provide better and competitive results in dealing with the considered truss design problems.

Conclusion: In summary, the CGO can provide better solutions in dealing with the considered real-size structural design problems with higher levels of complexity.

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Introduction

Over the past decades, human beings have put so much effort into maximizing the use of limited available resources. For example, one challenge is selecting design variables to consider design constraints in engineering designs and having the lowest construction and material costs. In fact, the main goal is to properly meet the basic and advanced design standards by considering the project's economic aspects. Recent advances in structural engineering reveal the need to consider greater accuracy, better performance, and higher construction speeds in the design of structural systems. Therefore, to address each of the above factors it is necessary to introduce new methods for design and optimization and implement them on complex and real-world systems. Optimization problems normally search for the minimum values of a cost function to systematically select the values for the variables that lead to the lowest cost. Metaheuristic algorithms are optimization methods that combine global and local search techniques to get the answers as close as possible to the optimal answer. Indeed, metaheuristic algorithms are types of approximate optimization algorithms capable of providing acceptable solutions and avoiding entrapment in local optimal points. Firefly Algorithm (FA) [1], Genetic Algorithm (GA) [2], Material Generation Algorithm (MGA) [3], Cuckoo Search Algorithm (CSA) [4], Chaos Game Optimization (CGO) [5,6], Slime Mould Algorithm (SMA) [7], Atomic Orbital Search (AOS) [8], Particle Swarm Optimizer (PSO) [9], and Crystal Structure Algorithm (CSA) [10] are some of the recently developed metaheuristic algorithms. Nevertheless, the application of these algorithms alongside the improved or hybrid versions has been investigated in different fields. Investigation of Lévy flight distribution for engineering optimization [11], optimum design of engineering problems with dynamic differential annealed optimization [12], optimum design of reinforced concrete footings with metaheuristic algorithms [13], investigation of nature-inspired algorithms for getting of bridge scour information [14], performance assessments of an artificial bee colony in optimal design of steel skeletal structures [15], design optimization of reinforced concrete building structures with metaheuristics [16], and estimation of solar photovoltaic cell parameters with a new stochastic slime mould metaheuristic algorithm [17], are some of the recent researches in this field.

Designing a well-established system is an iterative procedure in which the designer's experience, understanding, and artistry are essential for designing systems with better performance in most engineering fields. The iterative procedure aims to analyze several experimental systems one by one before an acceptable design can be obtained. Engineers strive to design the best systems, while the meaning of 'best' for different systems varies according to their characteristics. In general, the best system is a less expensive, more efficient, more reliable, and more durable system. In recent decades, these goals have been accomplished by utilizing metaheuristic algorithms in design procedures in which an iterative optimization procedure is conducted to achieve a system with better performance. Xia, Zhang, Xia and Shi [18] utilized a bi-directional evolutionary strategy for structural topology optimization of stress-based structures. Abd Elrehim, Eid and Sayed [19] investigated the optimization of concrete structural systems,

including the arch bridges through the GAs. Ho-Huu, Hartjes, Visser and Curran [20] developed a multi-objective optimization algorithm for structural optimization purposes. Sakata, Suzuki and Ben [21] utilized GA as an intelligent technique for optimum design of structural CFRP isogrid cylindrical shell systems. Brütting, Senatore, Schevenels and Fivet [22] developed mixed-integer linear programming for the optimal design of frame structures with stock constraints. Aydogdu, Carbas and Akin [23] discussed the overall efficiencies of Levy Flight as a stochastic procedure for performance improvements of the metaheuristic algorithms in structural optimization. Artar and Daloglu [24] utilized the Jaya algorithm to optimize steel space truss towers by considering the seismic effects. Fenu, Marano, Congiu and Briseghella [25] investigated the optimum design of an arched truss by applying horizontal and vertical multi-load cases. Kok, Lau, Phan and Ting [26] used GA for optimum steel residential roof truss design with cold-formed sections. Li and Xu [27] developed an improved wolf pack algorithm to optimize truss structures. Some of the recent challenges in the structural optimization area are modified subpopulation teaching-learning-based algorithms for topology optimization of truss problems [28], optimum structural design by the adaptive version of the symbiotic organisms search (SOS) method [29], topology optimization with different metaheuristics [30], Structural Optimization with plasma generation optimizer [31], and some other research [32–38]. Furthermore, some studies have considered this method in medicine. Jajarmi, Baleanu, Zarghami Vahid and Mobayen [39] examined immunogenic tumor dynamics' asymptotic behavior using a novel fractional model, which was built using the general fractional operators approach. The fulfillment of the control goal, according to the authors, is corroborated by certain simulation findings since the controlled variables follow the tumor-free steady state in all actual scenarios. Baleanu, Zibaei, Namjoo and Jajarmi [40] proposed and investigated a new fractional chaotic system with quadratic and cubic nonlinearities. To build the novel model and explore its chaotic behavior in both the time domain and the phase plane, the authors used an efficient nonstandard finite difference (NSFD) approach. In another study, Baleanu, Hassan Abadi, Jajarmi, Zarghami Vahid and Nieto [41] made an introduction for the COVID-19 pandemic where a broader version of fractional models was developed, which included the impacts of isolation and quarantine. Based on the findings, the authors concluded that a specific instance of the general fractional formula fit the actual data better than the other classical and fractional models.

In this paper, shape and size optimization of truss structures are considered through the Chaos Game Optimization (CGO) as one of the recently developed metaheuristic algorithms by Talatahari and Azizi [6]. The principles of chaos theory and the configuration of fractals are utilized as inspirational concepts. Metaheuristic algorithms have been utilized for optimization purposes in various areas, but the capability of these metaheuristics is entirely dependent on the preciseness of the mathematical presentation of the considered system problems. In most cases, the metaheuristic algorithms require a proper definition of the supposed problem, including a clear mathematical formulation. So considering these algorithms for optimum design purposes will lead to appropriate optimum design if the experts can interpret the system problem

and implement it in the optimization problem. However a proper selection of the metaheuristic algorithm is another issue. Most of the time, the optimum results are unsatisfactory because the selected metaheuristic algorithm is not well-formulated and is unable to conduct query through the search space. To summarize, the applicability of the metaheuristic algorithms in different problems could be deemed as a primary challenge by means of these two aspects that should be selected and determined wisely. Since most of the novel optimization algorithms are evaluated through mathematical test problems and simple engineering design, the capability of these methods should be assessed in dealing with complex optimization problems to have a better perspective on the overall capability of the algorithm. In this regard, the applicability of the CGO is evaluated in dealing with truss optimization problems. For the numerical purpose, the 10-bar, 37-bar, 52-bar, 72-bar, and 120-bar truss structures as five of the benchmark problems in this field are considered design examples. The frequency constraints are considered limits that must be dealt with during the optimization procedure. Multiple optimization runs are also conducted for having a comprehensive statistical analysis, while a comparative investigation is also conducted with other algorithms in the literature. The purpose of this paper is to evaluate the feasibility of using the CGO algorithm in dealing with intricate optimization problems such as the optimum design of truss structures, which due to the novelty of this algorithm, has not been used by researchers in recent years.

The rest of the paper is divided into the following sections. In section 2, the inspiration and mathematical model of the CGO algorithm are presented, and also the problem statement of the study is indicated in section 3. In sections 4 and 5, design examples, including 10, 37, 52, 72, 120-bar truss structures, alongside numerical investigations of the mentioned structures are illustrated. Finally in section 6, the core findings of this study are presented as concluding remarks.

Chaos Game optimization (CGO)

As one of the main branches of mathematics, chaos theory concerns the investigation of dynamic systems in which chaotic and unordered states are seemingly random, but in practice are governed by hidden patterns and definite rules with higher levels of sensitivity to the preliminary conditions of the considered system. Chaos theory refers to an interdisciplinary branch that states the interconnections, feedback loops, patterns, repetition, fractals, self-similarity, and self-organization in complex systems regardless of the apparent randomness in the general aspects of these systems. The butterfly effect is one of the basic principles of chaos theory, which denotes how small changes in a definite, nonlinear complex system can lead to significant differences in other subsystems. A metaphor for this behavior is a butterfly flying in Texas that could create a storm in China.

A fractal is a geometric structure obtained by enlarging each part of a structure in a predefined proportion to the original structure. Alternatively, a fractal is a structure with the same configuration as the whole and is seen the same from both far and near, which is called self-similarity. A Sierpiński fractal is generally an equilateral triangle that is inversely divided into smaller equilateral triangles. This fractal is one of the basic examples of the self-similar set, and it is named in honor of Polish mathematician Sierpiński. However, it was used as a decorative pattern centuries earlier.

The CGO is mathematically formulated based on the self-similarity aspects of fractals in chaos theory and the general aspects of generating the Sierpiński triangle. First, an initialization

procedure is configured as follows by determining the solution candidates' initial positions (X_i) inside the predefined search space, which is assumed to be a Sierpiński triangle:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} x_1^1 x_1^2 \cdots x_1^j \cdots x_1^d \\ x_2^1 x_2^2 \cdots x_2^j \cdots x_2^d \\ \vdots \\ x_i^1 x_i^2 \cdots x_i^j \cdots x_i^d \\ \vdots \\ x_n^1 x_n^2 \cdots x_n^j \cdots x_n^d \end{bmatrix}, \begin{cases} i = 1, 2, \dots, n. \\ j = 1, 2, \dots, d. \end{cases} \quad (1)$$

$$x_i^j(0) = x_{i,min}^j + rand. (x_{i,max}^j - x_{i,min}^j), \begin{cases} i = 1, 2, \dots, n. \\ j = 1, 2, \dots, d. \end{cases} \quad (2)$$

where d is the problem's dimension; n is the total number of initialized candidates inside the search space (Sierpinski triangle); $x_i^j(0)$ is the j th design variable in the i th point inside search space; $x_{i,min}^j$ and $x_{i,max}^j$ denote the lower and upper bounds of the decision variables; $rand$ represents a randomly created number in the range of $[0,1]$.

The leading search loop of the CGO is configured by considering that the initially created points are moved to change their positions to achieve the complete shape of a Sierpinski triangle. For this purpose, each of the points or solution candidates inside the triangle is joined by two other points to form a temporary triangle. These points are the Global Best (GB) vector which represents the best solution candidate found in the search space so far, and the Mean Group (MG_i) which is achieved by considering the means of a bunch of points selected randomly near the considered solution candidate (i th point).

For each of the temporary triangles inside the search space, a position updating process is conducted by employing three individual seeds positioned in the three points of the triangles. For the seed positioned in the point of i th candidate (X_i), a dice is utilized with three red and three green faces. When the dice is rolled, if the green face is shown, the seed is moved towards the global best solution as GB while for the red face, a movement towards the i th mean group as MG_i is determined. This aspect is mathematically modeled by generating two random integers between 0 and 1 in which the possibility of generating two equal integers is also determined, so the seed can also move along the connected line between the GB and MG_i . This aspect is shown in Fig. 1.a while the mathematical representation is as follows:

$$Seed_i^1 = X_i + \alpha_i \times (\beta_i \times GB - \gamma_i \times MG_i), i = 1, 2, \dots, n. \quad (3)$$

where $Seed_i^1$ is the seed positioned in the point of i th solution candidate; GB represents the global best; MG_i represents the i th candidate's mean group; α_i is the movement limitation factor; β_i , and γ_i are vectors including randomly created numbers in the range of $[0, 1]$.

For the seed positioned in the point of the global best solution as GB , a dice is utilized with three red and three blue faces. When the dice is rolled, if the blue face is shown, the seed is moved towards the i th candidate (X_i) while for the red face, a movement towards the i th mean group as MG_i is determined. By considering the possibility of generating two equal integers, the seed can also move along the connected line between the X_i and MG_i . This aspect is shown in Fig. 1.b while the mathematical representation is as follows:

$$Seed_i^2 = GB + \alpha_i \times (\beta_i \times X_i - \gamma_i \times MG_i), i = 1, 2, \dots, n. \quad (4)$$

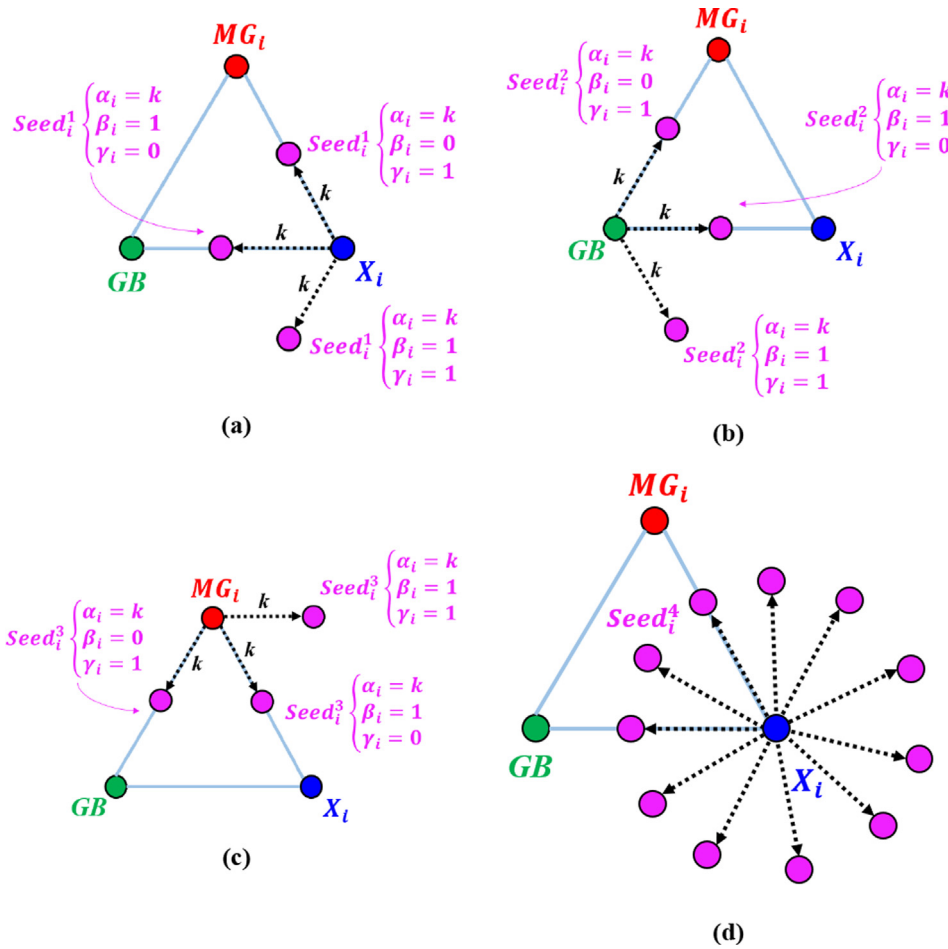


Fig. 1. Position updating process for the temporary triangles [6].

where $Seed_1^i$ is the seed positioned in the point of GB as the global best; MG_i represents the i th candidate's mean group (X_i); α_i is the movement limitation factor; β_i , and γ_i are vectors including randomly created numbers in the range of [0, 1].

For the seed positioned in the point of the mean group as MG_i , a dice is utilized with three blue and three green faces. When the dice is rolled, if the blue face is shown, the seed is moved towards the i th candidate (X_i) while for the green face, a movement towards the GB is determined. By considering the possibility of generating two equal integers, the seed can also move along the connected line between the X_i and GB . This aspect is shown in Fig. 1.c while the mathematical representation is as follows:

$$Seed_3^i = MG_i + \alpha_i \times (\beta_i \times X_i - \gamma_i \times GB), i = 1, 2, \dots, n. \quad (5)$$

where $Seed_3^i$ is the seed positioned at the point of MG_i as the i th candidate's mean group (X_i); GB is the global best; α_i is the movement limitation factor; β_i , and γ_i are vectors including randomly created numbers in the range of [0, 1].

To enhance the mutation phase of the CGO a fourth seed is considered, which is deemed for position updating purposes positioned in the point of the i th candidate (X_i) and is moved randomly and freely in the search space. This aspect is shown in Fig. 1.d while the mathematical representation is as follows:

$$Seed_4^i = X_i(x_i^k = x_i^k + R), k = [1, 2, \dots, d]. \quad (6)$$

where $Seed_4^i$ is the seed positioned in the point of i th candidate (X_i); R is a vector with random numbers in the range of [0, 1].

The movement limitation factor as α_i is delicately implemented into the position updating process to tune the exploration and exploitation rate of the CGO, which is determined randomly by choosing one of the following scenarios:

$$\alpha_i = \begin{cases} Rand \\ 2 \times Rand \\ (\delta \times Rand) + 1 \\ (\varepsilon \times Rand) + (\varepsilon) \end{cases} \quad (7)$$

where $Rand$ represent a random number which is distributed uniformly in the range of [0,1]; δ and ε are two random integers in the range of [0,1].

Problem statement

In this section, the general formulation of the structural design optimization problems is presented in which a weight minimization procedure is conducted by considering the frequency design constraints. For objective function, the overall weight of the structure is determined. At the same time, the cross-sectional areas of the structural elements are considered as the design variables in size optimization problems, and the nodal coordinates of structures are determined as the design variables in the shape optimization problems. In the problems in which the shape and size optimization procedures are considered simultaneously, both of these aspects are utilized as decision variables in the structural optimization procedure. The aspects are mathematically formulated as follows:

$$Weight(\mathbf{A}, \mathbf{X}) = \sum_{i=1}^e \rho_i L_i(x_i) A_i, i = 1, 2, \dots, n. \tag{8}$$

$$\omega_j \geq \omega_j^*, j = 1, 2, \dots, p. \tag{9}$$

$$\omega_k \leq \omega_k^*, k = 1, 2, \dots, p. \tag{10}$$

$$A_l^{low} \leq A_l \leq A_l^{up}, l = 1, 2, \dots, n. \tag{11}$$

$$x_m^{low} \leq x_m \leq x_m^{up}, m = 1, 2, \dots, r. \tag{12}$$

where \mathbf{A} represents a vector including the design variables for the cross-sectional areas of the structural elements with a total number of n structural elements; \mathbf{X} represents a vector including the design variables for the nodal coordinates of the structure with a total number of r nodes; ρ_i is the density of the utilized material; L_i is the length of the structural elements; ω_j and ω_k are the j th and k th natural frequency of the structure with a total number of p frequencies; ω_j^* and ω_k^* are the lower bound and upper bound of the j th and k th natural frequency of the structure, respectively; A_l^{low} and A_l^{up} are the lower bound and upper bound of the design variables regarding the cross-sectional area of the l th member (A_l), respectively; x_m^{low} and x_m^{up} are the lower and upper bounds of the design variables for nodal coordinates of the m th node (x_m) in the structure, respectively.

Since structural design optimization is a constraint optimization problem, a well-defined constraint handling approach should be utilized for conducting the optimization procedure. For this purpose, the penalty constraint handling method is utilized in this paper with a penalty function as follows:

$$f_{penalty}(\mathbf{A}) = (1 + \varepsilon_1 \cdot v)^{\varepsilon_2} \times Weight(\mathbf{A}, \mathbf{X}) \tag{13}$$

$$v = \sum_{i=1}^q \max\{0, g_i(\mathbf{A}, \mathbf{X})\} \tag{14}$$

where v is the summation of the violated design constraints; q represents the total number of design constraints; ε_1 and ε_2 denotes the control values for determining the penalty value during the process of optimization; and $g_i(\mathbf{A})$ represents the i th design constraint.

As presented in Eqs. (13) and (14), the penalty handling constraint method is utilized in this paper for constraint handling purposes, in which a summation of inequality constraint is considered. So multiple natural frequency constraints are determined in this paper by means of doing a summation for the supposed first five natural frequency of the structure.

Design examples

10-bar truss structure

This truss structure is the first size optimization problem in this paper which has ten members and six nodes with constraint limitations of 7, 15, and 20 Hz for the first three natural frequencies of the structure. $6.89 \times 10^{10} \text{N/m}^2$ is considered as the modulus of elasticity, and the density of the utilized steel material is 2770 kg/m^3 . The lower bound for the structural elements' cross-sectional area is set to $0.645 \times 10^{-4} \text{ m}^2$, while the upper bound is determined as $50 \times 10^{-4} \text{ m}^2$. The added mass to the free nodes is 454 kg. The schematic view of this structure is illustrated in Fig. 2.a.

37-bar truss structure

The second design example in this paper is a truss structure with 37 structural members and 20 nodes in which a simultaneous process of size and shape optimization in the structure is considered. The modulus of elasticity for the material is set to $2.1 \times 10^{11} \text{N/m}^2$ and the density of the utilized steel material is 7800 kg/m^3 . The lower bound for the structural elements' cross-sectional area is set to 0.0001 m^2 , while the upper bound is determined as 0.001 m^2 . The added mass to the free nodes is 10 kg. The constraint limitations of 20, 40, and 60 Hz for the first three natural frequencies of the structure are considered, while a total number of 19 design variables are determined for size the (14) and shape (5) optimization of the structure. The schematic view of this structure is illustrated in Fig. 2.b.

52-bar truss structure

This design example is the second shape and size optimization problem in this paper, which has 52 structural members and 21 nodes. The optimization processes are conducted simultaneously by considering five shape and eight sizing design variables. All of the free nodes in the structure are free to move within a maximum allowable tolerance of $\pm 2 \text{ m}$, while the constraint limitations of 15.961 and 28.648 Hz for the first two natural frequencies of the structure are considered. The modulus of elasticity is $2.1 \times 10^{11} \text{N/m}^2$, and the density of the utilized steel material is 7800 kg/m^3 . The lower bound is set to 0.0001 m^2 and the upper bound is determined as 0.001 m^2 for the cross-sectional area of the structural elements, while the added mass to the free nodes is 50 kg. The schematic view of this structure is illustrated in Fig. 2.c.

72-bar truss structure

This truss structure is the next size optimization problem in this paper which has 72 members and 20 nodes with constraint limitations of 4 and 6 Hz for the first and third natural frequencies of the structure. $6.89 \times 10^{10} \text{N/m}^2$ is determined as the modulus of elasticity, while the density of the utilized steel material is 2770 kg/m^3 . The lower bound is set to $0.645 \times 10^{-4} \text{ m}^2$ while the upper bound for the cross-sectional area of the structural elements is $20 \times 10^{-4} \text{ m}^2$. The added mass to the four top nodes of the structure is 2270 kg. The schematic view of this structure is illustrated in Fig. 2.d.

120-bar truss structure

The 120-bar truss problem is one of the complex size optimization problems in which node 1 has non-structural masses of 3000 kg, nodes 2 to 13 have 1500 kg, and 100 kg is determined for the rest of the nodes. The constraint limitations of 9 and 11 Hz are considered for the first two natural frequencies of the structure. $2.1 \times 10^{11} \text{N/m}^2$ is set as the modulus of elasticity, and the density of the utilized steel material is set to 7971.81 kg/m^3 . The lower bound for the cross-sectional area of the structural members is set to 0.0001 m^2 , while the upper bound is considered 0.01293 m^2 . The complete description of the loading scenario and other characteristics of this problem are provided in the literature, and the schematic view of this structure is illustrated in Fig. 2.e.

Numerical investigations

The specific structural details of the considered truss design problems are presented in this section alongside the best results

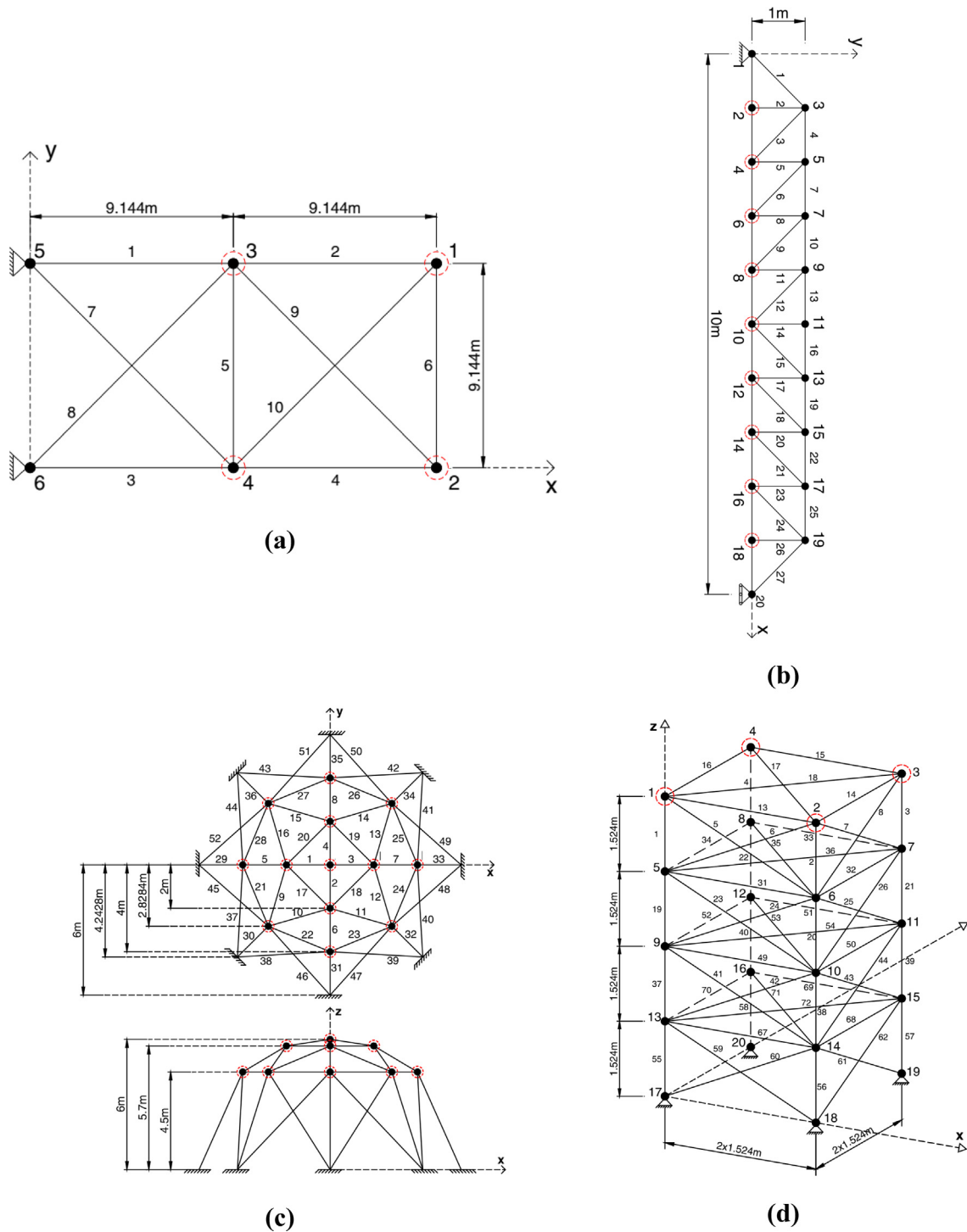


Fig. 2. Schematic presentation of the considered truss design examples.

and convergence histories of the optimization procedures. For statistical purposes, 30 independent optimization runs are conducted in each case. The results of the CGO are compared with other metaheuristic approaches in the literature for having a valid judgment.

10-bar truss structure

The convergence history of the CGO in dealing with the 10-bar truss design example is illustrated in Fig. 3.a, where the conver-

gence curves for the best and worst optimization runs alongside the mean of 30 independent runs.

In table 1, the best results of the CGO and other metaheuristic approaches regarding the conducted multiple optimization runs, optimum design variables, and the statistical results for the 10-bar truss problem are presented. The CGO can reach 524.4545 kg, which is better than the previously calculated weights of 524.4627 kg by Ho-Huu, Vo-Duy, Luu-Van, Le-Anh and Nguyen-Thoi [42] utilizing the Improved Differential Evolution (IDE) algo-

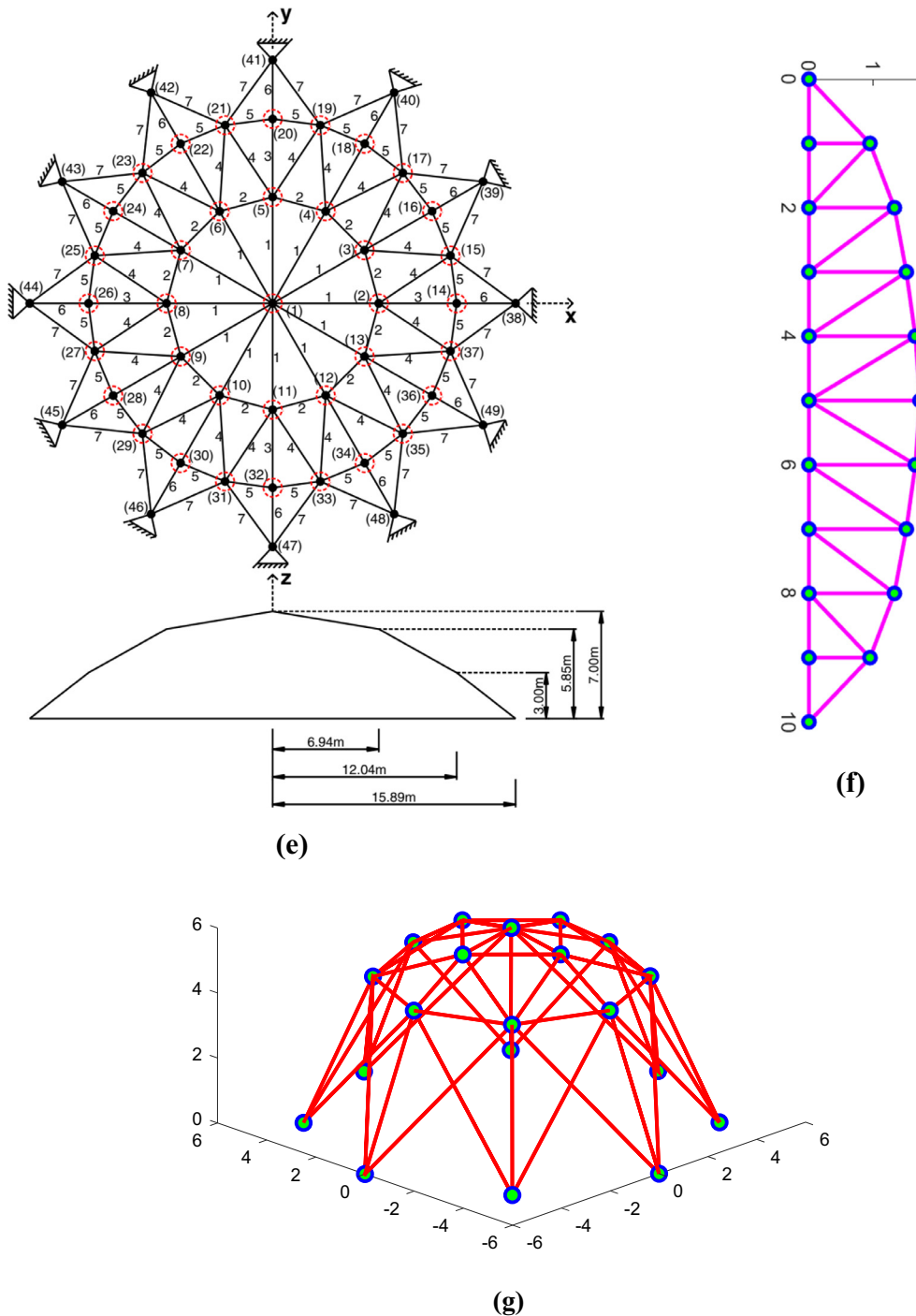


Fig. 2 (continued)

rithm. The CGO can provide 524.5099 kg as the mean of 30 independent runs with 524.7488 kg as the worst run, which are the best statistical results among other approaches.

Based on the fact that frequency constraints have to be satisfied during the truss optimization process, the first five natural frequencies of the CGO presented in Table 1 are derived from the best results of 30 conducted optimization runs. The results of other algorithms have also been provided from the literature. The capability of the CGO satisfying the constraints alongside other methods is demonstrated by considering the constraint limitations of 7, 15, and 20 Hz for the first three natural frequencies of the structure.

37-bar truss structure

In Fig. 3.b, the convergence results of the CGO are presented considering the 37-bar truss design example. In this figure, the best and worst optimization runs alongside the means of multiple runs are provided accordingly. Regarding the fact that a simultaneous procedure of shape and size optimization is conducted in this problem, the final optimal shape of the structure is illustrated in Fig. 2.f where the overall shape of the structure is different from the one presented in Fig. 2.b.

The best results of CGO in dealing with the 37-bar truss problem are presented in Table 2, where 30 independent optimization

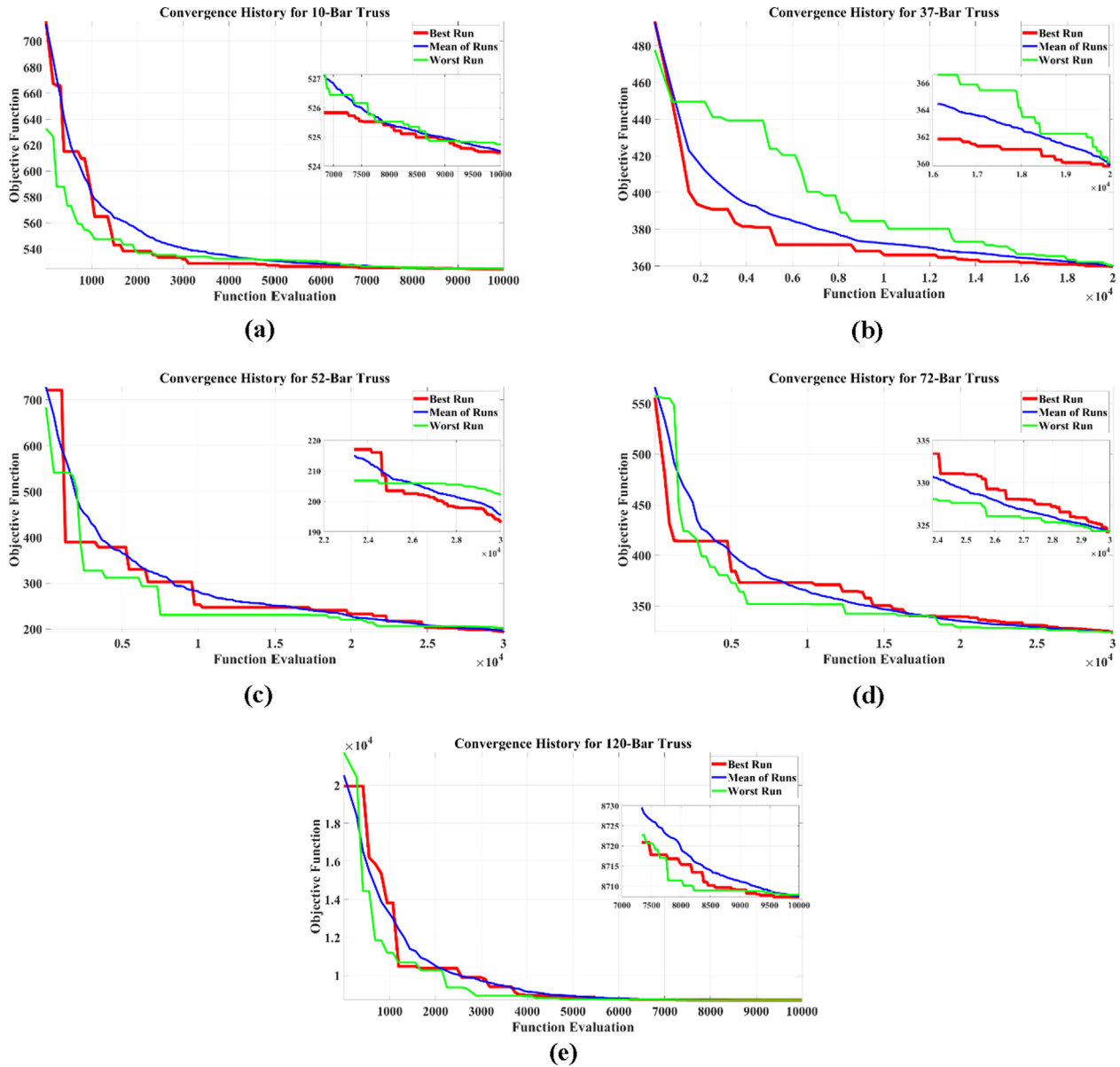


Fig. 3. Convergence history of CGO for 10-, 37-, 52-, 72- and 120-bar truss structures.

Table 1
Results of CGO and other approaches in dealing with the 10-bar truss problem.

Element Number	Gomes [43]	Miguel and Fadel Miguel [44]	Kaveh and Zolghadr [45]	Zuo et al. [46]	Ho-Huu et al. [42]	CGO
1	37.712	36.198	35.944	37.284	35.06057	35.1817
2	9.959	14.030	15.530	9.445	14.68508	14.6761
3	40.265	34.754	35.285	35.051	35.06875	35.0741
4	16.788	14.900	15.385	19.262	14.80946	14.6346
5	11.576	0.654	0.648	2.783	0.645136	0.6450
6	3.955	4.672	4.583	5.450	4.557799	4.5651
7	25.308	23.467	23.610	19.041	23.52708	23.8048
8	21.613	25.508	23.599	27.939	23.79982	23.7043
9	11.576	12.707	13.135	14.95	12.50381	12.3864
10	11.186	12.351	12.357	10.361	12.45989	12.4282
Weight (kg)	537.98	531.28	532.39	535.73	524.4627	524.4545
Worst weight (kg)	-	-	-	-	530.8448	524.7488
Average weight (kg)	540.89	535.07	537.8	-	525.6162	524.5099
Standard deviation	6.84	3.64	4.02	-	2.3041	0.0643
Frequency 1	7.0000	7.0002	7.0000	7.0007	7.0000	7.0000
Frequency 2	17.7860	16.1640	16.1870	17.030	16.1853	16.1896
Frequency 3	20.0000	20.0029	20.0000	20.156	20.0000	20.0000
Frequency 4	20.0630	20.0221	20.0210	-	20.0006	20.0001
Frequency 5	27.7760	28.5428	28.4700	-	28.5775	28.5625

Table 2
Results of CGO and other approaches in dealing with the 37-bar truss problem.

Variables	Lingyun et al. [47]	Gomes [43]	Kaveh and Zolghadr [45]	Kaveh and Ghazaan [48]	Ho-Huu et al. [42]	Tejani et al. [49]	CGO
Y3, Y19	1.1998	0.9637	0.9482	0.975	0.9564	0.9581	0.9494
Y5, Y17	1.6553	1.3978	1.3439	1.3577	1.3481	1.3614	1.3294
Y7, Y15	1.9652	1.5929	1.5043	1.5520	1.5308	1.5262	1.5149
Y9, Y13	2.0737	1.8812	1.6350	1.6920	1.6666	1.6429	1.6560
Y11	2.3050	2.0856	1.7182	1.7688	1.7402	1.7167	1.7281
A1, A27	2.8932	2.6797	2.6208	2.9652	2.8504	3.0754	2.9769
A2, A26	1.1201	1.1568	1.0397	1.0114	1.0043	1.0000	1.0000
A3, A24	1.0000	2.3476	1.0464	1.0090	1.0021	1.0001	1.0000
A4, A25	1.8655	1.7182	2.7163	2.4601	2.5221	2.7449	2.6348
A5, A23	1.5962	1.2751	1.0252	1.2300	1.2227	1.2446	1.1646
A6, A21	1.2642	1.4819	1.5081	1.2064	1.2618	1.2466	1.2288
A7, A22	1.8254	4.6850	2.3750	2.4245	2.5059	2.4648	2.5472
A8, A20	2.0009	1.1246	1.4498	1.4618	1.3466	1.3055	1.3477
A9, A18	1.9526	2.1214	1.4499	1.4328	1.5158	1.4983	1.5061
A10, A19	1.9705	3.8600	2.5327	2.5000	2.4482	2.5125	2.4786
A11, A17	1.8294	2.9817	1.2358	1.2319	1.2144	1.2255	1.2345
A12, A15	1.2358	1.2021	1.3528	1.3669	1.3663	1.2299	1.3295
A13, A16	1.4049	1.2563	2.9144	2.2801	2.4782	2.3904	2.4340
A14	1.0000	3.3276	1.0085	1.0011	1.0019	1.0025	1.0000
Weight (kg)	368.84	377.20	360.40	359.93	359.8194	359.9735	359.7893
Worst weight (kg)	-	-	-	-	360.3859	-	360.0873
Average weight (kg)	-	-	-	360.23	359.9767	361.3068	359.8842
Standard deviation	-	-	-	0.24	0.1473	1.0731	0.0662
Frequency 1	20.0013	20.0001	20.0194	20.0216	20.0001	20.0015	20.0000
Frequency 2	40.0305	40.0003	40.0113	40.0098	40.0037	40.0251	40.0012
Frequency 3	60.0000	60.0001	60.0082	60.0017	60.0001	60.1208	60.0000
Frequency 4	73.0444	73.0440	76.9896	76.7857	76.8632	76.4775	76.5596
Frequency 5	89.8244	89.8240	97.2222	96.3543	96.4625	95.9485	96.3753

runs were conducted for comparative purposes. Furthermore the results of other optimization methods are provided from the literature, while the statistical results are also presented to provide a fair judgment in this case. Based on the results, the CGO provides a best optimum weight of 359.7893 kg for this structure while the other attempts in this case, such as IDE, calculate 359.8194 kg, which demonstrates the capability of the CGO. Additionally, the CGO can provide 359.8842 kg as the mean of 30 independent runs with 360.0873 kg as the worst run, which are the best statistical results among other approaches.

In Table 2, the first five natural frequencies of the 37-bar truss problem are presented through the CGO and other literature approaches for a better perspective on the design constraints. It

is obvious that the CGO is capable of satisfying these constraints properly.

52-bar truss structure

The convergence results of CGO in dealing with the 52-bar truss design example are provided in Fig. 3.c. The convergence histories of the best and worst optimization runs alongside the mean of 30 independent runs are presented accordingly, to have a better schematic perspective on the performance of the CGO in dealing with this real-size optimization problem. Since the 52-bar truss problem is a simultaneous shape and size optimization procedure, the final optimum shape of the structure is demonstrated in Fig. 2.g in

Table 3
Comparative results of CGO and other approaches in dealing with the 52-bar truss problem.

Variables	Lingyun et al. [47]	Gomes [43]	Kaveh and Ghazaan [48]	Miguel and Fadel Miguel [44]	Ho-Huu et al. [42]	Tejani et al. [49]	CGO
Z _A	5.8851	5.5344	5.9362	6.4332	6.0052	5.8481	6.0139
X _B	1.7623	2.0885	2.2416	2.2208	2.3004	2.2609	2.3007
Z _B	4.4091	3.9283	3.7309	3.9202	3.7332	3.7000	3.7397
X _F	3.4406	4.0255	3.963	4.0296	4.0000	5.8481	4.0000
Z _F	3.1874	2.4575	2.500	2.5200	2.5000	3.9446	2.5000
A ₁	1.0000	0.3696	1.0001	1.0050	1.0001	2.5009	1.0000
A ₂	2.1417	4.1912	1.1654	1.3823	1.0875	1.0000	1.0821
A ₃	1.4858	1.5123	1.2323	1.2295	1.2135	1.1097	1.1977
A ₄	1.4018	1.5620	1.4323	1.2662	1.4460	1.2279	1.4358
A ₅	1.9110	1.9154	1.3901	1.4478	1.4315	1.5145	1.4150
A ₆	1.0109	1.1315	1.0001	1.0000	1.0000	1.4136	1.0000
A ₇	1.4693	1.8233	1.6024	1.5728	1.5623	1.0000	1.5713
A ₈	2.1411	1.0904	1.4131	1.4153	1.3724	1.6206	1.3863
Weight (kg)	236.046	228.381	194.85	197.53	193.2085	193.3465	193.1876
Worst weight (kg)	-	-	-	-	202.4215	-	202.2312
Average weight (kg)	-	234.3	196.85	212.8	196.0478	212.7035	195.4586
Standard deviation	-	5.22	2.38	17.98	4.1823	32.8481	3.8183
Frequency 1	12.8100	12.7510	11.4339	11.3119	11.6033	11.6331	11.6384
Frequency 2	28.6500	28.6490	28.6480	28.6529	28.6481	28.6492	28.6480
Frequency 3	28.6500	28.6490	28.6480	28.6529	28.6481	28.6492	28.6480
Frequency 4	29.5400	28.8030	28.6482	28.8030	28.6490	28.6495	28.6480
Frequency 5	30.2400	29.2300	28.6848	28.8030	28.6530	29.076	28.6482

Table 4
Results of CGO and other approaches in dealing with the 72-bar truss problem.

Variable	Gomes [43]	Kaveh and Zolghadr [45]	Khatibinia and Naseralavi [50]	Kaveh and Ghazaan [48]	Ho-Huu et al. [42]	Sedaghati [51]	Tejani et al. [49]	CGO
1–4	2.987	2.854	3.5142	3.3437	3.5863	3.499	3.3335	3.462923
5–12	7.849	8.301	7.9464	7.8688	7.8278	7.932	7.9054	7.849892
13–16	0.645	0.645	0.6450	0.6450	0.6450	0.645	0.6469	0.645
17–18	0.645	0.645	0.6450	0.6450	0.6450	0.645	0.6451	0.645
19–22	8.765	8.202	8.0641	8.1626	8.1052	8.056	7.9980	7.952499
23–30	8.153	7.043	8.0278	7.9502	7.8788	8.011	7.7682	7.9253
31–34	0.645	0.645	0.6450	0.6452	0.6451	0.645	0.6450	0.645
35–36	0.645	0.645	0.6450	0.6450	0.6450	0.645	0.6450	0.645
37–40	13.45	16.328	12.8493	12.2668	12.5157	12.812	12.8748	12.66778
41–48	8.073	8.299	8.0888	8.1845	8.0102	8.061	8.0855	7.974555
49–52	0.645	0.645	0.6450	0.6451	0.6450	0.645	0.6450	0.645
53–54	0.645	0.645	0.6450	0.6451	0.6452	0.645	0.6450	0.645017
55–58	16.684	15.048	17.317	17.9632	16.9997	17.279	17.0410	17.1044
59–66	8.159	8.268	8.1104	8.1292	8.0362	8.088	8.0003	8.001567
67–70	0.645	0.645	0.6450	0.6450	0.6451	0.645	0.6450	0.645
71–72	0.645	0.645	0.6450	0.6450	0.6453	0.645	0.6495	0.645
Weight (kg)	328.823	327.507	328.32	327.77	324.2441	327.605	324.3754	324.197
Worst weight (kg)	–	–	–	–	324.6444	–	–	324.2064
Average weight (kg)	–	–	329.12	327.99	324.3379	–	325.7494	324.1981
Standard deviation	–	–	1.496	0.19	0.1023	–	0.9186	0.002176
Frequency 1	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0011	4.0000
Frequency 2	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0011	4.0000
Frequency 3	6.0000	6.0040	6.0000	6.0000	6.0000	6.0000	6.0003	6.0000
Frequency 4	6.2190	6.2491	6.2410	6.2300	6.2779	6.2470	6.2625	6.267403
Frequency 5	8.9760	8.9726	9.0680	9.0410	9.1120	9.0740	9.0871	9.099443

which the overall shape of the structure is different from the one presented in Fig. 2.c.

By conducting 30 independent optimization runs, the best and statistical results of the CGO in dealing with the 52-bar truss problem are derived and presented in Table 3. Based on the results of other algorithms from the literature, CGO can reach 193.1876 kg which is the best among other approaches, while the IDE with 193.2085 kg is the following competitive result. The mean, worst, and standard deviation of the conducted runs demonstrate that CGO provides very stable results with a mean of 195.4586 kg and a standard deviation of 3.8183.

The first five natural frequencies of the 52-bar truss problem are presented in Table 3 for different methods alongside the CGO. It is evident that the CGO is capable of satisfying these constraints properly.

72-bar truss structure

In Fig. 3.d, the convergence history of the CGO in dealing with the 72-bar truss design example is illustrated by including the convergence histories for the best and worst optimization runs alongside the mean of 30 independent runs.

The best results of multiple optimization runs for the CGO and other metaheuristic approaches are prepared in Table 4 for the 75-bar truss problem. The statistical results alongside the optimum design variables are also provided. The CGO can reach 324.197 kg, which is the best among other methods. The CGO can also provide 324.1981 kg as the mean of 30 independent runs with 324.2064 kg as the worst run, which are the best statistical results among other approaches.

Based on the knowledge that frequency constraints have to be satisfied during the truss optimization process, the first five natural frequencies of the CGO are presented in table 4 and are derived from the best results of 30 conducted optimization runs. Furthermore, the results of other algorithms have also been provided from the literature. The capability of the CGO in satisfying the con-

straints alongside other methods is demonstrated by considering the constraint limitations of 4 and 6 Hz for the first and third natural frequencies of the considered truss structure.

120-bar truss structure

The best, worst and mean convergence history of the CGO in dealing with the 120-bar truss design example is presented in Fig. 3.e, where the convergence curves for the mean of 30 independent runs are also shown.

For the 120-bar truss problem, the best results of multiple optimization runs for the CGO and other metaheuristic approaches are provided in Table 5. Based on the obtained results, it can be concluded that CGO performs a much better optimization process and prepares 8707.2454 kg for the overall weight of this truss structure. In contrast, the results of other approaches are higher than this value. CGO also provides better statistical results, such as the mean (8707.3689 kg) and standard deviation (0.1510).

The first five natural frequencies of the CGO and other methods in dealing with the 120-bar truss problem are provided in table 5 for comparative purposes. The capability of the CGO in handling the constraints is in perspective.

Conclusion

Shape and size optimization of different large-scale truss structures are considered in this paper using the Chaos Game Optimization (CGO) as one of the recently proposed metaheuristic optimization algorithms. In this algorithm, the principles of chaos theory and the configuration of fractals are utilized as inspirational concepts. For the numerical purpose, the 10-bar, 37-bar, 52-bar, 72-bar and 120-bar truss structures as five of the benchmark problems in this field are considered design examples, in which the frequency constraints are considered as limits to be dealt with during the optimization procedure. Multiple optimization runs are also conducted for having a comprehensive statistical analysis, while

Table 5

Results of CGO and other approaches in dealing with the 120-bar truss problem.

Element number	Kaveh and Zolghadr [45]	Khatibinia and Naseralavi [50]	Kaveh and Ghazaan [48]	Ho-Huu et al. [42]	Tejani et al. [49]	CGO
1	19.607	20.263	19.8905	19.4670	19.4486	19.5089
2	41.290	39.294	40.4045	40.5004	40.3949	40.3698
3	11.136	9.989	11.2057	10.6136	10.6921	10.6034
4	21.025	20.563	21.3768	21.1073	21.3139	21.1148
5	10.060	9.603	9.8669	9.8417	9.8943	9.8343
6	12.758	11.738	12.7200	11.7735	11.7810	11.7734
7	15.414	15.877	15.2236	14.8264	14.5979	14.8415
Weight (kg)	8,890.48	8,724.97	8,889.96	8,707.2898	8708.729	8,707.2454
Worst weight (kg)	–	–	–	8,709.5109	–	8,707.8439
Average weight (kg)	–	8,745.58	8,900.39	8,707.8147	8734.74	8,707.3689
Standard deviation	–	1.183	6.38	0.5057	27.0503	0.1510
Frequency 1	9.0001	9.0020	9.0000	9.0000	9.0002	9.0000
Frequency 2	11.0007	11.0030	11.0000	11.0000	11.0000	11.0000
Frequency 3	11.0053	11.0030	11.0000	11.0000	11.0000	11.0000
Frequency 4	11.0129	11.0070	11.0100	11.0100	11.0006	11.0000
Frequency 5	11.0471	11.0760	11.0500	11.0669	11.0672	11.0669

a comparative investigation is also performed with other algorithms in the literature. Based on the results, the CGO can reach 524.4545 kg in dealing with the 10-bar truss problem, which is better than the previously calculated weights. The CGO can provide 524.5099 kg as the mean of 30 independent runs with 524.7488 kg as the worst run, which are the best statistical results among other approaches for this structure. The CGO provides a best optimum weight of 359.7893 kg for the 37-bar truss structure, while the other attempts in this case, such as IDE, calculate 359.8194 kg, which demonstrates the capability of the CGO. The CGO can provide 359.8842 kg as the mean of 30 independent runs with 360.0873 kg as the worst run, which are the best statistical results among other approaches. Based on the results of other algorithms from the literature for the 52-bar truss problem, CGO can reach 193.1876 kg which is the best among other approaches, while the IDE with 193.2085 kg is the next competitive result. The mean, worst and standard deviation of the conducted runs demonstrate that CGO provides very stable results with mean of 195.4586 kg and standard deviation of 3.8183. The CGO can reach 324.197 kg for the 72-bar truss structure, which is better than the results of other methods.

In comparison, this algorithm can provide 324.1981 kg as mean of 30 independent runs with 324.2064 kg as the worst run, which are the best statistical results among other approaches. It can be concluded that CGO performs a much better optimization process and prepares 8707.2454 kg for the overall weight of the 120-bar structure, while the results of other approaches are higher than this value. CGO also provides better statistical results, including the mean (8707.3689 kg) and standard deviation (0.1510). Finally, it can be concluded that CGO as a well-formulated metaheuristic algorithm can provide better and competitive results in dealing with all of the considered design examples. For future challenges however, the applicability of the CGO algorithm in different engineering fields can be deemed, including the optimum design of vibration control systems in buildings and other engineering structures. Furthermore, the standard formulation of the CGO algorithm can be modified by implementing a plethora of improvement techniques to the main loop of the CGO algorithm, including the differences between the discrete time concepts and continue time concepts, that can be exchanged for enhancing the overall behavior of this algorithm.

CRedit authorship contribution statement

Mahdi Azizi: Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft,

Visualization. **Uwe Aickelin:** Conceptualization, Investigation, Resources, Data curation, Writing – review & editing, Supervision, Project administration, Funding acquisition. **Hadi A. Khorshidi:** Investigation, Data curation, Writing – review & editing, Funding acquisition, Visualization. **Milad Baghalzadeh Shishehgharkhaneh:** Visualization, Investigation, Data curation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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