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Standardised approach to the reporting and presentation of refractive data: electronic patient record

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Refractive error and refractive power are core measures in ophthalmology, optometry and vision science. Refractive error measures what is missing, that is, a departure from emmetropia whereas refractive power for example, keratometry measures what is present. Refractive error is used to determine outcomes for many conditions, such as myopia or hypermetropia, amblyopia, anisometropia, outcomes of corneal transplants, progression of keratoconus and cataract and refractive surgery.

Sphere/cylinder x axis ($S/C \times A$) is one of the core measures used for paraxial optics (light rays close to the optical axis and so relatively independent of the angle of incidence) and thin Gaussian lenses and forms the basis of most clinical work.

$S/C \times A$is used to evaluate and report refractive data as the lenses in the trial lens case or phoropter are made as combinations of spheres and cylinders. $S/C \times A$ is a compound number, the components of which S, C and axis, are dependent on each other. Despite this, it is common for the components to be viewed and treated incorrectly as independent variables. For example, S is used to measure the spherical component of a person's refractive error and the cylinder (C) either on its own or in combination with the axis (A) to reflect and measure the person's astigmatism. For groups of patients and for comparison of outcomes, it is also common in the literature for the components of $S/C \times A$, that is S, C or $C \times A$ to be separately (independently) added together without transformation, and for statistical measures to be calculated to evaluate and report outcomes. For example, it is not uncommon to see reports in the literature where C has been used to measure the change in astigmatism before and after an intervention.

A spherical equivalent (SEq) is an often-used measure, because of its simplicity as a scalar variable and it is easily calculated as $SEq = S + C/2$. Despite its usefulness there is an obvious weakness of the SEq, as there are an infinite number of refractive errors having the same SEq value. It is important to note that although the term SEq is widely used, the preferable term is the nearest equivalent sphere[1] (NES) as there may be other formulae than $SEq = S + C/2$ used to provide a measure. SEq is also an insensitive measure. It has been shown following cataract surgery for example, that if $S/C \times A$ is used as a compound number to measure refractive outcome, many more patients are identified as refractive outliers than using the NES or the Cylinder, or an aggregation of NES and Cylinder.[2] Such patients are more likely to depend on spectacles or contact lenses for distance correction due to uncorrected astigmatic errors.[3]

For cataract surgery, refractive aims continue to evolve, from leaving the eye aphakic, inserting a standard intraocular lens (IOL), devising the first regression IOL power formulae to the current development of multi-variable formulae. Relying on NES to achieve better outcomes, however, results in a law of diminishing returns as we have achieved only a modest improvement in NES using multivariable formulae and highly sophisticated biometry machines, compared with immersion ultrasound and a third generation IOL formulae from 20 years ago. It is clear, therefore, that more sensitive measures such as the actual refractive error that is, $S/C \times A$ are needed.[4] In essence $S/C \times A$ provides a more tailored approach when planning cataract surgery whether using monofocal or toric IOLs.

As noted above, there have been many attempts to treat the components of the refractive error $S/C \times A$ as independent terms despite an abundance of evidence to show that these approaches are flawed and lose information.[5–10]

For example, consider the following two refractive (paraxial) powers: $+2/+2\times90$ and $+1/+1\times180$. If they were to be added together without transformation, what would be the result? There are three possibilities depending on whether each component is treated independently or dependently.[3]

(1) If they are treated independently as scalar values, this leads to the following situation,

$(+2/+2)+(+1/+1)=+3/+3$, which is incorrect.

(2) If the sphere and cylinder powers are added independently, this leads to

$$\left|\begin{array}{c}\textit{Sphere}\\2\\1\\\underline{\quad}\\3\end{array}\right| + \left|\begin{array}{c}\textit{Cylinder}\\2_{90}\\1_{180}\\\underline{\quad}\\1\frac{1}{90}\end{array}\right|$$

or $(+2/+2\times90)+(+1/+1\times180)=+3/+1\times90$, which again is incorrect.[3]

(3) If, however, they are treated dependently, then

$$\left|\begin{array}{cc}\textit{Sphere} & \textit{Cylinder}\\2^{1} & 2_{90}\\1 & 1_{180}\\\underline{\quad} & \underline{\quad}\\4 & 1\frac{1}{90}\end{array}\right|$$

or $(+2/+2\times90)+(+1/+1\times180)=+4/+1\times90$, which is the correct result.[3]

To avoid these errors, refractive data needs to be treated as a compound number, that is, $S/C\times A$.[7] Importantly, it has been shown that as a compound measure, the Sphere/Cylinder x Axis allows a measure of refractive blur as a global outcome and surrogate marker of vision.[11 12]

There are well established robust methods to measure differences and changes in refractive error and to present this information as $S/C\times A$.[7–11 13 14] These methods transform refractive error into components which are independent; equivalent to plotting a point using X, Y and Z axes (which are orthogonal to each other and therefore independent). Once transformed, compound number calculations such as differences, total, mean, SD, significance tests etc, can legitimately be undertaken[6 8] prior to the data being transformed back into the standard familiar format of $S/C\times A$. These methods provide accurate information needed clinically to decide whether there has been a meaningful change or progression in a condition.

The proposal, therefore, is not to disregard the NES, but to record and analyse refractive errors as Sphere/Cylinder x Axis ($S/C\times A$) *in addition to* current practice which uses ($NES=S+C/2$). Provision of both within an electronic medical or patient record has the potential to identify options to improve outcomes whether it be optimisation of IOL formula or a reduction in astigmatism.

Examples of clinical scenarios can be informative.

- A cataract surgeon may want to compare the expected to actual refractive outcome following surgery. If the expected or intended outcome following cataract surgery is $-0.75/+1.25\times5$ $(NES=-0.13)$ but the actual post-operative outcome is $-1/+1.75\times50$ ($NES=-0.13$), how should the difference be calculated that is, how far is the actual from the intended? Both the intended and actual have a *SEq* of −0.13 , so no apparent difference. Using the difference in cylinders is gives −0.50 (calculated as a scalar) or $-1.33/+2.15\times158$ if calculated treating the cylinder independently as a vector (both are incorrect). The actual difference in the refractive outcome, however, when treating the intended and actual outcome as compound numbers is $-1.08/+2.15\times158$ (for intended minus actual). Note the order is important that is, one needs to stipulate whether intended minus actual or actual minus intended as the axis of the difference will change by 90 degrees.
- A Paediatric Ophthalmologist will want to know the correct magnitude of a child's anisometropia and/or whether a child's refractive error has changed due to treatment of their amblyopia. As an example, if the refractive error of a child's right eye is $-1.5/+3.5\times80$ and left eye is $-0.5/+2\times120$, the anisometropia (right eye minus left eye) is $-2.11/+3.72\times64$. Note if using the NES this difference would be just 0.25 D.

Astigmatism is an important measure. Despite this, for the majority of patients undergoing cataract surgery, astigmatism does not form part of the outcomes used to measure the quality of clinical services.[15] Nevertheless, studies have shown that residual astigmatism has a far greater effect on unaided distance vision compared with residual spherical error.[3] In fact, with the current standards of refractive outcomes using monofocal non-toric IOLs in a non-selected population, patients are seven times more likely to need distance glasses due to uncorrected astigmatism, compared with uncorrected spherical refractive error.[16]

The cylinder component of $S/C\times A$ is often equated with astigmatism.[1] The cylinder (C) does contain or spans astigmatism but also contains a spherical equivalent of $C/2$ (if one accepts the concept of a spherical equivalent). For example, if the NES of a cylinder is subtracted from the cylinder, a Jackson cross-cylinder (JCC) results, that is, $(0/C\times A)-(\frac{C}{2})=-\frac{C}{2}/C\times A=JCC$.[1] In contrast a JCC always has a spherical equivalent of zero and is therefore, a pure paraxial measure of astigmatism. JCCs can be added but always result in a JCC.[1] Similarly, adding spherical equivalent, always results in a spherical equivalent. In contrast a cylinder (C) power when added to another cylinder C may result in a sphere e.g., $(0/+1\times90)+(0/+1\times180)=+1/0$ or a spherocylinder, for example, $(0/+1\times180)+(0+1\times120)=+0.5/+1\times150$, rather than always another cylinder.

The $S/C\times A$ representation of refractive data, therefore, can be transformed and decomposed into an NES and a

JCC, that is, $S/C \times A = NES + JCC$. For example, $-0.50/+2.00 \times 120$ has an NES of +0.50 and a JCC $-1.00/+2.00 \times 120$. This provides a measure of the amount of sphere present (NES) and the amount of astigmatism.

For example, if a patient's astigmatism changes from $-1.00/+2.00 \times 80\ (NES = 0)$ to $-0.50/+1.00 \times 120\ (NES = 0)$ this represents a change in astigmatism of $-1.04/+2.07 \times 156\ (NES = 0)$. A JCC lends itself to toric IOL calculations which predict a post op cross-cylindrical value at the corneal or spectacle plane.

The methods of analysing spherocylindrical outcomes using Long's formalism[13] have been available for decades but as they require more calculation, they have not been adopted in routine clinical practice. To analyse refractive and keratometric data, requires transformation into Long's formalism, following which any number of robust analyses can be undertaken.[2 4] Note, refractive powers such as $S/C \times A$ may be decomposed into a *NES* and a *JCC* and then added following transformation into Long's formalism as shown below.

In recent years, electronic patient records (EPRs) have replaced paper records, and this has greatly expanded the capabilities for performing audit and analysing outcomes. EPRs can carry out calculations automatically in the background that is, transform the data, analyse and then transform the data back into $S/C \times A$ thereby providing clinicians with the information needed to improve clinical practice.[16 17] In addition, there are many other derivatives that can be calculated once the data has been transformed into Long's formalism e.g., M, J_0, J_{45}, including blur strength which can be used as surrogate outcomes for unaided vision.[12 14 18 19] It should be noted that Fick introduced the ideas of applying matrices and linear algebra to dioptric power[20–22] while Long (1976)[13] and Keating (1980/1),[23 24] working independently, provided conversion equations from clinical notation to power matrices and back to clinical notation.

Rubin *et al*,[25] have reviewed issues which affect the analysis of refractive state such as data normality, transformations, outliers and anisometropia. They provide a review of the methods needed for analysing and representing dioptric power and concentrate on the optimal approach to understanding refractive state in addressing pertinent clinical and research questions. They demonstrate the need to use power matrices rather than power vectors for the analysis of refractive state. For example, identification of outliers in refractive data and the need to be able to transform samples of dioptric power towards normality.

In a second article, Evans and Rubin[26] review the basic principles of linear optics and discuss the use of linear optics in quantitative analysis with application to clinically important issues such as chromatic aberrations, positioning and design of IOL and magnification. They demonstrate how the application of linear optics is needed for the understanding of paraxial optical systems for example, astigmatic, tilted and decentred IOLs which clinicians commonly encounter.

The important issue is that all of these analyses can be undertaken within an EPR providing the clinician with robust systems for the analysis of refractive data

Encouraging EPR providers to analyse refractive and keratometric data in the correct format would facilitate many audit and research studies which would in turn bring benefits to patients.[2 4] Specifically, the routine compound number analysis of $S/C \times A$ refractive data can provide added value to patients through provision of clear clinically accessible information to eye health professionals when planning an intervention. By complimenting existing NES-based approaches there is potential to enhance the care of individual patients and the provision of accurate summary outcomes for tracking groups of patients. Moving forwards, it is highly desirable therefore that EPRs are upgraded to transform refractive data into Long's formalism for analysis, and then to transform and present the data back into $S/C \times A$ for eye health professionals. As the widespread use of EPRs has greatly expanded our ability to audit our outcomes, we should take advantage of automation of robust refractive calculations which could be included within EPRs.

IN SUMMARY

1. Present EPR do not allow the clinician to easily measure the change in a patient's refraction other than a change in the (NES, also known as spherical equivalent - *SEq*) and or (incorrectly) a change in cylinder. This is clinically relevant, as for example, there are innumerable large changes in refraction which would not register a change or be a minimal change in the NES. Measuring a change in refractive error is important, for example, when trying to decide if a patient with keratoconus has progressed or what might be significant anisometropia in a child or adult. Likewise, measuring a change in refraction is a basic measure for a cataract or refractive surgeon.
2. At present the EPR does not provide a cataract surgeon with the intended outcome other than as an NES. We have shown in a large study[2] on cataract outcomes using the national ophthalmic database data, that using the spherocylindrical method there were many patients (233=2.6%) with refractive outcomes outside of the three SD limits. Many of these errors would not have been identified using NES or cylinder. Of the 233 outliers, only 46 of 9020 (0.51%) would have been picked up with the NES and 76 (0.85%) using cylinder. Even by aggregating outliers detected by either NES or cylinder (or both), 111/233 or 47.6% of these >3 SD spherocylindrical outliers would have been missed. Since the 3 SD limits approximate to 99.8% limits, these 233 (2.6%) far exceeds expectation due to random variation alone.

3. There are three issues with using the cylinder as an independent measure.
   a. Although the cylinder is representative of astigmatism it also contains a non-zero spherical equivalent, that is, C/2. If one subtracts the C/2 from the $0/C\times A$ one obtains a CC, that is, $0/C\times A-\frac{C}{2}=-\frac{C}{2}/C\times A=JCC$ which has a zero spherical equivalent and is therefore, a pure paraxial astigmatic power. Paraxial astigmatism, therefore, needs to be analysed as a JCC. This is essential for example, in refractive surgery (forms the basis of paraxial laser surgery) or for inserting toric IOLs in cataract surgery.
   b. The space of cylinders is not mathematically closed under addition or subtraction. That is, if one adds or subtracts cylinders, one may end up with a sphere for example, $(0/+1\times 90)+(0/+1\times 180)=+1/0$ or a $S/C\times A$.
   c. In addition, if the component cylinder powers in $S/C\times A$ are added or subtracted independently of the sphere, this leads to an incorrect result (as shown below in this document)

   An EPR, therefore, needs to provide the user with the correct information that can be used for clinical management.
4. Decomposition of $S/C\times A$ into *NES* and a JCC.

The following is a proof for the addition of refractive measurements after decomposing $S/C\times A$ into *NES* and a JCC.

$$S/C\times a=NES+JCC,$$

$$NES=S+\frac{C}{2}$$

$$JCC=-\frac{C}{2}/C\times a$$

To prove $\sum_i^N[S/C\times a]_i\Leftrightarrow\sum_i^N[NES]_i+\sum_i^N[JCC]_i$

From Long's formalism,

$$[S/C\times a]_i\Leftrightarrow\begin{bmatrix}S_i+C_iSin^2a_i & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & S_i+C_iCos^2a_i\end{bmatrix}$$

$$[JCC]_i=\left[-\frac{C}{2}/C\times a\right]_i\Leftrightarrow\begin{bmatrix}-\frac{C_i}{2}+C_iSin^2a_i & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & -\frac{C_i}{2}+C_iCos^2a_i\end{bmatrix}$$

$$[NES]_i=\left[S_i+\frac{C_i}{2}\right]\Leftrightarrow\begin{bmatrix}S_i+\frac{C_i}{2} & 0\\ 0 & S_i+\frac{C_i}{2}\end{bmatrix}.$$

If these components are summed, then

$$\sum_i^N[S/C\times a]_i\Leftrightarrow\sum_i^N\begin{bmatrix}S_i+C_iSin^2a_i & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & S_i+C_iCos^2a_i\end{bmatrix}$$

$$\sum_i^N[NES]_i=\sum_i^N\left[S+\frac{C}{2}\right]_i=\sum_i^N\left[S_i+\frac{C_i}{2}\right]\Leftrightarrow\sum_i^N\begin{bmatrix}S_i+\frac{C_i}{2} & 0\\ 0 & S_i+\frac{C_i}{2}\end{bmatrix}$$

and

$$\sum_i^N[JCC]_i\Leftrightarrow\sum_i^N\begin{bmatrix}-\frac{C_i}{2}+C_iSin^2a_i & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & -\frac{C_i}{2}+C_iCos^2a_i\end{bmatrix}.$$

Then,

$$\sum_i^N[NES]_i+\sum_i^N[JCC]_i\Leftrightarrow\sum_i^N\begin{bmatrix}S_i+\frac{C_i}{2} & 0\\ 0 & S_i+\frac{C_i}{2}\end{bmatrix}+\sum_i^N\begin{bmatrix}-\frac{C_i}{2}+C_iSin^2a & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & -\frac{C_i}{2}+C_iCos^2a_i\end{bmatrix}$$

$$=\sum_i^N\begin{bmatrix}S_i+\frac{C_i}{2}-\frac{C_i}{2}+C_iSin^2a & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & S_i+\frac{C_i}{2}-\frac{C_i}{2}+C_iCos^2a_i\end{bmatrix}$$

$$=\sum_i^N\begin{bmatrix}S_i+C_iSin^2a & -C_iSina_iCosa_i\\ -C_iSina_iCosa_i & S_i+C_iCos^2a_i\end{bmatrix}$$

Therefore,

$$\sum_i^N[S/C\times a]_i\Leftrightarrow\sum_i^N[NES]_i+\sum_i^N[JCC]_i$$

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