# Dynamic and Static Nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 e)$ in the Selenanthrene System and Related Species Elucidated by QTAIM Dual Functional Analysis with QC Calculations 

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#### Abstract

The nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ of the ${ }^{\mathrm{B}} \mathrm{Br}-*_{-}{ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}-{ }^{-}-{ }^{\mathrm{B}} \mathrm{Br}$ form is elucidated for $\mathrm{SeC}_{12} \mathrm{H}_{8}(\mathrm{Br}) \mathrm{SeBr}--\mathrm{Br}-\mathrm{Br}--\mathrm{BrSe}(\mathrm{Br}) \mathrm{C}_{12} \mathrm{H}_{8} \mathrm{Se}$, the selenanthrene system, and the models with QTAIM dual functional analysis (QTAIM-DFA). Asterisks (*) are employed to emphasize the existence of bond critical points on the interactions in question. Data from the fully optimized structure correspond to the static nature of interactions. In our treatment, data from the perturbed structures, around the fully optimized structure, are employed for the analysis, in addition to those from the fully optimized one, which represent the dynamic nature of interactions. The ${ }^{A} \mathrm{Br}-*-{ }_{-}{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}{ }^{\mathrm{B}} \mathrm{Br}$ interactions are predicted to have the CT-TBP (trigonal bipyramidal adduct formation through charge transfer) nature and the typical hydrogen bond nature, respectively. The nature of $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ is also clarified typically, employing an anionic model of $\left[\mathrm{Br}-\mathrm{Se}\left(\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{Se}\right)-\mathrm{Br}--\mathrm{Br}--\mathrm{Br}-\mathrm{Se}\left(\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{Se}\right)-\mathrm{Br}\right]^{-}$, the 1,4 -diselenin system, rather than $\left(\mathrm{BrSeC}_{12} \mathrm{H}_{8}\right) \mathrm{Br}--\mathrm{Se}--\mathrm{Br}-\mathrm{Br}--\mathrm{Br}-\mathrm{Se}\left(\mathrm{C}_{12} \mathrm{H}_{8} \mathrm{Se}\right)$-Br, the selenanthrene system.


## 1. Introduction

We have been much interested in the behavior of the linear interactions of the $\sigma$-type, higher than $\sigma(3 \mathrm{c}-4 \mathrm{e}$ : three centerfour electron interactions) [1-6], constructed by the atoms of heavier main group elements. We proposed to call such linear interactions the extended hypervalent interactions, $\sigma(m c-n \mathrm{e}: 4 \leq m ; m<n<2 m)$, after the hypervalent $\sigma(3 \mathrm{c}-4 \mathrm{e})$. The linear alignments of four chalcogen atoms were first demonstrated in the naphthalene system, bis[8-(phenyl-chalcogenyl)naphthyl]-1,1'-dichalcogenides $\quad[\mathbf{I}: \quad 1-(8-$ $\left.\mathrm{Ph}^{\mathrm{B}} \mathrm{EC}_{10} \mathrm{H}_{6}\right){ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}\left(\mathrm{C}_{10} \mathrm{H}_{6}{ }^{\mathrm{B}} \mathrm{EPh}-8^{\prime}\right)-1^{\prime}\left({ }^{\mathrm{A}} \mathrm{E},{ }^{\mathrm{B}} \mathrm{E}=\mathrm{S}\right.$ and Se$\left.)\right]$ [7-12]. It was achieved through the preparation and the structural determination by the X-ray crystallographic analysis. The linear ${ }^{\mathrm{B}} \mathrm{E}--{ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}--{ }^{\mathrm{B}} \mathrm{E}$ interactions in I are proposed to be analysed as the ${ }^{\mathrm{A}} \mathrm{E}_{2}{ }^{\mathrm{B}} \mathrm{E}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$ model not by the double ${ }^{\mathrm{A}} \mathrm{E}^{\mathrm{B}} \mathrm{E}_{2} \sigma(3 \mathrm{c}-4 \mathrm{e})$ model. ${ }^{\mathrm{A}} \mathrm{E}_{2}{ }^{\mathrm{B}} \mathrm{E}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in I is characterized by the CT interaction of the
$n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{E}\right) \longrightarrow \sigma *\left({ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}\right) \leftarrow n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{E}\right)$ form [8, 10-12], where $n_{p}\left({ }^{B} \mathrm{E}\right)$ stands for the p-type nonbonding orbitals of ${ }^{\mathrm{B}} \mathrm{E}$ and $\sigma *\left({ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}\right)$ are the $\sigma *$ orbitals of ${ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}$. The novel reactivity of ${ }^{\mathrm{A}} \mathrm{E}_{2}{ }^{\mathrm{B}} \mathrm{E}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in I was also clarified [8].
$\sigma(4 \mathrm{c}-6 \mathrm{e})$ is the first member of $\sigma(m \mathrm{c}-n \mathrm{e}: 4 \leq m$; $m<n<2 m$ ) [7-13]. The $\sigma(4 \mathrm{c}-6 \mathrm{e})$ interactions are strongly suggested to play an important role in the development of high functionalities in materials and in the key processes of biological and pharmaceutical activities, recently. The bonding is applied to a wide variety of fields, such as crystal engineering, supramolecular soft matters, and nanosciences [4, 14-23]. The nature of ${ }^{\mathrm{B}} \mathrm{E}--{ }^{\mathrm{A}} \mathrm{E}$ and ${ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}$ in ${ }^{\mathrm{B}} \mathrm{E}---{ }^{\mathrm{A}} \mathrm{E}-$ ${ }^{\mathrm{A}} \mathrm{E}--{ }_{-}^{\mathrm{B}} \mathrm{E}$ of ${ }^{\mathrm{A}} \mathrm{E}_{2}{ }^{\mathrm{B}} \mathrm{E}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$ has been elucidated [24-27] using the quantum theory of atoms in molecules (QTAIM) approach, introduced by Bader [28-37]. The linear interactions of the $\sigma(4 \mathrm{c}-6 \mathrm{e})$ type will form if ${ }^{\mathrm{B}} \mathrm{E}$ in ${ }^{\mathrm{A}} \mathrm{E}_{2}{ }^{\mathrm{B}} \mathrm{E}_{2}$ is replaced by $X$, giving $\mathrm{E}_{2} \mathrm{X}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$. The nature of $\mathrm{E}_{2} \mathrm{X}_{2}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ in the naphthalene system of $1-\left(8-\mathrm{XC}_{10} \mathrm{H}_{6}\right) \mathrm{E}-$
$\mathrm{E}\left(\mathrm{C}_{10} \mathrm{H}_{6} \mathrm{X}-8^{\prime}\right)-1^{\prime}[\mathrm{II}(\mathrm{E}, \mathrm{X})=(\mathrm{S}, \mathrm{Cl}),(\mathrm{S}, \mathrm{Br}),(\mathrm{Se}, \mathrm{Cl})$, and $(\mathrm{Se}$, $\mathrm{Br})$ ] was similarly clarified very recently [38].

The $\sigma(4 \mathrm{c}-6 \mathrm{e})$ interaction will also be produced even if both ${ }^{\mathrm{B}} \mathrm{E}$ and ${ }^{\mathrm{A}} \mathrm{E}$ in ${ }^{\mathrm{A}} \mathrm{E}_{2}{ }^{\mathrm{B}} \mathrm{E}_{2}$ are replaced by $X . \mathrm{X}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ should also be stabilized through CT of the $n_{\mathrm{p}}(X) \longrightarrow \sigma *$ (X$\mathrm{X}) \leftarrow n_{\mathrm{p}}(X)$ form. The energy lowering of the system through the CT interaction must be the driving force for the formation of $\mathrm{X}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e}) . \mathrm{X}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ is the typical kind of halogen bonds, together with $\mathrm{E}_{2} \mathrm{X}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$, which are of current and continuous interest [39]. $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ has been clearly established in the selenanthrene system, $\mathrm{SeC}_{12} \mathrm{H}_{8}(\mathrm{Br})$ $\mathrm{SeBr}---\mathrm{Br}-\mathrm{Br}---\mathrm{BrSe}(\mathrm{Br}) \mathrm{C}_{12} \mathrm{H}_{8} \mathrm{Se}(\mathbf{1})$, through the preparation and the structural determination by the X-ray crystallographic analysis [39]. The atoms taking part in the linear interaction in question are shown in bold. The structure of $\left(\mathrm{BrSeC}_{12} \mathrm{H}_{8}\right) \mathbf{B r}---\mathrm{Se}---\mathrm{Br}-\mathrm{Br}---\mathrm{Br}-\mathrm{Se}\left(\mathrm{C}_{12} \mathrm{H}_{8} \mathrm{Se}\right)-\mathrm{Br}$ (2) was also reported, in addition to $\mathbf{1}$, which is suggested to contain $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ since the seven atoms of $\mathrm{Se}_{2} \mathrm{Br}_{5}$ align almost linearly in crystals. Figure 1 shows the structures of $\mathbf{1}$ and 2 determined by the X-ray analysis and the approximate MO model for $\sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\sigma(7 \mathrm{c}-10 \mathrm{e})$.

It is challenging to elucidate the nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ of the $n_{\mathrm{p}}(\mathrm{Br}) \longrightarrow \sigma *(\mathrm{Br}-\mathrm{Br}) \leftarrow n_{\mathrm{p}}(\mathrm{Br})$ form in 1 and $\mathrm{Se}_{2} \mathrm{Br}_{5}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$ in 2, together with the related species. Figure 2 illustrates the process assumed for the formation of $\mathbf{1}$ and $\mathbf{2}$ from selenanthrene ( $\left.\mathbf{S}: \mathrm{SeC}_{12} \mathrm{H}_{8} \mathrm{Se}\right)$. In this process, $\left(\mathrm{SeC}_{12} \mathrm{H}_{8}\right) \mathbf{B r}-\mathrm{Se}-\mathrm{Br}(3)$ should be formed first in the reaction of $\mathbf{S}$ with $\mathbf{B r}_{2}$, and then 3 reacts with $\mathbf{B r}_{2}$ to yield $\mathbf{B r}[\mathrm{Se}(\mathrm{Br})$ $\left.\left(\mathrm{C}_{12} \mathrm{H}_{8}\right)\right] \mathbf{S e}---\mathrm{Br}-\mathrm{Br}$ (4). The almost linear alignment of $\mathrm{Br}--\mathrm{Se}---\mathrm{Br}-\mathrm{Br}$ in 4 could be analysed by the $\mathrm{SeBr}_{3}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ model, where the $\mathbf{B r}$ and $\mathbf{S e}$ atoms in $\mathbf{4}$ are placed in close proximity in space. While $\mathbf{1}$ containing $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ forms in the reaction of $\left(3+\mathrm{Br}_{2}+3\right)$, the reaction of $3+4$ yields 2, consisting $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$. Both 1 and 2 are recognized as the $\mathrm{Br}_{2}$-included species. While $\mathrm{XC}_{4} \mathrm{H}_{4}(\mathrm{Br})$ SeBr---Br-Br---BrSe $(B r) \mathrm{C}_{4} \mathrm{H}_{4} \mathrm{X}(5(X=\mathrm{Se})$ and $6(X=\mathrm{S}))$, models of $\mathbf{1}$, also consisted of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e}), \mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ will appear typically in the anionic species, $\left[\mathrm{Br}-\mathrm{Se}\left(\mathrm{Me}_{2}\right)\right.$ $\left.\mathbf{B r}---\mathrm{Br}---\mathrm{Br}-\mathrm{Se}\left(\mathrm{Me}_{2}\right)-\mathrm{Br}\right]^{-}$(7) and $\left[\mathrm{Br}-\mathrm{Se}\left(\mathrm{SeC}_{4} \mathrm{H}_{4}\right)-\mathrm{Br}---\right.$ $\left.\mathbf{B r}---\mathrm{Br}-\mathrm{Se}\left(\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{Se}\right)-\mathrm{Br}\right]^{-}$(8), models of 2. Species, 5, 6, 7, and 8, are shown in Figure 2, where 5, 6, and $\mathbf{8}$ belong to the 1,4-diselenin system.

What are the differences and similarities between $\mathrm{X}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e}), \mathrm{E}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$, and $\mathrm{E}_{2} \mathrm{X}_{2} \sigma(4 \mathrm{c}-6 \mathrm{e})$ ? The nature of $\mathrm{X}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ in $\mathbf{1}(X=\mathrm{Br})$ is to be elucidated together with the models. Models, other than 5 and $\mathbf{6}$, are also devised to examine the stabilization sequence of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e}) . \mathrm{H}_{2} \mathrm{Br}_{4}$ $\left(C_{2 h}\right)$ and $\mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 h}\right)$ have the form of $\mathrm{R}-\mathrm{Br}--\mathrm{Br}-\mathrm{Br}--\mathrm{Br}-$ $\mathrm{R}\left(\mathrm{RBr}_{4} \mathrm{R}: \mathrm{R}=\mathrm{H}\right.$ and Me$)$, which are called the model group A (G(A)). The electronic efficiency to stabilize $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ seems small for $R$ in $G(\mathbf{A})$. $\mathrm{Br}_{6}\left(C_{2 h}\right)$ is detected as the partial structure in the crystals of $\mathrm{Br}_{2}$ [40]. $\mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)$ in the crystals is denoted by $\mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)_{\text {obsd. }}$. The optimized structure of $\mathrm{Br}_{6}\left(C_{2 h}\right)$ has one imaginary frequency, which belongs to $G(A)$, together with $\mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)_{\text {obsd }}$. The optimized structure of $\mathrm{Br}_{6}$ retains the $C_{2}$ symmetry, $\left(\operatorname{Br}_{6}\left(C_{2}\right)\right)$, which also belongs to $G(A)$. The CT interaction of the $n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{Br}\right) \longrightarrow \sigma *\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}\right) \longleftarrow n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{Br}\right)$ form in $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ will be much stabilized if the large negative charge is developed at the ${ }^{\mathrm{B}} \mathrm{Br}$ atoms in $\mathrm{Br}-\left(\mathrm{R}_{2}\right) \mathrm{Se}-{ }^{\mathrm{B}} \mathbf{B r}--{ }^{\mathrm{A}} \mathbf{B r}-$
${ }^{\mathrm{A}} \mathbf{B r}--{ }^{\mathrm{B}} \mathrm{Br}-\mathrm{Se}\left(\mathrm{R}_{2}\right)-\mathrm{Br}$, where the $\angle \mathrm{Se}^{\mathrm{B}} \mathrm{Br}^{\mathrm{A}} \mathbf{B r}$ is around $90^{\circ}$. The highly negatively charged ${ }^{\mathrm{B}} \mathrm{Br}$ in $\mathrm{Br}-\mathrm{Se}\left(\mathrm{R}_{2}\right)-{ }^{\mathrm{B}} \mathbf{B r}(\mathrm{R}=\mathrm{H}$ and Me ) of $\sigma(3 \mathrm{c}-4 \mathrm{e})$ is employed to stabilize $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$, in this case. The models form $\mathrm{G}(\mathbf{B})$. The nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in $\mathbf{5}$ and $\mathbf{6}$ is similarly analysed, which belongs to $\mathrm{G}(\mathbf{B}) . \mathrm{Br}_{4}^{2-}$ ( $D_{\infty}$ ) also belongs to $G(\mathbf{B})$ although one imaginary frequency was predicted for $\mathrm{Br}_{4}^{2-}$, if optimized at the MP2 level. Figure 3 illustrates the story for the stabilization of $\mathrm{Br}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ in the sequence of the species, starting from $\mathrm{G}(\mathbf{A})$ to 1, via $\mathrm{G}(\mathbf{B})$. Figure 3 also shows the ${ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{B}} \mathrm{Br}$ distances $\left(r\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}\right)\right.$ and $r\left({ }^{\mathrm{A}} \mathrm{Br}^{-}{ }^{\mathrm{B}} \mathrm{Br}\right)$, respectively), together with the charge developed at ${ }^{\mathrm{B}} \mathrm{Br}$ in the original species of $\mathrm{R}-{ }^{\mathrm{B}} \mathrm{Br}\left(\mathrm{Qn}\left({ }^{\mathrm{B}} \mathrm{Br}\right)\right)$, which construct $\mathrm{R}-{ }^{\mathrm{B}} \mathrm{Br}--{ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}--{ }^{\mathrm{B}} \mathrm{Br}-\mathrm{R}$.
$A$ chemical bond or interaction between atoms $A$ and $B$ is denoted by $\mathrm{A}-\mathrm{B}$, which corresponds to a bond path (BP) in the quantum theory of atoms in molecules (QTAIM) approach, introduced by Bader [28-37]. We will use A-*-B for BP , where the asterisk emphasizes the existence of a bond critical point (BCP, *) in A-B [28, 29]. (Dots are usually employed to show BCPs in molecular graphs. Therefore, A- - B would be more suitable to describe the BP with a BCP. Nevertheless, $\mathrm{A}-*-\mathrm{B}$ is employed to emphasize the existence of a BCP on the BP in question in our case. BCP is a point along BP at the interatomic surface, where $\rho(\mathbf{r})$ (charge density) reaches a minimum along the interatomic (bond) path, while it is a maximum on the interatomic surface separating the atomic basins). The chemical bonds and interactions are usually classified by the signs of Laplacian rho $\left(\nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\right)$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ at BCPs, where $\rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ are the charge densities and total electron energy densities at BCPs, respectively (see Scheme S1 in Supplementary File). The relations between $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right), \nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right), G_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ (the kinetic energy densities), and $V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ (the potential energy densities) are represented in equations (1) and (2):

$$
\begin{align*}
H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) & =G_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)+V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right),  \tag{1}\\
\left(\frac{\hbar^{2}}{8 m}\right) \nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) & =H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-\frac{V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)}{2}  \tag{2}\\
& =G_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)+\frac{V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)}{2} .
\end{align*}
$$

How can the nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\mathrm{Se}_{2} \mathrm{Br}_{5}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$ be clarified? For the characterization of interactions in more detail, we recently proposed QTAIM dual functional analysis (QTAIM-DFA) [42-47] for experimental chemists to analyze their own chemical bonds and interaction results based on their own expectations, according to the QTAIM approach [28-37]. $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ is plotted versus $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2\left(=\left(\hbar^{2} / 8 m\right) \nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\right)$ at BCPs in QTAIMDFA. The classification of interactions by the signs of $\nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ is incorporated in QTAIM-DFA. Data from the fully optimized structures correspond to the static natures of the interactions, which are analysed using the polar coordinate $(R, \theta)$, representation [42, 44-46]. Each interaction plot, containing data from both the perturbed structures and the fully optimized one include a specific curve that provides important information about the interaction. This plot is expressed by $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$, where $\theta_{\mathrm{p}}$


FIGURe 1: Structure of $\mathbf{1}$ determined by the X-ray crystallographic analysis (a) and the approximate MO model for $\sigma(4 \mathrm{c}-6 \mathrm{e})$ (b); structure of 2 (c) and the approximate MO model for $\sigma(7 \mathrm{c}-10 \mathrm{e})$ (d).
corresponds to the tangent line of the plot and $\kappa_{\mathrm{p}}$ is the curvature. The concept of the dynamic nature of interactions has been proposed based on $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ [42, 44]. $\theta$ and $\theta_{\mathrm{p}}$ are measured from the $y$-axis and the $y$-direction, respectively. We call $(R, \theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ QTAIM-DFA parameters, which are drawn in Figure 4, exemplified by $\operatorname{Br}_{4}^{2-}\left(D_{\text {ch }}\right)$. While ( $R$, $\theta)$ classifies the interactions, $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ characterizes them.

We proposed a highly reliable method to generate the perturbed structures for QTAIM-DFA very recently [48]. The method is called CIV, which employs the coordinates derived from the compliance force constants $C_{i j}$ for the internal vibrations. Compliance force constants $C_{i j}$ are defined as the partial second derivatives of the potential energy due to an external force, as shown in equation (3), where $i$ and $j$ refer to the internal coordinates and the force constants $f_{i}$ and $f_{j}$ correspond to $i$ and $j$, respectively. The $C_{i j}$ values and the coordinates corresponding to the values can be calculated using the compliance 3.0.2 program, released by Brandhorst and Grunenberg [49-52]. The dynamic nature of interactions based on the perturbed structures with CIV is described as the "intrinsic dynamic nature of interactions" since the coordinates are invariant to the choice of the coordinate system:

$$
\begin{equation*}
C_{i j}=\frac{\partial^{2} E}{\partial f_{i} \partial f_{j}} \tag{3}
\end{equation*}
$$

QTAIM-DFA has excellent potential for evaluating, classifying, characterizing, and understanding weak to strong interactions according to a unified form. The superiority of QTAIM-DFA to elucidate the nature of
interactions, employing the perturbed structures generated with CIV, is explained in the previous papers [48, 53] (see also Figure S2 and Table S2 in Supplementary File). QTAIMDFA is applied to standard interactions and rough criteria that distinguish the interaction in question from others which are obtained. QTAIM-DFA and the criteria are explained in Supplementary File using Schemes S1-S3, Figures S1 and S2, Table S1, and equations (S1)-(S7). The basic concept of the QTAIM approach is also explained.

We consider QTAIM-DFA, employing the perturbed structures generated with CIV, to be well suited to elucidate the nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in $\mathbf{1}, \mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ in 2, and the models derived from 1 and 2, together with the related linear interactions. The interactions in $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ are denoted by ${ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$, where the asterisk emphasizes the existence of a BCP in the interactions, so are those in $\mathrm{Se}_{2} \mathrm{Br}_{5}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$. Herein, we present the results of the investigations on the extended hypervalent interactions in the species, together with the structural feature. Each interaction is classified and characterized, employing the criteria as a reference.

## 2. Methodological Details in Calculations

Calculations were performed employing the Gaussian 09 programs package [54]. The basis sets employed for the calculations were obtained, as implemented from Sapporo Basis Set Factory [55]. The basis sets of the (621/31/2), (6321/ $621 / 3),(74321 / 7421 / 72)$, and ( $743211 / 74111 / 721 / 2+1 \mathrm{~s} 1 \mathrm{p}$ ) forms were employed for $\mathrm{C}, \mathrm{S}, \mathrm{Se}$, and Br , respectively, with the $(31 / 3)$ form for H . The basis set system is called BSS-A.


Figure 2: Process assumed for the formation of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in $\mathbf{1}$ from $\mathrm{Se}\left(\mathrm{C}_{12} \mathrm{H}_{8}\right) \mathrm{Se}(\mathbf{S})$ via $\mathbf{3}$ and $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ in $\mathbf{2}$ via $\mathbf{3}$ and $\mathbf{4} . \mathbf{5}$ and $\mathbf{6}$ with $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$, models of $\mathbf{1}$, and $\mathbf{7}$ and $\mathbf{8}$ with $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$, models of $\mathbf{2}$, are also shown. Atoms taking part in the linear interactions are shown by red.

All species were calculated employing BSS-A, and the Møller-Plesset second-order energy correlation (MP2) level [56-58] was applied for the optimizations. Optimized structures were confirmed by the frequency analysis. The results of the frequency analysis were used to calculate the $C_{i j}$ values and the coordinates $\left(\mathbf{C}_{i}\right)$ corresponding to the values. The DFT level of CAM-B3LYP [59] was also applied when necessary. The QTAIM functions were analysed with the AIM2000 [60] and AIMAll [61] programs.

The method to generate perturbed structures with CIV is the same as that explained in the previous papers [48,53]. As shown in equation (4), the $i$-th perturbed structure in question $\left(\mathbf{S}_{i w}\right)$ is generated by the addition of the $i$-th coordinates $\left(\mathbf{C}_{i}\right)$, derived from $C_{i j}$, to the standard orientation of a fully optimized structure $\left(\mathbf{S}_{\mathrm{o}}\right)$ in the matrix representation. The coefficient $f_{i w}$ in equation (4) controls the structural difference between $\mathbf{S}_{i w}$ and $\mathbf{S}_{0}: f_{i w}$ is determined to satisfy equation (5) for $r$, where $r$ and $r_{\mathrm{o}}$ stand for the interaction distances in question in the perturbed and fully optimized structures, respectively, with $a_{\mathrm{o}}=0.52918 \AA$ (Bohr radius). The $\mathbf{C}_{i}$ values of five digits are used to predict $\mathbf{S}_{i w}$ :

$$
\begin{align*}
\mathbf{S}_{i w}= & \mathbf{S}_{o}+f_{i w} \cdot C_{i}  \tag{4}\\
r= & r_{o}+w a_{o} \\
& \left(w=(0), \pm 0.05, \text { and } \pm 0.1 ; a_{o}=0.52918 \AA\right)  \tag{5}\\
& (w) \\
y= & c_{o}+c_{1} x+c_{2} x^{2}+c_{3} x^{3}
\end{align*}
$$

$$
\begin{equation*}
\left(R_{c}^{2}: \text { square of correlation coefficient }\right) \tag{6}
\end{equation*}
$$

In QTAIM-DFA, $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ is plotted versus $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-$ $V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ for data of $w=0, \pm 0.05$, and $\pm 0.10$ in equation (5). Each plot is analysed using a regression curve of the cubic function, as shown in equation (6), where $(x, y)=\left(H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)\right.$ $V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ and $\left.H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\right)\left(R_{c}^{2}\right.$ (square of correlation coefficient) $>0.99999$ in usual) [46].

## 3. Results and Discussion

3.1. Structural Optimizations. The structures of $\mathbf{1}\left(C_{i}\right)$ and $\mathbf{2}$ $\left(C_{1}\right)$ determined by the X-ray analysis are denoted by $\mathbf{1}$

(a) Ref. [40] (b) Ref. [41] (c) Ref. [39]

Figure 3: Sequence in the stabilization of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$, starting from those in $\mathrm{G}(\mathbf{A})$ to $\mathbf{1}$ via those of $\mathrm{G}(\mathbf{B})$.
$\left(C_{i}\right)_{\text {obsd }}$ and $2\left(C_{1}\right)_{\text {obsd }}$, respectively [39]. The structural parameters are shown in Tables S2 and S3 in Supplementary File, respectively. Figure 3 contains the selected structural parameters for $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$. The structures are optimized for $\mathrm{G}(\mathbf{A})$ of $\mathrm{H}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right), \mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right), \mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)$, and $\mathrm{Br}_{6}\left(C_{2}\right)$ and $\mathrm{G}(\mathbf{B})$ of $\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right), \mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right), \mathbf{5}$ $\left(C_{i}\right)$, and $6\left(C_{i}\right)$, together with $3\left(C_{s}\right), 4\left(C_{s}\right), 7\left(C_{2 \mathrm{~h}}\right), 8\left(C_{2 \mathrm{~h}}\right)$, and $\mathrm{Br}_{2}\left(D_{\infty h}\right)$. The optimized structural parameters are also collected in Tables S2 and S3 in Supplementary File. The frequency analysis was successful for the optimized structures, except for $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$ and $\mathrm{Br}_{6}\left(C_{2 h}\right)$. All positive frequencies were obtained for $\mathbf{1}\left(C_{i}\right)$, if calculated with CAM-B3LYP/BSS-A, which confirms the structure. The
$\mathrm{Br}---\mathrm{Br}$ distances of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in $\mathbf{1}\left(C_{i}\right)$ are somewhat longer if optimized at the CAM-B3LYP level, relative to 1 $\left(C_{i}\right)_{\text {obsd. }}$. While one imaginary frequency is detected in $\mathrm{Br}_{6}$ $\left(C_{2 \mathrm{~h}}\right), \mathrm{Br}_{6}\left(C_{2}\right)$ has all positive frequencies. The optimized structures are not shown in figures, instead, some of them can be found in Figures 3 and 5, where the molecular graphs are drawn on the optimized structures. Figure 3 contains the optimized $r\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}\right)$ and $r\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{B}} \mathrm{Br}\right)$ distances for the models and the charge developed at ${ }^{\mathrm{B}} \mathrm{Br}$ in the original $\mathrm{R}-{ }^{\mathrm{B}} \mathrm{Br}$ and $\mathrm{Br}-\left(\mathrm{R}_{2}\right) \mathrm{Se}-{ }^{\mathrm{B}} \mathrm{Br}\left(\mathrm{Qn}\left({ }^{\mathrm{B}} \mathrm{Br}\right)\right)$, which give the models of $\mathrm{G}(\mathbf{A})$ and $\mathrm{G}(\mathbf{B})$, respectively. The $r\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{B}} \mathrm{Br}\right)$ values become shorter in the order shown in equation (7), if evaluated with MP2/BSS-A:


Figure 4: QTAIM-DFA plots of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ versus $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ for ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}(\mathrm{a})$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}(\mathrm{b})$ in $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ of the species in Table 1, together with those of the perturbed structures generated with CIV. Marks and colours for the species are shown in the figure.

$$
\begin{align*}
& r\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{B}} \mathrm{Br}\right): \\
& \quad \mathrm{H}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right)>\mathbf{1}\left(C_{i}\right) \mathrm{CAM}>\mathrm{Br}_{6}\left(C_{2} \text { and } C_{2 \mathrm{~h}}\right) \\
& \quad>\mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)_{\mathrm{obsd}}[40]>\mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right)>\mathrm{Br}_{4}^{2-}\left(D_{\text {oh }}\right)>\mathbf{1}\left(C_{i}\right)_{\text {obsd }} \\
& \quad \geq \mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right) \geq \mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right)>\mathbf{5}\left(C_{i}\right) \geq \mathbf{6}\left(C_{i}\right) . \tag{7}
\end{align*}
$$

One imaginary frequency was also predicted for $\mathrm{Br}_{4}^{2-}$ ( $D_{\infty h}$ ) if optimized with MP2/BSS-A. $\mathrm{Br}_{4}^{2-}\left(D_{\infty \text { oh }}\right)$ seems to collapse to $\mathrm{Br}_{3}^{-}$and $\mathrm{Br}^{-}$, according to the imaginary frequency. The double negative charges in $\operatorname{Br}_{4}^{2-}\left(D_{\mathrm{oh}}\right)$ would be responsible for the results. The electrostatic repulsion between the double negative charges will operate to collapse it.

### 3.2. Energies for Formation of $\mathrm{Br}_{4} \sigma(4 c-6 e)$ and NBO Analysis.

 Energies for the formation of $\mathrm{R}^{\prime} \mathrm{Br}_{4} \mathrm{R}^{\prime}$ from the components $\left(2 \mathrm{R}^{\prime} \mathbf{B r}+\mathbf{B r}_{2}\right)(\Delta E)$ are defined by equation (8). The $\Delta E$ values evaluated on the energy surface are denoted by $\Delta E_{\mathrm{ES}}$, while those corrected with the zero-point energies are by $\Delta E_{\mathrm{ZP}}$. The $\Delta E_{\mathrm{ES}}$ and $\Delta E_{\mathrm{ZP}}$ values for the optimized structures are given in Table S2 in Supplementary File. $\Delta E_{\mathrm{ZP}}$ are excellently correlated to $\Delta E_{\mathrm{ES}} \quad\left(\Delta E_{\mathrm{ZP}}=0.99 \Delta E_{\mathrm{ES}}+1.93\right.$ : $R_{\mathrm{c}}^{2}=0.9998$, see Figure S3 in Supplementary File):$$
\begin{align*}
\Delta E\left(R_{2}^{\prime} \mathrm{Br}_{4}\right) & =E\left(R_{2}^{\prime} \mathrm{Br}_{4}\right)-\left[2 E\left(R^{\prime} \mathrm{Br}\right)+E\left(\mathrm{Br}_{2}\right)\right]  \tag{8}\\
E(2) & =q_{i} \times \frac{F(i, j)^{2}}{\left(\varepsilon_{j}-\varepsilon_{i}\right)} \tag{9}
\end{align*}
$$

NBO analysis [62] was applied to ${ }^{\mathrm{A}} \mathrm{Br}--{ }^{\mathrm{B}} \mathrm{Br}$ of the species to evaluate the contributions from CT to stabilize $\mathrm{R}^{\prime}-{ }^{\mathrm{B}} \mathrm{Br}--{ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}--{ }^{\mathrm{B}} \mathrm{Br}-\mathrm{R}^{\prime}$. For each donor NBO (i) and acceptor NBO $(j)$, the stabilization energy $E(2)$ is calculated based on the second-order perturbation theory in NBO, according to equation (9), where $q_{i}$ is the donor orbital occupancy, $\varepsilon_{i}$ and $\varepsilon_{j}$ are diagonal elements (orbital energies), and $F(i, j)$ is the off-diagonal NBO Fock matrix element. The results are collected in Table S4 in Supplementary File. The $\Delta E_{\mathrm{ES}}$ values are very well correlated to $E(2)$ for the optimized structures, except for $\mathrm{Br}_{4}^{2-}$ $\left(D_{\text {}}\right.$ h $) . \quad\left(\Delta E_{\mathrm{ES}}=-0.71(2 E(2))+7.17: \quad R_{c}^{2}=0.959, \quad\right.$ see Figure S4 in Supplementary File). $\mathrm{Br}_{4}^{2-}\left(D_{\mathrm{oh}}\right)$ is predicted to be less stable than the components.

Before application of QTAIM-DFA to $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$, molecular graphs were examined, as shown in the next section.
3.3. Molecular Graphs with Contour Plots for the Species Containing $\mathrm{Br}_{4} \sigma(4 c-6 e), \mathrm{Se}_{2} \mathrm{Br} 5 \sigma(7 c-10 e)$, and Related Linear Interactions. Figure 5 illustrates the molecular graphs of 5 $\left(C_{i}\right), 6\left(C_{i}\right), 7\left(C_{2 \mathrm{~h}}\right)$, and $8\left(C_{2 \mathrm{~h}}\right)$, drawn on the optimized structures, together with $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$ and $2\left(C_{1}\right)_{\text {obsd }}$. Figure 5 also shows the contour plots of $\rho(r)$ drawn on the suitable plane in the molecular graphs. BCPs are well demonstrated to locate on the (three-dimensional) saddle points of $\rho(r)$. Molecular graphs of $\mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right), \mathrm{Br}_{6}\left(C_{2}\right), \mathrm{Br}_{4}^{2-}\left(D_{\infty \mathrm{h}}\right)$, and $\operatorname{Br}\left(\mathrm{Me}_{2}\right) \mathrm{SeBr}_{4} \mathrm{Se}\left(\mathrm{Me}_{2}\right) \mathrm{Br}\left(C_{i}\right)$ are shown in Figure 3, which are drawn on the optimized structures.


Figure 5: Molecular graphs of $\mathbf{5}\left(C_{i}\right)(\mathrm{a}), \mathbf{6}\left(C_{i}\right)(\mathrm{b}), 7\left(C_{2 \mathrm{~h}}\right)(\mathrm{c})$, and $\mathbf{8}\left(C_{2 \mathrm{~h}}\right)(\mathrm{d})$ drawn on the structures optimized at the MP2 level, together with $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}(\mathrm{e})$ and $2\left(C_{1}\right)_{\text {obsd }}(\mathrm{f})$. Contour plots of $\rho(\mathbf{r})$ are also drawn on the planes containing the linear interactions. BCPs are denoted by red dots, RCPs (ring critical points) by yellow dots, and CCPs (cage critical points) by green dots. BPs (bond paths) are drawn as pink lines and the secondary ones as pink dots. They are associated with the BCPs. Carbon and hydrogen atoms are shown in black and gray, respectively. The contours $\left(e \mathrm{a}_{\mathrm{o}}{ }^{-3}\right)$ are at $2^{l}(l= \pm 8, \pm 7, \ldots$, and 0$)$.
3.4. Survey of $\mathrm{Br}_{4} \sigma(4 c-6 e)$ and $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 c-10 e)$. BPs in $\mathrm{Br}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\mathrm{Se}_{2} \mathrm{Br}_{6} \sigma(7 \mathrm{c}-10 \mathrm{e})$ seem straight, as shown in Figures 3 and 5. To show the linearity more clearly, the
lengths of $\mathrm{BPs}\left(r_{\mathrm{BP}}\right)$ for $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ are calculated. The values are collected in Table S5 in Supplementary File, together with the corresponding straight-line distances $\left(R_{\mathrm{SL}}\right)$.

The table contains the values for $\mathrm{Se}_{2} \mathrm{Br}_{6} \sigma(7 \mathrm{c}-10 \mathrm{e})$ in $7\left(\mathrm{C}_{2 \mathrm{~h}}\right)$ and $8\left(C_{2 h}\right)$. The differences between them $\left(\Delta r_{\mathrm{BP}}=r_{\mathrm{BP}}-R_{\mathrm{SL}}\right)$ are less than $0.003 \AA$. The $r_{\mathrm{BP}}$ values are plotted versus $R_{\mathrm{SL}}$, which are shown in Figure S5 in Supplementary File. The correlations are excellent, as shown in the figure. Therefore, $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\mathrm{Se}_{2} \mathrm{Br}_{6} \sigma(7 \mathrm{c}-10 \mathrm{e})$ in the species can be approximated by the straight lines.

QTAIM functions are calculated for $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ at BCPs. Table 1 collects the values for the interactions. $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ is plotted versus $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ for the data shown in Table 1, together with those from the perturbed structures generated with CIV. Figure 4 shows the plots for the ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-$ *- ${ }^{\mathrm{B}} \mathrm{Br}$ interactions in $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ of the bromine species. The plots for ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ appear in the region of $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) /$ $2>0$ and $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)<0$, for all species, except for the original $\mathrm{Br}_{2}$ $\left(D_{\infty h}\right)$, of which the plot appears in the region of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ $V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2<0$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)<0$. Therefore, the interactions are all classified by the regular-CS (closed shell) interactions, except for $\mathrm{Br}_{2}\left(D_{\infty \text { oh }}\right)$, which is classified by the SS (shard shell) interaction. On the contrary, data of ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ appear in the region of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2>0$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)>0$ for all species, except for those in $\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right), \mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right), \mathbf{5}\left(C_{i}\right)$, and $\mathbf{6}$ $\left(C_{i}\right)$, which appear in the region of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2>0$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)<0$. As a result, ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ is classified by the pure-CS interactions ( $p-\mathrm{CS}$ ) for all, except for the four species, of which ${ }^{\mathrm{A}} \mathrm{Br}-*{ }_{-}^{\mathrm{B}} \mathrm{Br}$ is classified by the regular-CS interactions ( $r-\mathrm{CS}$ ). The ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ interaction in $\mathrm{Br}_{4}^{2-}\left(D_{\infty h}\right)$ is very close to the borderline between $p$-CS and $r$-CS since $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)=0.0001$ au for $\operatorname{Br}_{4}^{2-}\left(D_{\infty}\right)$, which is very close to zero. QTAIM-DFA parameters of $(R, \theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ are obtained by analysing the plots of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ versus $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ in Figure 4, according to equations (S3)-(S6). Table 1 collects the QTAIM-DFA parameters for $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$. The classification of interactions will also be discussed based on the $(R, \theta)$ values.

QTAIM functions are similarly calculated for $\mathrm{Se}_{2} \mathrm{Br}_{6}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$ at BCPs , together with the related interactions. $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ is similarly plotted versus $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ although not shown in the figures. Then, QTAIM-DFA parameters of $(R, \theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ are obtained by analysing the plots, according to equations (S3)-(S6). Table 2 collects the QTAIM-DFA parameters of $(R, \theta)$ and $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ for $\mathrm{Br}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$.
3.5. Nature of $\mathrm{Br}_{4} \sigma(4 c-6 e)$. Interactions are characterized by $(R, \theta)$, which correspond to the data from the fully optimized structures. On the contrary, they are characterized employing $\left(\theta_{\mathrm{p}}, \kappa_{\mathrm{p}}\right)$ derived from the data of the perturbed structures around the fully optimized structures and the fully optimized ones. In this case, the nature of interactions is substantially determined based of the $\left(R, \theta, \theta_{\mathrm{p}}\right)$ values, while the $\kappa_{\mathrm{p}}$ values are used only additionally. It is instructive to survey the criteria before detail discussion. The criteria tell us that $180^{\circ}<\theta\left(H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2<0\right)$ for the SS interactions, $90^{\circ}<\theta<180^{\circ}\left(H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)<0\right)$ for the $r$-CS interactions, and $45^{\circ}<\theta<90^{\circ}\left(H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)>0\right)$ for $p$-CS interactions. The $\theta_{\mathrm{p}}$ value characterizes the interactions. In the $p$-CS region of $45^{\circ}<\theta<90^{\circ}$, the character of interactions will be the vdW type for $45^{\circ}<\theta_{\mathrm{p}}<90^{\circ}$, whereas it will be the typical HB type
without covalency ( $t-\mathrm{HB}_{\mathrm{nc}}$ ) for $90^{\circ}<\theta_{\mathrm{p}}<125^{\circ}$, where $\theta_{\mathrm{p}}=125^{\circ}$ is tentatively given for $\theta=90^{\circ}$. The CT interaction will appear in the $r$-CS region of $90^{\circ}<\theta<180^{\circ}$. The $t$-HB type with covalency $\left(t-\mathrm{HB}_{\mathrm{wc}}\right)$ appears in the region of $125^{\circ}<\theta_{\mathrm{p}}<150^{\circ}\left(90^{\circ}<\theta<115^{\circ}\right)$, where $\left(\theta, \theta_{\mathrm{p}}\right)=\left(115^{\circ}, 150^{\circ}\right)$ is tentatively given as the borderline between $t-\mathrm{HB}_{\mathrm{wc}}$ and the CT-MC nature. The borderline for the interactions between CT-MC and CT-TBP types is defined by $\theta_{\mathrm{p}}=180^{\circ} . \theta=150^{\circ}$ is tentatively given for $\theta_{\mathrm{p}}=180^{\circ}$. Classical chemical bonds of SS $\left(180^{\circ}<\theta\right)$ will be strong (Cov-s) when $R>0.15 \mathrm{au}$, whereas they will be weak (Cov-w) for $R<0.15 \mathrm{au}$. The classification and characterization of interactions are summarized in Table S1 and Scheme S3 in Supplementary File.

The ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{B}} \mathrm{Br}$ interactions of $\mathrm{Br}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ will be classified and characterized based on the ( $R$, $\theta, \theta_{\mathrm{p}}$ ) values, employing the standard values as a reference (see Scheme S2 in Supplementary File). $R<0.15$ au for all interactions in Table 1; therefore, no Cov-s were detected in this work. The $\left(\theta, \theta_{\mathrm{p}}\right)$ values are $\left(180.1^{\circ}, 191.8^{\circ}\right)$ for the original $\mathrm{Br}_{2}\left(D_{\text {oh }}\right)$ if evaluated with MP2/BSS-A. Therefore, the nature of $\mathrm{Br}-*-\mathrm{Br}$ in $\mathrm{Br}_{2}\left(D_{\infty}\right)$ is classified by the SS interactions and characterized as the Cov-w nature, which is denoted by SS/Cov-w. The ( $\theta, \theta_{\mathrm{p}}$ ) values are (170.6-179.0 ${ }^{\circ}$, $190.6-191.7^{\circ}$ ) for ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in the optimized structures in Table 1, of which nature is $r$-CS/CT-TBP. The $\left(\theta, \theta_{\mathrm{p}}\right)$ values are (78.0-84.1 ${ }^{\circ}, 94.7-105.1^{\circ}$ ) for ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}{ }^{\mathrm{B}} \mathrm{Br}$ in the optimized structures of $\mathrm{Br}_{6}\left(C_{2}\right), \mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)$, and $\mathrm{R}_{2} \mathrm{Br}_{4}$ $\left(C_{2 h}\right)(\mathrm{R}=\mathrm{H}$ and Me$)$; therefore, the nature is predicted to be $r$ - $\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}$. The nature of ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ in $\mathrm{R}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right)$ ( $\mathrm{R}=\mathrm{H}$ and Me ), $5\left(C_{i}\right)$ and $6\left(C_{i}\right)$, is $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}$, judging from the $\left(\theta, \theta_{\mathrm{p}}\right)$ values of $\left(90.9-92.8^{\circ}, 116.4-122.5^{\circ}\right)$. The calculated ( $\theta, \theta_{\mathrm{p}}$ ) values of ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ for the optimized structure of $\mathrm{Br}_{4}^{2-}\left(D_{\infty h}\right)$ are $\left(170.6^{\circ}, 190.6^{\circ}\right)$ and $\left(89.5^{\circ}, 118.2^{\circ}\right.$ ), respectively. In this case, ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-$ $*-{ }^{\mathrm{B}} \mathrm{Br}$ are predicted to have the nature of $r$-CS/CT-TBP and $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}$, respectively. However, ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ is just the borderline region to the $r$-CS interactions with $\theta=89.5^{\circ}$. The characteristic nature of the ${ }^{\mathrm{B}} \mathrm{E}--{ }^{\mathrm{A}} \mathrm{E}-{ }^{\mathrm{A}} \mathrm{E}--{ }^{\mathrm{B}} \mathrm{E}$ interactions in $\operatorname{Br}_{4}^{2-}\left(D_{\mathrm{oh}}\right)$ would be controlled by the double negative charges in the species.

The results in Table 1 show that the ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ interaction in $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ becomes weaker, as the strength of the corresponding ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ increases. The strength of ${ }^{\mathrm{A}} \mathrm{Br}-$ *- ${ }^{\mathrm{A}} \mathrm{Br}$ becomes weaker in the order shown in equation (10), if evaluated by $\theta$, while that of ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ increases in the order shown in equation (11), if measured by $\theta$. Very similar results were obtained by $\theta_{\mathrm{p}}$ :

$$
\begin{gather*}
\theta \text { for }{ }^{\mathrm{A} \mathrm{Br}-*-{ }^{A} \mathrm{Br}:} \\
\mathrm{Br}_{2}\left(D_{\infty h}\right)>\mathrm{H}_{2} \mathrm{Br}_{4}\left(C_{2 h}\right) \geq \mathrm{Br}_{6}\left(C_{2} \text { and } C_{2 h}\right)  \tag{10}\\
>\mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 h}\right)>\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right) \geq \mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right) \\
\geq \mathbf{1}\left(C_{i}\right)_{\text {obsd }}>\mathbf{5}\left(C_{i}\right)>\mathbf{6}\left(C_{i}\right)>\mathrm{Br}_{6}\left(C_{2 h}\right)_{\text {obsd }}, \\
\theta \text { for }{ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}: \\
\mathrm{H}_{2} \mathrm{Br}_{4}\left(C_{2 h}\right)>\mathrm{Br}_{6}\left(C_{2 h} \text { and } C_{2}\right) \geq \mathrm{Br}_{6}\left(C_{2 h}\right)_{\text {obsd }}  \tag{11}\\
>\mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 h}\right)<\mathbf{1}\left(C_{i}\right)_{\text {obsd }}<\mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right) \\
<\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right)<\mathbf{5}\left(C_{i}\right) \approx \mathbf{6}\left(C_{i}\right) .
\end{gather*}
$$



| Species (symmetry) | Interaction $\mathrm{X}-*-\mathrm{Y}$ | $\rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\left(e \mathrm{a}_{\mathrm{o}}{ }^{-3}\right)$ | $c \nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)^{\mathrm{b})}(\mathrm{au})$ | $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)(\mathrm{au})$ | $k_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)^{\text {c }}$ | $R^{\text {d) }}(\mathrm{au})$ | $\theta^{\mathrm{e})}\left({ }^{\circ}\right.$ ) | $C_{i j}\left(\AA \mathrm{mdyn}^{-1}\right)$ | $\theta_{\text {p:CIV }}{ }^{\text {f }}\left({ }^{\circ}\right.$ ) | $\kappa_{\mathrm{p}: \mathrm{CIV}{ }^{\mathrm{g})}\left(\mathrm{au}^{-1}\right)}$ | Predicted nature |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Br}_{2}\left(D_{\infty}\right)^{\mathrm{h})}$ | $\mathrm{Br}-*-\mathrm{Br}$ | 0.1130 | -0.0001 | -0.0497 | -2.005 | 0.0497 | 180.1 | 0.4 | 191.8 | 1.8 | SS/Cov-w ${ }^{\text {i }}$ ) |
| $\mathrm{Br}^{2-}\left(D_{\text {l }}\right)^{\mathrm{j}}$ ) | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.0922 | 0.0052 | -0.0313 | -1.751 | 0.0317 | 170.6 | 0.8 | 190.6 | 3.6 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
| $\mathrm{Br}_{4}^{2}\left(D_{\text {¢ }}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\text {B }} \mathrm{Br}$ | 0.0198 | 0.0058 | 0.0001 | -0.995 | 0.0058 | 89.5 | -19.6 | 118.2 | 146 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{\text {l }}$ ) |
| $\mathrm{Br}_{6}\left(\mathrm{C}_{2}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1099 | 0.0010 | -0.0466 | -1.960 | 0.0467 | 178.8 | 0.4 | 191.3 | 2.3 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0131 | 0.0049 | 0.0009 | -0.899 | 0.0050 | 79.6 | 14.3 | 97.8 | 105 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{\text {l }}$ ) |
| $\mathrm{Br}_{6}\left(C_{2 h}\right)^{\mathrm{m})}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1099 | 0.0010 | -0.0466 | -1.961 | 0.0467 | 178.8 | 0.4 | 191.7 | 1.7 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0131 | 0.0049 | 0.0009 | -0.899 | 0.0050 | 79.6 | 14.3 | 97.6 | 97 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{1)}$ |
| $\left.\operatorname{Br}_{6}\left(C_{2 h}\right)_{\text {obsd }}{ }^{\mathrm{n}}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.0765 | 0.0053 | -0.0200 | -1.654 | 0.0207 | 165.2 |  |  |  | $r$-CS |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0156 | 0.0055 | 0.0007 | -0.929 | 0.0055 | 82.5 |  |  |  | $p$-CS |
| $\mathrm{H}_{2} \mathrm{Br}_{4}\left(\mathrm{C}_{2 \mathrm{~h}}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1101 | 0.0008 | -0.0468 | -1.966 | 0.0468 | 179.0 | 0.4 | 191.7 | 2.0 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0118 | 0.0045 | 0.0010 | -0.881 | 0.0046 | 78.0 | 15.5 | 94.7 | 100 | $p-\mathrm{CS} / t-\mathrm{HB}^{\text {nc }}{ }^{1)}$ |
| $\mathrm{Me}_{2} \mathrm{Br}_{4}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1076 | 0.0016 | -0.0444 | -1.932 | 0.0445 | 177.9 | 0.4 | 191.4 | 1.8 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0164 | 0.0057 | 0.0006 | -0.945 | 0.0057 | 84.1 | 10.5 | 105.1 | 100 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{1)}$ |
| $\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1028 | 0.0031 | -0.0400 | -1.866 | 0.0401 | 175.6 | 0.5 | 191.5 | 2.5 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0220 | 0.0068 | -0.0002 | -1.016 | 0.0068 | 91.9 | 9.9 | 117.4 | 75 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\text {o }}$ |
| $\mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1028 | 0.0032 | -0.0400 | -1.862 | 0.0402 | 175.4 | 0.5 | 191.4 | 3.7 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0212 | 0.0067 | -0.0001 | -1.008 | 0.0067 | 90.9 | 9.9 | 116.4 | 101 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\text {o }}$ ) |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\text {A }} \mathrm{Br}$ | 0.1016 | 0.0036 | -0.0389 | -1.844 | 0.0391 | 174.7 | 0.5 | 191.5 | 4.4 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
| $5\left(C_{i}\right)^{\mathrm{p}}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0226 | 0.0070 | -0.0003 | -1.023 | 0.0070 | 92.7 | 6.8 | 118.0 | 557 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\text {o }}$ ) |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}^{\mathrm{q}}{ }^{\text {( }}$ | 0.0226 | 0.0070 | -0.0003 | -1.023 | 0.0070 | 92.7 | 6.8 | 118.0 | 551 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\text {o }}$ ) |
| $5\left(C_{i}\right)^{\mathrm{r})}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1047 | 0.0020 | -0.0383 | -1.905 | 0.0384 | 177.0 | 0.5 | 191.3 | 2.5 | $r$-CS/CT-TBP ${ }^{\text {k }}$ ) |
|  | ${ }^{\mathrm{A} r} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0145 | 0.0048 | 0.0008 | -0.904 | 0.0048 | 80.1 | 15.9 | 97.4 | 102 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{\text {o }}$ ) |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1014 | 0.0037 | -0.0388 | -1.841 | 0.0389 | 174.6 | 0.5 | 191.5 | 36 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
| $6\left(C_{i}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}^{\mathrm{q}}{ }^{\text {( }}$ | 0.1014 | 0.0037 | -0.0388 | -1.841 | 0.0389 | 174.6 | 0.5 | 191.6 | 36 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}^{\mathrm{q}}$ | 0.0227 | 0.0070 | -0.0004 | -1.024 | 0.0071 | 92.8 | 42.1 | 122.5 | 2474 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\text {o }}$ ) |
| $6\left(C_{i}\right)^{\mathrm{r}}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1044 | 0.0021 | -0.0380 | -1.901 | 0.0381 | 176.9 | 0.5 | 191.3 | 2.6 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.0147 | 0.0048 | 0.0008 | -0.907 | 0.0049 | 80.3 | 16.6 | 97.8 | 103 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{\text {o }}$ ) |
| $1\left(C_{i}\right)^{\mathrm{r})}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1063 | 0.0013 | -0.0398 | -1.939 | 0.0398 | 178.1 | 0.5 | 191.7 | 2.3 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\text {B }} \mathrm{Br}$ | 0.0123 | 0.0043 | 0.0010 | -0.868 | 0.0044 | 76.9 | 18.3 | 91.8 | 100 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{\text {l }}$ |
| $1\left(C_{i}\right)_{\text {obsd }}{ }^{\text {s }}$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ | 0.1019 | 0.0032 | -0.0393 | -1.860 | 0.0394 | 175.3 |  |  |  | $r-\mathrm{CS}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ | 0.0200 | 0.0066 | 0.0003 | -0.979 | 0.0066 | 87.7 |  |  |  | $p-\mathrm{CS}$ |




 with covalency. ${ }^{\mathrm{p})}$ With one imaginary frequency for the vibration mode of the AU symmetry. ${ }^{\mathrm{q}} \mathbf{w}=(0), \pm 0.025$, and $\pm 0.05$. ${ }^{\mathrm{r})} \mathrm{At}$ the CAM-B3LYP level. ${ }^{\mathrm{s})}$ See ref. [39].
 $\left(C_{\mathrm{s}}\right)$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Se}-*-{ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\mathrm{C}} \mathrm{Br}-*-{ }^{\mathrm{D}} \mathrm{Br}$ in $4\left(\mathrm{C}_{\mathrm{s}}\right)$, evaluated with MP2 BSS-A ${ }^{\text {a }}$

| Species (symmetry) | Interaction $\mathrm{X}-*-\mathrm{Y}$ | $\rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\left(e \mathrm{a}_{\mathrm{o}}^{-3}\right)$ | $c \nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)^{\mathrm{b})}(\mathrm{au})$ | $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)(\mathrm{au})$ | $\left.k_{b}\left(\mathbf{r}_{\mathrm{c}}\right)^{\mathrm{c}}\right)$ | $R^{\text {d) }}$ (au) | $\theta^{e}\left({ }^{\circ}\right.$ ) | $C_{i j}\left(\AA \mathrm{mdyn}^{-1}\right)$ | $\theta_{\mathrm{p}: \operatorname{CIV}}{ }^{\text {f }}\left({ }^{\circ}\right.$ ) | $\kappa_{\mathrm{p}: \mathrm{CIV}^{\text {g }}}{ }^{\mathrm{g}}\left(\mathrm{au}^{-1}\right)$ | Predicted nature |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $7\left(C_{2 h}\right)$ | ${ }^{\text {A }} \mathrm{Se}-*-{ }^{\text {A }} \mathrm{Br}^{\text {h }}{ }^{\text {( }}$ | 0.0423 | 0.0080 | -0.0056 | -1.258 | 0.0098 | 124.8 | 6.3 | 169.9 | 55 | $r$-CS/CT-MC ${ }^{\text {i }}$ ) |
|  | ${ }^{\text {A }} \mathrm{Se}-*-{ }^{\text {B }} \mathrm{Br}^{\text {j }}$ | 0.0825 | 0.0043 | -0.0264 | -1.753 | 0.0267 | 170.7 | 1.2 | 192.2 | 2.2 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\text {C }} \mathrm{Br}^{\text {l }}$ | 0.0335 | 0.0086 | -0.0022 | -1.115 | 0.0088 | 104.6 | 9.4 | 145.5 | 102 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\mathrm{m})}$ |
|  | ${ }^{\text {A }} \mathrm{Se}-*-{ }^{\text {A }} \mathrm{Br}^{\text {h }}{ }^{\text {a }}$ | 0.0492 | 0.0085 | -0.0079 | -1.318 | 0.0116 | 133.0 | 2.3 | 172.7 | 53 | $r$-CS/CT-MC ${ }^{\text {i }}$ |
| $8\left(C_{2 h}\right)$ | ${ }^{\text {A }} \mathrm{Se}-*-{ }^{\text {B }} \mathrm{Br}^{\text {j }}$ ) | 0.0662 | 0.0075 | -0.0158 | -1.513 | 0.0175 | 154.6 | 2.2 | 187.7 | 17 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\text {C }} \mathrm{Br}^{\text {l }}$ | 0.0398 | 0.0092 | -0.0038 | -1.171 | 0.0100 | 112.5 | 4.2 | 151.5 | 54 | $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}{ }^{\mathrm{m})}$ |
|  | ${ }^{\text {A }} \mathrm{Br}-*-{ }^{\text {A }} \mathrm{Se}$ | 0.0219 | 0.0065 | -0.0005 | -1.039 | 0.0065 | 94.6 |  |  |  | $r$-CS |
|  | ${ }^{\text {A }} \mathrm{Se}-*-{ }^{\text {B }} \mathrm{Br}$ | 0.0576 | 0.0102 | -0.0113 | -1.356 | 0.0152 | 137.8 |  |  |  | $r$-CS |
| $2\left(C_{1}\right)_{\text {obsd }}{ }^{\text {n) }}$ | ${ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\text {C }} \mathrm{Br}$ | 0.0952 | 0.0068 | -0.0337 | -1.713 | 0.0343 | 168.6 |  |  |  | $r$-CS |
|  | ${ }^{\mathrm{C}} \mathrm{Br}-*-{ }^{\mathrm{D}} \mathrm{Br}$ | 0.0183 | 0.0062 | 0.0005 | -0.961 | 0.0062 | 85.7 |  |  |  | $p-\mathrm{CS}$ |
|  | ${ }^{\mathrm{D}} \mathrm{Br}-*-{ }^{\text {B }} \mathrm{Se}$ | 0.0818 | 0.0063 | -0.0271 | -1.682 | 0.0278 | 166.9 |  |  |  | $r$-CS |
|  | ${ }^{\mathrm{B}} \mathrm{Se}-*-{ }^{\text {E }} \mathrm{Br}$ | 0.0753 | 0.0072 | -0.0220 | -1.604 | 0.0231 | 161.9 |  |  |  | $r$-CS |
| $3\left(C_{s}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\text {A }} \mathrm{Se}$ | 0.0737 | 0.0061 | -0.0214 | -1.636 | 0.0223 | 164.0 | 0.8 | 185.8 | 8.4 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
|  | ${ }^{\mathrm{A}} \mathrm{Se}-*-{ }^{\text {B }} \mathrm{Br}$ | 0.0678 | 0.0069 | -0.0177 | -1.562 | 0.0189 | 158.7 | 1.0 | 183.0 | 18 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
| $4\left(C_{s}\right)$ | ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\text {A }} \mathrm{Se}$ | 0.0131 | 0.0042 | 0.0004 | -0.945 | 0.0042 | 84.0 | 6.4 | 105.1 | 84 | $p-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}{ }^{\text {o }}$ ) |
|  | ${ }^{\text {A }} \mathrm{Se}-*-{ }^{\text {B }} \mathrm{Br}$ | 0.0425 | 0.0088 | -0.0052 | -1.229 | 0.0103 | 120.6 | 4.6 | 163.2 | 63 | $r$-CS/CT-MC ${ }^{\text {i }}$ |
|  | ${ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\text {C }} \mathrm{Br}$ | 0.0933 | 0.0059 | -0.0321 | -1.732 | 0.0326 | 169.6 | 0.9 | 192.1 | 5.6 | $r$-CS/CT-TBP ${ }^{\text {k }}$ |
| ${ }^{\text {a) }}$ See the text for B $(y / x) .{ }^{\mathrm{f})} \theta_{\mathrm{p}}=90^{\circ}-\tan ^{-1}$ CS interaction of the C | S. ${ }^{\mathrm{b})} c \nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)=H_{\mathrm{b}}$ <br> d $y / \mathrm{d} x) .{ }^{\mathrm{g})} \kappa_{\mathrm{p}}=\left\|\mathrm{d}^{2} y / \mathrm{d} x^{2}\right\|$ <br> -TBP nature. ${ }^{1)}$ The sam | $\begin{aligned} & \mathrm{c})-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2, \\ & \left.1+(\mathrm{d} y / \mathrm{d} x)^{2}\right]^{3 / 2} . \\ & \text { as }{ }^{\mathrm{C}} \mathrm{Br}-*-{ }^{\mathrm{D}} \mathrm{Br} . \mathrm{n} \end{aligned}$ | here $c=\hbar^{2} / 8 m$ ecause it has $C_{i}$ sym The pure-CS intera | $k_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)=V_{\mathrm{l}}$ <br> etry, it is the on of the H | $\left(\mathbf{r}_{\mathrm{c}}\right) / G_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right.$ ame as ${ }^{\mathrm{B}} \mathrm{S}$ nature wi | d) $R=$ <br> - ${ }^{\mathrm{E}} \mathrm{Br}$. ${ }^{\text {i) }}$ <br> no cova | $\begin{aligned} & { }^{2}+y^{2} \\ & \text { regular } \\ & \text { cy. }{ }^{\text {n) }} \mathrm{Se} \end{aligned}$ | /2, where ( $x, y$ CS interaction of th ref. [39]. ${ }^{\circ}$ The reg | $\begin{aligned} & \left(H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-\right. \\ & \text { TT-MC natur } \\ & r \text { - } \mathrm{CS} \text { intera } \end{aligned}$ | $V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2, H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right.$ <br> ${ }^{\mathrm{j})}$ The same as ${ }^{\mathrm{B}} \mathrm{S}$ on the HB nat | ${ }^{e)} \theta=90^{\circ}-\tan ^{-1}$ <br> ${ }^{\mathrm{D}} \mathrm{Br} .{ }^{\mathrm{k}}$ The regular <br> e with no covalency |



Figure 6: Molecular orbitals for $\sigma(7 \mathrm{c}-10 \mathrm{e}) . \psi_{184}(\mathrm{HOMO})$ and $\psi_{185}$ (LUMO) of $8\left(C_{2 h}\right)$.

The orders shown in equations (10) and (11) seem to reasonably explain the characteristic behavior of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$. The results must be the reflection of the $n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{Br}\right) \longrightarrow \sigma *\left({ }^{\mathrm{A}} \mathrm{Br}-\right.$ $\left.{ }^{\mathrm{A}} \mathrm{Br}\right) \leftarrow n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{Br}\right)$ form of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$, where ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ become weaker and stronger, respectively, as the CT interaction increases. $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ will be stabilized more effectively, if the negative charge is developed more at ${ }^{B} \mathrm{Br}$. However, the two $\mathrm{Br}^{-}$ligands in $\mathrm{Br}_{4}^{2-}\left(D_{\mathrm{oh}}\right)$ seem not so effective than that expected. This would come from the electrostatic repulsive factor between the double negative charges in $\mathrm{Br}_{4}^{2-}\left(D_{\infty h}\right)$, as mentioned above.

The $\theta$ values for ( ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and $\left.{ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{B}} \mathrm{Br}\right)$ in $\mathrm{Br}_{6}$ $\left(C_{2 \mathrm{~h}}\right)_{\text {obsd }}$ and $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$ are $\left(165.2^{\circ}, 82.5^{\circ}\right)$ and $\left(175.3^{\circ}, 87.7^{\circ}\right)$, respectively. Therefore, ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{B}} \mathrm{Br}$ are classified by $r$-CS and $p-\mathrm{CS}$, respectively. Both ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-$ *- ${ }^{\mathrm{B}} \mathrm{Br}$ in $\mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)_{\text {obsd }}$ are predicted to be weaker than those in $1\left(C_{i}\right)_{\text {obsd }}$, respectively. The results would be curious at the first glance, since ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ will be weaker, if ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{B}} \mathrm{Br}$ in ${ }^{\mathrm{B}} \mathrm{Br}-$ *- ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{B}} \mathrm{Br}$ becomes stronger, as mentioned above. They would be affected from the surrounding, such as the crystal packing effect. A $\mathrm{Br}_{2}$ molecule interacts with four bromine atoms adjacent to the $\mathrm{Br}_{2}$ molecule on the $b c$-plane in crystals, equivalently with $3.251 \AA$ [40].

Similar investigations were carried out for $\mathrm{I}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$, which will be discussed elsewhere (it is demonstrated that $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ is predicted to be somewhat stronger than $\mathrm{I}_{4}$ $\sigma(4 c-6 e))$.
3.6. Nature of $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 c-10 e)$. The nature of $\mathrm{Se}_{2} \mathrm{Br}_{5}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$ in $7\left(C_{2 \mathrm{~h}}\right)$ and $\mathbf{8}\left(C_{2 \mathrm{~h}}\right)$ is elucidated, together with $\mathrm{SeBr}_{2} \sigma(3 \mathrm{c}-4 \mathrm{e})$ in 3 and $\mathrm{SeBr}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ in 4 . The results are collected in Table 2. Figure 6 shows symmetric $\psi_{184}$ (HOMO) and antisymmetric $\psi_{185}$ (LUMO) of $8\left(C_{2 h}\right)$, which correspond to $\psi_{5}$ and $\psi_{6}$ in $\sigma(7 \mathrm{c}-10 \mathrm{e})$, illustrated in Figure 1 although the Se atoms are contained in the linear $\mathrm{Se}_{2} \mathrm{Br}_{5}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$ in $8\left(C_{2 \mathrm{~h}}\right)$. The linear seven atomic orbitals on $\mathrm{Se}_{2} \mathrm{Br}_{5}$ are shown to construct $\psi_{184}$ (HOMO) and $\psi_{185}$ (LUMO) of $8\left(C_{2 h}\right)$, which can be analysed as the $\mathrm{Se}_{2} \mathrm{Br}_{5}$ $\sigma(7 \mathrm{c}-10 \mathrm{e})$ [39], so can the linear interaction in $7\left(C_{2 \mathrm{~h}}\right)$, although not shown. The pseudolinear interaction of the seven atoms of $\mathbf{1}\left(C_{1}\right)_{\text {obsd }}$ could also be explained by the $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ model.

The results demonstrate that $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ stabilize well $7\left(C_{2 \mathrm{~h}}\right)$ and $\mathbf{8}\left(C_{2 \mathrm{~h}}\right)$ although $\mathbf{1}\left(C_{1}\right)_{\text {obsd }}$ seems not so effective. The negative charge developed at the Br atom in 3 would not be sufficient to stabilize $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ in $\mathbf{1}$ $\left(C_{1}\right)_{\text {obsd }}$, relative to the case of the $\mathrm{Br}^{-}$anion in $7\left(C_{2 \mathrm{~h}}\right)$ and 8 $\left(C_{2 \mathrm{~h}}\right.$ ), irrespective of the highly negatively charged Br atoms in $\mathrm{SeBr}_{2} \sigma(3 \mathrm{c}-4 \mathrm{e})$ of 3 .

## 4. Conclusion

The intrinsic dynamic and static nature of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ is elucidated for $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$ and the related species with QTAIM-DFA, employing the perturbed structures generated with CIV. The ${ }^{\mathrm{A}} \mathrm{Br}^{-}{ }^{\mathrm{A}} \mathrm{Br}$ interactions in ${ }^{\mathrm{B}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}-$ *- ${ }^{\mathrm{A}} \mathrm{Br}-*{ }_{-}{ }^{\mathrm{B}} \mathrm{Br}$ of $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ are weaker than $\mathrm{Br}-*-\mathrm{Br}$ in the optimized structure of $\mathrm{Br}_{2}\left(D_{\infty h}\right)$, which is predicted to have the $\mathrm{SS} / \mathrm{Cov}-\mathrm{w}$ nature. The ${ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}$ interactions in $\mathrm{Br}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$ of the models are predicted to have the $r$-CS/CTTBP nature, if optimized with MP2/BSS-A. The ${ }^{A} B r-{ }^{A} B r$ interaction in $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$ also appears in the $r$-CS region. On the contrary, the ${ }^{\mathrm{A}} \mathrm{Br}^{-}{ }^{\mathrm{B}} \mathrm{Br}$ interactions in $\mathrm{Br}_{6}\left(C_{2}\right), \mathrm{Br}_{6}\left(C_{2 \mathrm{~h}}\right)$, $\mathrm{H}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right)$, and $\mathrm{Me}_{2} \mathrm{Br}_{4}\left(C_{2 \mathrm{~h}}\right)$ are predicted to have the $p$ $\mathrm{CS} / t-\mathrm{HB}_{\mathrm{nc}}$ nature, whereas those in $\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{4}\left(C_{i}\right)$, $\mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{4}\left(C_{i}\right), 5\left(C_{i}\right)$, and $6\left(C_{i}\right)$ have the $r-\mathrm{CS} / t-\mathrm{HB}_{\mathrm{wc}}$ nature, if evaluated with MP2/BSS-A. The ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}{ }^{\mathrm{B}} \mathrm{Br}$ interactions become stronger in the order of $\mathrm{H}_{2} \mathrm{Br}_{4}\left(\mathrm{C}_{2 \mathrm{~h}}\right)<\mathrm{Br}_{6}$ $\left(C_{2 \mathrm{~h}}\right) \leq \mathrm{Br}_{6} \quad\left(C_{2}\right)<\mathrm{Me}_{2} \mathrm{Br}_{4} \quad\left(C_{2 \mathrm{~h}}\right) \ll \mathrm{Me}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6} \quad\left(C_{i}\right) \leq$ $\mathrm{H}_{4} \mathrm{Se}_{2} \mathrm{Br}_{6}\left(C_{i}\right) \leq \mathbf{5}\left(C_{i}\right)<\mathbf{6}\left(C_{i}\right)$, which is the inverse order for ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$, as a whole. The results are in accordance with the CT interaction of the $n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{Br}\right) \longrightarrow \sigma *\left({ }^{\mathrm{A}} \mathrm{Br}-{ }^{\mathrm{A}} \mathrm{Br}\right) \leftarrow n_{\mathrm{p}}\left({ }^{\mathrm{B}} \mathrm{Br}\right)$ form derived from $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$. The decreased binding force of ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{A}} \mathrm{Br}$ must be transferred to ${ }^{\mathrm{A}} \mathrm{Br}-*{ }_{-}{ }^{\mathrm{B}} \mathrm{Br}$ in $\mathrm{Br}_{4}$ $\sigma(4 \mathrm{c}-6 \mathrm{e})$. Namely, it is demonstrated that $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ is stabilized as the strength of ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }_{-}^{\mathrm{B}} \mathrm{Br}$ in $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ increases, while ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ becomes weakened relative to that in the original $\mathrm{Br}_{2}\left(D_{\text {chh }}\right)$. In this process, $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ is totally stabilized. The ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{A}} \mathrm{Br}$ and ${ }^{\mathrm{A}} \mathrm{Br}-*-{ }^{\mathrm{B}} \mathrm{Br}$ interactions in $\operatorname{Br}_{6}\left(C_{2 \mathrm{~h}}\right)_{\text {obsd }}$ and $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$ are classified by the $r$ - CS and $p$-CS interactions, respectively, where the interactions in $\mathrm{Br}_{6}$ $\left(C_{2 \mathrm{~h}}\right)_{\text {obsd }}$ seem somewhat weaker than those in $\mathbf{1}\left(C_{i}\right)_{\text {obsd }}$. The $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ interactions are similarly elucidated for $2\left(C_{1}\right)_{\text {obsd }}$ and the anionic models of $7\left(C_{2 \mathrm{~h}}\right)$ and $8\left(C_{2 \mathrm{~h}}\right)$. The $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$ nature is clearly established for the optimized structures of $7\left(C_{2 \mathrm{~h}}\right)$ and $8\left(C_{2 \mathrm{~h}}\right)$, rather than 2 $\left(C_{1}\right)_{\text {obsd }}$. Extended hypervalent interactions of the $\sigma(m c-n e$ : $4 \leq m ; m<n<2 m)$ type are shown to be well analysed and evaluated with QTAIM-DFA, employing the perturbed structures generated with CIV, exemplified by $\mathrm{Br}_{4} \sigma(4 \mathrm{c}-6 \mathrm{e})$ and $\mathrm{Se}_{2} \mathrm{Br}_{5} \sigma(7 \mathrm{c}-10 \mathrm{e})$.

## Data Availability

The data used to support the findings of this study are available in the supplementary information files.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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## Supplementary Materials

Scheme S1: classification of interactions by the signs of $\nabla^{2} \rho_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ and $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$, together with $G_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$ and $V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)$. Scheme S2: QTAIM-DFA: a plot of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)$ versus $H_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ for weak to strong interactions. Scheme S3: rough classification and characterization of interactions by $\theta$ and $\theta_{\mathrm{p}}$, together with $k_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\left(=V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / G_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right)\right)$. QTAIM-DFA approach, computational data (Tables $\mathrm{S} 2-\mathrm{S} 5$ and Figures S3-S5), computation information and geometries of compounds, and graphical abstract. Figure S1: polar $(R, \theta)$ coordinate representation of $H_{b}\left(\mathbf{r}_{c}\right)$ versus $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$, with $\left(\theta_{p}, \kappa_{p}\right)$ parameters. Figure S2: plot of $H_{b}\left(\mathbf{r}_{c}\right)$ versus $w$ in $r\left({ }^{1} \mathrm{Cl}-{ }^{2} \mathrm{Cl}\right)=r_{\mathrm{o}}\left({ }^{1} \mathrm{Cl}-{ }^{2} \mathrm{Cl}\right)+w a_{o}$ for ${ }^{1} \mathrm{Cl}^{2}{ }^{2} \mathrm{Cl}^{-}{ }^{3} \mathrm{Cl}^{-}$(a) with the magnified picture of (a) (b) and that of $H_{b}\left(\mathbf{r}_{\mathrm{c}}\right)-V_{\mathrm{b}}\left(\mathbf{r}_{\mathrm{c}}\right) / 2$ versus $w$ (c). Typical hydrogen bonds without covalency and typical hydrogen bonds with covalency are abbreviated as $t$ HB without cov. and $t$-HB with cov., respectively, whereas Cov-w and Cov-s stand for weak covalent bonds and strong covalent bonds, respectively. Table S1: proposed definitions for the classification and characterization of interactions. (Supplementary Materials)

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