

On randomly changing conformity bias in cultural transmission

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Humans and nonhuman animals display conformist as well as anticonformist biases in cultural transmission. Whereas many previous mathematical models have incorporated constant conformity coefficients, empirical research suggests that the extent of (anti)conformity in populations can change over time. We incorporate stochastic time-varying conformity coefficients into a widely used conformity model, which assumes a fixed number n of “role models” sampled by each individual. We also allow the number of role models to vary over time (n_t). Under anticonformity, nonconvergence can occur in deterministic and stochastic models with different parameter values. Even if strong anticonformity may occur, if conformity or random copying (i.e., neither conformity nor anticonformity) is expected, there is convergence to one of the three equilibria seen in previous deterministic models of conformity. Moreover, this result is robust to stochastic variation in n_t . However, dynamic properties of these equilibria may be different from those in deterministic models. For example, with random conformity coefficients, all equilibria can be stochastically locally stable simultaneously. Finally, we study the effect of randomly changing weak selection. Allowing the level of conformity, the number of role models, and selection to vary stochastically may produce a more realistic representation of the wide range of group-level properties that can emerge under (anti)conformist biases. This promises to make interpretation of the effect of conformity on differences between populations, for example those connected by migration, rather difficult. Future research incorporating finite population sizes and migration would contribute added realism to these models.

random conformity | stochastic local stability | convergence | random selection

Cavalli-Sforza and Feldman (1) studied the finite population dynamics of a trait whose transmission from one generation to the next depended on the mean value of that trait in the population. This “group transmission” constrained the within-group variability but could lead to increasing variance in the average trait value between groups. Other analyses of cultural transmission biases have incorporated characteristics of trait variation, such as the quality, and characteristics of transmitters, including success and prestige (2). Another class of transmission biases is couched in terms of the frequencies of the cultural variants in the population (3). These “frequency-dependent” biases include conformity and anticonformity, which occur when a more common variant is adopted at a rate greater or less than its population frequency, respectively (4).

Humans have exhibited conformity in mental rotation (5), line discrimination (6), and numerical discrimination tasks (7). Anticonformity has been exhibited by young children performing numerical discrimination (7). Unbiased frequency-dependent transmission, known as random copying (8), has been suggested to account for choices of dog breeds (9), Neolithic pottery motifs, patent citations, and baby names (10, 11). However, baby name distributions appear more consistent with frequency-dependent (8, 12) and/or other (13, 14) biases.

In nonhuman animals, conformity has been observed in nine-spined sticklebacks choosing a feeder (15) and great tits solving a puzzle box (16, 17) (but see ref. 18). Fruit flies displayed both conformist and anticonformist bias with respect to mate choice (19) (but these authors used a different definition of anticonformity from that of ref. 4, which we use, and therefore did not consider these behaviors to be anticonformist).

Asch (20, 21) used a different definition of conformity from ref. 4, namely “the overriding of personal knowledge or behavioral dispositions by countervailing options observed in others” (ref. 22, p. 34). Aschian conformity (22) has been observed in chimpanzees (23, 24), capuchin monkeys (25, 26) (but see ref. 27), vervet monkeys (28), and great tits (16). It has also been empirically tested in at least 133 studies of humans and, in the United States, has declined from the 1950s to the 1990s (29).

Temporal variation may also occur in forms of conformity other than Aschian. In ref. 12, popular US baby names from 1960 to 2010 show a concave turnover function indicative of negative frequency-dependent bias, but male baby names from earlier decades (1880 to 1930) show a convex turnover indicative of positive frequency-dependent or direct bias. However, most previous mathematical models of conformity have incorporated constant, rather than time-dependent, conformity coefficients.

Significance

Animals, from humans to *Drosophila*, display conformity and anticonformity. Population dynamics under (anti)conformity may explain emergent properties of groups including fads, norms, and collective behavior. Although empirical evidence suggests that a population’s level of conformity can vary over time, most mathematical models have not included time-varying conformity coefficients. To potentially improve applicability to real-world systems, we allow conformity coefficients, numbers of sampled “role models,” and weak selection to vary stochastically in an established conformity model. Novel dynamics are possible, including simultaneous stochastic local stability of monomorphisms and polymorphism. Interpreting real-world population differences in terms of (anti)conformity may therefore not be straightforward. Under some conditions, however, the deterministic model provides a useful approximation to the stochastic model.

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Cavalli-Sforza and Feldman (ref. 3, chap. 3) and Boyd and Richerson (ref. 4, chap. 7) studied models of frequency-dependent transmission of a cultural trait with two variants. Boyd and Richerson (4) incorporated conformist and anticonformist bias through a conformity coefficient denoted by D . In their simplest model, if the frequency of variant A is p and that of variant B is $1 - p$, then the frequency of variant A in the offspring generation, p' , is

$$p' = p + Dp(1 - p)(2p - 1), \quad [1]$$

where $D > 0$ entails conformity (A increases if its frequency is $p > \frac{1}{2}$), $D < 0$ entails anticonformity, $D = 0$ entails random copying, and $-2 < D < 1$. In this model, each offspring samples the cultural variants of $n = 3$ members of the parental generation (hereafter, role models). Sampling $n > 3$ role models requires different constraints and, if $n > 4$, there are multiple conformity coefficients (Eq. 19).

Many subsequent models have built upon Boyd and Richerson's (4) simplest model (Eq. 1). These have incorporated individual learning, information inaccuracy due to environmental change (30–34), group selection (35), and other transmission biases, including payoff bias (36), direct bias, and prestige bias (37). Other models, which include a single conformity coefficient and preserve the essential features of Eq. 1, incorporate individual learning, environmental variability (32, 38), group selection (39), and multiple cultural variants (38).

In agent-based statistical physics models, the up and down spins of an electron are analogous to cultural variants A and B (40, 41). Individuals are nodes in a network and choose among a series of actions with specified probabilities, such as independently acquiring a spin, or sampling neighboring individuals and adopting the majority or minority spin in the sample. The number of sampled role models can be greater than three (42, 43). (Anti)conformity may occur if all (42–47), or if at least r (40, 48), sampled individuals have the same variant. In contrast, Boyd and Richerson's (4) general model (Eq. 19) allows, for example, stronger conformity to a 60% majority of role models and weaker conformity or anticonformity to a 95% majority (in humans, this might result from a perceived difference between “up-and-coming” and “overly popular” variants).

In Boyd and Richerson's (4) general model, individuals sample n role models, which is more realistic than restricting n to 3 (as in Eq. 1); individuals may be able to observe more than three members of the previous generation. With $n > 4$, different levels of (anti)conformity may occur for different samples j of n role models with one variant. In addition to the example above with 60 and 95% majorities, other relationships between the level of conformity and the sample j of n are possible. For example, the strength of conformity might increase as the number of role models with the more common variant increases. In a recent exploration of Boyd and Richerson's (4) general model, we found dynamics that departed significantly from those of Eq. 1 (49). If conformity and anticonformity occur for different majorities j of n role models (i.e., $j > \frac{n}{2}$), polymorphic equilibria may exist that were not possible with Eq. 1. In addition, strong enough anticonformity can produce nonconvergence: With as few as 5 role models, stable cycles in variant frequencies may arise, and with as few as 10 role models, chaos is possible. Such complex dynamics may occur with or without selection.

Here, we extend both Boyd and Richerson's (4) simplest (Eq. 1) and general (Eq. 19) models to allow the conformity coefficient(s) to vary randomly across generations, by sampling them from probability distributions. Although some agent-based models allow individuals to switch between “conformist” and

“non-” or “anticonformist” states over time (40, 42, 47, 50, 51), to our knowledge, random temporal variation in the conformity coefficients themselves has not been modeled previously. In reality, the degree to which groups of individuals conform may change over time, as illustrated by the finding that young children anti-conformed while older children conformed in a discrimination task (7); thus, it seems reasonable to expect that different generations may also exhibit different levels of conformity. Indeed, generational changes have occurred for Aschian conformity (29) and possibly in frequency-dependent copying of baby names (12). Our stochastic model may therefore produce more realistic population dynamics than previous deterministic models, and comparisons between the two can suggest when the latter is a reasonable approximation to the former.

We also allow the number of role models, n_t , to vary over time. Agent-based conformity models have incorporated temporal (43) and individual (43, 45, 46) variation in the number of sampled individuals, whereas here, all members of generation t sample the same number n_t of role models. Causes of variation in n_t are not explored here, but there could be several. For instance, different generations of animals may sample different numbers of role models due to variation in population density. In humans, changes in the use of social media platforms or their features may cause temporal changes in the number of observed individuals. For example, when Facebook added the feature “People You May Know,” the rate of new Facebook connections in a New Orleans dataset nearly doubled (52).

In the stochastic model without selection, regardless of the fluctuation in the conformity coefficient(s), if there is conformity on average, the population converges to one of the three equilibria present in Boyd and Richerson's (4) model with conformity ($D(j) > 0$ for $\frac{n}{2} < j < n$ in Eq. 19). These are $p^* = 1$ (fixation on variant A), $p^* = 0$ (fixation on variant B), and $p^* = \frac{1}{2}$ (equal representation of A and B). However, their stability properties may differ from those in the deterministic case. In Boyd and Richerson's (4) model with random copying, every initial frequency p_0 is an equilibrium. Here, with random copying expected and independent conformity coefficients, there is convergence to $p^* = 0, \frac{1}{2}$, or 1. In this case, and in the case with conformity expected, convergence to $p^* = 0, \frac{1}{2}$, or 1 also holds with stochastic variation in the number of role models, n_t . With either stochastic or constant weak selection in Boyd and Richerson's (4) simplest model (Eq. 1) and random copying expected, there is convergence to a fixation state ($p^* = 0$ or 1). Finally, with anticonformity in the deterministic model or anticonformity expected in the stochastic model, nonconvergence can occur.

Generalizing the Deterministic Model

Here we generalize Eq. 1 to allow the coefficient D to change randomly at each generation. If t denotes the generation number ($t = 0, 1, 2, \dots$), then Eq. 1 becomes

$$p_{t+1} = p_t + D_t p_t (1 - p_t) (2p_t - 1). \quad [2]$$

In Eq. 2 we assume that the random variables D_t for $t = 0, 1, 2, \dots$ are identically distributed with $-2 < D_t < 1$ for $n = 3$ role models, $E(D_t) = d$, $\text{Var}(D_t) = \sigma^2$, and D_t are independent of p_t in each generation t . We focus on the possible convergence of p_t to equilibrium as $t \rightarrow \infty$.

Eq. 1 entails that $\sigma^2 = 0$, and we now summarize the known results for this model. If the conformity coefficients are constant over time, i.e., $D_t \equiv d \neq 0$ for all t , then

- 1) There are three possible equilibria, $p^* = 0$, $p^* = 1$, and $p^* = \frac{1}{2}$, where $p^* = 0$ and $p^* = 1$ correspond to fixation in type B and type A , respectively, and $p^* = \frac{1}{2}$ is a polymorphic equilibrium where both A and B are equally represented.
- 2) As $t \rightarrow \infty$ there is always convergence of p_t to an equilibrium.
- 3) If $d > 0$, namely with conformity bias, if $0 \leq p_0 < \frac{1}{2}$, then $p_t \xrightarrow{t \rightarrow \infty} 0$, whereas if $\frac{1}{2} < p_0 \leq 1$, then $p_t \xrightarrow{t \rightarrow \infty} 1$.
- 4) If $d < 0$, namely with anticonformity bias, if $0 < p_0 < 1$, then $p_t \xrightarrow{t \rightarrow \infty} \frac{1}{2}$.

In generalizing Eq. 1, we investigate whether, if D_t changes randomly over generations, there is also global convergence of the process $\{p_t\}_{t=0}^\infty$ and, if so, what can be said about the limit distribution. Our analysis is based on the following observations for the process $\{p_t\}_{t=0}^\infty$ ($t = 0, 1, 2, \dots$) specified in Eq. 2:

Observation 1.

- 1) If $0 < p_0 < \frac{1}{2}$, then $0 < p_t < \frac{1}{2}$ with probability 1.
- 2) If $\frac{1}{2} < p_0 < 1$, then $\frac{1}{2} < p_t < 1$ with probability 1.

Observation 2.

- 1) If $d = E(D_t) = 0$, then $E(p_{t+1} | p_t) = p_t$.
- 2) If $d = E(D_t) > 0$, then if $0 < p_0 < \frac{1}{2}$, $E(p_{t+1} | p_t) < p_t$ and if $\frac{1}{2} < p_0 < 1$, then $E(p_{t+1} | p_t) > p_t$. If $d = E(D_t) < 0$, then if $0 < p_0 < \frac{1}{2}$, $E(p_{t+1} | p_t) > p_t$, and if $\frac{1}{2} < p_0 < 1$, then $E(p_{t+1} | p_t) < p_t$.

Proof of Observation 1. From Eq. 2 we see that

$$(p_{t+1} - \frac{1}{2}) = (p_t - \frac{1}{2})[1 + 2D_t p_t(1 - p_t)]. \quad [3]$$

Since D_t satisfies $-2 < D_t < 1$, and $0 \leq x \leq 1$ entails that $0 \leq x(1-x) \leq \frac{1}{4}$, with probability 1 we have

$$1 + 2D_t p_t(1 - p_t) \geq 1 - 4p_t(1 - p_t) \geq 0, \quad [4]$$

with equality only when $p_t = \frac{1}{2}$. Therefore the signs of $(p_t - \frac{1}{2})$ and $(p_{t+1} - \frac{1}{2})$ coincide.

Proof of Observation 2. For all $t = 0, 1, 2, \dots$

- 1) If $d = E(D_t) = 0$, then Eq. 2 gives $E(p_{t+1} | p_t) = p_t$.
- 2) If $0 < p_0 < \frac{1}{2}$, then by Observation 1 we have $0 < p_t < \frac{1}{2}$ and $p_t(1 - p_t)(2p_t - 1) < 0$. Hence, if $d > 0$, Eq. 2 implies that $E(p_{t+1} | p_t) < p_t$ and if $d < 0$, then $E(p_{t+1} | p_t) > p_t$. If $\frac{1}{2} < p_0 < 1$, then with probability 1 we have $\frac{1}{2} < p_t < 1$, and a similar calculation shows that $E(p_{t+1} | p_t) > p_t$ if $d > 0$ and $E(p_{t+1} | p_t) < p_t$ if $d < 0$.

Using the two observations, we have the following result.

Result 1. For any starting $0 \leq p_0 \leq 1$, the process $\{p_t\}_{t=0}^\infty$ converges. That is, $\lim_{t \rightarrow \infty} p_t = p_\infty$ with probability 1.

Proof: From Observation 2 and the fact that $0 \leq p_t \leq 1$ with probability 1 for all $t = 0, 1, 2, \dots$, and any $0 \leq p_0 \leq 1$, the process $\{p_t\}_{t=0}^\infty$ is a bounded martingale, submartingale, or supermartingale. Therefore, the martingale convergence theorem ensures that there is a random variable p_∞ such that $p_t \xrightarrow{t \rightarrow \infty} p_\infty$ with probability 1.

As the process $\{p_t\}_{t=0}^\infty$ is bounded by 0 and 1, the Lebesgue convergence theorem gives the following result:

Result 2. For any $0 \leq p_0 \leq 1$, $\lim_{t \rightarrow \infty} E(p_t^\ell) = E(p_\infty^\ell)$ for $\ell = 1, 2, \dots$

The constant equilibria $p^* = 0$, $p^* = \frac{1}{2}$, and $p^* = 1$ are also equilibria of the process $\{p_t\}_{t=0}^\infty$ determined by Eq. 2. Is there a relation between these constant equilibria and the random variable p_∞ ? The following result gives the answer:

Result 3. If $p_t \xrightarrow{t \rightarrow \infty} p_\infty$, then $p_\infty(1 - p_\infty)(2p_\infty - 1) = 0$ with probability 1; namely,

$$P(p_\infty = 0 \text{ or } p_\infty = \frac{1}{2} \text{ or } p_\infty = 1) = 1. \quad [5]$$

Proof: From Eq. 2, since D_t is independent of p_t ,

$$E(p_{t+1} | p_t) = p_t + E(D_t)p_t(1 - p_t)(2p_t - 1), \quad [6]$$

which implies that

$$E(p_{t+1}) = E(p_t) + dE[p_t(1 - p_t)(2p_t - 1)]. \quad [7]$$

By Result 2, $E(p_t) \xrightarrow{t \rightarrow \infty} E(p_\infty)$ and $E[p_t(1 - p_t)(2p_t - 1)] \xrightarrow{t \rightarrow \infty} E[p_\infty(1 - p_\infty)(2p_\infty - 1)]$. Therefore from Eq. 7

$$dE[p_\infty(1 - p_\infty)(2p_\infty - 1)] = 0. \quad [8]$$

If $d \neq 0$, then Eq. 8 implies $E[p_\infty(1 - p_\infty)(2p_\infty - 1)] = 0$. Now if $0 < p_0 < \frac{1}{2}$, then also $0 \leq p_\infty \leq \frac{1}{2}$ with probability 1, and $p_\infty(1 - p_\infty)(2p_\infty - 1) \leq 0$. Thus in this case, $p_\infty(1 - p_\infty)(2p_\infty - 1) = 0$ with probability 1. A similar argument applies if $\frac{1}{2} < p_0 < 1$.

If $d = 0$, we use Eq. 2 to see that

$$\begin{aligned} E(p_{t+1} | p_t) &= p_t, \\ \text{Var}(p_{t+1} | p_t) &= \text{Var}(D_t)p_t^2(1 - p_t)^2(2p_t - 1)^2. \end{aligned} \quad [9]$$

Now $\text{Var}(D_t) = \sigma^2$, and

$$\text{Var}(p_{t+1}) = E[\text{Var}(p_{t+1} | p_t)] + \text{Var}[E(p_{t+1} | p_t)] \quad [10]$$

$$= \sigma^2 E[p_t^2(1 - p_t)^2(2p_t - 1)^2] + \text{Var}(p_t). \quad [11]$$

Applying Result 2, $\text{Var}(p_t) \xrightarrow{t \rightarrow \infty} \text{Var}(p_\infty)$ and

$$E[p_t^2(1 - p_t)^2(2p_t - 1)^2] \xrightarrow{t \rightarrow \infty} E[p_\infty^2(1 - p_\infty)^2(2p_\infty - 1)^2].$$

Therefore, since $\sigma^2 > 0$, Eq. 11 implies that

$$E[p_\infty^2(1 - p_\infty)^2(2p_\infty - 1)^2] = 0. \quad [12]$$

But $p_\infty^2(1 - p_\infty)^2(2p_\infty - 1)^2 \geq 0$ with probability 1. Hence, Eq. 12 is possible if and only if $p_\infty(1 - p_\infty)(2p_\infty - 1) = 0$ with probability 1.

Result 3 entails that the random variable p_∞ can take only the three values 0, 1, and $\frac{1}{2}$ corresponding to the constant equilibria $p^* = 0$, $p^* = 1$, and $p^* = \frac{1}{2}$. What are the probabilities that p_∞ takes each of these values?

Result 4. If $d = E(D_t) = 0$, then

- 1) If $0 < p_0 < \frac{1}{2}$,

$$P(p_\infty = 0) = 1 - 2p_0, \quad P(p_\infty = \frac{1}{2}) = 2p_0. \quad [13]$$

- 2) If $\frac{1}{2} < p_0 < 1$,

$$P(p_\infty = \frac{1}{2}) = 2(1 - p_0), \quad P(p_\infty = 1) = 2p_0 - 1. \quad [14]$$

Proof: If $d = E(D_t) = 0$, then Eq. 2 implies that $E(p_{t+1} | p_t) = p_t$, so $E(p_{t+1}) = E(p_t)$ for all $t = 0, 1, 2, \dots$. Therefore $E(p_t) = p_0$ for all $t = 0, 1, 2, \dots$, and $E(p_\infty) = p_0$. If $0 < p_0 < \frac{1}{2}$, then $0 \leq p_\infty \leq \frac{1}{2}$ and by Eq. 5, $P(p_\infty = 0 \text{ or } p_\infty = \frac{1}{2}) = 1$. Hence the random variable p_∞ takes two values 0 and $\frac{1}{2}$, and as $E(p_\infty) = p_0$, the probabilities of 0 and $\frac{1}{2}$ are as in Eq. 13. The argument is similar if $\frac{1}{2} < p_0 < 1$.

If $d = E(D_t) \neq 0$, the analysis is more complicated because in this case it is not clear what $E(p_\infty)$ is. We address this case using the notion developed by Karlin and Liberman (53) of stochastic local stability (SLS) of a constant equilibrium. Let $\{x_t\}_{t=0}^\infty$ be a sequence of random variables satisfying $x_{t+1} = f(x_t)$ for $t = 0, 1, 2, \dots$. Then x^* is a constant equilibrium of this sequence if x^* satisfies $x^* = f(x^*)$.

Definition: SLS. A constant equilibrium x^* is said to be SLS if for any $\varepsilon > 0$ there is a $\delta > 0$ such that

$$|x_0 - x^*| < \delta \implies P\left(\lim_{t \rightarrow \infty} x_t = x^*\right) \geq 1 - \varepsilon. \quad [15]$$

In our case, the sequence $\{p_t\}_{t=0}^\infty$ determined by Eq. 2 has three constant equilibria: $p^* = 0$, $p^* = 1$, and $p^* = \frac{1}{2}$. Following ref. 53, to determine whether a constant equilibrium p^* is SLS we examine the linear approximation of the transformation in Eq. 2 “near” p^* . The linear approximations of Eq. 2 near $p^* = 0$ and $p^* = 1$ for small ν_t are both

$$\nu_{t+1} = \nu_t(1 - D_t), \quad [16]$$

while that near $p^* = \frac{1}{2}$ for small η_t is

$$\eta_{t+1} = \eta_t(1 + \frac{D_t}{2}). \quad [17]$$

In Eq. 16, the frequency of a rare variant increases under anti-conformity ($D_t < 0$), whereas in Eq. 17, the frequency of a common variant ($\eta_t > \frac{1}{2}$) increases under conformity ($D_t > 0$). Thus, we have the following:

Result 5.

- 1) If $E[\log(1 - D_t)] < 0$, then both $p^* = 0$ and $p^* = 1$ are SLS. If $E[\log(1 - D_t)] > 0$, then $P(\lim_{t \rightarrow \infty} p_t = 0) = P(\lim_{t \rightarrow \infty} p_t = 1) = 0$.
- 2) If $E[\log(1 + \frac{D_t}{2})] < 0$, then $p^* = \frac{1}{2}$ is SLS, whereas if $E[\log(1 + \frac{D_t}{2})] > 0$, then $P(\lim_{t \rightarrow \infty} p_t = \frac{1}{2}) = 0$.

As both functions $y = \log(1 - x)$ and $y = \log(1 + \frac{x}{2})$ are concave (for $x < 1$ and $x > -2$, respectively), we apply Jensen’s inequality to obtain

$$\begin{aligned} E[\log(1 - D_t)] &< \log[E(1 - D_t)], \\ E[\log(1 + \frac{D_t}{2})] &< \log[E(1 + \frac{D_t}{2})], \end{aligned} \quad [18]$$

since $-2 < D_t < 1$ with probability 1 and $\sigma^2 = \text{Var}(D_t) > 0$.

Applying inequalities in Eq. 18 and Result 5, we have the following:

Result 6.

- 1) If $d = E(D_t) \geq 0$, then $p^* = 0$ and $p^* = 1$ are SLS.
- 2) If $d = E(D_t) \leq 0$, then $p^* = \frac{1}{2}$ is SLS.

Therefore, if $d = E(D_t) = 0$, all three constant equilibria, $p^* = 0$, $p^* = 1$, and $p^* = \frac{1}{2}$ are SLS. Hence p_∞ can take the three values as stated in Result 4.

The dynamics with $n = 3$ role models are illustrated in Figs. 1 A, D, and G; 2A; and 3A. D_t is denoted by $D_t(2)$ because it corresponds to the case where $j = 2$ of 3 role models have the same variant [the reason for this $D_t(j)$ notation becomes clear with $n > 4$ and is described under Eq. 19]. Consistent with Observation 1, runs that start in $(0, \frac{1}{2})$ or $(\frac{1}{2}, 1)$ remain in $(0, \frac{1}{2})$ or $(\frac{1}{2}, 1)$, respectively. Equilibria are $p^* = 0, \frac{1}{2}$, and 1. It is possible that all three equilibria are SLS with $E[D_t(2)] = 0$, $E[D_t(2)] < 0$, or $E[D_t(2)] > 0$ (Fig. 1 A, D, and G, respectively). For $p^* = \frac{1}{2}$ to be unstable (Fig. 2A), it is necessary that $E[D_t(2)] > 0$. In this case, the dynamics are similar to those of deterministic Eq. 1 with conformity ($D > 0$). For $p^* = 0$ and 1 to be unstable (Fig.

3A), it is necessary that $E[D_t(2)] < 0$, in which case the dynamics are similar to Eq. 1 with anticonformity ($D < 0$). With $n = 3$ and identically distributed conformity coefficients, there is always convergence of the population to an equilibrium.

If $D_t(2)$ are not identically distributed, nonconvergence can occur with $n = 3$ role models. In Fig. 4A, $D_t(2)$ alternates between two fixed values, producing a stable, two-generation cycle in the frequency of A. This cycle is confined to a region in $(0, \frac{1}{2})$ or $(\frac{1}{2}, 1)$, depending on whether the initial frequency p_0 is below or above $p = \frac{1}{2}$, respectively. [Recall that Observation 1 does not rely on the assumption of identically distributed $D_t(2)$.]

More Role Models. The above results apply when the number of role models is $n = 3$. Do such results for global convergence hold for any number n of role models? To address this, we follow Boyd and Richerson (4) and Eq. 2 above is replaced by

$$p_{t+1} = p_t + \sum_{j=0}^n \frac{D_t(j)}{n} \binom{n}{j} p_t^j (1 - p_t)^{n-j}. \quad [19]$$

Eq. 19 with the subscript t removed from $D_t(j)$ is the general form of Boyd and Richerson’s (4) deterministic model. The sum contains the probability of each possible sample j of n role models with variant A (a binomial, due to random sampling) multiplied by the scaled conformity coefficient for that sample, $\frac{D_t(j)}{n}$. For any generation $t = 0, 1, 2, \dots$ the vector $\mathbf{D}_t = [D_t(0), D_t(1), \dots, D_t(n)]$ of conformity coefficients $D_t(j)$ has the following properties for $j = 0, 1, 2, \dots, n$:

$$D_t(0) = D_t(n) = 0, \quad D_t(n - j) = -D_t(j), \quad [20a]$$

$$\text{and for } j = 1, 2, \dots, n - 1, \quad -j < D_t(j) < n - j. \quad [20b]$$

Eq. 20b ensures that probabilities of acquiring A and B are in $(0, 1)$. The left-hand side of Eq. 20a entails that if a sample contains A(B) at frequency 1, an individual acquires A(B) with probability 1, and the right-hand side entails “equal treatment” of A and B. If an individual samples j of type A and $n - j$ of type B, the amount by which it (anti)conforms to A, namely $D_t(j)$, must be equal and opposite to the amount by which it (anti)conforms to B, $-D_t(j)$, for all probabilities to sum to 1. Therefore, when A rather than B is present in $n - j$ role models, its conformity coefficient $D_t(n - j)$ must equal that for B in the same scenario: $-D_t(j)$. Therefore, Eq. 19 is equivalent to

$$p_{t+1} = p_t + \sum_{j=k}^n \frac{D_t(j)}{n} \binom{n}{j} \left[p_t^j (1 - p_t)^{n-j} - p_t^{n-j} (1 - p_t)^j \right], \quad [21]$$

where $k = \frac{n}{2} + 1$ if n is even and $k = \frac{n+1}{2}$ if n is odd.

Assume that the conformity coefficients change randomly at each generation t , and for any $t = 0, 1, 2, \dots$, \mathbf{D}_t is a vector of random variables such that the \mathbf{D}_t s are identically distributed and independent of the population state p_t . The entries of \mathbf{D}_t , $D_t(j)$, need not be identically distributed. Since probabilities of different samples j of n , with corresponding $D_t(j)$, depend on p_t , it is possible that higher or lower p_t corresponds to greater or smaller differences between p_{t+1} and p_t .

Assume first that $E(\mathbf{D}_t) = \mathbf{0}$; i.e., $E(D_t(j)) = 0$ for all $t = 0, 1, 2, \dots$ and $j = 0, 1, 2, \dots, n$. In this case we have the following result:

Result 7.

- 1) If $E(D_t) = \mathbf{0}$, then starting from any initial state $0 \leq p_0 \leq 1$ the process $\{p_t\}_{t=0}^\infty$ converges; that is, $\lim_{t \rightarrow \infty} p_t = p_\infty$ with probability 1.
- 2) If, in addition, $D_t(k), D_t(k+1), \dots, D_t(n)$ for any t are independent random variables, then p_∞ takes the values $0, \frac{1}{2}, 1$ with probability 1.

Proof: If D_t is independent of p_t and $E(D_t) = \mathbf{0}$ for $t = 0, 1, 2, \dots$, Eq. 19 implies that $E(p_{t+1} | p_t) = p_t$. Hence $\{p_t\}_{t=0}^\infty$ is a bounded martingale and the martingale convergence theorem implies that $\lim_{t \rightarrow \infty} p_t = p_\infty$ with probability 1.

Let $\sigma_j^2 = \text{Var}(D_t(j))$ be the positive variance of $D_t(j)$ for $j = 1, 2, \dots, n-1$ [$\sigma_0 = \sigma_n = 0$ as $D_t(0) = D_t(n) = 0$]. Then, if $D_t(k), D_t(k+1), \dots, D_t(n-1)$ are independent

random variables, Eq. 21 implies for any $t = 0, 1, 2, \dots$ that

$$\begin{aligned} \text{Var}(p_{t+1} | p_t) &= \sum_{j=k}^{n-1} \frac{\sigma_j^2}{n^2} \binom{n}{j}^2 \left[p_t^j (1-p_t)^{n-j} - p_t^{n-j} (1-p_t)^j \right]^2. \end{aligned} \quad [22]$$

Since $\text{Var}(p_{t+1}) = \text{Var}(E(p_{t+1} | p_t)) + E(\text{Var}(p_{t+1} | p_t))$, we have

$$\begin{aligned} \text{Var}(p_{t+1}) &= \text{Var}(p_t) \\ &+ \sum_{j=k}^{n-1} \frac{\sigma_j^2}{n^2} \binom{n}{j}^2 E \left\{ \left[p_t^j (1-p_t)^{n-j} - p_t^{n-j} (1-p_t)^j \right]^2 \right\}. \end{aligned} \quad [23]$$

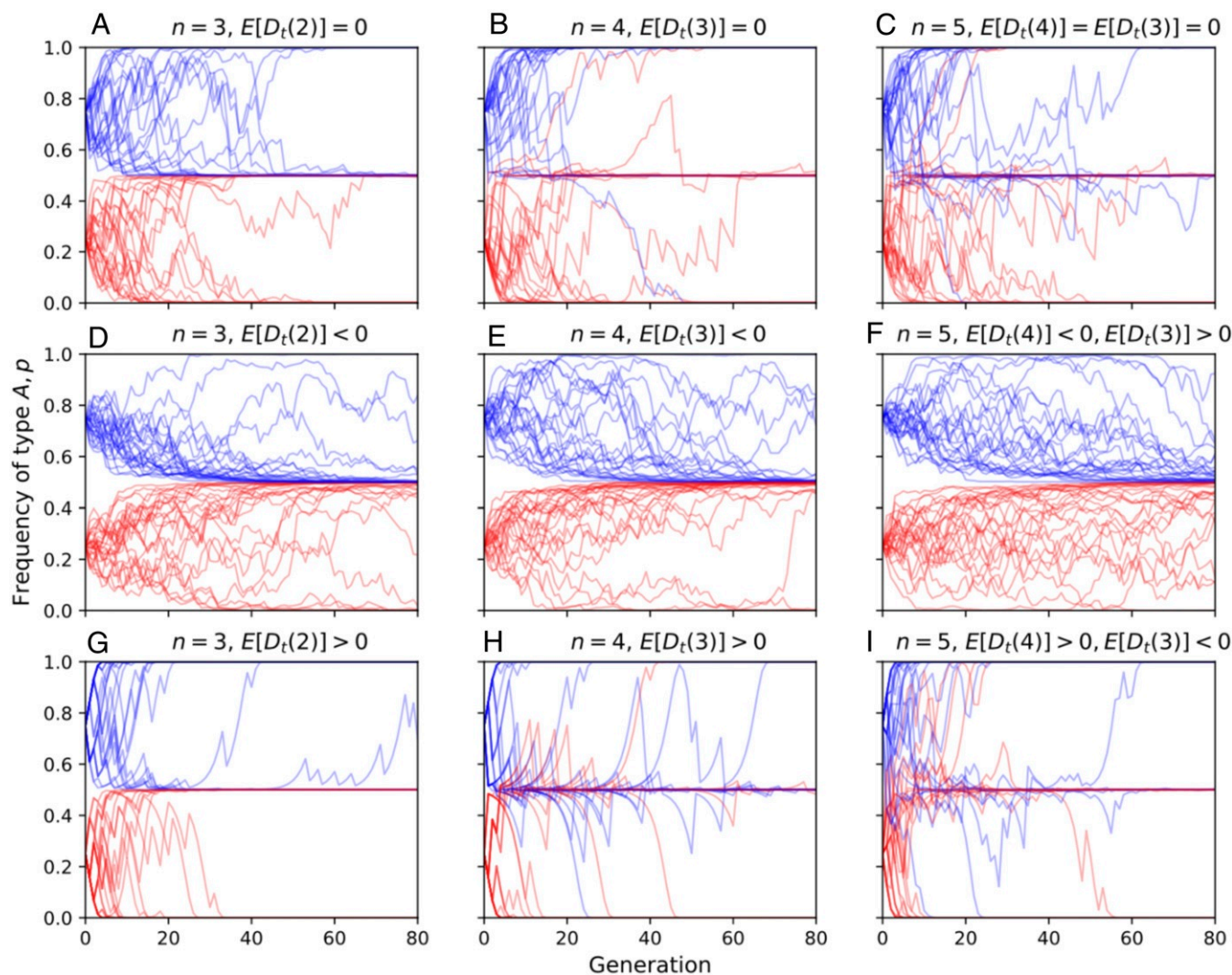


Fig. 1. Stochastic local stability of $p^* = 0, \frac{1}{2}, 1$. Each plot shows 25 runs with initial $p_0 = 0.25$ (red) and 25 runs with $p_0 = 0.75$ (blue). Eighty generations are shown, although equilibria $p^* = 0, \frac{1}{2}, 1$ may take longer to be reached; how often each equilibrium was reached is in *SI Appendix, section A* along with $E(D_t(j))$, the mean of $\log[1 - D_t(n-1)]$, and the mean of $\log\left[1 + \frac{D_t(2)}{n}\right]$ for $n = 3$. (A) $n = 3$, $D_t(2) \sim U[0, 1]$ with probability $\frac{2}{3}$ and $D_t(2) \sim U(-2, 0]$ with probability $\frac{1}{3}$. (B) $n = 4$, $D_t(3) \sim U[0, 1]$ with probability $\frac{3}{4}$ and $D_t(3) \sim U(-3, 0]$ with probability $\frac{1}{4}$. (C) $n = 5$, $D_t(4) \sim U[0, 1]$ with probability $\frac{4}{5}$ and $D_t(4) \sim U(-4, 0]$ with probability $\frac{1}{5}$. $D_t(3) \sim U[0, 2)$ with probability $\frac{2}{5}$ and $D_t(3) \sim U(-3, 0]$ with probability $\frac{2}{5}$. (D) $n = 3$, $D_t(2)$ is sampled from a truncated normal distribution on $(-2, 1)$ with mean (before truncation) -0.05 and SD 0.5 . (E) $n = 4$, $D_t(3)$ is sampled from a truncated normal distribution on $(-3, 1)$ with mean (before truncation) -0.05 and SD 0.5 . (F) $n = 5$, $D_t(4)$ is sampled from a truncated normal distribution on $(-4, 1)$ with mean (before truncation) -0.05 and SD 0.5 . $D_t(3)$ is sampled from a truncated normal distribution on $(-3, 2)$ with mean (before truncation) 0.05 and SD 0.5 . (G) $n = 3$, $D_t(2) = 0.99$ with probability $\frac{2}{3}$ and $D_t(2) = -1.5$ with probability $\frac{1}{3}$. (H) $n = 4$, $D_t(3) = 0.99$ with probability $\frac{3}{4}$ and $D_t(3) = -2.5$ with probability $\frac{1}{4}$. (I) $n = 5$, $D_t(4) = 0.9$ with probability $\frac{4}{5}$ and $D_t(4) = -3$ with probability $\frac{1}{5}$. $D_t(3) = 1.5$ with probability $\frac{2}{5}$ and $D_t(3) = -2.85$ with probability $\frac{2}{5}$.

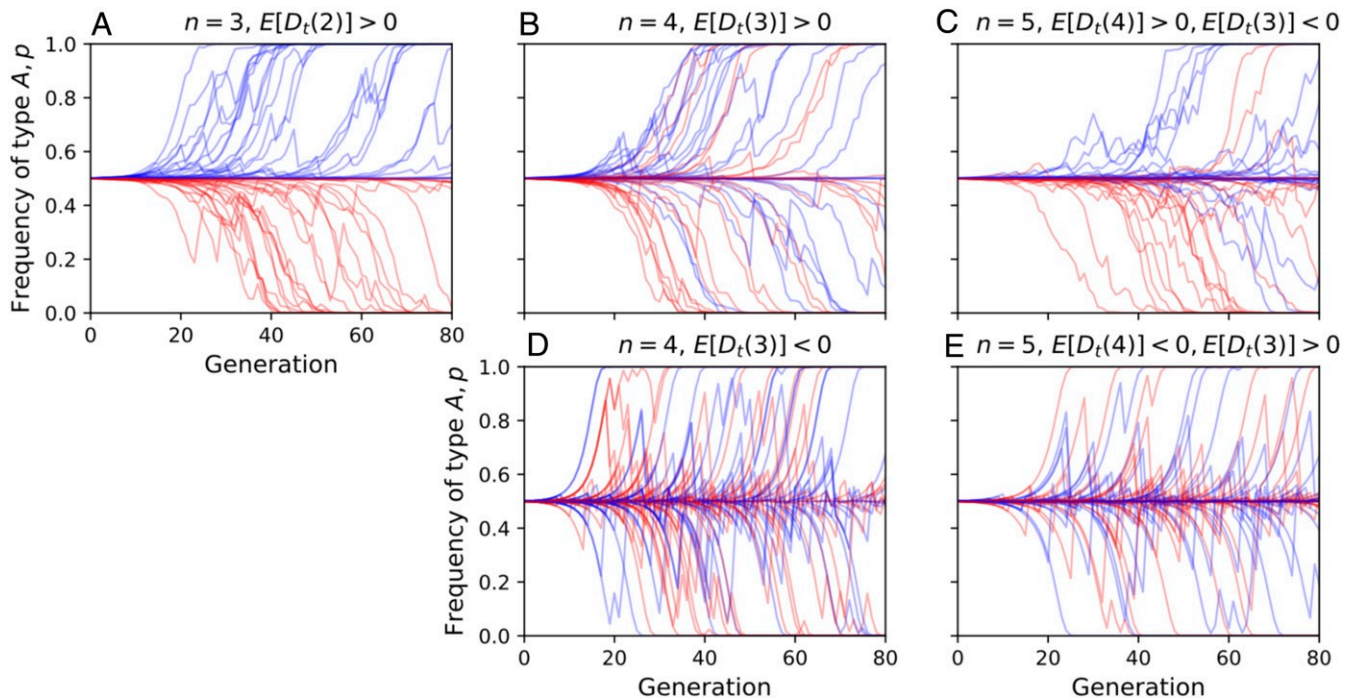


Fig. 2. Stochastic local stability of $p^* = 0, 1$ and instability of $p^* = \frac{1}{2}$. Each plot shows 25 runs with initial $p_0 = 0.499$ (red) and 25 runs with $p_0 = 0.501$ (blue). The first 80 generations are shown, although equilibria may take longer to be reached; how often each equilibrium was reached is in [SI Appendix, section A](#), along with $E[D_t(j)]$, the mean of $\log[1 - D_t(n-1)]$, and the mean of $\log\left[1 + \frac{D_t(2)}{2}\right]$ for $n=3$. (A) $n=3$, $D_t(2) \sim U[0, 1]$ with probability 0.9 and $D_t(2) \sim U(-2, 0]$ with probability 0.1. (B) $n=4$, $D_t(3) \sim U[0, 1]$ with probability 0.925 and $D_t(3) \sim U(-3, 0]$ with probability 0.075. (C) $n=5$, $D_t(4) \sim U[0, 1]$ with probability 0.98 and $D_t(4) \sim U(-4, 0]$ with probability 0.02. $D_t(3) \sim U[0, 2]$ with probability 0.55 and $D_t(3) \sim U(-3, 0]$ with probability 0.45. (D) $n=4$, $D_t(3) = 0.99$ with probability $\frac{3}{4}$ and $D_t(3) = -2.99$ with probability $\frac{1}{4}$. (E) $n=5$, $D_t(4) = 0.99$ with probability $\frac{3}{4}$ and $D_t(4) = -3.99$ with probability $\frac{1}{4}$. $D_t(3) = 0.1$ with probability $\frac{3}{4}$ and $D_t(3) = -0.1$ with probability $\frac{1}{4}$.

Now as $p_t \xrightarrow[t \rightarrow \infty]{} p_\infty$ with probability 1, by the Lebesgue convergence theorem $E(p_t^\ell) \xrightarrow[t \rightarrow \infty]{} E(p_\infty^\ell)$ for any $\ell = 1, 2, \dots$, and Eq. 23 implies that

$$\sum_{j=k}^{n-1} \frac{\sigma_j^2}{n^2} \binom{n}{j}^2 E \left\{ \left[p_\infty^j (1-p_\infty)^{n-j} - p_\infty^{n-j} (1-p_\infty)^j \right]^2 \right\} = 0. \quad [24]$$

Hence $p_\infty^j (1-p_\infty)^{n-j} = p_\infty^{n-j} (1-p_\infty)^j$ for $j = k, k+1, \dots, n-1$ and $p_\infty = 0, p_\infty = 1$, or $p_\infty = 1-p_\infty$ in which case $p_\infty = \frac{1}{2}$. Thus, with probability 1, p_∞ takes values $0, \frac{1}{2}, 1$.

The following result concerning the SLS of $p^* = 0$ and $p^* = 1$ holds for $E(D_t) = \mathbf{0}$ as well as $E(D_t) \neq \mathbf{0}$.

Result 8.

- 1) If $E[\log(1 - D_t(n-1))] < 0$, then $p^* = 0$ and $p^* = 1$ are SLS.
- 2) If $E[\log(1 - D_t(n-1))] > 0$, then $P(\lim_{t \rightarrow \infty} p_t = 0) = 0$ and $P(\lim_{t \rightarrow \infty} p_t = 1) = 0$.

The proof is similar to that of *Result 5* based on the linear approximation of Eq. 19 near $p^* = 0$ and $p^* = 1$ in equation B2 of Denton et al. (49) and the assumption that $D_t(n-1) < 1$ by Eq. 20b. If $E(D_t(n-1)) > 0$, then using Jensen's inequality, $E[\log(1 - D_t(n-1))] < 0$, so $p^* = 0$ and $p^* = 1$ are SLS.

We are unable to obtain corresponding results for the SLS of $p^* = \frac{1}{2}$. Consider the linear approximation of Eq. 19

near the constant equilibrium $p^* = \frac{1}{2}$ given in ref. 49, equation B8:

$$\nu' = \nu \left[1 + \left(\frac{1}{2}\right)^{n-2} \sum_{j=k}^{n-1} \frac{D(j)}{n} \binom{n}{j} ((2j-n)) \right] = \nu \phi_n. \quad [25]$$

Ref. 49's table S1 shows that depending on n , ϕ_n can be negative, in which case $\log \phi_n$ is not defined and conditions for SLS of $p^* = \frac{1}{2}$ cannot be determined. Also, *Observation 1* does not hold.

With $n > 3$ role models and $E(D_t) \neq \mathbf{0}$, there need not be convergence to an equilibrium (Fig. 5). Below we determine conditions for convergence to an equilibrium with $n = 4$ role models and, subsequently, a general number n of role models.

Example: The Case of $n = 4$ Role Models. Recall that the recursion with fixed conformity coefficients is the same for $n = 3$ and 4 (4). However, with random conformity coefficients the case $n = 4$ exhibits different evolutionary dynamics from those with $n = 3$ role models. When $n = 4$, we have five conformity coefficients $D_t(j)$ for $j = 0, 1, \dots, 4$, Eq. 20a and inequality in Eq. 20b,

$$D_t(0) = D_t(2) = D_t(4) = 0, \quad D_t(1) = -D_t(3), \quad -3 < D_t(3) < 1. \quad [26]$$

Hence, from Eq. 21 with $n = 4$ and $k = 3$, we have

$$p_{t+1} = p_t + D_t p_t (1-p_t)(2p_t - 1), \quad [27]$$

where D_t are identically distributed and correspond to $D_t(3)$ in Eq. 26. Therefore in this case $[1 + 2D_t p_t (1-p_t)]$ with $-3 < D_t < 1$ can be negative, and by Eq. 3, $(p_{t+1} - \frac{1}{2})$ and

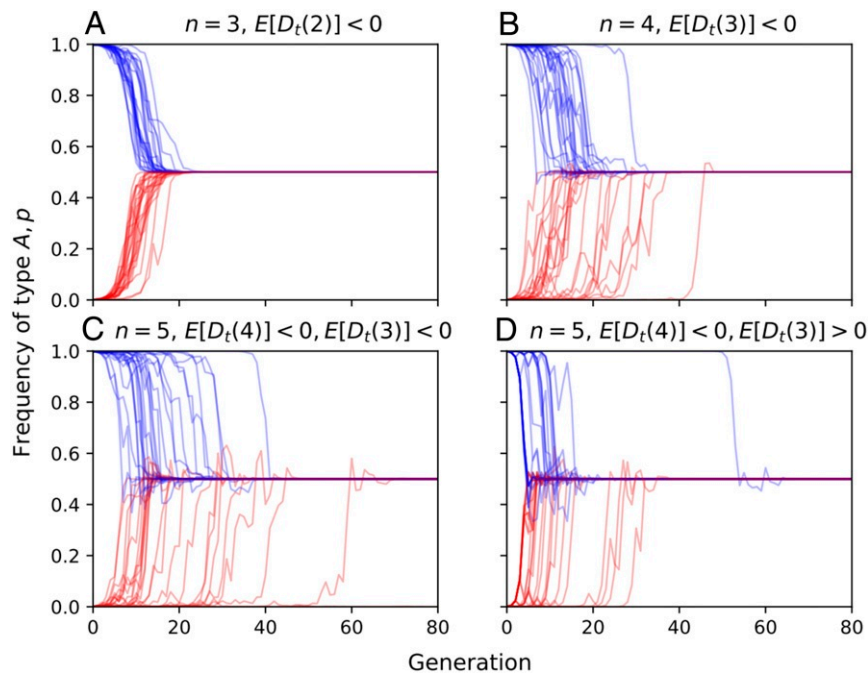


Fig. 3. Stochastic local stability of $p^* = \frac{1}{2}$ and instability of $p^* = 0, 1$. Each plot shows 25 runs with initial $p_0 = 0.001$ (red) and 25 runs with $p_0 = 0.999$ (blue). The equilibrium $p^* = \frac{1}{2}$ is reached in every case, although only the first 80 generations are shown and this equilibrium may take longer to be reached. *SI Appendix, section A* reports $E[D_t(j)]$, the mean of $\log[1 - D_t(n-1)]$, and the mean of $\log[1 + \frac{D_t(2)}{2}]$ for $n=3$. (A) $n=3$, $D_t(2) \sim U[0, 1]$ with probability 0.05 and $D_t(2) \sim U[-2, 0]$ with probability 0.95. (B) $n=4$, $D_t(3) \sim U[0, 1]$ with probability 0.25 and $D_t(3) \sim U[-3, 0]$ with probability 0.75. (C) $n=5$, $D_t(4) \sim U[0, 1]$ with probability 0.4 and $D_t(4) \sim U[-4, 0]$ with probability 0.6. $D_t(3) \sim U[0, 2]$ with probability 0.1 and $D_t(3) \sim U[-3, 0]$ with probability 0.9. (D) $n=5$, $D_t(4) = -3.9$ with probability $\frac{3}{4}$, $D_t(4) = 0.9$ with probability $\frac{1}{4}$, $D_t(3) = 0.9$ with probability $\frac{3}{4}$, and $D_t(3) = -1.9$ with probability $\frac{1}{4}$.

$(p_t - \frac{1}{2})$ can have different signs. Therefore *Observation 1*, which was a key factor in the proof of global convergence when $n=3$, does not hold. Will there still be global convergence of $\{p_t\}$ when $d = E(D_t) \neq 0$ with $n=4$?

Since $(p_{t+1} - \frac{1}{2})$ and $(p_t - \frac{1}{2})$ can have different signs, from Eq. 3 we focus on $Q_t = (p_t - \frac{1}{2})^2$. Then

$$(p_{t+1} - \frac{1}{2})^2 = (p_t - \frac{1}{2})^2 [1 + 2D_t p_t (1 - p_t)]^2. \quad [28]$$

Now $p_t(1 - p_t) = \frac{1}{4} - Q_t$, and Eq. 28 is equivalent to

$$Q_{t+1} = Q_t [1 + 2D_t (\frac{1}{4} - Q_t)]^2, \quad t = 0, 1, 2, \dots \quad [29]$$

The following result applies to the process $\{Q_t\}_{t=0}^{\infty}$:

Result 9. If $d = E(D_t) > 0$, then for any $0 \leq Q_0 \leq \frac{1}{4}$ the process $\{Q_t\}_{t=0}^{\infty}$ converges with probability 1, and if $Q_{\infty} = \lim_{t \rightarrow \infty} Q_t$, then Q_{∞} can take only two values, $Q_{\infty} = 0$ or $Q_{\infty} = \frac{1}{4}$.

Proof: Rewrite Eq. 29 as

$$Q_{t+1} = Q_t [1 + 4D_t (\frac{1}{4} - Q_t) + 4D_t^2 (\frac{1}{4} - Q_t)^2]. \quad [30]$$

Since D_t is independent of p_t and so also of Q_t , if $d = E(D_t)$ and $\sigma^2 = \text{Var}(D_t)$, we have

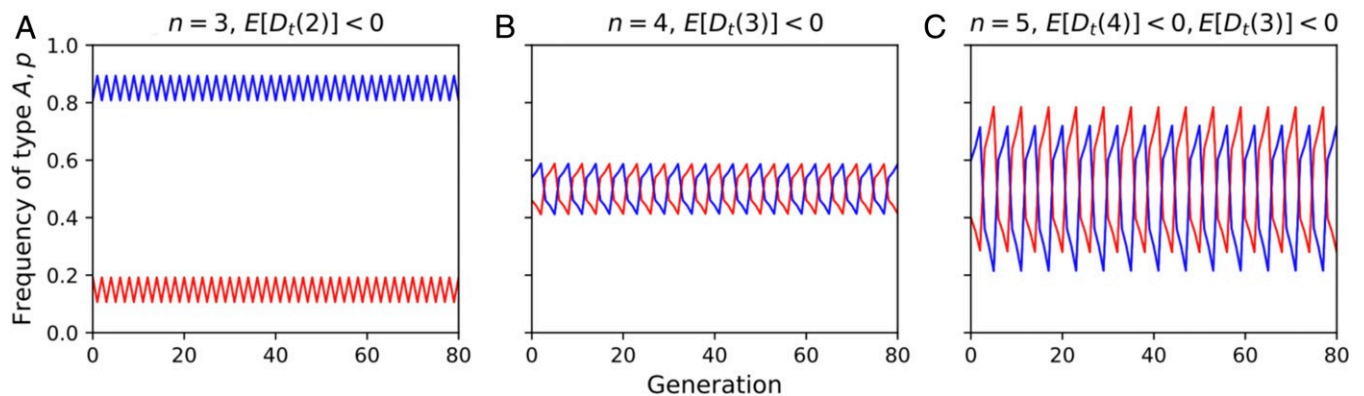


Fig. 4. Exact cycles. Each plot shows 25 runs with initial $p_0 < \frac{1}{2}$ (red) and 25 runs with $p_0 > \frac{1}{2}$ (blue). Eighty generations are shown, although simulations ran for 10^6 generations; no equilibria were reached. Descriptions of the dynamics, exact initial frequencies, and $E[D_t(j)]$ are reported in *SI Appendix, section A*. (A) $n=3$, $D_t(2)$ alternates between -1.15 and 0.9 every other generation. (B) $n=4$, $D_t(3) = -2.99$ every third generation and $D_t(3) = 0.99$ in the other generations. (C) $n=5$, $D_t(4) = -3.99$ and $D_t(3) = -2.1$ every third generation and in the other generations $D_t(4) = 0.99$ and $D_t(3) = 0.5$.

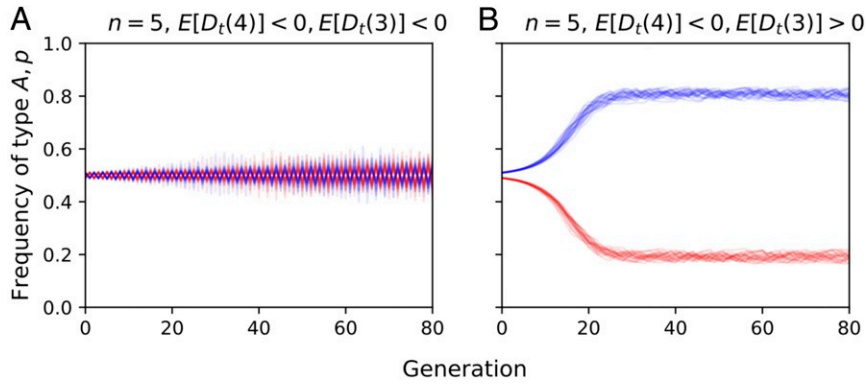


Fig. 5. Stochastic fluctuation. Each plot shows 25 runs with initial $p_0 = 0.49$ (red) and 25 runs with $p_0 = 0.51$ (blue). Eighty generations are shown, although simulations ran for 10^6 generations; no equilibria were reached. Mean frequencies and $E[D_t(j)]$ are reported in *SI Appendix, section A*. (A) $n = 5$, $D_t(4) = -3.9$ with probability $\frac{1}{2}$, $D_t(4) = -3.4$ with probability $\frac{1}{2}$, $D_t(3) = -2.9$ with probability $\frac{1}{2}$, and $D_t(3) = -2.4$ with probability $\frac{1}{2}$. (B) $n = 5$, $D_t(4) = -0.6$ with probability $\frac{1}{2}$, $D_t(4) = -0.8$ with probability $\frac{1}{2}$, $D_t(3) = 1.85$ with probability $\frac{1}{2}$, and $D_t(3) = 1.95$ with probability $\frac{1}{2}$.

$$E(Q_{t+1} | Q_t) = Q_t [1 + 4d(\frac{1}{4} - Q_t) + 4(\sigma^2 + d^2)(\frac{1}{4} - Q_t)^2]. \quad [31]$$

If $d > 0$, then $E(Q_{t+1} | Q_t) \geq Q_t$ for $t = 0, 1, 2, \dots$. Thus $\{Q_t\}_{t=0}^\infty$ is a submartingale and since $0 \leq Q_t \leq \frac{1}{4}$, we can apply the martingale convergence theorem to deduce that with probability 1, $\{Q_t\}_{t=0}^\infty$ converges to Q_∞ . Also applying the Lebesgue convergence theorem, $E(Q_t^\ell) \xrightarrow{t \rightarrow \infty} E(Q_\infty^\ell)$ for any $\ell = 1, 2, \dots$.

Thus Eq. 31 entails that

$$E(Q_\infty) = E(Q_\infty) + 4dE[Q_\infty(\frac{1}{4} - Q_\infty)] + 4(\sigma^2 + d^2)E[Q_\infty(\frac{1}{4} - Q_\infty)^2]. \quad [32]$$

Since $d > 0$, we deduce from Eq. 32 that $E[Q_\infty(\frac{1}{4} - Q_\infty)] = 0$ and $E[Q_\infty(\frac{1}{4} - Q_\infty)^2] = 0$. But $0 \leq Q_\infty \leq \frac{1}{4}$ with probability 1. Hence $Q_\infty(\frac{1}{4} - Q_\infty) = 0$ with probability 1 and Q_∞ can take only the two values $Q_\infty = 0$ or $Q_\infty = \frac{1}{4}$.

If $Q_\infty = 0$, then $\lim_{t \rightarrow \infty} (\frac{1}{2} - p_t)^2 = 0$ and $\lim_{t \rightarrow \infty} p_t = \frac{1}{2}$. If $Q_\infty = \frac{1}{4}$, then $\lim_{t \rightarrow \infty} p_t(1 - p_t) = 0$. Therefore for t large, either p_t is near $p^* = 0$ or p_t is near $p^* = 1$. But if $d > 0$, both $p^* = 0$ and $p^* = 1$ are SLS. Therefore, if $Q_\infty = \frac{1}{4}$, either $\lim_{t \rightarrow \infty} p_t = 0$ or $\lim_{t \rightarrow \infty} p_t = 1$.

Conclusion. If $E(D_t) > 0$, then the process $\{p_t\}_{t=0}^\infty$ converges with probability 1. If $p_\infty = \lim_{t \rightarrow \infty} p_t$, then p_∞ can assume only the three values 0, 1, $\frac{1}{2}$.

The situation is more complicated when $d = E(D_t) < 0$. Eq. 31 can be rewritten as $E(Q_{t+1} | Q_t) = Q_t(1 + u_t)$ with

$$u_t = 4d(\frac{1}{4} - Q_t) + 4(\sigma^2 + d^2)(\frac{1}{4} - Q_t)^2. \quad [33]$$

Since $0 \leq \frac{1}{4} - Q_t \leq \frac{1}{4}$, we have

$$u_t \leq 4(\frac{1}{4} - Q_t) \left[d(1 + \frac{d}{4}) + \frac{\sigma^2}{4} \right]. \quad [34]$$

If $4d + d^2 + \sigma^2 \leq 0$, or equivalently, if $4d + E(D_t^2) \leq 0$, then $u_t \leq 0$ and $E(Q_{t+1} | Q_t) \leq Q_t$, making the process a bounded supermartingale so that $Q_t \xrightarrow{t \rightarrow \infty} Q_\infty$ with probability 1. For example, this occurs if $D_t \in [-3, 0]$ so that $\sigma^2 \leq -d^2 - 3d$ by the Bhatia–Davis inequality (54). Therefore, if $u_t \leq 0$, a similar argument to the proof of *Result 9* (Eq. 32) gives

$$4dE[Q_\infty(\frac{1}{4} - Q_\infty)] + 4(\sigma^2 + d^2)E[Q_\infty(\frac{1}{4} - Q_\infty)^2] = 0. \quad [35]$$

As $0 \leq \frac{1}{4} - Q_\infty \leq \frac{1}{4}$ and $d(1 + \frac{d}{4}) + \frac{\sigma^2}{4} < 0$, we have

$$0 \leq 4E[Q_\infty(\frac{1}{4} - Q_\infty)] \left[d(1 + \frac{d}{4}) + \frac{\sigma^2}{4} \right] \leq 0. \quad [36]$$

Inequalities in Eq. 36 imply that $E[Q_\infty(\frac{1}{4} - Q_\infty)] = 0$ and $Q_\infty(\frac{1}{4} - Q_\infty) = 0$ so that $Q_\infty = 0$ or $Q_\infty = \frac{1}{4}$ with probability 1. Dynamics with $n = 4$ role models are illustrated in Figs. 1B, E, and H; 2B and D; and 3B. The conformity coefficient, D_t , is denoted by $D_t(3)$ (Eqs. 26 and 27). Because *Observation 1* no longer holds with $n > 3$, runs that start in $(0, \frac{1}{2})$ (red) or $(\frac{1}{2}, 1)$ (blue) can cross the $p = \frac{1}{2}$ line (e.g., Fig. 1B). Fig. 1B, E, and H shows examples in which all three equilibria $p^* = 0, \frac{1}{2}$, and 1 are SLS with $E[D_t(3)] = 0$, $E[D_t(3)] < 0$, and $E[D_t(3)] > 0$, respectively. Fig. 2B and D shows that $p^* = \frac{1}{2}$ can be unstable with either $E[D_t(3)] > 0$ or $E[D_t(3)] < 0$. However, $E[D_t(3)] < 0$ is necessary for instability of $p^* = 0$ and 1 (Fig. 3B and *Result 8*). Here, the dynamics are similar to those of Eq. 1 with anticonformity ($D < 0$), apart from fluctuations across $p^* = \frac{1}{2}$ before reaching this equilibrium.

Removing the assumption that $D_t(3)$ are identically distributed, Fig. 4B shows an example in which $D_t(3)$ takes one fixed value every third generation and another fixed value in the other generations. This system produces an exact six-generation cycle in the frequency p_t . Unlike in the case of $n = 3$ (Fig. 4A), this cycle occurs around $p = \frac{1}{2}$ because *Observation 1* does not hold.

Convergence for Any Number of Role Models. *Result 9* can be generalized to apply for any number n of role models. Recall that with n role models we have

$$p_{t+1} = p_t + \sum_{j=k}^n \frac{D_t(j)}{n} \binom{n}{j} \left[p_t^j (1 - p_t)^{n-j} - p_t^{n-j} (1 - p_t)^j \right], \quad [37]$$

where $k = \frac{n}{2} + 1$ if n is even and $k = \frac{n+1}{2}$ if n is odd. Denton et al. (49) showed that for any $j \geq k$

$$p_t^j (1 - p_t)^{n-j} - p_t^{n-j} (1 - p_t)^j = p_t(1 - p_t)(2p_t - 1)G_j(p_t(1 - p_t)), \quad [38]$$

where G_j is a polynomial in the variable $p_t(1-p_t)$ for $n > 4$. Simple algebra shows that for any $0 \leq p_t \leq 1$

$$G_j(p_t(1-p_t)) = [p_t(1-p_t)]^{n-j-1} \times \left[\sum_{i=0}^{2j-n-1} (1-p_t)^i p_t^{2j-n-1-i} \right] \geq 0. \quad [39]$$

Moreover, when $j = k$, if n is even and $k = \frac{n}{2} + 1$, then

$$p_t^k (1-p_t)^{n-k} - p_t^{n-k} (1-p_t)^k = [p_t(1-p_t)]^{\frac{n}{2}-1} (2p_t - 1), \quad [40a]$$

while if n is odd and $k = \frac{n+1}{2}$,

$$p_t^k (1-p_t)^{n-k} - p_t^{n-k} (1-p_t)^k = [p_t(1-p_t)]^{\frac{n-1}{2}} (2p_t - 1). \quad [40b]$$

As $n \geq 3$, in both Eqs. 40a and 40b the power of $[p_t(1-p_t)]$ is at least 1.

We are now ready to prove a convergence result for any $n \geq 3$ role models with identically distributed D_t .

Result 10. *If $E[D_t(j)] > 0$ for all $k \leq j < n$, where $k = \frac{n}{2} + 1$ if n is even and $k = \frac{n+1}{2}$ if n is odd, the process $\{p_t\}_{t=0}^{\infty}$ converges with probability 1. Moreover, if $p_{\infty} = \lim_{t \rightarrow \infty} p_t$, then p_{∞} can assume only the values 0, 1, and $\frac{1}{2}$.*

The proof of Result 10 is similar to the proof of Result 9 and is given in *SI Appendix, section B*.

To illustrate dynamics with $n > 4$ role models, examples with $n = 5$ are shown in Figs. 1 C, F, and I; 2 C and E; 3 C and D; and 5. With $n = 5$ there are two $D_t(j)$ parameters: $D_t(4)$ and $D_t(3)$ correspond to the cases where $j = 4$ or 3, respectively, of the 5 role models have the same variant. Simultaneous SLS of $p^* = 0, \frac{1}{2}$, and 1 can occur with $E[D_t(4)] = E[D_t(3)] = 0$, $E[D_t(4)] < 0$ and $E[D_t(3)] > 0$, or $E[D_t(4)] > 0$ and $E[D_t(3)] < 0$ (Fig. 1 C, F, and I, respectively). Note that all three equilibria can also be SLS when both $E[D_t(4)], E[D_t(3)] < 0$ or both $E[D_t(4)], E[D_t(3)] > 0$ (not shown in Fig. 1). Fixation states $p^* = 0$ and 1 may be SLS while $p^* = \frac{1}{2}$ is unstable if $E[D_t(4)] > 0$ and $E[D_t(3)] < 0$ or if $E[D_t(4)] < 0$ and $E[D_t(3)] > 0$ (Fig. 2 C and E). This is also possible with both $E[D_t(4)], E[D_t(3)] > 0$ or both $E[D_t(4)], E[D_t(3)] < 0$ (not shown in Fig. 2). The condition $E[D_t(4)] < 0$ is necessary but not sufficient for instability of $p^* = 0$ and 1 (Result 8). Fig. 3 C and D shows examples in which $p^* = 0$ and 1 are unstable and $p^* = \frac{1}{2}$ is SLS with $E[D_t(4)] < 0$ and either $E[D_t(3)] < 0$ or $E[D_t(3)] > 0$. These dynamics are similar to those in the deterministic model with $n = 5$, $D(4), D(3) < 0$, and sufficiently weak anticonformity or in some cases with $D(4) < 0$ and $D(3) > 0$. In other cases with $D(4) < 0$ and $D(3) > 0$ in the deterministic model, stable polymorphic equilibria with $p^* \neq \frac{1}{2}$ can occur (figure 1 of ref. 49). Fig. 5B incorporates these specific values, namely $E[D_t(4)] = D(4)$ and $E[D_t(3)] = D(3)$ from ref. 49's figure 1, and shows stochastic fluctuation around frequencies $p \neq \frac{1}{2}$. Finally, nonconvergence is also possible in the stochastic model with $E[D_t(4)], E[D_t(3)] < 0$ (Fig. 5A) and in the deterministic model with strong anticonformity (49).

Stable cycles can also occur when the coefficients D_t cycle between fixed values (and are therefore not identically distributed). In Fig. 4C, $D_t(4)$ and $D_t(3)$ each take one fixed value every third generation and another fixed value in the other generations. As in Fig. 4B, an exact six-generation cycle is produced, but unlike in Fig. 4B, the red and blue runs cycle between different frequencies.

Random Number of Role Models. Do the global convergence properties described in Results 7 and 10 for any fixed constant number n ($n \geq 3$) of role models also hold when the number of role models can change randomly at each generation? In this case, the frequency p_{t+1} of the variant A at generation $t + 1$ for any $t = 0, 1, 2, \dots$ depends on p_t, n_t , and D_{n_t} , where

- 1) For each t , the number of role models n_t is chosen, independently of p_t , from a probability distribution on the values $3, 4, 5, \dots, N$, which is the same for all t ;
- 2) Given n_t , the conformity coefficients $D_{n_t}(0), D_{n_t}(1), \dots, D_{n_t}(n_t)$ are random variables independent of p_t . They are subject to the constraints for $j = 0, 1, \dots, n_t$

$$D_{n_t}(0) = D_{n_t}(n_t) = 0, \quad D_{n_t}(j) = -D_{n_t}(n_t - j) \quad [41a]$$

$$\text{and for } j = 1, 2, \dots, n_t, -1, -j < D_{n_t}(j) < n_t - j. \quad [41b]$$

For any generation $t = 0, 1, 2, \dots$ we then write p_{t+1} as (Eqs. 19 and 21)

$$p_{t+1} = p_t + \sum_{j=0}^{n_t} \frac{D_{n_t}(j)}{n_t} \binom{n_t}{j} p_t^j (1-p_t)^{n_t-j} \quad [42a]$$

$$= p_t + \sum_{j=k_t}^{n_t-1} \frac{D_{n_t}(j)}{n_t} \binom{n_t}{j} \left[p_t^j (1-p_t)^{n_t-j} - p_t^{n_t-j} (1-p_t)^j \right]. \quad [42b]$$

From Eqs. 42a and 42b we have the following results that generalize Results 7 and 10 to the case where the number of role models n_t is chosen randomly at each generation.

Result 11.

- 1) If $E[D_{n_t}(j) | n_t] = 0$ for all $j = 0, 1, \dots, n_t$, then $\lim_{t \rightarrow \infty} p_t = p_{\infty}$ with probability 1.
- 2) If, in addition, given n_t the conformity coefficients $D_{n_t}(k_t), D_{n_t}(k_t + 1), \dots, D_{n_t}(n_t - 1)$ are independent random variables (where $k_t = \frac{n_t}{2} + 1$ if n_t is even and $k_t = \frac{n_t+1}{2}$ if n_t is odd), then p_{∞} assumes the values 0, 1, $\frac{1}{2}$ with probability 1.

Result 12. *If $E[D_{n_t}(j) | n_t] > 0$ for $j = k_t, k_t + 1, \dots, n_t - 1$ (where $k_t = \frac{n_t}{2} + 1$ if n_t is even and $k_t = \frac{n_t+1}{2}$ if n_t is odd), then $\lim_{t \rightarrow \infty} p_t = p_{\infty}$ with probability 1 and p_{∞} can assume the values 0, 1, $\frac{1}{2}$. The proofs of Results 11 and 12 are very similar to the proofs of Results 7 and 10, respectively, and are given in *SI Appendix, section B*.*

Weak Random Selection. To incorporate natural selection on variants A and B into the Boyd and Richerson (4) model with $n = 3$, we assume that the fitness of A is $1 + s$ relative to 1 for B . Then after selection, the frequency of A is given by

$$p' = \frac{(1+s)p[1+(1-p)(2p-1)D]}{1+sp[1+(1-p)(2p-1)D]}. \quad [43]$$

Assume that selection is weak so that $|s|$ is small. We can approximate Eq. 43 by

$$p' \simeq (1+s)p[1+(1-p)(2p-1)D] - sp^2[1+(1-p)(2p-1)D]^2. \quad [44]$$

Suppose now that for all t ($t = 0, 1, 2, \dots$), s_t are identically distributed random variables and that, as before, D_t are identically distributed random variables. Then

$$p_{t+1} \simeq (1+s_t)p_t[1+(1-p_t)(2p_t-1)D_t] - s_t p_t^2[1+(1-p_t)(2p_t-1)D_t]^2. \quad [45]$$

For $|s_t|$ small, we have the following result:

Result 13. For any $t = 0, 1, 2, \dots$, suppose that s_t and D_t are independent of each other, both are independent of p_t , $E(D_t) = 0$, and $E(s_t) \neq 0$. Then with probability 1, $\lim_{t \rightarrow \infty} p_t = p_\infty$ and $p_\infty = 0$ or $p_\infty = 1$.

Proof: Let $E(s_t) = \mu$ and $\text{Var}(D_t) = \sigma^2$. Then, as

$$\begin{aligned} & [1 + (1 - p_t)(2p_t - 1)D_t]^2 \\ &= 1 + 2(1 - p_t)(2p_t - 1)D_t + (1 - p_t)^2(2p_t - 1)^2D_t^2, \end{aligned} \quad [46]$$

Eq. 45 implies that

$$E(p_{t+1} | p_t) \approx p_t \{1 + \mu(1 - p_t) [1 - p_t(1 - p_t)(2p_t - 1)^2\sigma^2]\}. \quad [47]$$

Since $-2 < D_t < 1$, we have $\sigma^2 = E(D_t^2) < 4$, and as $0 \leq p_t \leq 1$ we have

$$p_t(1 - p_t)(2p_t - 1)^2\sigma^2 < \frac{1}{4} \cdot 1 \cdot 4 = 1. \quad [48]$$

Thus if $\mu > 0$, then $E(p_{t+1} | p_t) \geq p_t$, and if $\mu < 0$, then $E(p_{t+1} | p_t) \leq p_t$, and the process $\{p_t\}_{t=0}^\infty$ is either a bounded submartingale or a bounded supermartingale. The martingale convergence theorem then ensures that $\lim_{t \rightarrow \infty} p_t = p_\infty$ with probability 1.

We can also classify p_∞ . For example, if $\mu = E(s_t) > 0$, then Eq. 47 implies that

$$E(p_{t+1}) \geq E(p_t) + \hat{\mu} E\{p_t(1 - p_t) [1 - p_t(1 - p_t)(2p_t - 1)^2\sigma^2]\} \quad [49]$$

for any $0 < \hat{\mu} < \mu$. Since $p_t \xrightarrow{t \rightarrow \infty} p_\infty$ and $0 \leq p_t \leq 1$, for all $t = 0, 1, \dots$, $E(p_t^\ell) \xrightarrow{t \rightarrow \infty} E(p_\infty^\ell)$ for any $\ell = 1, 2, \dots$. Then from Eq. 49,

$$E\{p_\infty(1 - p_\infty) [1 - p_\infty(1 - p_\infty)(2p_\infty - 1)^2\sigma^2]\} \leq 0. \quad [50]$$

But Eq. 48 entails that

$$p_\infty(1 - p_\infty) [1 - p_\infty(1 - p_\infty)(2p_\infty - 1)^2\sigma^2] \geq 0, \quad [51]$$

which means that $p_\infty(1 - p_\infty) = 0$ with probability 1. We therefore infer that with probability 1, either $p_\infty = p^* = 0$ or $p_\infty = p^* = 1$. It should be pointed out that *Result 13* also applies to the case when $s_t = s = \mu$ is constant over time.

Observe that both $p^* = 0$ and $p^* = 1$ are constant equilibria even though s_t and D_t are random variables. Following ref. 49, the SLS of these equilibria is determined as follows:

$$\begin{aligned} p^* = 0 \text{ is SLS if } E\{\log[(1 + s_t)(1 - D_t)]\} < 0 \\ p^* = 1 \text{ is SLS if } E\left[\log \frac{1 - D_t}{1 + s_t}\right] < 0. \end{aligned} \quad [52]$$

Thus, using Jensen's inequality, if $E(D_t) \geq 0$ and $E(s_t) \leq 0$, both $E[\log(1 + s_t)]$ and $E[\log(1 - D_t)]$ are negative and $p^* = 0$ is SLS. If s_t are small for $t = 0, 1, 2, \dots$, then $E\left[\log \frac{(1 - D_t)}{(1 + s_t)}\right] \approx E\{\log[(1 - D_t)(1 - s_t)]\}$, and $p^* = 1$ is SLS if $E(D_t) \geq 0$ and $E(s_t) \geq 0$. If $s_t = s$ is constant, then if $E(D_t) \geq 0$,

$$\begin{aligned} p^* = 0 \text{ is SLS if } -1 \leq s \leq 0 \\ p^* = 1 \text{ is SLS if } s \geq 0. \end{aligned} \quad [53]$$

Discussion

Previous analytical models of conformity have included constant rather than variable conformity coefficient(s). Many of these include a single conformity coefficient and dynamics that follow Eq. 1 (4, 30–33, 35–37) or a similar formulation (32, 38, 39, 55). These, dynamics are represented by an *S*-shaped curve and there

are three equilibria at $p^* = 0, \frac{1}{2}$, and 1, with convergence to one of these. Others have implemented qualitatively different formulations of conformity that also include a single, constant conformity coefficient and have found more complex dynamics (56, 57).

A model of “strong anticonformity” was explored in ref. 56, where individuals acquire a variant that is sampled at a frequency of 0 with probability 1 (i.e., the left-hand side of Eq. 20a does not hold). In that model, stable cycles of variant frequencies can occur with $n = 3$ role models. In refs. 56 and 57, a model of conformity was studied in which there is one cultural variant of interest, such as an item of learned information. Conformity or anticonformity occurs if this variant is present in the majority of sampled role models, but if the variant is present in a minority of role models, conformity or anticonformity to the opposite type (absence of information) does not occur (i.e., the right-hand side of Eq. 20a does not hold). In ref. 57, social and individual learners are in environments subject to change, and stable cycles in variant frequencies as well as chaos may occur. Finally, ref. 58 included a modified version of Eq. 1 in a susceptible, infectious, susceptible (SIS) epidemic-type model and found polymorphic equilibria with $p^* \neq \frac{1}{2}$. They also found that under some conditions, a polymorphic equilibrium and a fixation state could be stable simultaneously.

We previously showed that complex dynamics including cycles, chaos, and stable polymorphic equilibria with $p^* \neq \frac{1}{2}$ are possible under Boyd and Richerson's (4) general model of conformity specified by Eqs. 19 and 20 (49). This model includes conformity coefficients that are fixed over time but can differ in sign and magnitude depending on the number of role models sampled that carry the same variant. For example, in ref. 19, fruit flies that observed 60% of conspecifics copulating with pink (or green) males showed a greater than 60% chance of copulating with the more commonly chosen male (i.e., conformist bias), whereas those that observed 83% of conspecifics copulating with pink (or green) males showed a less than 83% chance of copulating with the more commonly chosen male (i.e., anticonformist bias). Including both conformity and anticonformity that depend on the numbers of *A* and *B* in the sample of role models is necessary to allow polymorphic equilibria with $p^* \neq \frac{1}{2}$, but is not necessary to produce cycles or chaos. Cycles or chaos can occur when $n \geq 5$ or $n \geq 10$, respectively, and anticonformity is sufficiently strong (49).

The present analysis generalizes previous models by allowing the conformity coefficient(s) to vary over time. Temporal variation in conformity may account for differences in baby name turnover in recent versus older decades (12), and individuals' tendency to exhibit a related bias, namely Aschian conformity, has decreased over time (29). Thus, conformity coefficients may be better regarded as random variables than fixed parameters. At each generation, *t*, conformity coefficient(s) are sampled from a continuous or discrete probability distribution. With $n \geq 5$ role models there are multiple conformity coefficients in the model (Eq. 19), which may be sampled from the same or from different probability distributions, whereas with $n = 3$ or 4 there is a single conformity coefficient. All members of generation *t* display the same levels of conformity, D_t . We have not considered individual variation in conformity, which may be important in real populations. We assume that D_t is sampled independently of the population frequency, p_t [although the probability of sampling *j* of *n* role models with conformity coefficient $D_t(j)$ does depend on p_t]. Association between D_t and p_t , such as increasing all conformity coefficients $D_t(j)$ with increasing p_t , could be incorporated in future models to determine whether these relationships affect the presence or stochastic local stability of equilibria.

In addition, we analyzed the role of variation in the number of role models, n_t , which is always finite and greater than or equal to 3. In agent-based models of conformity on networks, the number of sampled role models can vary by individual (43, 45, 46)

and over time (43). In the present model, the number of role models can change over time, but is the same for all individuals in the population at a given time. For example, members of different generations may encounter different numbers of role models due to changes in the dispersion of individuals in a population or changes in modes of information transmission (e.g., social media).

In the deterministic model (4) with conformity [$D(j) > 0$ for all $\frac{n}{2} < j < n$ in Eq. 19], the frequency of a common variant ($p > \frac{1}{2}$) increases to 1 and the frequency of an uncommon variant ($p < \frac{1}{2}$) decreases to 0. Thus, stable equilibria are $p^* = 1$ (variant *A* fixed) and $p^* = 0$ (variant *B* fixed), whereas the equilibrium $p^* = \frac{1}{2}$ equal frequencies of *A* and *B* is unstable and separates the two domains of attraction. In the stochastic model with conformity expected [i.e., $E[D_t(j)] > 0$ for $\frac{n}{2} < j < n$ and all $t = 0, 1, \dots$], convergence to one of the equilibria $p^* = 0, \frac{1}{2}$, or 1 is also ensured. This is true whether the number of role models, n_t , is fixed or stochastically variable. Unlike in the deterministic conformity model, here $p^* = \frac{1}{2}$ need not be unstable, and thus all three equilibria can be SLS (Fig. 1 *G* and *H*). Moreover, if $n > 3$, $p^* = \frac{1}{2}$ need not separate the domains of attraction: A population with initial frequency $p_0 < \frac{1}{2}$ ($p_0 > \frac{1}{2}$) can reach $p^* = 1$ ($p^* = 0$) (Fig. 1*H*). On the other hand, if conformity is expected, $n = 3$, and $p^* = \frac{1}{2}$ is unstable (Fig. 2*A*), Boyd and Richerson's (4) equation 1 with conformity ($D > 0$) provides a reasonable approximation to the stochastic model.

In the deterministic model with random copying, where individuals do not favor a common or an uncommon variant [$D(j) = 0$ for all j in Eq. 19], there is no change in variant frequencies and any initial p_0 is an equilibrium. In the stochastic model, however, if $E[D_t(j)] = 0$ for all j and all $t = 0, 1, \dots$, and $D_t(j)$ are independent random variables for $j > \frac{n}{2}$, then there is convergence to one of $p^* = 0, \frac{1}{2}$, or 1 with probability 1. In this model, fluctuations in (anti)conformity can cause one variant to increase relative to the other; hence, there cannot be equilibria where the frequencies are nonzero and unequal (i.e., $0 < p < \frac{1}{2}$ and $\frac{1}{2} < p < 1$). As in the model with conformity expected, this convergence result holds with a fixed or time-varying number of role models. Fig. 1*A–C* shows that if $n = 3, 4$, or 5 (respectively) and random copying is expected, $p^* = 0, \frac{1}{2}$, and 1 are all SLS.

With $n = 3$ role models, $E(D_t) = 0$, and weak selection with $E(s_t) \neq 0$, convergence to $p^* = \frac{1}{2}$ does not occur. This is intuitive, as $E(s_t) \neq 0$ entails that one variant is selectively favored, on average, relative to the other. In this case, convergence to a fixation state ($p^* = 0$ or 1) occurs with probability 1. With $n = 3$ and either random copying or conformity expected, if weak selection is expected to favor variant *A*, $p^* = 1$ is SLS; if it is expected to favor variant *B*, $p^* = 0$ is SLS; and if $E(s_t) = 0$, both $p^* = 0$ and 1 are SLS. Finally, all of these results for selection also hold if selection is constant, rather than randomly varying.

In the deterministic model with anticonformity [$D(j) < 0$ for all $\frac{n}{2} < j < n$ in Eq. 19], convergence to an equilibrium is guaranteed if $n = 3$ or $n = 4$ (49), in which case $p^* = 0$ and 1 are unstable and $p^* = \frac{1}{2}$ is globally stable, opposite to the deterministic dynamics with conformity. In the stochastic model with anticonformity expected, dynamics of Fig. 3*A* ($n = 3$) and *B* ($n = 4$) are reasonably approximated by Boyd and Richerson's (4) anticonformity model (apart from the fluctuations across $p^* = \frac{1}{2}$ prior to convergence in the case $n = 4$). On the other hand, Fig. 1*D* and *E* shows that all three equilibria $p^* = 0, \frac{1}{2}$, and 1 can be SLS with $n = 3$ or 4 and $E[D_t(2)] < 0$ or $E[D_t(3)] < 0$, respectively. Moreover, with $n = 4$ and $E[D_t(3)] < 0$, $p^* = \frac{1}{2}$ can be unstable while $p^* = 0$ and 1 are SLS (Fig. 2*D*), a result more consistent with Boyd and Richerson's (4) model of conformity than anticonformity.

In both deterministic and stochastic models with anticonformity, nonconvergence can also occur. A common variant with

frequency $p_t > \frac{1}{2}$ may be strongly disfavored due to anticonformity such that p_{t+1} is well below $p = \frac{1}{2}$; this rare variant may then be strongly favored such that p_{t+2} is well above $p = \frac{1}{2}$, and so on. Figure 2 in ref. 49 illustrates a two-generation cycle around $p^* = \frac{1}{2}$ with $n = 5$. In the stochastic model with $n = 5$, fluctuation around $p^* = \frac{1}{2}$ is shown in Fig. 5*A*.

Finally, with $n > 4$, both conformity and anticonformity are possible for different samples j of n role models with one variant. In figure 1 of ref. 49, with $n = 5$, $D(4) = -0.7$ (anticonformity) and $D(3) = 1.9$ (conformity), there are two stable polymorphic equilibria: $p^* = 0.1927$ and $1 - p^* = 0.8073$. Here, in the analogous stochastic case $E[D_t(4)] = -0.7$ and $E[D_t(3)] = 1.9$, in Fig. 5*B* we see stochastic fluctuation around average frequencies $p = 0.1929$ (if $p_0 < \frac{1}{2}$) or $p = 0.8072$ (if $p_0 > \frac{1}{2}$). Thus, although polymorphic equilibria with $p^* \neq \frac{1}{2}$ were not found in the stochastic model, fluctuation occurred around average frequencies close to the stable polymorphic equilibria with $p^* \neq \frac{1}{2}$ from the constant conformity model.

Thus far, our discussion has pertained to cases with identically distributed D_t . This assumption is removed in Fig. 4, where D_t instead cycles between fixed values. Here, unlike in the case of identically distributed D_t , nonconvergence is possible with $n = 3$ role models. Fig. 4*A* shows an example of an exact two-generation cycle with $n = 3$, where if the initial frequency $p_0 > \frac{1}{2}$ or $p_0 < \frac{1}{2}$, the cycle occurs above or below $p = \frac{1}{2}$ (respectively). With $n = 4$, cycles can occur around $p = \frac{1}{2}$ (Fig. 4*B*). These dynamics are more complex than in Boyd and Richerson's (4) model with constant conformity coefficients, where nonconvergence is possible only for $n \geq 5$ (49).

Future empirical research could explore the level of conformity exhibited by members of a fixed age group over generations, allowing the amount of temporal variation in conformity to be estimated and hence which D_t distributions might be appropriate. Such research may also reveal deterministic trends in conformity over time, stimulating new models. For example, just as Aschian conformity in line discrimination tasks decreased over time (29), Boyd and Richerson (4) conformity might also decrease over time in similar mental discernment contexts. In much of our analysis, the vector of conformity coefficients (D_t) was sampled at each generation from the same multivariate distribution, and the assumption of identically distributed D_t was removed only in examples with fixed, cycling D_t (Fig. 4). Additional research could explore how variation in the number of role models, n_t , is associated with the level of conformity. In our model, for a given $n_t = n$, an n -dimensional conformity vector (D_t) is sampled from a probability distribution, but we do not include any known relationships between D_t and n_t , which may exist and be important. For example, (anti)conformity might be stronger if 70 of 100 sampled individuals share a variant compared to 7 of 10 sampled individuals, as the smaller sample might be perceived as a less reliable indicator of population-level variation.

Other extensions of our analysis could model finite rather than infinite populations, subpopulations with migration among them, and individual-level variation in conformity. Finite populations experience greater stochastic fluctuation in variant frequencies than infinite populations, resulting in elimination of polymorphic equilibria. Migration may also greatly alter population dynamics. For example, incorporating migration into Boyd and Richerson's (4) deterministic conformity model with infinite population sizes can produce surprising polymorphic equilibria ($p^* \neq \frac{1}{2}$) (49). In infinite, multipopulation models, it would be interesting to determine whether incorporating stochastic variation in conformity would eliminate or allow stochastic local stability of these polymorphic equilibria. Finally, levels of conformity need not be the same for all individuals in a population (41, 56), which we have assumed. Incorporating individual-level variation in conformity

could produce more realistic dynamics, as well as enable a more direct application to empirical research on individual biases.

In conclusion, if conformity coefficients vary over time, evolutionary dynamics are seen that are not possible with constant conformity coefficients. In the constant conformity model, at least one equilibrium must be unstable (49), whereas here all equilibria can be stochastically locally stable simultaneously. We find that irrespective of whether n is constant or time varying, if conformity is expected [$E[D_i(j)] > 0$ for all $\frac{n}{2} < j < n$] or random copying is expected [$E[D_i(j)] = 0$ for all j and $D_i(j)$ are independent], convergence to an equilibrium $p^* = 0, \frac{1}{2}$, or 1 is ensured. If random copying is expected, $n = 3$, and there is weak selection that favors one variant over the other on aver-

age, only fixation states $p^* = 0$ or 1 are reached as $t \rightarrow \infty$. With anticonformity expected, nonconvergence is possible under different conditions from those in previous deterministic models (49). Finally, when conformity coefficients are not identically distributed, exact cycles can arise with as few as $n = 3$ role models. Thus, temporal fluctuations in the patterns of conformity can cause significant departures from the conclusions that have been drawn from models with constant levels of conformity.

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