Heliyon 10 (2024) e32895

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Improved fuzzy multi-objective transportation problem with Triangular fuzzy numbers

A. Kokila [a], G. Deepa [a,*]

[a] Department of Mathematics, SAS, Vellore Institute of Technology, Vellore, Tamil Nadu, India

ARTICLE INFO

Keywords:
Fuzzy set
Triangular fuzzy number
Fuzzy multi-objective transportation problem
Harmonic mean
Fuzzy optimal solution

ABSTRACT

The present study investigates a Multi-Objective Transportation Problem within a fuzzy environment. The cost of transportation, supply, and demand data are assumed to be inaccurate due to market variations. As a result, the parameters are imprecise or fuzzy data. We offer a multi-objective, balanced transportation problem during this work, where all the parameters are fuzzy numbers. Following a mathematical formulation, fuzzy arithmetic will be used to divide the Fuzzy MOTP into three levels MOTP (lower, medium, upper). After reducing the problem to a crisp MOTP and applying a harmonic mean to each objective function, a suggested solution procedure is presented. Determining the optimal solutions for the FMOTP under unknown situations is, thus, the most important objective of this research.

1. Introduction

Hitchcock presented the fundamental transportation problem initially in 1941 [1]. In various real-world scenarios, goods have to be transported from various locations (factories) to various destinations (warehouses). The purpose of the decision-maker is to determine how many products to order. Nowadays, everyday distribution problems, like those that arise within business and industrial environments, are frequently erroneous because of parameter uncertainties. All of these features of the TP may not, however, be completely comprehended in real time due to unexpected circumstances. L.A. Zadeh [2] developed a fuzzy set theory in 1965 and successfully used it in various fields. The fuzzy decision-making approach was created in 1978 for TP and FTP, Zimmermann [3] developed a number of fuzzy optimization approaches. As it happens, not every transportation problem is the same. A lot of everyday transportation problems involve scenarios in which various objectives must be taken into consideration to be simultaneously optimized. These kinds of problems are known as MOTPs. The MOTP handles the transport of products and considers many factors at once, including delivery time, cost, and the amount of goods provided. As a result of inaccurate data, insufficient proof, and other factors, MOTP data may not be precise; rather, it may be fuzzy, arbitrary, or a mix of the two. An FMOTP is a MOTP that has a minimum of one parameter represented by fuzzy numbers.

Some authors, such as Gupta, Kumar and Kaur (2011) [4] suggested Mehar's approach for determining the precise fuzzy optimum solution to a complete FMOTP. To deal with the MOTP, Waiel et al. [5] introduced an interactive fuzzy goal programming method. Zangiabadi et al. [6] developed a fuzzy goal programming approach to find the best compromise solution for the MOTP. Pitam Singh et al. (2017) [7] developed a useful method for resolving MOTP by combining goal programming, fuzzy programming, and interactive programming approaches.Vidhya et al. [8] discussed efficient solution of a FMOTP. Dhanaseker et al. [9] proposed the

* Corresponding author.
E-mail addresses: kokilaa0206@gmail.com (A. Kokila), deepa.g@vit.ac.in (G. Deepa).

https://doi.org/10.1016/j.heliyon.2024.e32895
Received 16 October 2023; Received in revised form 5 June 2024; Accepted 11 June 2024
Available online 14 June 2024
2405-8440/

Hungarian-Modi approach to deal with the fully FTP. Kundu et al. (2013) [10] offered two approaches to tackling multi-objective, multi-item solid transportation problems using fuzzy parameters. Ebrahimnejad and Ali (2015) [11] proposed a two-step strategy for dealing with FTP.

The initial step involves converting the FTP to an LPP, which includes both crisp constraints as well as fuzzy costs, utilizing fuzzy arithmetic. In the next phase, we will split the resulting LPP into three different bounded TPs. Bharati et al. [12] created a new distance function that measures the difference between two trapezoidal numbers to solve an FFMOTP. Osuji et al. [13] created a new approach for finding a MOTP via a fuzzy programming algorithm. A new two-stage fuzzy multi-objective linear programming approach is proposeded by Jiu-ying Dong et al. (2019) [14] and used to the selection of engineering project portfolios. Shu-Ping Wan et al. [15] developed a new hesitant fuzzy mathematical programming technique to perform hybrid multi-criteria group decision-making (MCGDM) using hesitant fuzzy truth values. A novel technique for the fuzzy linear programme was proposed by Jiu-ying Dong et al. (2018) [16], in which all the resources, technological coefficients, and objective coefficients are trapezoidal fuzzy number. An investment problem and a fuzzy knapsack problem are used to verify this method. A unique interval-valued intuitionistic fuzzy (IVIF) mathematical programming technique for hybrid MCGDM was developed by Shu-Ping Wan et al. [17] [2015]. A Pythagorean fuzzy (PF) mathematical programming technique was developed by Shu-Ping Wan et al. (2018) [18] to address multi-attribute group decision-making problems in a PF environment. Using TrFNs, Shu-Ping Wan et al. [19] [2014] created a novel approach to linear programming. Mahmoodirad et al. (2018) [20] suggested an approach to address the problems identified in previous methods. Using an extended reduced gradient method (GRT), Baidya et al. (2019) [21] built a fully fuzzy solid TP with extra restrictions on the overall budget of every destination. Anukokila and Radhakrishnan (2019) [22] used goal programming to handle the fractional FTP. Mishra and Kumar (2020) [23] introduced a novel way of tackling fully FTPs while also addressing a limitation of Ebrahimnejad's method, presented in 2015 [11]. A new fuzzy DEA-based method was developed by Bagheri et al. (2020) [24] for solving FFMOTP. Ghosh and Roy (2020) [25] identified product-combining requirements for the transportation of raw materials with various levels of purity for customer fulfilment as an essential part of MOTPs. Gowthami et al. (2019) [26] investigated the Solution of multi objective transportation problem under fuzzy environment TFNs may be used to resolve a FFMOFTP without converting it to a crisp version of the suggested solutions, according to Krishnaveni and Ganesan (2020) [27]. Kokila et al. [28] proposed the Row-Column Maxima Method to deal with FTP. Mishra and Kumar (2021) [29] created a JMD method for handling unbalanced, fully trapezoidal IFTPs. MOFCSTPs were studied by Ghosh et al. [30]. Yadvendra Kache et al. (2022) [31] introduced a novel method for handling FFMOTPs, the harmonic mean methodology. In this study, Fathy et al. (2022) [32] examine a fully fuzzy, multi-level, multi-objective LPP in their work, making use of the HM technique to combine several goals into a single goal at a particular stage. To acquire the fuzzy compromise solution, the whole single-objective LPP is finally solved. A unique method for the completely intuitionistic FMOFTP was presented by Sayed et al. (2021) [33]. Using a fuzzy linear membership function, Sharif Uddin et al. (2021) [34] described a goal programming strategy for unclear MOTP. Shivani et al. (2022) [35] created an effective approach for solving an unbalanced fully multi-objective fixed-charge TP under fuzzy rough environment. Using the extension principle, Harish Garg et al. (2021) [36] addressed the fractional two-stage transshipment problem under uncertainty. Esmaiel Keshavarz et al. (2023) [37], consider a fixed-charge transportation problem using fuzzy shipping charges and propose a membership function based on objective values. A new crisp nonlinear mixed-integer programming problem is developed that makes use of Bellman-Zadeh's max-min criteria rather than the original problem. A fuzzy fixed-charge transportation problem under uncertainty was examined by Ali Mahmoodirad and Sadegh Niroomand (2023) [38]. We provide a novel approach that yields both a lower as well as an upper bound for a fuzzy optimal value for the fuzzy fixed-charge transportation issue, using representations for the fixed cost and the transportation cost. Zhihao Peng et al. (2024) [39] are discussed a fully interval-valued fuzzy transportation problems with development and prospects. Kokila et al. [40] discussed an FMOTP that utilized a TFN. This content is offered for pre-print online. In Table 1, all necessary abbreviations are included.

1.1. Highlights of the motivation and novelty for the proposed work

The motives for the study are described as follows: (1) The goal of this work is, to the greatest of our expertise, to solve FMOTPs utilizing the proposed method; (2) the suggested strategy gives a quick and effective solution to transform FMOTP into a TL- CMOTP.

Table 1
Abbreviations.

Abbreviation	Full name
TP	Transportation Problem
FTP	Fuzzy Transportation Problem
MOTP	Multi-Objective Transportation Problem
FMOTP	Fuzzy Multi-Objective Transportation Problem
TFN	Triangular Fuzzy Number
LPP	linear programming problem
FFMOTP	Fully Fuzzy Multi-Objective Transportation Problem
FFMOFTP	Fully fuzzy Multi-Objective fractional Transportation Problem
MOFCSTP	Multi-objective fixed-charge stochastic transportation problem
TL-CMOTP	Three Level - Crisp Multi-Objective Transportation Problems
HM	Harmonic Mean
TC,TT,TD	Transportation Cost, Transportation Time, and Transportation Distance

Reducing the number of iterations required to solve the FMOTP is our major effort in this work. We compare our proposed method to solve FMOTPs obtained by Malihe Niksirat [41], Gowthami, Prabhakaran [26], and Yadvendra Kacher and Pitam Singh [31]. Consequently, the following is a summary of this paper's novelties.

- Fuzzy multi-objective transportation problems are considered.
- The parameters of FMOTPs are considered to be TFNs.
- The suggested technique simplifies the problem to a TL-CMOTP. Then, in only one step, it simplifies to TL-CMOTP using fuzzy harmonic mean.
- The suggested approach converts unbalanced FMOTPs to balanced ones without the requirement for a fake origin or destination, which gives an optimal solution.

The remaining sections of this research are structured as follows: In Section 2, the concepts of fuzzy set theory and fuzzy harmonic mean are presented, and after the section, the mathematical formulas for an FMOTP are developed. In Section 3, we describe a new strategy for dealing with the FMOTP. Section 4 provides numerical examples of the existing techniques. A real-world FMOTP solution and a comparison utilizing the proposed method are provided in Section 5. The results and a discussion of them are given in Section 6. Advantages and disadvantages are given in section 7 and also the managerial insights are given in section 7.1.The conclusion is eventually given in Section 8.

2. Preliminaries

The fundamental ideas of fuzzy set theory, as well as additional terminology that will help clarify the suggested strategy are presented in this section.

2.1. *Fuzzy set [28]*

A fuzzy set $\overline{P}$ is a group of ordered pairs in, where $\mathbb{U}$ is a universal set.

$\mathbb{U} : \overline{P} = \{(\widetilde{\alpha}, \mu_{\overline{P}}(\widetilde{\alpha}))\,|\, \widetilde{\alpha} \in \mathbb{U}\},$

The grade of membership for $\widetilde{\alpha}$ in $\overline{P}$ is $\mu_{\overline{P}}(\widetilde{\alpha}) : \mathbb{U} \longrightarrow [0,1]$.

2.2. *Fuzzy number [28]*

Consider the fuzzy set $\overline{P}$, which is defined on the universal set of real numbers. If R is a fuzzy number, its membership function satisfies the following conditions:

i. The membership function $\mu_{\overline{P}}(\widetilde{\alpha})$ is a piecewise continuous.
ii. $\overline{P}$ is a convex fuzzy set, i.e. $\mu_{\overline{P}}(\rho\widetilde{\alpha}_1 + (1-\rho)\widetilde{\alpha}_2) \geq \min(\mu_{\overline{P}}(\widetilde{\alpha}_1), \mu_{\overline{P}}(\widetilde{\alpha}_2)),\ \forall\, \widetilde{\alpha}_1, \widetilde{\alpha}_2 \in$ R and $\forall\, \rho \in [0,1]$.
iii. $\overline{P}$ is a Normal fuzzy set, i.e. $\exists\, \widetilde{\alpha} \in$ R, s.t $\mu_{\overline{P}}(\widetilde{\alpha}) = 1$.

2.3. *Triangular fuzzy number (TFN) [28]*

A fuzzy number $\overline{P} = (\widetilde{p}_1, \widetilde{p}_2, \widetilde{p}_3)$ on R is a TFN if its membership function $\overline{P}$: R → [0,1] satisfies the following conditions. Fig. 1 illustrates the TFN membership function graphically.

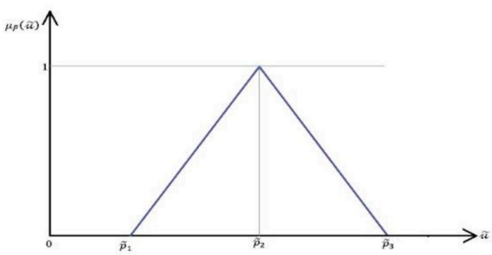

Fig. 1. Triangular fuzzy number.

$$\overline{P}(\widetilde{\mathscr{u}})=\begin{cases}\dfrac{\widetilde{\mathscr{u}}-\widetilde{p}_1}{\widetilde{p}_2-\widetilde{p}_1}, for\ \widetilde{p}_1\leq\widetilde{\mathscr{u}}\leq\widetilde{p}_2\\ 1, for\ \widetilde{\mathscr{u}}=\widetilde{p}_2\\ \dfrac{\widetilde{p}_3-\widetilde{\mathscr{u}}}{\widetilde{p}_3-\widetilde{p}_2}, for\ \widetilde{p}_2\leq\widetilde{\mathscr{u}}\leq\widetilde{p}_3\\ 0, elsewhere\end{cases}$$

2.4. Positive Triangular fuzzy number (PTFN) [31]

A PTFN is denoted as $(\widetilde{p}_1,\widetilde{p}_2,\widetilde{p}_3)$ where all $\widetilde{p}_i>0,\forall\, i=1,2,3$.

2.5. Negative Triangular fuzzy number (NTFN) [31]

A NTFN is denoted as $(\widetilde{p}_1,\widetilde{p}_2,\widetilde{p}_3)$ where all $\widetilde{p}_i<0,\forall\, i=1,2,3$.

2.6. Arithmetic operation on Triangular fuzzy numbers [28]

Let $\overline{P}=(\widetilde{p}_1,\widetilde{p}_2,\widetilde{p}_3)$ and $\overline{Q}=(\widetilde{q}_1,\widetilde{q}_2,\widetilde{q}_3)$ are two TFNs. Then operations are stated as follows:

(i) Addition: $\overline{P}+\overline{Q}=(\widetilde{p}_1+\widetilde{q}_1,\widetilde{p}_2+\widetilde{q}_2,\widetilde{p}_3+\widetilde{q}_3)$
(ii) Subtraction: $\overline{P}-\overline{Q}=(\widetilde{p}_1-\widetilde{q}_3,\widetilde{p}_2-\widetilde{q}_2,\widetilde{p}_3-\widetilde{q}_1)$
(iii) Multiplication: If $\overline{P}=(\widetilde{p}_1,\widetilde{p}_2,\widetilde{p}_3)$ is an arbitrary TFN and $\overline{Q}=(\widetilde{q}_1,\widetilde{q}_2,\widetilde{q}_3)$ is a non-negative TFN, the operations are given as follows:

$$\overline{P}\odot\overline{Q}=\overline{PQ}=\left\{\begin{array}{c}(\widetilde{p}_1\widetilde{q}_1,\widetilde{p}_2\widetilde{q}_2,\widetilde{p}_3\widetilde{q}_3)\ \ if\ \widetilde{p}_1\geq0,\\ (\widetilde{p}_1\widetilde{q}_3,\widetilde{p}_2\widetilde{q}_2,\widetilde{p}_3\widetilde{q}_3)\ \ if\ \widetilde{p}_1<0,\widetilde{p}_3\geq0,\\ (\widetilde{p}_1\widetilde{q}_3,\widetilde{p}_2\widetilde{q}_2,\widetilde{p}_3\widetilde{q}_1)\ \ if\ \widetilde{p}_3<0,\end{array}\right\}$$

2.7. Ranking of a fuzzy number [31]

The final answer is frequently a fuzzy number in problems involving fuzzy decision-making. To offer these fuzzy solutions a preference order, we need a strategy or procedure that can deliver crisp preference ordering. Ranking of a fuzzy number is the term used to describe this procedure. The arithmetic mean of the TFN's elements is used in this study to determine the TFN's ranking; for example, if $\overline{P}=(\widetilde{p}_1,\widetilde{p}_2,\widetilde{p}_3)$ is a fuzzy number, then $\overline{P}$'s ranking is given by

$$R(\overline{P})=\frac{\widetilde{p}_1+\widetilde{p}_2+\widetilde{p}_3}{3}.$$

2.8. Harmonic Mean [31]

Data aggregation for central tendency is frequently performed using the harmonic mean. Generally, this is described as being the reciprocal of the arithmetic mean of the reciprocal of observations. For example, if $\overline{P}=(\widetilde{p}_1,\widetilde{p}_2,\widetilde{p}_3)$ there are three observations, the HM of these 3 observations is given as

$$HM(\overline{P})=\frac{3}{\left(\frac{1}{\widetilde{p}_1}+\frac{1}{\widetilde{p}_2}+\frac{1}{\widetilde{p}_3}\right)}.$$

2.9. Mathematical formulation of a fuzzy multi-objective transportation problem [31]

A FMOTP can be expressed mathematically as follows:

$$\text{Min}\ \mathscr{Z}_k(\overline{\mathscr{X}})=\sum_{i=1}^{m}\sum_{j=1}^{n}C_{ij}^{k}\overline{\mathscr{X}}_{ij}(k=1,2,.....r) \tag{1}$$

Subject to the constraints

$$\sum_{j=1}^{n}\overline{\mathscr{X}}_{ij}=\mathscr{s}_i, i=1,2,.........,\widetilde{m} \tag{2}$$

$$\sum_{i=1}^{m} \overline{\mathscr{X}}_{ij} = \mathscr{d}_j, j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{3}$$

$$\overline{\mathscr{X}}_{ij} \geq 0, i = 1, 2, \ldots\ldots\ldots, \widetilde{m} \text{ and } j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{4}$$

Where $\widetilde{m}$: total number of sources, $\widetilde{n}$: total number of destinations

Notations:

$\mathscr{s}_i$: Fuzzy supply of the commodity at i^{th} the origin.

$\mathscr{d}_j$: Fuzzy demand of the commodity at j^{th} the destination.

C_{ij}^{k}: The cost of transporting one unit of the commodity from i^{th} the source to j^{th} the destination associated with the k^{th} objective function.

$\overline{\mathscr{X}}_{ij}$: A fuzzy quantity is supplied from i^{th} the source to the j^{th} destination in order to minimize the amount of overall fuzzy transportation.

$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k}\overline{\mathscr{X}}_{ij}$: The cost of transporting single unit of the commodity from i^{th} the source to the j^{th} destination.

The previous problem (equations (1)–(4)) may be stated as follows:

$$\text{Min } \mathscr{Z}_k(\overline{\mathscr{X}}) = \min\left(h_{11}(\overline{\mathscr{X}}), h_{12}(\overline{\mathscr{X}}), \ldots, h_{1r}(\overline{\mathscr{X}})\right) \tag{5}$$

Where,

$$h_{11}(\overline{\mathscr{X}}) = ((v_{11}^{l}, v_{11}^{m}, v_{11}^{u}) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(C_{ij}^{1l}, C_{ij}^{1m}, C_{ij}^{1u}\right) \otimes \left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right)$$

$$h_{12}(\overline{\mathscr{X}}) = (v_{12}^{l}, v_{12}^{m}, v_{12}^{u}) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(C_{ij}^{2l}, C_{ij}^{2m}, C_{ij}^{2u}\right) \otimes \left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right)$$

$$h_{1r}(\overline{\mathscr{X}}) = (v_{1r}^{l}, v_{1r}^{m}, v_{1r}^{u}) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(C_{ij}^{rl}, C_{ij}^{rm}, C_{ij}^{ru}\right) \otimes \left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right)$$

$$\text{s.t} \sum_{j=1}^{n}\left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right) = \left(s_i^{l}, s_i^{m}, s_i^{u}\right), i = 1, 2, \ldots\ldots\ldots, \widetilde{m} \tag{6}$$

$$\sum_{i=1}^{m}\left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right) = \left(d_j^{l}, d_j^{m}, d_j^{u}\right), j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{7}$$

$$\left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right) \geq 0, i = 1, 2, \ldots\ldots\ldots, \widetilde{m} \ \& \ j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{8}$$

3. Proposed approach to a fuzzy multi-objective transportation problem

Theorem 3.1:

If $\overline{P} = (\widetilde{p}_1, \widetilde{p}_2, \widetilde{p}_3)$ be the TFN, then the fuzzy harmonic mean

$$HM(\overline{P}) = \frac{3\widetilde{p}_1\widetilde{p}_2\widetilde{p}_3}{(\widetilde{p}_1\widetilde{p}_2 + \widetilde{p}_2\widetilde{p}_3 + \widetilde{p}_3\widetilde{p}_1)}$$

Proof:

In order to determine the harmonic mean formula for a Triangular fuzzy number $\overline{P} = (\widetilde{p}_1, \widetilde{p}_2, \widetilde{p}_3)$, Cheng [42] provided the following centroid formula to modify the HM formula, using Triangular membership formula we have

$$f^{L}(\overline{P}) = \frac{\widetilde{u} - \widetilde{p}_1}{\widetilde{p}_2 - \widetilde{p}_1}, f^{R}(\overline{P}) = \frac{\widetilde{p}_3 - \widetilde{u}}{\widetilde{p}_3 - \widetilde{p}_2}, \text{then we get}$$

$$HM(\overline{P}) = \frac{\int_{\tilde{p}_1}^{\tilde{p}_2} f^{L}(\overline{P})du + \int_{\tilde{p}_2}^{\tilde{p}_3} f^{R}(\overline{P})du}{\int_{\tilde{p}_1}^{\tilde{p}_2} uf^{L}(\overline{P})du + \int_{\tilde{p}_2}^{\tilde{p}_3} uf^{R}(\overline{P})du}$$

$$HM(\overline{P}) = \frac{\int_{\tilde{p}_1}^{\tilde{p}_2} \frac{\widetilde{u} - \widetilde{p}_1}{\widetilde{p}_2 - \widetilde{p}_1} du + \int_{p_2}^{p_3} \frac{\widetilde{p}_3 - \widetilde{u}}{\widetilde{p}_3 - \widetilde{p}_2} du}{\int_{p_1}^{p_2} \frac{\widetilde{u} - \widetilde{p}_1}{\widetilde{p}_2 - \widetilde{p}_1} udu + \int_{p_2}^{p_3} \frac{\widetilde{p}_3 - \widetilde{u}}{\widetilde{p}_3 - \widetilde{p}_2} udu}$$

$$HM(\overline{P}) = \frac{\frac{1}{(\widetilde{p}_2 - \widetilde{p}_1)}\int_{\widetilde{p}_1}^{\widetilde{p}_2}(u-\widetilde{p}_1)du + \frac{1}{(\widetilde{p}_3-\widetilde{p}_2)}\int_{\widetilde{p}_2}^{\widetilde{p}_3}(\widetilde{p}_3 - u)du}{\frac{1}{(\widetilde{p}_2 - \widetilde{p}_1)}\int_{\widetilde{p}_1}^{\widetilde{p}_2}(u-\widetilde{p}_1)udu + \frac{1}{(\widetilde{p}_3-\widetilde{p}_2)}\int_{\widetilde{p}_2}^{\widetilde{p}_3}(\widetilde{p}_3 - u)udu}$$

$$HM(\overline{P}) = \frac{(\widetilde{p}_3 - \widetilde{p}_2)\int_{\widetilde{p}_1}^{\widetilde{p}_2}(u-\widetilde{p}_1)du + (\widetilde{p}_2 - \widetilde{p}_1)\int_{\widetilde{p}_2}^{\widetilde{p}_3}(\widetilde{p}_3 - u)du \Big/ (\widetilde{p}_2 - \widetilde{p}_1)(\widetilde{p}_3 - \widetilde{p}_2)}{(\widetilde{p}_3 - \widetilde{p}_2)\int_{\widetilde{p}_1}^{\widetilde{p}_2}(u-\widetilde{p}_1)udu + (\widetilde{p}_2 - \widetilde{p}_1)\int_{\widetilde{p}_2}^{\widetilde{p}_3}(\widetilde{p}_3 - u)udu \Big/ (\widetilde{p}_2 - \widetilde{p}_1)(\widetilde{p}_3 - \widetilde{p}_2)}$$

$$HM(\overline{P}) = \frac{(\widetilde{p}_3 - \widetilde{p}_2)\left[\frac{u^2}{2} - \widetilde{p}_1 u\right]_{\widetilde{p}_1}^{\widetilde{p}_2} + (\widetilde{p}_2 - \widetilde{p}_1)\left[\widetilde{p}_3 u - \frac{u^2}{2}\right]_{\widetilde{p}_2}^{\widetilde{p}_3}}{(\widetilde{p}_3 - \widetilde{p}_2)\left[\frac{u^3}{3} - \widetilde{p}_1 \frac{u^2}{2}\right]_{\widetilde{p}_1}^{\widetilde{p}_2} + (\widetilde{p}_2 - \widetilde{p}_1)\left[\widetilde{p}_3 \frac{u^2}{2} - \frac{u^3}{3}\right]_{\widetilde{p}_2}^{\widetilde{p}_3}} \tag{9}$$

Take the numerator in equation (9),

$$\Longrightarrow (\widetilde{p}_3 - \widetilde{p}_2)\left[\frac{u^2}{2} - \widetilde{p}_1 u\right]_{\widetilde{p}_1}^{\widetilde{p}_2} + (\widetilde{p}_2 - \widetilde{p}_1)\left[\widetilde{p}_3 u - \frac{u^2}{2}\right]_{\widetilde{p}_2}^{\widetilde{p}_3}$$

$$= (\widetilde{p}_3 - \widetilde{p}_2)\left[\frac{\widetilde{p}_2{}^2}{2} - \widetilde{p}_1\widetilde{p}_2 - \frac{\widetilde{p}_1^2}{2} + \widetilde{p}_1^2\right] + (\widetilde{p}_2 - \widetilde{p}_1)\left[\widetilde{p}_3{}^2 - \frac{\widetilde{p}_3^2}{2} - \widetilde{p}_3\widetilde{p}_2 + \frac{\widetilde{p}_2^2}{2}\right]$$

$$= \left[\frac{\widetilde{p}_2^2\widetilde{p}_3}{2} - \widetilde{p}_1\widetilde{p}_2\widetilde{p}_3 - \frac{\widetilde{p}_1^2\widetilde{p}_3}{2} + \widetilde{p}_1^2\widetilde{p}_3 - \frac{\widetilde{p}_2^3}{2} + \widetilde{p}_1\widetilde{p}_2^2 + \frac{\widetilde{p}_1^2\widetilde{p}_2}{2} - \widetilde{p}_1^2\widetilde{p}_2\right] + \left[\widetilde{p}_2\widetilde{p}_3^2 - \frac{\widetilde{p}_2\widetilde{p}_3^2}{2} - \widetilde{p}_3\widetilde{p}_2^2 + \frac{\widetilde{p}_2^3}{2} - \widetilde{p}_1\widetilde{p}_3^2 + \frac{\widetilde{p}_1\widetilde{p}_3^2}{2} + \widetilde{p}_1\widetilde{p}_3\widetilde{p}_2 - \frac{\widetilde{p}_1\widetilde{p}_2^2}{2}\right]$$

$$= \frac{1}{2}\left[\widetilde{p}_3\widetilde{p}_2^2 - \widetilde{p}_1^2\widetilde{p}_3 + 2\widetilde{p}_1^2\widetilde{p}_3 + 2\widetilde{p}_1\widetilde{p}_2^2 + \widetilde{p}_1^2\widetilde{p}_2 - 2\widetilde{p}_1^2\widetilde{p}_2 + 2\widetilde{p}_2\widetilde{p}_3^2 - \widetilde{p}_2\widetilde{p}_3^2 - 2\widetilde{p}_3\widetilde{p}_2^2 - 2\widetilde{p}_1\widetilde{p}_3^2 + \widetilde{p}_1\widetilde{p}_3^2 - \widetilde{p}_1\widetilde{p}_2^2\right]$$

$$= \frac{1}{2}\left[\widetilde{p}_1^2(\widetilde{p}_3 - \widetilde{p}_2) + \widetilde{p}_2^2(\widetilde{p}_1 - \widetilde{p}_3) + \widetilde{p}_3^2(\widetilde{p}_2 - \widetilde{p}_1)\right]$$

$$= \frac{1}{2}\left[-\widetilde{p}_2^2\widetilde{p}_3 + \widetilde{p}_3\widetilde{p}_1^2 + \widetilde{p}_1\widetilde{p}_2^2 - \widetilde{p}_1^2\widetilde{p}_2 + \widetilde{p}_2\widetilde{p}_3^2 - \widetilde{p}_1\widetilde{p}_3^2\right]$$

$$= \frac{1}{2}\left[\widetilde{p}_1^2(\widetilde{p}_3 - \widetilde{p}_2) + \widetilde{p}_2^2(\widetilde{p}_1 - \widetilde{p}_3) + \widetilde{p}_3^2(\widetilde{p}_2 - \widetilde{p}_1)\right] \tag{10}$$

Take the denominator in equation (9)

$$\Longrightarrow (\widetilde{p}_3 - \widetilde{p}_2)\left[\frac{u^3}{3} - \widetilde{p}_1\frac{u^2}{2}\right]_{\widetilde{p}_1}^{\widetilde{p}_2} + (\widetilde{p}_2 - \widetilde{p}_1)\left[\widetilde{p}_3\frac{u^2}{2} - \frac{u^3}{3}\right]_{\widetilde{p}_2}^{\widetilde{p}_3}$$

$$= (\widetilde{p}_3 - \widetilde{p}_2)\left[\frac{\widetilde{p}_2^3}{3} - \frac{\widetilde{p}_1\widetilde{p}_2^2}{2} - \frac{\widetilde{p}_1^3}{3} + \frac{\widetilde{p}_1^3}{2}\right] + (\widetilde{p}_2 - \widetilde{p}_1)\left[\frac{\widetilde{p}_3^3}{2} - \frac{\widetilde{p}_3^3}{3} - \frac{\widetilde{p}_3\widetilde{p}_2^2}{2} + \frac{\widetilde{p}_2^3}{3}\right]$$

$$= \left[\frac{\widetilde{p}_3\widetilde{p}_2^3}{3} - \frac{\widetilde{p}_1\widetilde{p}_2^2\widetilde{p}_3}{2} - \frac{\widetilde{p}_1^3\widetilde{p}_3}{3} + \frac{\widetilde{p}_1^2\widetilde{p}_3}{2} - \frac{\widetilde{p}_2^4}{3} + \frac{\widetilde{p}_1\widetilde{p}_2^3}{2} + \frac{\widetilde{p}_1^3\widetilde{p}_2}{3} - \frac{\widetilde{p}_1^3\widetilde{p}_2}{2} + \frac{\widetilde{p}_2\widetilde{p}_3^3}{2} - \frac{\widetilde{p}_2\widetilde{p}_3^3}{3} - \frac{\widetilde{p}_3\widetilde{p}_2^3}{2} + \frac{\widetilde{p}_2^4}{3} - \frac{\widetilde{p}_1\widetilde{p}_3^3}{2} + \frac{\widetilde{p}_1\widetilde{p}_3^3}{3} + \frac{\widetilde{p}_1\widetilde{p}_3\widetilde{p}_2^2}{2} - \frac{\widetilde{p}_1\widetilde{p}_2^3}{3}\right]$$

$$= \frac{1}{6}\left[2\widetilde{p}_3\widetilde{p}_2^3 - 2\widetilde{p}_1^3\widetilde{p}_3 + 3\widetilde{p}_1^3\widetilde{p}_3 + 3\widetilde{p}_1\widetilde{p}_2^3 + 2\widetilde{p}_1^3\widetilde{p}_2 - 3\widetilde{p}_1^3\widetilde{p}_2 + 3\widetilde{p}_2\widetilde{p}_3^3 - 2\widetilde{p}_2\widetilde{p}_3^3 - 3\widetilde{p}_3\widetilde{p}_2^3 - 3\widetilde{p}_1\widetilde{p}_3^3 + 2\widetilde{p}_1\widetilde{p}_3^3 - 2\widetilde{p}_1\widetilde{p}_2^3\right]$$

$$= \frac{1}{6}\left[\widetilde{p}_1^3\widetilde{p}_3 - \widetilde{p}_1^3\widetilde{p}_2 + \widetilde{p}_1\widetilde{p}_2^3 + \widetilde{p}_2\widetilde{p}_3^3 - \widetilde{p}_1\widetilde{p}_3^3 - \widetilde{p}_3\widetilde{p}_2^3\right]$$

$$= \frac{1}{6}\left[\widetilde{p}_1^3(\widetilde{p}_3 - \widetilde{p}_2) + \widetilde{p}_2^3(\widetilde{p}_1 - \widetilde{p}_3) + \widetilde{p}_3^3(\widetilde{p}_2 - \widetilde{p}_1)\right] \tag{11}$$

Cyclic formula,

$$\widetilde{p}_1^3(\widetilde{p}_2 - \widetilde{p}_3) + \widetilde{p}_2^3(\widetilde{p}_3 - \widetilde{p}_1) + \widetilde{p}_1^3(\widetilde{p}_1 - \widetilde{p}_2) = -(\widetilde{p}_1 + \widetilde{p}_2 + \widetilde{p}_3)(\widetilde{p}_3 - \widetilde{p}_1)(\widetilde{p}_2 - \widetilde{p}_3)(\widetilde{p}_1 - \widetilde{p}_2)$$

$$\widetilde{p}_1^2(\widetilde{p}_2 - \widetilde{p}_3) + \widetilde{p}_2^2(\widetilde{p}_3 - \widetilde{p}_1) + \widetilde{p}_1^2(\widetilde{p}_1 - \widetilde{p}_2) = -(\widetilde{p}_1 - \widetilde{p}_2)(\widetilde{p}_2 - \widetilde{p}_3)(\widetilde{p}_3 - \widetilde{p}_1)$$

From equations (10) and (11) substituted in equation (9), then we get

$$HM(\overline{P}) = \frac{\frac{1}{2}\left[\widetilde{p}_1^2(\widetilde{p}_3 - \widetilde{p}_2) + \widetilde{p}_2^2(\widetilde{p}_1 - \widetilde{p}_3) + \widetilde{p}_3^2(\widetilde{p}_2 - \widetilde{p}_1)\right]}{\frac{1}{6}\left[\widetilde{p}_1^3(\widetilde{p}_3 - \widetilde{p}_2) + \widetilde{p}_2^3(\widetilde{p}_1 - \widetilde{p}_3) + \widetilde{p}_3^3(\widetilde{p}_2 - \widetilde{p}_1)\right]}$$

$$HM(\overline{P}) = \frac{6}{2}\left[\frac{[(\widetilde{p}_1 - \widetilde{p}_2)(\widetilde{p}_2 - \widetilde{p}_3)(\widetilde{p}_3 - \widetilde{p}_1)]}{[(\widetilde{p}_1 + \widetilde{p}_2 + \widetilde{p}_3)(\widetilde{p}_3 - \widetilde{p}_1)(\widetilde{p}_2 - \widetilde{p}_3)(\widetilde{p}_1 - \widetilde{p}_2)]}\right]$$

$$HM(\overline{P}) = \frac{3}{(\widetilde{p}_1 + \widetilde{p}_2 + \widetilde{p}_3)} \tag{12}$$

Put, $\widetilde{p}_1 = \frac{1}{\widetilde{p_1}}$, $\widetilde{p}_2 = \frac{1}{\widetilde{p_2}}$ and $\widetilde{p}_3 = \frac{1}{\widetilde{p_3}}$ in equation (12), then we get

$$HM(\overline{P}) = \frac{3}{\left(\frac{1}{\widetilde{p_1}} + \frac{1}{\widetilde{p_2}} + \frac{1}{\widetilde{p_3}}\right)}$$

$$HM(\overline{P}) = \frac{3\widetilde{p}_1\widetilde{p}_2\widetilde{p}_3}{(\widetilde{p}_1\widetilde{p}_2 + \widetilde{p}_2\widetilde{p}_3 + \widetilde{p}_3\widetilde{p}_1)} \tag{13}$$

Hence equation (13) denotes the Fuzzy Harmonic Mean.

3.1. Algorithm for the proposed approach

The adopted method of FMOTPs allows us to locate the optimal solution more efficiently. This includes the following steps.

Step 1. Verify whether the provided FMOTP is balanced or not.

Case 1. If $\sum_{i=1}^{m}(s_i^l, s_i^m, s_i^u) = \sum_{j=1}^{n}\left(d_j^l, d_j^m, d_j^u\right)$. Proceed to step 3 after that.

Case 2. If $\sum_{i=1}^{m}(s_i^l, s_i^m, s_i^u) \neq \sum_{j=1}^{n}\left(d_j^l, d_j^m, d_j^u\right)$, possible, never utilizing a dummy row or column when convert to balanced. We have, (i) $w = \sum_{i=1}^{m}(s_i^l, s_i^m, s_i^u) - \sum_{j=1}^{n}\left(d_j^l, d_j^m, d_j^u\right)$, *if* $\sum_{j=1}^{n}\left(d_j^l, d_j^m, d_j^u\right) < \sum_{i=1}^{m}(s_i^l, s_i^m, s_i^u)$. (ii) $w = \sum_{j=1}^{n}\left(d_j^l, d_j^m, d_j^u\right) - \sum_{i=1}^{m}(s_i^l, s_i^m, s_i^u)$, *if* $\sum_{i=1}^{m}(s_i^l, s_i^m, s_i^u) < \sum_{j=1}^{n}\left(d_j^l, d_j^m, d_j^u\right)$.

Step 2. The difference $w = (\omega_l, \omega_m, \omega_u)$ which is adding to the minimal supply (s^l, s^m, s^u) or demand $\left(d^l, d^m, d^u\right)$. Construct the supplied Fuzzy transportation table utilizing $(s^l + \omega_l, s^m + \omega_m, s^u + \omega_u)/\left(d^l + \omega_l, d^m + \omega_m, d^u + \omega_u\right)$.

Step 3. In this step, the complete problem (equations (1)–(8)) is divided into TL-CMOTP).

3.1.1. L - lower level (L-MOTP)

$$\min(v_{11}^l, v_{12}^i, \ldots, v_{1r}^l)$$

$$= \min\left(\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{1l} \otimes \widetilde{x}_{ij}^l, \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{2l} \otimes \widetilde{x}_{ij}^l, \ldots, \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{rl} \otimes \widetilde{x}_{ij}^l\right) \tag{14}$$

$$\text{s.t} \ \sum_{j=1}^{n} \widetilde{x}_{ij}^l = s_i^l, i = 1, 2, \ldots\ldots\ldots, \widetilde{m} \tag{15}$$

$$\sum_{i=1}^{m} \widetilde{x}_{ij}^l = d_j^l, j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{16}$$

$$\widetilde{x}_{ij}^l \geq 0, i = 1, 2, \ldots\ldots\ldots, \widetilde{m}\ ; \ j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{17}$$

3.1.2. M − middle level (M-MOTP)

$$\min(v_{11}^m, v_{12}^m, \ldots, v_{1r}^m)$$

$$= \min\left(\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{1m} \otimes \widetilde{x}_{ij}^m, \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{2m} \otimes \widetilde{x}_{ij}^m, \ldots, \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{rm} \otimes \widetilde{x}_{ij}^m\right) \tag{18}$$

$$\text{s.t} \sum_{j=1}^{n} \widetilde{x}_{ij}^{m} = s_i^m, i = 1, 2, \ldots\ldots\ldots, \widetilde{m} \tag{19}$$

$$\sum_{i=1}^{m} \widetilde{x}_{ij}^{m} = d_j^m, j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{20}$$

$$\widetilde{x}_{ij}^{m} \geq 0, i = 1, 2, \ldots\ldots\ldots, \widetilde{m}\ ;\ j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{21}$$

3.1.3. U - upper level (U-MOTP)

$$\min(v_{11}^u, v_{12}^u, \ldots, v_{1r}^u)$$

$$= \min\left(\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{1u} \otimes \widetilde{x}_{ij}^{u}, \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{2u} \otimes \widetilde{x}_{ij}^{u}, \ldots, \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{ru} \otimes \widetilde{x}_{ij}^{u}\right) \tag{22}$$

$$\text{s.t} \sum_{j=1}^{n} \widetilde{x}_{ij}^{u} = s_i^u, i = 1, 2, \ldots\ldots\ldots, \widetilde{m} \tag{23}$$

$$\sum_{i=1}^{m} \widetilde{x}_{ij}^{u} = d_j^u, j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{24}$$

$$\widetilde{x}_{ij}^{u} \geq 0, i = 1, 2, \ldots\ldots\ldots, \widetilde{m}\ ;\ j = 1, 2, \ldots\ldots\ldots, \widetilde{n} \tag{25}$$

Step 4. Using equation (13), $HM(\overline{P}) = \frac{3\widetilde{p}_1\widetilde{p}_2\widetilde{p}_3}{(\widetilde{p}_1\widetilde{p}_2+\widetilde{p}_2\widetilde{p}_3+\widetilde{p}_3\widetilde{p}_1)}$ determine the harmonic mean of the relevant cells in the TL-CMOTPs (L-MOTP, M-MOTP, and U-MOTP). Find the harmonic mean of the respective cells of the three objectives like cost, time and distance. Here by first choosing L-MOTP.

Step 5. After step 4, subtract the whole matrix's components by their smallest element.

Step 6. After step 5, subtract each row entry from the corresponding row minimum in the following transportation table.

Step 7. In the reduced matrix obtained from step 6, subtract every single column entry of the given transportation table from the associated column minimum.

Step 8. Select the row's or columns minimum value for the fuzzy cost. Next, identify and give the lowest supply and demand values.

Step 9. After completing step 8, remove the row or column when supply and demand have hit their maximum.

Step 10. Continue steps 8 to 9 until $(\widetilde{m} + \widetilde{n} - 1)$ cells are allotted.

Step 11. We may obtain the values of $\widetilde{x}_{ij}, (i = 1, 2, \ldots\ldots\ldots, \widetilde{m}, j = 1, 2, \ldots\ldots\ldots, \widetilde{n})$ for all three levels $\left(\widetilde{x}_{ij}^{l}, \widetilde{x}_{ij}^{m}, \widetilde{x}_{ij}^{u}\right)$, by solving these distinct level crisps MOTPs. This results in a combined fuzzy optimal solution for the necessary problem. The optimal solution for the problems in equations (14)–(22)–(25) in this case ensures that the fuzzy optimal solution to FMOTP (eqns (6)–(9)) is a non-negative TFN.

Step 12. The fuzzy compromise solution from step 11 is defuzzified in this phase to produce crisp values using the ranking function from Section 2.7. This will enable us to compare the results to those of previous studies.

Now, we provide the solution methodology in the flowchart below.

3.2. Flow diagram for the suggested approach

.

Table 2
Transportation cost for FMOTP.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	(60, 65, 70)	(55, 60, 75)	(90, 95, 100)	(72, 80, 90)	(50, 80, 90)	(80, 90, 110)	(90, 120, 150)
T_2	(30, 60, 90)	(40, 55, 75)	(15, 50, 70)	(110, 120, 140)	(60, 70, 80)	(64, 70, 78)	(75, 95, 105)
T_3	(80, 85, 90)	(130, 150, 170)	(70, 95, 105)	(100, 120, 140)	(80, 90, 110)	(70, 93, 110)	(45, 60, 75)
T_4	(105, 120, 140)	(115, 140, 150)	(50, 75, 95)	(60, 75, 90)	(75, 85, 95)	(40, 50, 60)	(60, 75, 90)
Demand	(35, 42, 55)	(50, 65, 75)	(45, 60, 70)	(60, 75, 85)	(42, 58, 70)	(38, 50, 65)	

4. Numerical example 1

A tile company has four factories that manufacture tiles, which are transported to six distribution centers. The cost of transportation, supply and demand criteria, like minimizing overall transportation cost, time, and distance, as well as every one of the problem parameters, which are referred to as per-unit product quantity of supply, needed demand are considered in TFNs. Tables 2–4 provide the cost matrices for the objective functions.

Using the suggested algorithm:

Step 1: $\sum_{i=1}^{m} \mathscr{s}_i = (270, 350, 420)$, $\sum_{j=1}^{n} \mathscr{d}_j = (270, 350, 420)$.

$$\Longrightarrow \sum_{i=1}^{m} \mathscr{s}_i = \sum_{j=1}^{n} \mathscr{d}_j.$$

Since the given FTP is balanced, proceed to step 3 now.

Step 3: Using the equations (14)–(25), the complete problem is divided into TL-CMOTPs (lower, middle, and upper), which are given in Tables 5, 10 and 12, respectively.

Step 4: The HM of corresponding cells in the lower-level Crisp multi-objective (cost, time, and distance) transportation problem is presented in Table 6.

Step 5: As shown in Table 7, at this stage, all the matrix's elements are subtracted by the matrix's smallest element.

Step 6: After step 5, subtract every single row entry from the transportation table provided by the equivalent row minimum shown in Table 8.

Step 7: After using steps 7–10, we get the final allocation given in Table 9.

From Table 9, the objective values are T C ($\mathscr{Z}_1$), TT ($\mathscr{Z}_2$) and TD ($\mathscr{Z}_3$) for the lower- MOFTP is

$$\mathscr{Z}_1 = 55x50 + 72x40 + 30x35 + 110x2 + 64x38 + 100x3 + 42x80 + 50x45 + 60x15$$

$$= 2750 + 2880 + 1050 + 220 + 2432 + 300 + 3360 + 2250 + 90 = 16142$$

$$\mathscr{Z}_2 = 4x50 + 3x40 + 2x35 + 2x6 + 2x38 + 2x3 + 1x42 + 2x45 + 6.5x15$$

$$= 200 + 120 + 70 + 12 + 76 + 6 + 42 + 90 + 97.5 = 713.5$$

$$\mathscr{Z}_3 = 5x50 + 7x40 + 3x35 + 8.5x2 + 4.5x38 + 6x3 + 3x42 + 3x45 + 10x15$$

$$= 250 + 280 + 105 + 17 + 171 + 18 + 126 + 135 + 150 = 1252$$

After using steps 4–10, we get the final allocation of the middle-level crisp MOTP given in Table 11.

From the Table 11, we get the objective values $\mathscr{Z}_1, \mathscr{Z}_2$ and $\mathscr{Z}_3$ for Middle-level MOTP (by Table 10) are

$$\mathscr{Z}_1 = 80x75 + 90x45 + 60x42 + 55x53 + 90x58 + 93x2 + 140x12 + 75x60 + 50x3$$
$$\mathscr{Z}_1 = 6000 + 4050 + 102 + 2915 + 5220 + 186 + 1680 + 4500 + 150$$
$$\mathscr{Z}_1 = 24803$$

$$\mathscr{Z}_2 = 4x75 + 7.5x45 + 3.5x42 + 4x53 + 2.5x58 + 10x2 + 5x12 + 3x60 + 3x9$$
$$\mathscr{Z}_2 = 300 + 337.5 + 147 + 212 + 145 + 20 + 60 + 180 + 27$$
$$\mathscr{Z}_2 = 1428.5$$

$$\mathscr{Z}_3 = 8x75 + 6x45 + 6x42 + 8x53 + 4x58 + 11.5x2 + 7.5x12 + 5x60 + 11x3$$
$$\mathscr{Z}_3 = 600 + 270 + 252 + 424 + 223 + 23 + 90 + 300 + 33$$
$$\mathscr{Z}_3 = 2224$$

After using steps 4-10, we get the final allocation Table 13 for Upper - level MOTP of Table 12.

From the above Table 13, the objective values $\mathscr{Z}_1, \mathscr{Z}_2$ and $\mathscr{Z}_3$ for the Upper-level MOTP are

Table 3
Transportation time for FMOTP.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	(7, 8.5, 9)	(4, 5.5, 6)	(5.5, 6, 7)	(3, 4, 5.5)	(8, 8.5, 9)	(6, 7.5, 8)	(90, 120, 150)
T_2	(2, 3.5, 5)	(3.5, 4, 6)	(5, 6.5, 8)	(6, 5, 7)	(7, 8.5, 9)	(2, 4.5, 6)	(75, 95, 105)
T_3	(3, 5, 6)	(8, 9, 10)	(3, 5, 6.5)	(2, 4, 5)	(1, 2.5, 4)	(9, 10, 11)	(45, 60, 75)
T_4	(7, 8, 10)	(4, 5, 6)	(2, 3, 4)	(6.5, 8, 10)	(8.5, 10, 11)	(8, 9, 10.5)	(60, 75, 90)
Demand	(35, 42, 55)	(50, 65, 75)	(45, 60, 70)	(60, 75, 85)	(42, 58, 70)	(38, 50, 65)	

Table 4
Transportation distance for FMOTP.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	$(10,15,20)$	$(5,10,15)$	$(10,12,14)$	$(7,8,9)$	$(11,14,17)$	$(4,6,8)$	$(90,120,150)$
T_2	$(3,6,9)$	$(6,8,10)$	$(4.5,5,6)$	$(8.5,10,12)$	$(10.5,12,13)$	$(4.5,6,8)$	$(75,95,105)$
T_3	$(5,6.5,8)$	$(7,9,10.5)$	$(4,5,6.5)$	$(6,8.5,9)$	$(3,4,5.5)$	$(10,11.5,13)$	$(45,60,75)$
T_4	$(8,8.5,10)$	$(6,7.5,9)$	$(3,5,6.5)$	$(10,12,14)$	$(9,11,15)$	$(9.5,11,13)$	$(60,75,90)$
Demand	$(35,42,55)$	$(50,65,75)$	$(45,60,70)$	$(60,75,85)$	$(42,58,70)$	$(38,50,65)$	

Table 5
Lower-level crisp multi-objective transportation problems (L-CMOTP).

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	60	55	90	72	50	80	90
	7	4	5.5	3	8	6	
	10	5	10	7	11	4	
T_2	30	40	15	110	60	64	75
	2	3.5	5	6	7	2	
	3	6	4.5	8.5	10.5	4.5	
T_3	80	130	70	100	80	70	45
	3	8	3	2	1	9	
	5	7	4	6	2	10	
T_4	105	115	50	60	75	40	60
	7	4	2	6.5	8.5	8	
	8	6	3	10	9	9.5	
Demand	35	50	45	60	42	38	

Table 6
HM of L-CMOTP

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	11.55	6.42	10.24	6.12	12.71	6.99	90
T_2	3.46	6.28	6.13	10.22	11.77	4.1	75
T_3	5.4	10.88	5.01	4.43	2.22	13.31	45
T_4	5.46	7.05	3.51	11.09	10.39	11.75	60
Demand	35	50	45	60	42	38	

Table 7

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	9.33	4.19	8.02	3.9	10.49	4.77	90
T_2	1.24	4.06	3.91	8	9.55	1.88	75
T_3	3.18	8.66	2.79	2.21	0	11.09	45
T_4	3.27	4.83	1.29	8.87	10.17	9.53	60
Demand	35	50	45	60	42	38	

Table 8

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	5.43	0.29	4.12	0	6.59	0.87	90
T_2	0	2.82	2.67	6.67	6.76	8.31	75
T_3	3.18	8.66	2.79	2.21	0	11.09	45
T_4	1.98	3.54	0	7.58	8.88	8.24	60
Demand	35	50	45	60	42	38	

$$\mathscr{Z}_1 = 75x75 + 75x90 + 90x55 + 78x50 + 140x5 + 110x70 + 95x70 + 90x5 + 60x15$$
$$\mathscr{Z}_1 = 5625 + 6750 + 4950 + 3900 + 700 + 7700 + 6650 + 450 + 900$$
$$\mathscr{Z}_1 = 37625$$

$$\mathscr{Z}_2 = 6x75 + 5.5x75 + 5x55 + 6x50 + 5x5 + 4x70 + 4x70 + 10x5 + 10.5x15$$
$$\mathscr{Z}_2 = 450 + 412.5 + 275 + 300 + 25 + 280 + 280 + 50 + 157.5$$
$$\mathscr{Z}_2 = 2230$$

Table 9
Final allocation table of L-CMOTP.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	5.43	**50** 0	4.12	**40** 0	6.59	0.21	90
T_2	**35** 0	2.53	2.67	**2** 6.76	8.31	**38** 0	75
T_3	3.18	8.37	2.79	**3** 2.21	**42** 0	10.43	45
T_4	1.98	3.25	**45** 0	**15** 7.58	8.88	7.58	60
Demand	35	50	45	60	42	38	

Table 10
Middle-level crisp multi-objective transportation problems (M-CMOTP).

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	65 8.5 15	60 55 10	95 6 12	80 4 8	80 8.5 14	90 7.5 6	120
T_2	60 3.5 6	55 4 8	50 6.5 5	120 5 10	70 8.5 12	70 4.5 6	95
T_3	85 5 6.5	150 9 9	95 5 5	120 4 8.5	10 2.5 4	93 10 11.5	60
T_4	120 8 8.5	140 5 7.5	75 3 5	75 8 12	85 10 11	50 9 11	75
Demand	**42**	**65**	**60**	**75**	**58**	**50**	

Table 11
Final allocation table of M-CMOTP.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	7.28	1.07	3.77	**75** 0	7.14	**45** 0.85	120
T_2	**42** 0	**53** 0	1.63	3.33	7.54	0	95
T_3	3.71	7.36	2.81	3.41	**58** 0	**2** 9.62	60
T_4	6.48	**12** 2.08	**60** 0	8.65	9.32	**3** 6.98	75
Demand	42	65	60	75	58	50	

Table 12
Upper-level crisp multi-objective transportation problems (U-CMOTP).

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	70 9 20	75 6 15	100 7 14	90 5.5 9	90 9 17	110 8 8	150
T_2	90 5 9	75 6 10	70 8 6	140 7 12	80 9 13	78 6 8	105
T_3	95 6 8	170 10 10.5	105 6.5 6.5	140 5 9	110 4 5.5	110 11 13	75
T_4	140 10 10	150 6 9	95 4 6.5	90 10 14	95 11 15	60 10.5 13	90
Demand	55	75	70	85	70	65	

Table 13
Final allocation table of U-CMOTP.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply
T_1	9.21	**75** 0	5.48	**75** 0	8.68	3.14	150
T_2	**55** 0	1.4	0.49	1.57	5.65	**50** 0	105
T_3	3.12	8.11	2.65	**5** 0.65	**70** 0	9.61	75
T_4	7.25	3.31	**70** 0	**5** 7.23	10.61	**15** 8.11	90
Demand	55	75	70	85	70	65	

$$\mathscr{Z}_3 = 15x75 + 9x75 + 9x55 + 8x50 + 9x5 + 5.5x70 + 6.5x70 + 14x5 + 13x15$$
$$\mathscr{Z}_3 = 1125 + 675 + 495 + 400 + 45 + 380 + 455 + 70 + 195 = 3840$$

Step 11&12: By completing these different level crisp MOTPs, we obtain the values of $\mathscr{x}_{ij}, (i = 1, 2, \ldots\ldots\ldots, \widetilde{m}\ \&\ j = 1, 2, \ldots\ldots\ldots, \widetilde{n})$ for all the three-level $\left(\mathscr{x}_{ij}^{l}, \mathscr{x}_{ij}^{m}, \mathscr{x}_{ij}^{u}\right)$, this results in a combined fuzzy optimum solution to the relevant problem.

Fuzzy transportation cost $\mathscr{Z}_1 = (16142, 24803, 37625), R(\mathscr{Z}_1) = 26190.$

Fuzzy transportation time $\mathscr{Z}_2 = (713.5, 1428.5, 2230), R(\mathscr{Z}_2) = 1457.33.$

Fuzzy transportation distance $\mathscr{Z}_3 = (1252, 2224, 3840), R(\mathscr{Z}_3) = 2438.66.$

4.1. Physical analysis of the results

1. For the given FTP, its fuzzy TC $\mathscr{Z}_1$ is a TFN, shown as follows: $\mathscr{Z}_1 = (16142, 24803, 37625)$.

In Fig. 2, it is depicted how the overall transportation cost differs depending on chance.

Therefore, the lowest overall transportation cost is going to be greater than 16142 but less than 24803, with the likelihood that it will be 37625 being the greatest.

2. For the given FTP, its fuzzy transportation time $\mathscr{Z}_2$ is a TFN, shown as follows:

$$\mathscr{Z}_2 = (713.5, 1428.5, 2230)$$

In Fig. 3, it is depicted how the overall transportation time differs depending on chance.

Therefore, the lowest total transportation time is likely to be greater than 713.5 and less than 1428.5, with the likelihood that it will be 2230 being the greatest.

3. For the given FTP, its fuzzy transportation distance $\mathscr{Z}_3$ is a TFN, shown as follows: $\mathscr{Z}_3 = (1252, 2224, 3840)$.

In Fig. 4, it is depicted how the overall transportation distance differs depending on chance.

Therefore, the lowest total transportation distance is likely to be greater than 1252 and less than 2224, with the likelihood that it will be 3840 being the greatest.

5. The proposed approach is used to solve a real-life fuzzy multi-objective transportation problem

Example 2: The example presented in Ref. [41] is utilized to compare the outcomes of the suggested approach to those of other approaches. Look for an online dairy store in three cities for fresh dairy products. Distributors have to deliver items within 12 h after an online purchase in eight locations. The major goal is to maximize profits and minimize TT and loss along a particular route. TFNs are provided for supply, demand, TT, loss of transportation, and overall transportation profit per unit.

Solution: Following the processes outlined above in our suggested technique, the final optimum values for all fuzzy three-objective functions (delivery time, loss, and profit) are (1945.2, 2265, 3048.5), (322, 842, 967.4), and (21908, 29195, 34151), respectively. Table 14 compares the fuzzy optimal solutions with the crisp solution achieved in Example 2.

Example 3: Consider an FFMOTP in which transportation cost and time, as well as supply and demand, are expressed in TFNs [26, 31].

Solution: The final answer can be achieved by following the procedures outlined above that are included in our suggested approach.

Fuzzy Transportation cost $\mathscr{Z}_1 = (107, 168, 230)$.

Fuzzy Transportation time $\mathscr{Z}_2 = (154, 268.5, 349)$.

In Table 15 provides a comparison of the fuzzy optimal solutions to the crisp solutions found in Example 3.

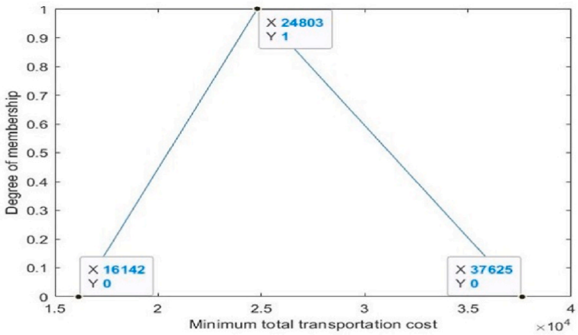

Fig. 2. Membership function of Triangular fuzzy number representing the minimum total transportation cost.

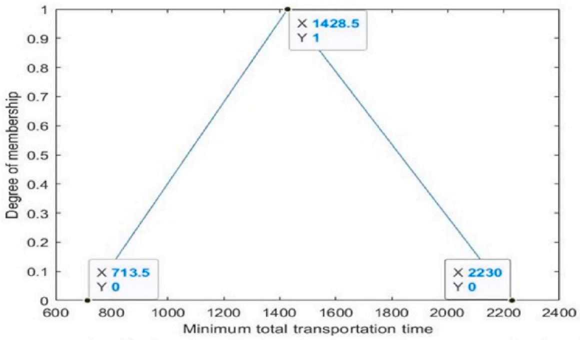

Fig. 3. Membership function of Triangular fuzzy number representing the minimum total transportation time.

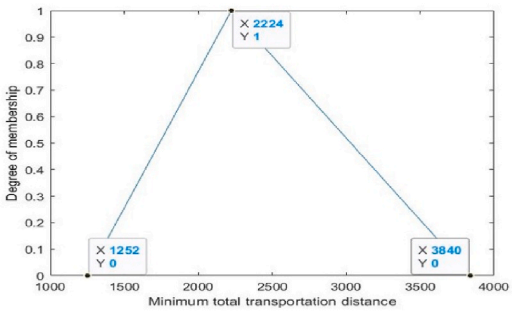

Fig. 4. Membership function of Triangular fuzzy number representing the minimum total transportation distance.

Table 14
Comparison of fuzzy optimal solution.

Objectives	Malihe Niksirat	Proposed Approach
Delivery Time	(1877.32, 2377.9, 2655.28)	(1945.2, 2265,3048.5)
Crisp solution.	2303.5	2419.5
Loss	(543.16, 798.1, 987.07)	(322, 842,967.4)
Crisp solution.	776.11	710.46
Profit	(16402.4, 21449.12, 23998.56)	(21908, 29195,34151)
Crisp solution.	20616.69	28418

6. Results and discussion

The present section discusses the findings found for Examples 2 & 3, utilizing the suggested technique with the other methods that were previously employed by Malihe Niksirat [41], R. Gowthami and K. Prabhakaran [26], and Yadvendra Kacher and Pitam Singh

Table 15
Comparison of fuzzy optimal solution.

Objectives	Gowthami, Prabakaran	Yadvendra Kacher, Pitam Singh	Proposed Approach
Cost	(187.5, 189.5,191.5)	(114, 174.5, 244)	(107, 168, 230)
Crisp solution.	189.5	177.5	168.33
Time	(176.5, 178.5, 180.5)	(126, 193.5, 271)	(154, 268.5, 349)
Crisp solution.	178.5	196.8	257.16

[31]. However, in order to better understand the final results, the fuzzy optimal solutions provided by the suggested approaches and the methods in Refs. [26,41] are defuzzified using the ranking function established in Section 2.7. Table 14 shows the crisp and final fuzzy optimal solutions obtained in Example 2 using Malihe Niksirat's [41] algorithm with our suggested method.

Similarly, Table 15 compares the fuzzy compromise solution as well as crisp solutions produced by our suggested technique with those obtained by Gowthami et al. [26] and Yadvendra Kacher et al. [31]. This also applies to Example 3.

It is clear from Table 16 that the proposed technique provided an optimal solution in examples 1, 2, and 3 that was better than those provided by Malihe Niksirat [41], Gowthami & Prabakaran [26], and Yadvendra Kacher and Pitam Singh [31] methods. The time target is a little high because of the competing nature of numerous purposes, however, both profit and cost objectives are comparatively better optimized than the other approaches now in use.

7. Advantages and disadvantages

- This technique simplifies calculations, minimizes iterations, and is useful for dealing with ambiguous information in real-world scenarios.
- The whole problem is divided into a TL-CMOLPP using the fuzzy HM. On the other hand, the whole problem is divided into a TL-CMOTP in our suggested approach. It then reduces to TL-CMOTP using fuzzy HM in a single step. As a result, our suggested approach has reduced computational complexity throughout the solution process as it requires fewer steps for calculation.
- The suggested technique solves both balanced and unbalanced FMOTPs. It converts an unbalanced TP towards a balanced TP not requiring a fake origin or destination.
- The suggested technique applies to solving FTP problems with fuzzy numbers for all parameters.
- The disadvantage of the proposed method is that it requires converting all problem objectives into a minimization type.
- The suggested approach efficiently solves FFMOTP with non-negative TFNs.

7.1. The managerial insights

In addressing a transportation challenge, the pivotal factors of cost, time, and distance come to the forefront. In the first numerical illustration, the decision-making process hinges on evaluating time, distance, and cost metrics. The scenario involves six distribution centers receiving tiles from four company-owned facilities. For the procurement of fresh dairy products, consideration extends to three cities and an online dairy outlet. With a time constraint of 12 h post-online transaction, eight sites are tasked with efficient delivery management. The overarching aim is to maximize profit and minimize distribution time as well as losses along designated routes. Transportation fuzzy numbers (TFNs) are employed to quantify supply and demand, time to delivery, transportation losses, and overall transportation profit per unit of production. Example 3 explores a fuzzy multi-objective transportation problem (FFMOTP), where transportation cost and time data, along with supply and demand parameters, are expressed in TFNs.

8. Conclusions and future scope

A new approach to handling an FFMOTP was presented in this paper. This method converts the FMOTP into a TL-CMOTP using fuzzy arithmetic. To determine the best values to use in calculating the FHM of these values, the objective functions for each of these crisp problems can be individually addressed. To obtain the necessary fuzzy compromise solution, we finally independently solved this TL-CMOTP. Two more examples are provided to examine the effectiveness of our suggested approach. Table 16 shows that the proposed method produces a more optimum solution for Examples 1, 2, and 3 than the methods of Malihe Niksirat [41], Gowthami and

Table 16
Comparative analyses of the crisp solution of MOFTP.

Problems/Methods	Example 1	Example 2	Example 3
Gowthami, Prabakaran	(29456, 1438.43, 3578.33)	(2458.5, 786.57, 24567)	(189.5, 178.5)
Yadvendra Kacher, Pitam Singh	(28858,1448.35, 2456.56)	(2339.6, 871.56, 32911)	(177.5, 196.8)
Malihe Niksirat	(28678,1564.55,2543)	(2303.5, 776.11, 20616.69)	(92.20,102)
Proposed Approach	(26190,1457.33, 2438.66)	(2419.5, 710.46, 28418)	(168.33, 257.16)

Prabakaran [26], and Yadvendra Kacher and Pitam Singh [31]. Moreover, examples are provided to assess the reliability of our suggested technique, and we discovered that it delivers a superior fuzzy compromise solution for these situations.

The proposed approach for resolving TL-CMOTPs without converting to FMOTPs may be updated in the future scope of this research. It is possible to use this suggested technique to solve FMOTP for different kinds of fuzzy parameters as well.

Funding

The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Data availability

Not applicable.

CRediT authorship contribution statement

A. Kokila: Writing – original draft. **G. Deepa:** Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

The authors wish to thank the management of Vellore Institute of Technology, Vellore – 632 014, India. for their continuous support and encouragement to carry out this research work.

References

[1] F.L. Hitchcock, The distribution of a product from several sources to numerous localities, J. Math. Phys. 20 (1941) 224–230.
[2] L.A. Zadeh, Fuzzy Sets, Information and Control, vol. 8, 1965, pp. 338–353.
[3] H.J. Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets Syst 1 (1) (1978) 45–55.
[4] A. Gupta, A. Kumar, A. Kaur, Mehar's method to find exact fuzzy optimal solution of unbalanced fully fuzzy multi-objective transportation problems, Optim. Lett. 6 (8) (2012) 1737–1751.
[5] F.A.E.W. Waiel, S.M. Lee, Interactive fuzzy goal programming for multi-objective transportation problems, Omega 34 (2006) 158–166.
[6] M. Zangiabadi, H.R. Maleki, Fuzzy goal programming for multi-objective transportation problems, J.Appl. Math. Comput. 24 (1–2) (2007) 449–460.
[7] Pitam Singh, Saru Kumari, Priyamvada Singh, Fuzzy efficient interactive goal programming approach for multi-objective transportation problems, Int. J. Appl. Comput. Math. 3 (2) (2017) 505–525.
[8] V. Vidhya, K. Ganesan, Efficient solution of a multi objective fuzzy transportation problem, J. Phys. Conf. Ser. 1000 (1) (2018) 012132, https://doi.org/10.1088/1742-6596/1000/1/012132.
[9] S. Dhansekar, S. Hariaran, Seker, Fuzzy Hungarian MODI Algorithem to solve fully fuzzy transportation problems, Int. J. Fuzzy Syst. 19 (5) (2017), 1479-149.
[10] P. Kundu, S. Kar, M. Maiti, Multi-objective multi-item solid transportation problem in fuzzy environment, Appl. Math. Model. 37 (4) (2013) 2028–2038.
[11] A. Ebrahimnejad, An improved approach for solving fuzzy transportation problem with triangular fuzzy numbers, J. Intell. Fuzzy Syst. 29 (2) (2015) 963–974.
[12] S.K. Bharati, S.R. Singh, A computational algorithm for the solution of fully fuzzy multi-objective linear programming problem, Int. J. Dyn. Control 6 (3) (2018) 1384–1391.
[13] George A. Osuji, Cecilia N. Okoli, Jude Opara, Solution of multi-objective transportation problem via fuzzy programming algorithm, Sci. J. Appl. Math. Stat. 2 (4) (2014) 71–77, https://doi.org/10.11648/j.sjams.20140204.11.
[14] J.Y. Dong, S.P. Wan, A new method for solving fuzzy multi-objective linear programming problems, Iranian Journal of Fuzzy Systems 16 (Number 3) (2019) 145–159.
[15] Shu-Ping Wan, Ying-Li Qin, Jiu-Ying Dong, A hesitant fuzzy mathematical programming method for hybrid multi-criteria group decision making with hesitant fuzzy truth degrees, Knowl. Base Syst. 138 (2017) 232–248.
[16] Jiu-ying Dong, Shu-Ping Wan, A new trapezoidal fuzzy linear programming method considering the acceptance degree of fuzzy constraints violated, Knowl. Base Syst. 148 (2018) 100–114.
[17] Shu-Ping Wan, Jiu-Ying Dong, Interval-valued intuitionistic fuzzy mathematical programming method for hybrid multi-criteria group decision making with interval-valued intuitionistic fuzzy truth degrees, Inf. Fusion 26 (2015) 49–65.
[18] Shu-Ping Wan, Zhen Jin, Jiu-Ying Dong, Pythagorean fuzzy mathematical programming method for multi-attribute group decision making with Pythagorean fuzzy truth degrees, Knowl. Inf. Syst. 55 (2018) 437–466, https://doi.org/10.1007/s10115-017-1085-6.
[19] Shu-Ping Wan, Jiu-Ying Dong, Possibility linear programming with trapezoidal fuzzy numbers, Appl. Math. Model. 38 (2014) 1660–1672.
[20] A. Mahmoodirad, T. Allahviranloo, S. Niroomand, A new effective solution method for fully intuitionistic fuzzy transportation problem, Soft Comput. 23 (12) (2019) 4521–4530.
[21] A. Baidya, U.K. Bera, Solid transportation problem under fully fuzzy environment, Int. J. Math. Oper. Res. 15 (4) (2019) 498–539.
[22] P. Anukokila, B. Radhakrishnan, Goal programming approach to fully fuzzy fractional transportation problem, J. Taibah Univ. Sci. 13 (1) (2019) 864–874.
[23] A. Mishra, A. Kumar, Mehar method to find a unique fuzzy optimal value of balanced fully triangular fuzzy transportation problems, in: Aggregation Operators for Various Extensions of Fuzzy Set and its Applications in Transportation Problems, Springer, 2021, pp. 87–118.
[24] M. Bagheri, A. Ebrahimnejad, Solving the fully fuzzy multi-objective transportation problem based on the common set of weights in DEA, J. Intell. Fuzzy Syst. 39 (3) (2020) 3099–3124.
[25] S. Ghosh, S.K. Roy, Fuzzy-rough multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station, Oper. Res. 55 (2021) S2923–S2952.
[26] R. Gowthami, K. Prabakaran, Solution of multi objective transportation problem under fuzzy environment, J. Phys. Conf. Ser. 1377 (1) (2019) 012038, https://doi.org/10.1088/1742-6596/1377/1/012038.

[27] G. Krishnaveni, K. Ganesan, A fully fuzzy multi objective fuzzy transportation problem under fuzzy environment, AIP Conf. Proc. 2277 (1) (2020) 090009, https://doi.org/10.1063/5.0025267.
[28] A. Kokila, G. Deepa, Fuzzy transportation problem by implementing the row-column Maxima method, Reliability Theory & Applications 17 (71) (December 2022), 4.
[29] A. Mishra, A. Kumar, JMD approach for solving unbalanced fully trapezoidal intuitionistic fuzzy transportation problems, in: Aggregation Operators for Various Extensions of Fuzzy Set and its Applications in Transportation Problems, Springer, 2021, pp. 143–203.
[30] S. Ghosh, S.K. Roy, A. Ebrahimnejad, J.L. Verdegay, Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem, Complex Intell. Syst. 7 (2) (2021) 1009–1023.
[31] Yadvendra Kacher, Pitam Singh, Fuzzy harmonic mean technique for solving fully fuzzy multi-objective transportation problem, Journal of Computational Science 63 (2022) 101782.
[32] E. Fathy, A.E. Hassanien, Fuzzy harmonic mean technique for solving fully fuzzy multilevel multi-objective linear programming problems, Alex. Eng. J. 61 (10) (2022) 8189–8205.
[33] M.A. El Sayed, A. Mahmoud, Abo-sinna A novel approach for fully intuitionistic fuzzy multi-objective fractional transportation problem, Alex. Eng. J. 60 (1) (February 2021) 1447–1463.
[34] Md Sharif Uddin, Musa Miah, Md Al-Amin Khan, Ali AlArjani, Goal programming tactic for uncertain multi-objective transportation problem using fuzzy linear membership function, Alex. Eng. J. 60 (2) (April 2021) 2525–2533.
[35] Deepika Rani Shivani, Ebrahimnejad Ali, An approach to solve an unbalanced fully rough multi-objective fixed-charge transportation problem, Comput. Appl. Math. 41 (4) (2022) 1–27.
[36] Harish Garg, Mahmoodirad Ali, Sadegh Niroomand, Fractional two-stage transshipment problem under uncertainty: application of the extension principle approach, Complex and Intelligent Systems 7 (2) (2021) 807–822.
[37] Esmaiel Keshavarz, Mahmoodirad Ali, Sadegh Niroomand, A transportation problem considering fixed charge and fuzzy shipping costs, Decision Making Advances 1 (1) (2023) 115–122.
[38] Mahmoodirad Ali, Sadegh Niroomand, A heuristic approach for fuzzy fixed charge transportation problem, Journal of Decision Analytics and Intelligent Computing 3 (1) (2023) 139–147.
[39] Zhihao Peng, Maryam Nikbakht, Ali Ebrahimnejad, Farhad Hosseinzadeh Lotfi, Tofigh Allahviranloo, Fully interval-valued fuzzy transportation problems: development and prospects, Comput. Appl. Math. 43 (1) (2024).
[40] A. Kokila and G. Deepa, Modified Fuzzy Multi-Criteria Transportation Problem by Using Triangular Fuzzy Number. Available at SSRN: https://ssrn.com/abstract=4622076 or https://doi.org/10.2139/ssrn.4622076.
[41] Malihe Niksirat, A new approach to solve fully fuzzy multi-objective transportation problem, Fuzzy Information and Engineering (2022), https://doi.org/10.1080/16168658.2022.2152836.
[42] C.H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Set Syst. 95 (1998) 307–317.