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# Estimation of parameter of fractional order COVID-19 SIQR epidemic model

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## ABSTRACT

Epidemic model have been broadly used in different forms for studying and forecasting epidemiological processes the spread of dengue, zika virus, HIV, SARS and recently, the 2019–20 corona virus which is an ongoing pandemic of corona virus disease (COVID-19). In the present paper, an inverse problem to find the parameters for the single term (multi term) fractional order system of an outbreak of COVID-19 is considered. In the starting, we propose a numerical method for fractional order corona virus system based on the Gorenflo-Mainardi-Moretti-Paradisi (GMMP) scheme, and then to find the parameters we use GMMP method and the modified hybrid Nelder-Mead Simplex search and particle swarm optimization algorithm. With the new fractional orders and parameters our fractional order corona virus system is capable to providing numerical results that agree well with the real data.

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## 1. Introduction

In the middle of March of 2020 World health organization announced Covid-19 as a public health emergency of international concern. The origin of this disease is China. In December 2019 China has betimes desolation around the world as well as in India. The first patient of Covid-19 found on 23 February 2020 in India [1]. The virus has been affected most of the countries and taken the lives of several crore of people worldwide. In the march 2020 the World Health Organization (WHO) declared the disease a pandemic, first of its kind in our generation. Many countries and regions have been lockdown and applied strict social distancing measures to spread of virus. From a strategic and health care management are observing pattern of the disease and the prediction of its spread over time is great importance, to save the lives and to minimize the social and economic consequences of the disease.

Fractional calculus has been the subject of worldwide attention in the last decades [2] due to its broad range of application in chemistry [3] biology [4] physics [5] engineering [6] image pro-

cessing [7]. Hence, fractional derivative based on epidemic system have also been used to deal with some epidemic behaviors [8–10]. The fractional derivative can provide a better epidemic model than an integer-order derivative [11]. The main advantage of fractional order differential equation is to provides a powerful instrument for incorporation of memory and hereditary properties of the systems as opposed to the integer order differential equation in which such effects are neglected, or difficult to incorporate. An important problem concerned with the fractional constitutive models is to determine the unknown parameters from real data, which leads to the so-called fractional inverse problem [12]. In the present paper, we mainly consider the fractional order CORONA virus system with the fractional derivatives defined in Caputo sense. This is a general system with different fractional orders.

This paper is organized as follows : in section 2, we describe fractional derivative and fractional order CORONA virus system in brief. The numerical solution of fractional order CORONA virus system is obtained using the GMMP scheme and the Newton method in section 3. In section 4, the MH-NMSS-PSO algorithm is described for parameter estimation in fractional differential equation. The parameter estimation techniques two term model is described in section 5. The conclusion of our work is presented in section 6.

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## 2. Fractional derivative and the system of fractional order CORONA virus

### 2.1. Fractional derivatives

Fractional calculus plays an important role in modern science [2,13]. In this study we propose the use of Caputo definition of the fractional derivative. Another two commonly is fractional derivative definitions are the Grunewald- Letnikov and Riemann–Liouville derivative definitions [14].

**Definition 1:** The Caputo derivative with the order  $\alpha$  of function  $f(t)$  is given as follows:

$${}_aCD_t^\alpha f(t) = {}_0D_t^{-(h-\alpha)} \frac{d^h}{dt^h} f(t) = \frac{1}{\sqrt{h-\alpha}} \int_a^t (t-i)^{h-\alpha-1} f^{(h)}(t)$$

$$n-1 < \alpha < n, n \in \mathbb{Z}^+$$

**Definition 2:** The Riemann-Liouville derivative with order  $\alpha$  of a function  $f(t)$  is defined as follows:

$${}_aRLD_t^\alpha f(t) = \frac{d^h}{dt^h} D_t^{-(h-\alpha)} f(t) = \frac{1}{\sqrt{h-\alpha}} \frac{d^h}{dt^h} \int_a^t (t-i)^{h-\alpha-1} f^{(h)}(t)$$

$$n-1 < \alpha < n, n \in \mathbb{Z}^+$$

**Definition 3:** The Grünwald-Letnikov derivative of order  $\alpha$  of a function  $f(t)$  is defined as follows:

$${}_aGLD_t^\alpha f(t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a) t^{k-\alpha}}{\Gamma(k+1-\alpha)} + \frac{1}{\sqrt{h-\alpha}} \frac{d^h}{dt^h} \int_a^t (t-i)^{h-\alpha-1} f^{(h)}(t)$$

It follows from the definition of fractional derivatives We may define the Caputo derivative in terms of the Riemann - Lowville derivative definition in the following way.

$${}_aCD_t^\alpha f(t) = {}_aRLD_t^\alpha f(t) \sum_{k=0}^{n-1} \frac{(t-a)^{k-\alpha} f^{(k)}(a)}{\sqrt{k+1-\alpha}}$$

By letting

$$h(t) = f(t) - \sum_{k=0}^{n-1} \frac{(t-a)^k f^{(k)}(a)}{k!}$$

We get

$${}_aCD_t^\alpha f(t) = {}_aRLD_t^\alpha h(t) = {}_aRLD_t^\alpha f(t) - \sum_{k=0}^{n-1} \frac{(t-a)^k f^{(k)}(a)}{k!}$$

This may be simplified down to the following expression

$${}_aCD_t^\alpha h(t) = {}_aCD_t^\alpha f(t)$$

We will consider the caputo derivative, since it may be combined with classical initial conditions. the Riemann - Lowville derivative, for example is no suitable to be combined with classical initial condition.

The original model used for this epidemic is the integer order SIQR model extended form of basic SIR model [15]. The classical model of COVID –19 epidemic considers that the total human population  $N$  is divided in to four subpopulation :  $S$  susceptible,  $I$  infected,  $Q$  quarantined and  $R$  recovered. The classical model consists of 4 ordinary differential equation for four independent functions is as follows:

$$\begin{aligned} \frac{dS}{dt} &= -\frac{\beta SI}{N(1-I)} \\ \frac{dI}{dt} &= \frac{\beta SI}{N(1-I)} - (\mu + \eta)I \\ \frac{dQ}{dt} &= \eta I - \gamma Q \\ \frac{dR}{dt} &= \gamma Q + \alpha I \end{aligned}$$

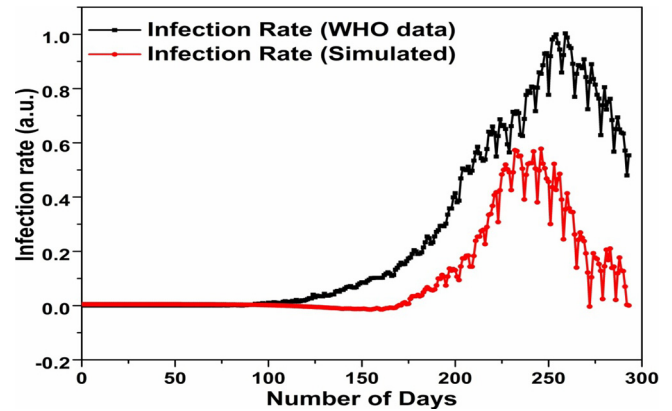


Fig. 1. Integer order Model, Simulation overlayed on real data.

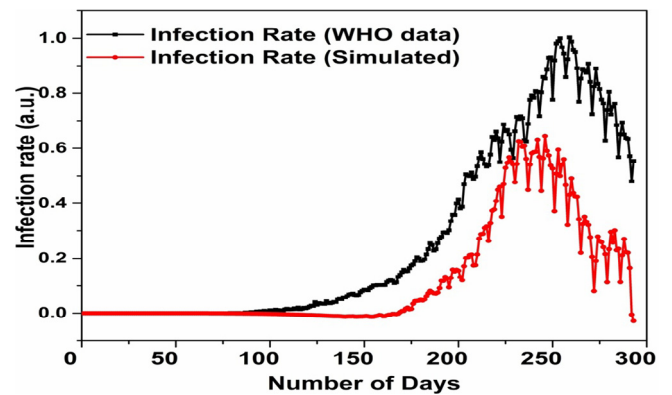


Fig. 2. Fractional order model.

Where  $\beta$  is the infectious rate,  $\eta$  indicates the rate at which new cases are recognized from the infected population.  $\gamma$  indicates the rate at which isolation are become removed (recovered or died).  $\mu$  indicating the removal rate of infected individuals who are asymptomatic and didn't became quarantined,  $N$  consider as population size. The factor  $(1-I)$  to  $N$  with 'I' being the fraction of population following lockdown, is used to consider lockdown. But we assume that everyone not following lockdown so every person has equal probability to become infected in to contact with every other person. After obtaining the numerical solutions of the differential equation system (7). The results are shown in the figure 0.1, which demonstrates that the solution of the system of differential equations provides a poor match with the real number of the infected humans.

This integer order simulated model provider a poor fit to the measured date shown in Fig. 1. We now consider replacing the integer order derivative with fractional order derivative terms. Fig. 2.

### 3. Fractional order model

$$\begin{aligned} {}_aCD_t^{\alpha_1} &= -\frac{\beta SI}{N(1-I)} \\ {}_aCD_t^{\alpha_2} &= \frac{\beta SI}{N(1-I)} - (\mu + \eta)I \\ {}_aCD_t^{\alpha_3} &= \eta I - \gamma Q \\ {}_aCD_t^{\alpha_4} &= \gamma Q + \alpha I \end{aligned}$$

This model now allows for extra flexibility [8–9] where the fractional order of each term may now be adjusted to a non-integer

value. Diethelm [16] proposes the following choices for the fraction order.

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.7$  or may another value. The result for this specific choice of the alphas reduces the root-mean-square relative error is  $g = 0.98$ . This exhibit a major improvement from the integer model, however noticeable error in figure encourages further exploration into the choice of the fractional order terms and 4 other parameters in the model. Let us discuss the numerical solver and proposed parameter estimation techniques [13–14,17].

**4. Numerical solver for the fractional order model**

Since the system of equation is non-linear, Newton-GMRES [18] has been chosen to solve the system numerically. Compared to other conventional numerical solvers, using GMMP (Gorenflo-Mainardi Moretti-Paradisi) [17] scheme and Newton method is very time efficient.

We generate a uniform grid  $t_j = a + jh, j = 0, 1, 2, \dots, Nh = t - a$  and  $\alpha > 0$ .

The Caputo derivative is approximated as

$${}^aCD_t^\alpha f(t) \approx \frac{1}{h^\alpha} \sum_{k=0}^N C_k^\alpha (f(t_{n-k})) - \sum_{j=D}^{n-1} \frac{(t-a)^j f^j(a)}{j!}$$

where

$$C_k^\alpha = (-1)^k \binom{\alpha}{j}$$

consider the general case for our fractional order non-linear system.

$${}^aCC_k^\alpha x(t) = f(t, x(t))$$

Where the right hand side in one of the 4th equations is in the dynamical system. The Caputo derivative is now submitted in to this expression to receive.

$$\sum_{k=0}^N C_k^\alpha \left( x(t_{n-k}) - \sum_{j=1}^{n-1} \frac{(t-1)^j t x^{(j)}(a)}{j} \right) = h^\alpha f(t_n, x(t_n))$$

Now by bringing  $x(t_n)$  to the left hand side

$$x(t_n) = h^\alpha f(t_n, x(t_n)) + \sum_{j=1}^{n-1} \frac{(t-a)^j x^{(j)}(a)}{j!} - \sum_{k=1}^N C_k^\alpha \left( x(t_{n-k}) - \sum_{k=1}^N C_k^\alpha - \sum_{j=1}^{n-1} \frac{(t-a)^j x^{(j)}(a)}{j!} \right)$$

However when  $0 < \alpha < 1$  the expression simplifies to

$$x(t_n) = h^\alpha f(t_n, x(t_n)) + x(a) - \sum_{j=1}^{n-1} \frac{(t-a)^j x^{(j)}(a)}{j!} - \sum_{k=1}^N C_k^\alpha ((x(t_{n-k}) - (a)))$$

When we plough through the algebra for each equation, we generate the solution of equations.

$$S(t_N) = h^{\alpha_1} f(t_N, S(t_N)) + S(a) - \sum_{k=1}^N C_k^{\alpha_1} [S(t_{N-k}) - S(a)]$$

$$I(t_N) = h^{\alpha_2} f(t_N, I(t_N)) + I(a) - \sum_{k=1}^N C_k^{\alpha_2} [I(t_{N-k}) - I(a)]$$

$$Q(t_N) = h^{\alpha_3} f(t_N, Q(t_N)) + Q(a) - \sum_{k=1}^N C_k^{\alpha_3} [Q(t_{N-k}) - Q(a)]$$

$$R(t_N) = h^{\alpha_4} f(t_N, R(t_N)) + R(a) - \sum_{k=1}^N C_k^{\alpha_4} [R(t_{N-k}) - R(a)]$$

Newton GMRES methods is an effective method to solving non-linear equations. Newton method is given as follows:

$$x_{n+1} = x_n - J_F(x_n)^{-1} F(x_n), \quad n = 0, 1, 2, \dots$$

Where  $J_F(x_n)$  denote the Jacobian matrix.

**5. Inverse problems parameter estimation techniques single term model**

For this study to solve the inverse problem both the Nelder-Mead Simplex Search (NMSS) [19] and the Particle Swarm Optimization (PSO) [20] were used. Both of these technique have different traits when it comes to estimating parameter for example in the NMSS methods, the initial points is pre-defined and the method moves the parameter away from paints with poorer function values. The PSO methods has a set of randomly chosen points and move towards points better with function values. The Modified Hybrid Nelder-Mead Simplex-Search and Particle Swarm Optimization (MHNMSS-PSO) was adopted from these methods. Steps of this methods are as follows

Define the mean square error (MSE) by g:

$$g(P^*) = \min_{PED} g(P) = \min_{PCD} \sqrt{\frac{\sum_{j=0}^n (x(t_j) - x_j)^2}{n+1}}$$

1. Create a Population of size  $3m + 1$ . From  $a$  the simpler of  $m$  dimensions.

$$P_i = (P_{1,i}, P_{2,i}, \dots, P_{m,i}) \quad ED_i = 1, 2, \dots, m + 1$$

Here

$$P_{ji} = P_j^{(Min)} + \frac{(i-1) \times (P_j^{(Min)} - P_j^{(Min)})}{m+1}$$

$$j = 1, 2, \dots, m, \quad i = 1, 2, \dots, m + 1$$

A pair of particles are created from the PSO methods

$$P_i = (P_{1,i}, P_{2,i}, \dots, P_{m,i}) \quad ED, \quad i = M + L, -3m + 1$$

Where

$$P_{j,i} = P_j^{(Min)} + Rand \times (P_j^{(man)} - P_j^{(min)}), \quad j = 1, 2, \dots, m$$

$$i = m + 2, \dots, 3M + 1$$

Lastly their velocities are calculated

$$V_{j,i} = (V_j^{man} - V_j^{min}) / L_j, \quad j = 1, 2, \dots, m$$

$$i = M + 2, \quad 3m + 1$$

2. Evaluate 'g' at each particle 'p' order them from smallest to longest.

$$g(P_1) \leq g(P_2) \leq \dots \leq g(P_{3M+1})$$

3. Incorporate the NMSS, Calculate center of gravity.

$$P_0 = (P_{1,0}, P_{2,0}, \dots, P_{m,0}) \quad ED$$

$$P_{j,0} = \frac{\sum_{i=1}^m P_{ji}}{m}, \quad i = 1, 2, \dots, m$$

$$\text{Calculate } P_r = (1 + \alpha)P_0 - \alpha P_{m+1}$$

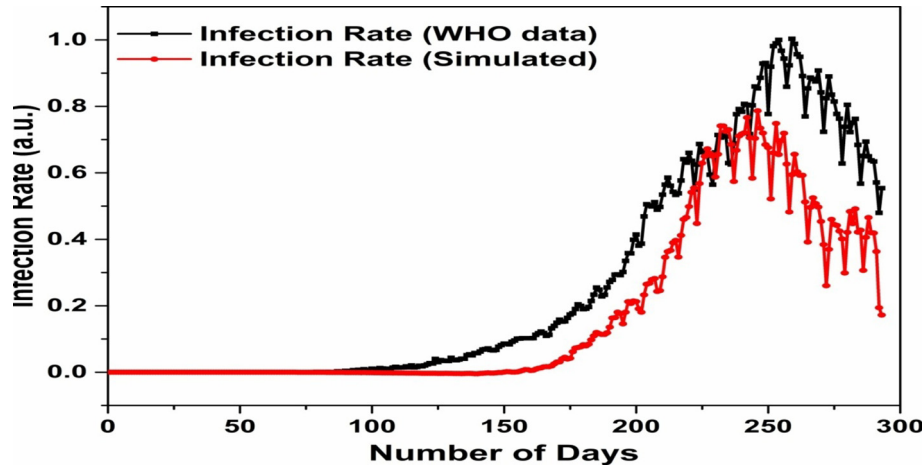


Fig. 3. Fractional order model with estimated parameter.

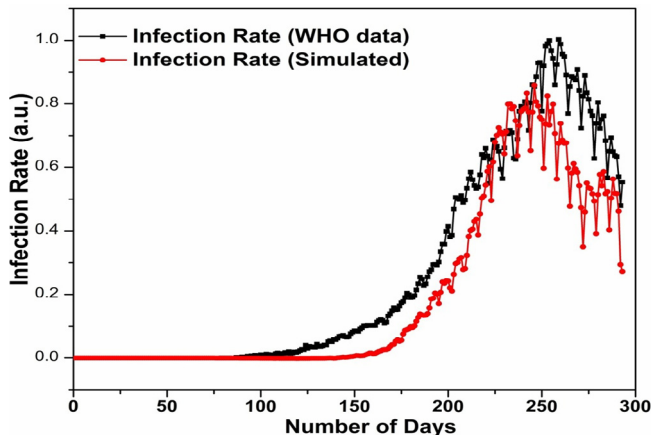


Fig. 4. Plot of two term fractional order model with parameter estimation.

$\alpha > 0$  recommended that  $\alpha > 1$

Case – 1 : if  $g(P_1) \leq g(P_r) \leq g(P_m)$  then  $P_{m+1} = P_r$

Case – 2 : if  $g(P_r) \leq g(P_1)$  then compute  $rP_r + (1 - r)P_0$

where  $r = 2$  if  $g(P_e) \leq g(P_1)$ ,  $P_{m+1} = P_e$  else  $P_{m+1} = P_r$

Case – 3 : if  $g(P_r) \geq g(P_m)$  and  $g(P_r) \leq g(P_{m+1})$ ,  $P_{m+1} = P_r$  compute

$$P_c = \beta P_{m+1} + (1 - \beta)P_c$$

if  $g(P_c) \leq g(P_{m+1})$ ,  $P_{m+1} = P_r$  else let

$$P_i = \sigma P_i + (P_1 - \sigma)P_1, \quad i = 1, 2, \dots, m + 1 \text{ chose}$$

$$\beta = 0.5 \text{ and } \sigma = 0.5$$

4. In corporate PSO update 2 m Particles with the poorest MSE function value.

5. If  $S_c < \epsilon$  stop. otherwise return to step 2

$$S_c = \left[ \sum_{i=1}^{m+1} \frac{(\bar{g} - \sqrt{g_i})^2}{m + 1} \right]^{1/2}$$

$$\bar{g} = \sum_{i=1}^{m+1} g_i \text{ and } g_i^* = \sqrt{g_i} = \sqrt{g_i(P_1, P_2, \dots, P_m)}$$

From this parameter estimation method. The model now fits much better.

### 6. Inverse problem estimation techniques two term model

We propose the two term fractional order model as following.

$$\lambda_1 \text{ } {}_a^c D_t^{\alpha_1} x(t) + \lambda_2 \text{ } {}_a^c D_t^{\alpha_2} x(t) = f(t_n, x(t_n))$$

Apply our approximated definition for the Caputo derivative

$$\frac{\lambda_1}{h^{\alpha_1}} \sum_{k=0}^N C_k^{\alpha_1} [x(t_{n-k}) - x(a)] + \frac{\lambda_2}{h^{\alpha_2}} \sum_{k=0}^N C_k^{\alpha_2} [x(t_{n-k}) - x(a)] = f(t_n, x(t_n))$$

Omit  $k = 0$  term and solve for  $x(t_n)$

$$\left[ \frac{\lambda_1}{h^{\alpha_1}} - \frac{\lambda_2}{h^{\alpha_2}} \right] [x(t_n) - x(a)] + \frac{\lambda_1}{h^{\alpha_1}} \sum_{k=0}^N C_k^{\alpha_1} [x(t_{n-k}) - x(a)] + \frac{\lambda_2}{h^{\alpha_2}} \sum_{k=0}^N C_k^{\alpha_2} [x(t_{n-k}) - x(a)] = f(t_n, x(t_n))i$$

Simplify by letting  $r = \frac{\lambda_1}{h^{\alpha_1}} - \frac{\lambda_2}{h^{\alpha_2}}$

$$A_1 = \frac{\lambda_1}{h^{\alpha_1}} - \sum_{k=1}^N C_k^{\alpha_1} (x(t_{n-k}) - x(a))$$

$$A_2 = \frac{\lambda_2}{h^{\alpha_2}} - \sum_{k=1}^N C_k^{\alpha_2} (x(t_{n-k}) - x(a))$$

Thus simplify and rearranged expression is given is  $x(t_n) = x(0) + \frac{1}{k} [f(t_n, x(t_n)) - (A_1 + A_2)]$

From here, the GMMP scheme [18,21] and Newton –GMRES method can be applied to obtain a numerical solution for x. So the new two term fractional order dynamical system is

$$\begin{aligned} \lambda_1 \text{ } {}_a^c D_t^{\alpha_1} S + \lambda_2 \text{ } {}_a^c D_t^{\alpha_2} S &= -\frac{\beta SI}{N(1-I)} \\ \lambda_3 \text{ } {}_a^c D_t^{\alpha_3} I + \lambda_4 \text{ } {}_a^c D_t^{\alpha_4} I &= \frac{\beta SI}{N(1-I)} - (\mu + \eta)I \\ \lambda_5 \text{ } {}_a^c D_t^{\alpha_5} Q + \lambda_6 \text{ } {}_a^c D_t^{\alpha_6} Q &= \eta I - \gamma Q \\ \lambda_7 \text{ } {}_a^c D_t^{\alpha_7} R + \lambda_8 \text{ } {}_a^c D_t^{\alpha_8} R &= \gamma Q + \lambda I \end{aligned}$$

Following the some methodology from the single term counterpart, we can numerically, solve this two term fractional order non-linear system of equations.

## 7. Conclusion

The final result of two term technique shows a drastic MSE test, demonstrating great effectiveness in modeling this epidemic using the two term fractional order model with parameter estimation. However, there is noticeable error between 125th –225th day and 250th–275th day of the model. However, despite this poor fitting in the section, the rest of the model agrees very well with the real data, showing this model's effectiveness. Perturbations in each parameter was attempted, while keeping all other parameters fixed, but the only significant change to the model was in the middle region, between days 1st –120th and 225th –250th. So this contribution of error could not be reduced in this two term model.

For the purpose of using this model to make predictions on the behavior of this disease, the single term model in Fig. 3 would suffice. Despite its greater error than two term model in the Fig. 4, and also sufficiently models the entire span of the real data. This paper has proposed a new two term method to achieve more accurate model, Indian data of CORONA virus epidemic has been simulated from 11/11/2019 to 18/11/2020.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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