

# 1                    **Synergy and antagonism in the integration of BCR and CD40 signals** 2                    **that control B-cell proliferation**

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## 9 10 11                    **ABSTRACT**

12                    In response to infection or vaccination, a successful antibody response must enrich high-affinity  
13                    antigen-reactive B-cells through positive selection, but eliminate auto-reactive B-cells through  
14                    negative selection. B-cells receive signals from the B-cell receptor (BCR) which binds the  
15                    antigen, and the CD40 receptor which is stimulated by neighboring T-cells that also recognize  
16                    the antigen. How BCR and CD40 signaling are integrated quantitatively to jointly determine B-  
17                    cell fate decision and proliferation remains unclear. To investigate this, we developed a  
18                    differential-equations-based model of the BCR and CD40 signaling networks activating NFκB.  
19                    Our model accurately recapitulates the NFκB dynamics of B-cells stimulated through their BCR  
20                    and CD40 receptors, correctly predicting that costimulation induces more NFκB activity.  
21                    However, when linking it to established cell fate decision models of cell survival and cell cycle  
22                    control, it predicted potentiated population expansion that was not observed experimentally. We  
23                    found that this discrepancy was due to a time-dependent functional antagonism exacerbated by  
24                    BCR-induced caspase activity that can trigger apoptosis in founder cells, unless NFκB-induced  
25                    survival gene expression protects B-cells in time. Guided by model predictions, sequential co-  
26                    stimulation experiments revealed how the temporal dynamics of BCR and CD40 signaling  
27                    control the fate decision between negative and positive selection of B-cell clonal expansion. Our  
28                    quantitative findings highlight a complex non-monotonic integration of BCR and CD40 signals  
29                    that is controlled by a balance between NFκB and cell-death pathways, and suggest a  
30                    mechanism for regulating the stringency of B-cell selection during an antibody response.

## 31 32 33                    **Keywords**

34                    Germinal center reaction, B-cell selection, CD40 signaling, BCR signaling, NFκB, activation-  
35                    induced cell death, mathematical modeling, systems immunology

36

## 37 **Highlights**

- 38 • CD40 and BCR signaling in B-cells synergize to potentiate NF $\kappa$ B cRel activation
- 39 • BCR-apoptotic signaling may enhance or antagonize CD40-driven proliferation
- 40 • BCR-induced apoptosis may be rescued by CD40 within a temporal window
- 41 • A mathematical model reveals regulators of the dose-dependent selection stringency

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## 45 **INTRODUCTION**

46 A critical component of the immune response is the generation of antibody. In order to generate  
47 antigen-reactive antibodies, B-cells evolve in the germinal center (GC), where they mutate their  
48 B-cell receptor (BCR) sequences, and are subjected to selection based on their interactions with  
49 antigens and helper T-cells (Nowosad, Spillane and Tolar, 2016). Based on clonal selection  
50 theory, high-affinity antigen-reactive B cells receive a stronger stimulus and hence proliferate to  
51 a greater extent, while low-affinity B cells receive a weaker stimulus and do not proliferate as  
52 much (Burnet, 1957; Victora and Nussenzweig, 2022). But a successful response must not only  
53 enrich high-affinity antigen-reactive B-cells through positive selection, but must also eliminate  
54 autoreactive B-cells by negative selection. Thus, the stimuli acting on GC B-cells determine the  
55 outcome between positive and negative selection.

56

57 B-cells receive two stimuli: they are stimulated briefly through their BCRs while picking up  
58 antigens from neighboring follicular dendritic cells (FDCs), followed by a period when they  
59 search for a T follicular helper cell (Tfh) that will stimulate them through their CD40 receptor.  
60 The BCR is a dual-purpose receptor, functioning both to endocytose antigen and present it to T-  
61 cells and to initiate signaling. However, how the signaling functions of the BCR are relevant to  
62 the selection of B-cells in the GC remains unclear. Prior studies demonstrated that BCR signal  
63 transduction was short-circuited in GC B-cells (Khalil, Cambier and Shlomchik, 2012) and that  
64 the CD40 signal alone may be sufficient for B-cell clonal expansion (Shulman *et al.*, 2014), while  
65 others suggested that BCR signaling is necessary for the survival and priming of GC B-cells for  
66 their positive selection (Chen *et al.*, 2023). Overall, although both BCR and CD40 signaling  
67 have been profiled experimentally (Damdinsuren *et al.*, 2010; Akkaya *et al.*, 2018), there is a  
68 lack of quantitative understanding of how these two signaling events are integrated within the  
69 dynamic sequence of those interactions. A mathematical model is thus needed to understand  
70 how the two signals interact and jointly determine the appropriate B-cell survival and  
71 proliferation to maintain a balance in positive and negative selection.

72

73 Both BCR and CD40 signaling pathways converge on the nuclear factor kappa B (NF $\kappa$ B)  
74 signaling system. While BCR stimulation activates the canonical NF $\kappa$ B pathway only transiently,  
75 CD40 stimulates both the canonical and noncanonical pathways, resulting in prolonged NF $\kappa$ B  
76 activity (Sen, 2006). Mathematical models of the NF $\kappa$ B signaling system have been established  
77 (Mitchell *et al.*, 2023) and they have been integrated with models of cell fate decision circuits to

78 recapitulate *in vitro* B-cell population dynamics resulting from the toll-like receptor ligand CpG, a  
79 T-independent stimulus (Shokhirev *et al.*, 2015; Mitchell *et al.*, 2018; Roy *et al.*, 2019). Another  
80 set of models explored the feedback mechanisms within the BCR molecular network, which  
81 involves protein kinase C  $\beta$  (PKC $\beta$ ), CARD containing MAGUK protein1 (CARMA1),  
82 transforming growth factor  $\beta$ -activated kinase 1 (TAK1) and I $\kappa$ B kinase  $\beta$  (IKK $\beta$ ) (Shinohara *et al.*  
83 *et al.*, 2014, 2016; Inoue *et al.*, 2016). However, despite many studies of the CD40 signaling  
84 pathway (Dadgostar *et al.*, 2002; Elgueta *et al.*, 2009; Akiyama, Shinzawa and Akiyama, 2012),  
85 there is no mathematical model or quantitative understanding of the dynamics of CD40  
86 signaling. Further, no work has been done to combine the BCR and CD40 receptor signaling  
87 knowledge and explore how the two signals combine quantitatively to control NF $\kappa$ B signaling  
88 and resulting cell fate decisions such that the B-cell population dynamics in response to T-  
89 dependent stimulation may be understood or predicted.

90  
91 Here, we undertook quantitative studies to develop mathematical models for the receptor  
92 activation modules of the BCR and CD40. We then tested the reliability of these models by  
93 linking them to established cell survival and cell cycle models for quantitative studies of the B-  
94 cell population dynamics in response to BCR and CD40 receptor stimulation. The combined  
95 model correctly recapitulated the population dynamics data of B-cells stimulated with either  
96 stimulus, but simulating the combined stimulus conditions revealed discrepancies with  
97 experimental data. Our investigations revealed an unexpected time-dependent functional  
98 antagonism that modulates the expected synergy between BCR and CD40 signaling. It is  
99 exacerbated by BCR-induced caspase activity that can trigger apoptosis in founder cells, unless  
100 NF $\kappa$ B-induced survival gene expression protects B-cells in time. Model-guided sequential co-  
101 stimulation studies then revealed how temporal signaling dynamics regulate the control of cell  
102 fate decisions that underlie negative vs. positive selection of B-cell clonal expansion.

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104

## 105 RESULTS

106

### 107 **A mathematical model of B-cell signaling during T-dependent stimulus responses**

108 To understand how BCR and CD40 signaling are mechanistically integrated during T-dependent  
109 (TD) immune responses, we developed a mathematical model of the molecular interaction  
110 network that downstream of these receptors. We built on an established mathematical model of  
111 the BCR signaling pathway (Inoue *et al.*, 2016) and formulated a new CD40 model to include  
112 key mechanistic features of its known signaling pathway (Elgueta *et al.*, 2009; Akiyama,  
113 Shinzawa and Akiyama, 2012). Both BCR and CD40 pathways culminate in canonical and non-  
114 canonical IKK activation, defining a T-dependent B-cell signaling network model that includes 37  
115 mass-action equations (Fig. 1A). We parameterized the T-dependent B-cell signaling network  
116 model by adopting parameter values from established models, using half-lives and synthesis  
117 rates from biochemical experiments in the literature, or manually fitting to published time course  
118 data (Table 1: parameter values).

119

120 Stimulation of B cells with TD ligands activates the multi-dimeric NF $\kappa$ B signaling system to  
121 regulate downstream cell fate response. Therefore, once the receptor model outputs appeared

122 to fit the published activation dynamics of IKK induced by TD stimuli (Shinohara *et al.*, 2016), we  
123 connected it to the latest mathematical model of the NF $\kappa$ B signaling network that accounts for  
124 the time-dependent activity of multiple NF $\kappa$ B dimers (Mitchell *et al.*, 2023) (Fig. 1B). We then  
125 introduced heterogeneity to the signaling dynamics by sampling parameter sets from parameter  
126 distributions (see more details in “Computational modeling of the T-dependent receptor  
127 signaling pathway” section of the Methods). To test if the BCR/CD40-NF $\kappa$ B model recapitulates  
128 the NF $\kappa$ B dynamics induced by TD stimulation, we stimulated naïve B cells for 7, 24, and 48  
129 hours with different stimulation conditions – low  $\alpha$ -CD40 concentration (1 $\mu$ g/mL), high  $\alpha$ -CD40  
130 concentration (10 $\mu$ g/mL), and costimulation with high concentration of both  $\alpha$ -CD40 and  $\alpha$ -BCR  
131 (10 $\mu$ g/mL) – to quantify their NF $\kappa$ B signaling activity by immunoblotting the nuclear fraction for  
132 RelA (p65) and cRel level (Fig. S1A), following live cell normalization (Fig. S1B).

133  
134 The optimal range of experimental doses of  $\alpha$ -CD40 were chosen based on prior literature that  
135 carefully examined the B cell proliferation response to defined  $\alpha$ -CD40 concentrations (Rush  
136 and Hodgkin, 2001; Turner, Hawkins and Hodgkin, 2008; Hawkins *et al.*, 2013). The chosen  
137 experimental doses were determined to be equivalent to model simulations with 6nM of stimulus  
138 for low dose and 30nM for high dose. We then undertook a systematic dose-response analysis  
139 focusing on NF $\kappa$ B, and found that peak nuclear RelA:p50 (Fig. 1C) and nuclear cRel:p50 (Fig.  
140 1D) reach saturation with the high dose, but that cRel:p52 does not. This indicates that the  
141 canonical and non-canonical NF $\kappa$ B activities have differential dose responses in response to  
142 CD40 stimulation. Thereby the low dose of  $\alpha$ -CD40 allows us to examine how BCR and CD40  
143 signals are integrated in unsaturated NF $\kappa$ B conditions; while the high dose of  $\alpha$ -CD40 allows us  
144 to test their integration under saturated canonical NF $\kappa$ B condition.

145  
146 The integrated model recapitulated the amplitude, dose-responsiveness, and speed of RelA and  
147 cRel dynamics in response to various stimulation schemes (Fig. 1E-H). For example, in both  
148 simulated and experimental data, nuclear RelA level in B-cells stimulated with high and low  
149 doses of  $\alpha$ -CD40 increased 10- and 5-fold, respectively, after 7 hours of stimulation (Fig. 1E-F),  
150 and nuclear cRel was induced to around 13- and 4-fold relative to its steady-state level (Fig. 1G-  
151 H). The model further captured the steeper gradient of downregulation of nuclear cRel than  
152 RelA from 7hrs to 24hrs after stimulation. Although both model simulation and immunoblot  
153 results showed a decrease in nuclear NF $\kappa$ B levels at 24hrs after stimulation, *in vitro* data  
154 indicated a slight rebound at 48hrs (Fig. 1F,H), whereas the *in silico* levels continued to  
155 decrease (Fig. 1E,G). This discrepancy in late-phase NF $\kappa$ B dynamics could be due to *in vitro*  
156 cells undergoing cell death or proliferation by this timepoint (Fig. S1B), which this signaling-only  
157 model does not account for. In sum, the model was able to recapitulate early B-cell NF $\kappa$ B  
158 dynamics in response to TD stimulation, but failed to capture late activity in the absence of  
159 accounting for cell fate decisions.

### 160 161 **Combining models of signaling and cell fate decisions**

162 Given that NF $\kappa$ B dynamics are critical determinants of B-cell fate decisions (Shokhirev *et al.*,  
163 2015; Mitchell *et al.*, 2018; Roy *et al.*, 2019), we next asked whether linking the CD40-NF $\kappa$ B  
164 signaling model to cell fate decision models would correctly predict the population dynamics in  
165 response to TD stimulation. After connecting these modules (Fig. 2A left), we could simulate the

166 time-dependent dynamics of successive generations of B-cells that results from division and  
167 death decisions. To generate experimental data for comparison, we stained naïve B cells with  
168 the Cell Trace Far Red (CTFR) dye, cultured them under various anti-CD40 stimulus conditions  
169 to observe their proliferation kinetics *via* dye dilution (Fig. S1C), and quantified the number of  
170 cells under each generation using FlowJo's generation deconvolution feature (Roederer, 2011)  
171 (Fig. 2A right). Four conditions were used in the experiment and model simulation, respectively:  
172 no stimulus, low concentration (1 $\mu$ g/mL in experiment, which corresponds to 6nM in model  
173 simulation), medium concentration (3 $\mu$ g/mL / 20nM), and high concentration (10 $\mu$ g/mL / 60nM)  
174 of  $\alpha$ -CD40 stimulus.

175  
176 Inspecting the data, we found that increasing doses of CD40 affect both the time to first division  
177 (Tdiv0) and the total number of divisions a B-cell can reach, while T-independent (TI) ligands  
178 CpG and LPS, which were used prior studies, show fast Tdiv0 even at low doses (Hawkins *et*  
179 *al.*, 2013). Our published cell fate module that was tuned to B-cells stimulated with the TI ligand  
180 CpG (Shokhirev *et al.*, 2015; Mitchell *et al.*, 2018; Roy *et al.*, 2019) qualitatively fit the TD ligand  
181 CD40 data, but the simulated responses were faster than observed (Fig. S2A). Meanwhile, the  
182 later division times (Tdiv1+) of the CD40 experimental data were shorter than predicted by the  
183 model (Fig. S2A), while the proportion of dividers was lower. To improve the model fit, we  
184 identified locally sensitive parameters in the cell cycle module that contribute to Tdiv0 and  
185 Tdiv1+ by calculating the standard deviation in division times when scaling each parameter from  
186 0.2 to 5.0 times the original values (Fig. S2C, see more details in "Local sensitivity analysis to  
187 tune CD40-activated cell fates" section of the Methods). After fine-tuning the sensitive  
188 parameters, the model appeared to recapitulate key aspects of B-cell population dynamics in  
189 response to all tested CD40 doses (Fig. 2B-C). For example, the fold change of live cell counts  
190 and the proportion of generation 0 cells (non-dividers) relative to generation 1+ cells (dividers)  
191 appeared to be consistent between model simulation and experimental results at most time  
192 points; both features were also captured by the model in a dose-dependent manner.

193  
194 To quantitatively evaluate the model fit at the population dynamics level, we calculated the root  
195 mean square deviation (RMSD) of the population expansion index (Fig. 2D) and generational  
196 composition (Fig. 2E) between model and experimental outputs at each experimental timepoint  
197 (0, 24, 36, 48, 72, and 96 hrs). Total RMSDs were evaluated between each model and  
198 experiment pair, regardless of matching and mismatching CD40 doses. As a permutation null,  
199 the dose-mismatched pairs demonstrated high RMSD values of around or above 1.0, while the  
200 dose-matched pairs exhibited much lower RMSD values of around or below 0.5 for population  
201 expansion index, indicating great fit in all CD40 doses (Fig. 2D, right). This marked an  
202 improvement from the RMSD values before tuning the cell fate parameters. For example, before  
203 tuning, population expansion of B-cells stimulated with a medium dose *in silico* had a 1.00  
204 RMSD from its matched *in vitro* medium dose, higher than its 0.75 RMSD from the mismatched  
205 high dose (Fig. S2B, top). After tuning, medium dose *in silico* data had a 0.45 RMSD from the  
206 matched *in vitro* medium dose data, much lower than its 1.33 RMSD from the mismatched high  
207 dose (Fig. 2D, right). In sum, the multi-scale model was able to reliably predict B-cell population  
208 dynamics over 96hrs in terms of heterogeneous receptor-induced NF $\kappa$ B signaling dynamics and  
209 ensuing cell death and division decisions.

210

### 211 **BCR and CD40 costimulation show both synergy and antagonism**

212 As the model captured B-cell population dynamics in response to various doses of CD40  
213 stimulation, we next asked if it can accurately predict the dynamics in response to BCR and  
214 CD40 costimulation. We followed the same workflow as in Fig. 2A to generate model simulation  
215 and dye dilution data in response to two BCR and CD40 costimulation conditions: first, high  $\alpha$ -  
216 IgM (10 $\mu$ g/mL) and low  $\alpha$ -CD40 (1 $\mu$ g/mL) (co-low) and second, high  $\alpha$ -IgM (10 $\mu$ g/mL) and high  
217  $\alpha$ -CD40 (10 $\mu$ g/mL) (co-high). In both cases, the multi-scale model predicted more population  
218 expansion in costimulation (Fig. 3A and 3C) than the corresponding high and low CD40 single-  
219 ligand stimulation (Fig. 2B). Because both BCR and CD40 stimuli activate the pro-survival and  
220 pro-proliferative NF $\kappa$ B pathway, the model simulation results were consistent with our  
221 expectation of synergistic population expansion.

222

223 However, experimental results of matching stimulus conditions revealed that the two TD stimuli  
224 synergized only in a dose-dependent manner. While the dye dilution data (Fig. 3B) showed a  
225 synergistic population expansion in co-low condition as predicted (Fig. 3A), there was an  
226 unexpected antagonistic effect of  $\alpha$ -IgM costimulation when combined with high CD40  
227 stimulation (Fig. 3D). Indeed, when we calculated the RMSDs between simulated and  
228 experimental data for the 2 costimulation conditions, the co-low condition had a score of 0.93,  
229 indicating a good fit that's comparable to the CD40 single-ligand stimulation conditions (that  
230 ranged from 0.32 to 0.90), but the co-high condition had a bigger deviation of 1.58, suggesting a  
231 poorer fit (Fig. 3E). Notably, the poor RMSD in co-high condition was mainly attributed to the  
232 population expansion index, which had an RMSD of 0.89 that's much higher than its 0.25-to-  
233 0.45 range in CD40 single stimulation conditions (Fig. 2D, right), while the generation  
234 composition RMSD was 0.68, only slightly above its 0.06 to 0.53 range in CD40 single  
235 stimulation conditions (Fig. 2E, right).

236

237 To better understand the source of discrepancy between simulated and experimental population  
238 dynamics, we next examined the experimental effects of high  $\alpha$ -IgM on the background of CD40  
239 stimulation. High  $\alpha$ -IgM costimulation seemed to have a positive effect on B-cells stimulated with  
240 a low concentration of CD40, increasing both the population expansion index (Fig. 3F) and  
241 proliferative capacity (Fig. 3G). The RMSD scores between these two experimental conditions  
242 also highlighted that high  $\alpha$ -IgM costimulation deviated in both population expansion and  
243 generation composition (Fig. 3H). In the context of high CD40 stimulation, however, the addition  
244 of high  $\alpha$ -IgM had a less straightforward effect, causing the B-cell population to expand less at  
245 96 hours than without  $\alpha$ -IgM (Fig. 3I). Conversely, the generation composition chart showed a  
246 similar proliferation profile in both conditions, and even slightly earlier Tdiv0 values in  
247 costimulation (Fig. 3J). Consistent with figures 3I-J, the RMSD between these two experimental  
248 conditions suggested that the addition of high  $\alpha$ -IgM to high CD40 predominantly altered B-cell  
249 population expansion, without significantly changing its proliferative capacity (Fig. 3K). High  $\alpha$ -  
250 IgM thus inflicted opposite effects on B-cell population dynamics, depending on a background of  
251 low or high dose of CD40. In sum, the model appeared to accurately predict the NF $\kappa$ B-  
252 dependent synergistic signaling interaction between BCR and CD40 at low dose of CD40, but  
253 failed to reproduce an NF $\kappa$ B-independent antagonistic interaction at co-high dose.

254

### 255 **Considering BCR-induced apoptosis and NF $\kappa$ B saturation**

256 Since the model did not accurately predict the population dynamics in response to BCR-CD40  
257 costimulation, we searched for a mechanistic explanation. Population expansion is a result of  
258 both cell proliferation and cell survival, and BCR stimulation (through  $\alpha$ -IgM or *in vivo* antigen)  
259 can have pro-proliferative effects on B-cells (Shokhirev and Hoffmann, 2013; Chen *et al.*, 2023).  
260 Therefore, the reduced population expansion in the high co-stimulation condition seemed to  
261 suggest that  $\alpha$ -IgM stimulation had an NF $\kappa$ B-independent anti-survival effect that overrides its  
262 pro-survival effect through NF $\kappa$ B signaling. Indeed, it was reported that ligation of the BCR  
263 induces cell death in some B cells (Chen *et al.*, 1999; Graves, Craxton and Clark, 2004) due to  
264 activation of Bcl-2 Interacting Mediator of cell death (Bim) (Gao, Kazama and Yonehara, 2012),  
265 caspase-2 or -8 (Chen *et al.*, 1999), mitochondrial dysfunction (Akkaya *et al.*, 2018), or other  
266 pathways. Based on the signaling mechanisms that may mediate activation-induced cell death  
267 (AICD) and the available species in the existing cell death module, we revised the T-dependent  
268 multi-scale B-cell model to include a simplified pathway from activated BCR to caspase-8  
269 processing (Fig. 4A, see more details in “Computational modeling of BCR-induced cell death”  
270 section of the Methods). This processing of pre-caspase-8 into caspase-8 then triggers B-cell  
271 death by initiating the cleavage of downstream effector caspases in the cell death module.

272

273 To test if the revised model could capture the population dynamics in costimulatory conditions,  
274 we re-simulated the virtual B-cell population. With the addition of BCR-induced caspase  
275 processing, the simulated cell population in response to co-high stimulation exhibited more cell  
276 death and resulted in an overall reduction in population expansion (Fig. 4B, left), which is more  
277 consistent with the experimental data (Fig. 4B, right). Meanwhile, Fig. 4C shows slightly less  
278 synergy in co-low than previously predicted (Fig. 4A), resulting in more concordance with  
279 experimental data as well. A decreased RMSD further confirmed the improvement in model fit  
280 (Fig. 4D). Overall, this indicates that the functional antagonism may be mediated by BCR-  
281 induced caspase activity triggering apoptosis in founder cells.

282

283 To further test the model in which BCR stimulation is pro-proliferative due to NF $\kappa$ B signaling and  
284 pro-apoptotic due to AICD, we then asked why the two TD stimuli manifested synergy at co-low  
285 stimulation but exhibited antagonism at co-high stimulation. We examined nuclear RelA and  
286 cRel dynamics, this time with the involvement of cell fate states, and noticed a much stronger  
287 early NF $\kappa$ B signaling synergy in the co-low condition (Fig. 4F) than the co-high stimulation (Fig.  
288 4E). In the context of high CD40 stimulation, the additional high  $\alpha$ -IgM costimulation had  
289 minimal effects on nuclear RelA and cRel levels in the first 24 hours (Fig. 4E). This lack of early  
290 synergy suggested NF $\kappa$ B signaling saturation in high  $\alpha$ -CD40 stimulation, such that BCR signal  
291 cannot contribute more. On the other hand, in B-cells stimulated with the low, sub-saturating  
292 CD40 dose,  $\alpha$ -IgM could amplify nuclear RelA and cRel levels (Fig. 4F). In sum, the combination  
293 of AICD and NF $\kappa$ B signaling saturation explained the dose-dependent interaction in BCR-CD40  
294 costimulation. These results also demonstrated the model’s capacity to capture both early and  
295 late B-cell NF $\kappa$ B dynamics in response to TD stimuli when simulations account for cell death  
296 and proliferation decisions.

297

## 298 **BCR-induced apoptosis can override BCR-induced population growth**

299 As the BCR has the potential to activate both pro-survival signaling via NF $\kappa$ B and anti-survival  
300 via caspase-8, we next examined the response relationships of these two pathways and  
301 whether the net outcome may be dose-dependent. We first validated that the simulated  
302 population dynamics fit the experimental data for costimulation with high dose CD40 combined  
303 with three doses of BCR (Fig. 5A). We found consistent population dynamics in experimental  
304 and simulation studies, in which the dose of BCR stimulus had only subtle effects. We observed  
305 that in both model-simulated and experimental populations, the number of non-proliferating cells  
306 (lightest gray) is the lowest at 96hrs when costimulated with high BCR, while the proliferating  
307 cell populations (darker grays) remain comparable across conditions (Fig. 5B). This indicates  
308 that when costimulated with a high CD40 dose, cell survival and overall population expansion  
309 monotonically decrease with increasing BCR dose from zero, low, to high.

310  
311 We next focused on how these stimuli potentially affect cell survival which could censor the  
312 proliferation module in the multi-scale model. Simulating B-cells stimulated with various doses of  
313 either BCR or CD40, we observed distinct dose-response patterns. For CD40, the higher the  
314 dose, the shallower the Kaplan-Meier survival curve is, indicating a monotonic pro-survival  
315 effect of CD40 stimulation (Fig. 5C). On the other hand, the survival dose response to BCR  
316 stimulation appeared non-monotonic (Fig. 5D), with low-BCR-stimulation increasing the  
317 probability of survival over unstimulated cells, but high-BCR-stimulation actually reducing the  
318 probability of survival. When we quantified the number of surviving cells in the first 24hrs (Fig.  
319 5E), we clearly observed the difference between the two distinct dose-response patterns in  
320 terms of monotonicity for CD40 vs BCR agonists.

321  
322 To gain a systematic understanding of the effects of BCR-induced apoptosis on B-cell response  
323 in all combinations of BCR and CD40 doses, we simulated 25 single or costimulation scenarios,  
324 each with 1000 founder B-cells. We then used locally estimated scatterplot smoothing (LOESS)  
325 to fit a smooth curve through this scatterplot of 25 data points to generate heatmaps of cell  
326 survival rate, proliferation capacity, and population fold-change. Without AICD, we observed  
327 monotonic increase in all metrics with respect to both BCR and CD40 doses (Figure 5F-H).  
328 When we incorporated AICD in the model, all the metrics still monotonically increased with  
329 respect to increasing CD40 doses (5I-K), and the divided cells percentages remained  
330 unchanged (5G, J) with little difference in the percentage dividers (5M), as expected. However,  
331 with increasing BCR doses at a low (or zero) CD40 dose, the cell survival rate first increases  
332 then decreases (5I, left 2 columns). When the CD40 dose is medium or high, increasing doses  
333 of BCR monotonically decreased the cell survival rate (5I, right 3 columns). Examining the  
334 resulting population fold-change (5H,K), we observed that BCR-induced apoptosis prevented  
335 BCR stimulation from promoting population growth, and rendered the B-cell response  
336 independent of BCR signaling.

## 337 338 **A limited temporal window of opportunity to acquire CD40 signals**

339 In TD activation, a B-cell first experiences a BCR signal when binding the antigen, and then a  
340 CD40 signal when it has found a T-cell that also recognizes the antigen (Bretscher and Cohn,  
341 1970; Parker, 1993). The time delay between the two signals is determined by the B-cell



342 searching for T-cell help (Okada *et al.*, 2005). Given that our multi-scale model captured B-cell  
343 NF $\kappa$ B and population dynamics in response to all tested doses of CD40 stimulation as well as  
344 BCR-CD40 costimulation doses, we asked if it could provide some insights on how the two  
345 signals combine in the more physiological TD stimulation scenario of sequential BCR and CD40  
346 stimulation. To simulate this scenario, a one-hour BCR signal was initiated at 0hr, and the CD40  
347 signal was initiated at 1, 3, 5, or 8hrs. In this stimulation scenario using high BCR + low CD40,  
348 the multi-scale model predicted that the B-cell population decreases over time, with a steeper  
349 decrease when the gap is longer (Fig. 6B, left). In contrast, in the low BCR + high CD40  
350 sequential stimulation condition, the B-cell population increased drastically after 48hrs despite  
351 an initial decrease in the first 24 hrs. That initial decrease was faster when the time gap was 8hr  
352 than 1hr gap but the resulting population size at 96hrs was similar (Fig. 6B, right). However,  
353 when we simulated the high BCR + high CD40 stimulation scenario, the model simulations  
354 exhibited a larger variation in population size at 96hrs, where an 8hr gap resulted in less than  
355 half the number of live cells than a 1hr or 3hr gap (Fig. 6B, middle). This indicated that there is a  
356 limited temporal window of opportunity for B-cells to acquire CD40 signal that rescues a  
357 crashing cell population following BCR stimulation.

358  
359 To test these model predictions, we undertook experiments with these stimulation scenarios.  
360 We stained naïve B cells with the Cell Trace Far Red (CTFR) dye, stimulated them with low  
361 (1 $\mu$ g/mL) or high (10 $\mu$ g/mL) dose of anti-BCR stimulus for 1hr, washed, and cultured them  
362 under low (1 $\mu$ g/mL) or high (10 $\mu$ g/mL) anti-CD40 stimulus conditions at 1, 3, 5, or 8hrs after  
363 BCR pre-activation for 4 days (Fig. 6A). We then observed and analyzed their proliferation  
364 kinetics *via* dye dilution using the same workflow as in Fig. 2A. Experimental results  
365 demonstrated distinct effects of time-gaps on B-cell population dynamics across sequential  
366 stimulation conditions (Fig. 6C). Specifically, increasing the time gaps between BCR and CD40  
367 stimulation had relatively small effects on B-cell population fold-change in both high BCR + low  
368 CD40 (Fig. 6C, left) and low BCR + high CD40 sequential stimulation (Fig. 6C, right), as the  
369 colored lines representing different time-gaps followed each other closely. On the other hand,  
370 larger time-gaps (5-8hrs) significantly diminished B-cell population compared to smaller time-  
371 gaps (1-3hrs) in high BCR + high CD40 sequential stimulation (Fig. 6C, middle). Surprisingly,  
372 the time-gap also appeared to have a non-monotonic effect on B-cell population expansion,  
373 where cells stimulated 3hr apart (orange) resulted in higher fold-change than cells stimulated  
374 1hr-apart (red), both *in silico* (Fig. 6B) and *in vitro* (Fig. 6C, middle & right). This may be due to  
375 reduced NF $\kappa$ B signaling saturation when the two stimuli were further apart, while being still  
376 close enough to allow for rescue from AICD. In sum, the experimental results were consistent  
377 with the model simulation, confirming the existence of a limited window of opportunity at the  
378 high BCR + high CD40 sequential stimulation regime (Fig. 6C, middle).

379  
380 Overall, our *in silico* and experimental investigations of the temporal relationship between these  
381 antagonistic signals revealed a limited time window within which CD40 signaling may effectively  
382 rescue cell death triggered by BCR signaling. The size of the temporal window depends on the  
383 strength of the BCR and CD40 signals, but when the time gap exceeds a threshold of about 5  
384 hours, the opportunity to trigger B-cell population expansion is severely diminished.

385

### 386 **Noisy BCR-induced Bcl-xL expression determines the window of opportunity**

387 We next asked what may determine the window of opportunity and the heterogeneous survival  
388 outcomes in single B-cells that are a prerequisite for subsequent population expansion. As BCR  
389 stimulation was shown to be both pro-survival due to NF $\kappa$ B-induced Bcl-xL activity and anti-  
390 survival due to AICD (Fig. 7A) resulting in a non-linear dose response (Fig. 5), we examined  
391 how this paradoxical signaling affects single B-cell responses *in silico* model simulations. In the  
392 apoptosis pathway, activation of caspase-8 leads to downstream activation and oligomerization  
393 of Bax to the mitochondrial outer membrane, forming Bax pores that trigger mitochondrial outer  
394 membrane permeabilization (MOMP). The prominent anti-apoptotic Bcl-xL protects cells from  
395 MOMP by sequestering Bax from oligomerization or by retrotranslocating Bax to the cytosol  
396 (Dou *et al.*, 2021).

397  
398 We first examined Bcl-xL and caspase-8 trajectories in 3 stimulation conditions (Fig. 7B).  
399 Noticeably, a decline in free Bcl-xL level (thin lines) correlated with a substantial increase in  
400 caspase-8 activity (denoted by a quick color transition from deep blue to pink), the timing of both  
401 corresponds to a decline in cell survival (thick line). While CD40 stimulation induces Bcl-xL in a  
402 homogeneous manner (Fig. 7B, left), BCR stimulation introduces more heterogeneity in Bcl-xL  
403 level among different cells (Fig. 7B, middle and right). Because Bcl-xL is induced by NF $\kappa$ B  
404 transcription factors, we also reported RelA and cRel nuclear activity and found that B-cells with  
405 higher NF $\kappa$ B activity induced their Bcl-xL levels faster and to a higher extent (Fig. 7C),  
406 protecting the cells until the onset of CD40 stimulation. On the other hand, cells with lower NF $\kappa$ B  
407 activity could not generate enough anti-apoptotic Bcl-xL and ceased to live (indicated by  
408 discontinued lines). Cells that survived the first 12 hours had significantly higher peak RelA,  
409 cRel, and Bcl-xL activity than cells that died (Fig. 7D). The variability in BCR-induced nuclear  
410 NF $\kappa$ B level was consistent with a previous report (Shinohara *et al.* 2014), where the TAK1-IKK2  
411 positive feedback resulted in a switch-like behavior in BCR activation.

412  
413 To gain a systematic understanding of the effects of the time gap between CD40 and BCR  
414 stimulation on B-cell response in all combinations of BCR and CD40 doses, we again simulated  
415 25 single or sequential scenarios, each with 1000 founder B-cells, to generate maps of cell  
416 survival rate, proliferation capacity, and population fold-change. With a 1hr pulse in BCR  
417 stimulation, we observed a cell survival trend (Fig. 7E) similar to that with coincident  
418 costimulation than the scenarios in Fig. 5J. With an 8hr staggered stimulation (Fig. 7F), the  
419 effect of AICD on cell survival was much stronger than in coincident costimulation (Fig. 5J),  
420 showing heightened cell death within the first 24hrs at medium and high BCR doses. The two  
421 population size maps in Fig. 7H-I showed very little difference at low CD40 doses but  
422 demonstrated the biggest differences in the upper and lower right corners, where virtual B-cells  
423 were stimulated with high CD40 doses and either high or no BCR doses (Fig. 7J). Overall, these  
424 results clearly illustrated the importance of not only BCR and CD40 stimulation doses but their  
425 temporal relationships in determining cell fates and ultimately population expansion.

426  
427  
428  
429

## 428 **DISCUSSION**

430 In this work, we investigated how the BCR-mediated signal I and the CD40-mediated signal II  
431 are integrated in the B-cell fate decision process to clarify their roles in T-dependent B-cell  
432 selection. Prior work demonstrated that BCR and CD40 signaling synergize at the level of NF $\kappa$ B  
433 activation (Damdinsuren *et al.*, 2010), but did not determine how these signals combine to  
434 determine the subsequent B-cell fate decisions and thus the emergent population dynamics.  
435 Here, we presented a mechanistic mathematical model of B-cell signaling and fate decision in  
436 response to T-dependent stimulation scenarios that recapitulates experimental observations  
437 (Fig. 1-4) and could be used to explore the biological consequences of the dose and temporal  
438 relationship between type I and II signals (Fig. 5-7). We showed that while BCR signaling has  
439 the potential to prime B-cells for positive selection by synergizing with CD40 on NF $\kappa$ B signaling  
440 (Fig. 3A-B, F-H), it could also initiate negative selection by functionally antagonizing CD40  
441 signaling through AICD (Fig. 3-4). Our work suggests that BCR signaling is the key to tuning the  
442 balance between positive and negative selection in mature B-cells.

443  
444 To construct a tractable mathematical model, we took a parsimonious approach to abstract the  
445 signaling pathway initiated by the T-dependent stimuli leading to NF $\kappa$ B. For example, CD40  
446 ligand engagement recruits adapter proteins, which include several tumor necrosis factor  
447 receptor-associated factor (TRAF)s, such as TRAF1, TRAF2, TRAF3, TRAF5, TRAF6, and a  
448 combination of their complexes (Elgueta *et al.*, 2009). To avoid the complexity of combinatorial  
449 biochemical reactions among the TRAF complexes, we used TRAF3 to represent the TRAF2-  
450 TRAF3 complex that constitutively inhibits the noncanonical NF $\kappa$ B pathway, and TRAF6 to  
451 represent the TRAF1-TRAF2, TRAF3-TRAF5, and TRAF6-TRAF2 complexes that all activate  
452 the canonical NF $\kappa$ B pathway. The construction of the CD40 signaling model further included  
453 parameters extracted from a substantial literature of experimental studies. For example, the  
454 degradation rate of NIK was calculated from its half-life (3hrs) estimated in a pulse-chase assay  
455 for B-cells stimulated with BAFF and anti-CD40 (Qing, Qu and Xiao, 2005). The differential  
456 degradation rates of CD40 receptor (CD40R) and ligated CD40R (CD40LR) were obtained from  
457 cell surface biotinylation assay at the surface of 9HTEo- epithelial cells (Tucker and Schwiebert,  
458 2008); similar parametrization applied to the internalization rates of BCR and antigen-ligated  
459 BCR (ABCR) (Coulter *et al.*, 2018). Furthermore, the rates of association and dissociation  
460 between CD40 and anti-CD40 were derived from the  $K_a$  and  $K_d$  values determined by surface  
461 plasmon resonance (SPR) binding analysis (Ceglia *et al.*, 2021). Having incorporated  
462 substantial molecular details and biochemical data, the model serves as a framework for an *in-*  
463 *silico* laboratory that could be expanded and revised iteratively with wet-lab experiments for  
464 mechanistically investigating the effects of various genetic and pharmacological perturbations  
465 on T-dependent-activated B-cell NF $\kappa$ B dynamics.

466  
467 Our previous work identified NF $\kappa$ B as a key determinant of B-cell population dynamics, and  
468 quantified the relative contributions of cRel- and RelA-containing NF $\kappa$ B dimers to downstream  
469 cell fate effector functions (Shokhirev *et al.*, 2015; Roy *et al.*, 2019). Here, we present an  
470 extended mathematical model, which demonstrated that NF $\kappa$ B-induced survival and proliferation  
471 as well as BCR-activation-induced apoptosis are sufficient to explain the survival and  
472 proliferation kinetics of B-cells in the explored conditions. Although other signaling pathways  
473 that are induced by BCR and CD40, such as PI3K and MAPK, could also play a role, without

474 perturbation studies of these pathways their roles are not quantifiable and are only implicit in the  
475 current mathematical model. Previous work concluded that the NF $\kappa$ B signaling pathway, but not  
476 PI3K, dominates the primary response to CD40 stimulation in GC B-cells (Luo, Weisel and  
477 Shlomchik, 2018). In response to TD stimuli, the roles of the MAPK p38 and ERK pathways are  
478 thought to be minor and generally cooperative with NF $\kappa$ B (Dadgostar *et al.*, 2002). We used  
479 naive B-cells isolated from the spleen for experimental studies that allowed us to obtain granular  
480 datasets on signaling dynamics and cell fate decisions with the dynamic population response.  
481 The possibility that B-cells may behave differently in the *in vivo* lymph node GC  
482 microenvironment (Young and Brink, 2021) could affect the reliability of extrapolating our  
483 conclusions from the present study to the *in vivo* phenomena of positive and negative selection.  
484 Still, Silva *et al.* showed that treatment with antigen bearing only the immunodominant epitope  
485 during the early GC response selectively suppressed those GC B-cells, and in turn promoted  
486 subdominant GC B-cells in mice immunized with both antigens (Silva *et al.*, 2017). This  
487 confirmed the ability of BCR signaling to modulate positive and negative selection in GC B-cells,  
488 and is consistent with our observations of BCR-CD40 antagonism.

489  
490 To fit the temporal dynamics of the model-simulated proliferative response to TD stimulation, we  
491 tuned a few parameters in the cell cycle model that was adopted from our previous work on TI  
492 ligand CpG (Fig. S2). CD40 generally stimulates a stronger NF $\kappa$ B activity than CpG at  
493 saturating doses as CD40 activates also the non-canonical pathway, thereby relieving the I $\kappa$ B $\delta$   
494 brake on canonical signaling (Rodriguez *et al.*, 2024). However, the proliferative response to  
495 CpG is faster and stronger, suggesting that another pathway, such as MAPK which is strongly  
496 induced by the MyD88 adaptor (Caldwell *et al.*, 2014; Cheng *et al.*, 2017), may be responsible  
497 for boosting cell cycle entry in response to some TI stimuli. Our revised model based on the  
498 CD40 stimulus that does not activate much MAP p38 pathway is thus more accurate in  
499 recapitulating the control of cell growth and cell cycle in response to NF $\kappa$ B.

500  
501 Our work revealed a non-monotonic integration of BCR and CD40R signals in the proliferative  
502 responses of B-cells, due to BCR-induced apoptosis and NF $\kappa$ B signaling saturation (Fig. 4).  
503 Consistent with previous research which suggested that CD40 signaling alone was sufficient for  
504 B-cell affinity maturation (Victora *et al.*, 2010; Shulman *et al.*, 2014), we found that BCR-induced  
505 apoptosis prevented BCR stimulation from promoting additional population growth, rendering  
506 population expansion primarily CD40-dependent (Fig. 4-5). However, BCR signaling provides  
507 an important modulatory role in T-dependent selection of B-cells. Chen *et al.* found BCR  
508 signaling to facilitate positive selection by prolonging B-cell survival and by priming B cells to  
509 receive synergistic Tfh cell signals (Chen *et al.*, 2023). What we found was consistent with this  
510 observation but completed with another part of the story: when costimulated with CD40, BCR  
511 signaling modulates the dose-response curve of CD40 by boosting less-stimulated cells with its  
512 pro-proliferative effects yet dampening proliferative responses of more-highly stimulated cells  
513 with its anti-survival effects (Fig. 3, 5). When stimulated alone, BCR regulates B-cell survival in  
514 a non-monotonic dose response curve, thereby potentially eliminating cells encoding self-  
515 reactive BCRs that elicit strong signals. Indeed, Shih *et al.* found both more cell division and  
516 increased cell death in higher-affinity B1-8<sup>hi</sup> B-cells compared to lower-affinity B1-8<sup>lo</sup> cells in

517 post-immunized mice spleen, highlighting again the paradoxical role of BCR stimulation (Shih,  
518 Roederer and Nussenzweig, 2002).

519

520 Considering the temporal dynamics of the process, our work delineated a narrow temporal  
521 window of opportunity for B-cells to receive CD40 signals following BCR activation. Proper  
522 timing (3hrs) of the two signals can maximize B-cell survival and proliferative response, while  
523 longer delays (8hrs) can lead to significant apoptosis and thus reduced population growth (Fig.  
524 6). Consistent with this temporal window, Akkaya et al. also found that BCR signaling activated  
525 a metabolic program that imposed a limited time frame (9hrs) during which B-cells either receive  
526 a second signal (CD40 or CpG) or are eliminated due to mitochondrial dysfunction (Akkaya *et*  
527 *al.*, 2018). In contrast, Tan et al. showed that BCR-induced NR4A nuclear receptors were the  
528 key mediators of the restraint on B cell responses to antigen when the cell fails to receive signal  
529 2 within a defined time window, by repressing MYC, and even T-cell chemokines (Tan *et al.*,  
530 2020). Overall, regardless of the exact molecular mediator of the temporal window of  
531 opportunity, our model captured the phenotypes described in a large body of literature, and  
532 resolved apparently conflicting literature into a more unified systematic model.

533

534 Previous work that distinguished the phenotypes between naïve and GC B-cells often examined  
535 late GC B-cells 10-14 days after immunization (Luo, Weisel and Shlomchik, 2018). However,  
536 this strategy overlooked the early GC B-cells or the progression of the GC B-cell phenotype  
537 necessary for affinity-based selection under different dynamic range. In the early GC phase,  
538 where the average antigen-affinity is low, B-cells with mediocre-affinity BCRs need to survive  
539 and proliferate. On the other hand, in the late GC phase, where average antigen-affinity is high,  
540 the same B-cells with mediocre BCRs would need to avoid proliferating such that B-cells with  
541 the highest affinity could be distinguished appropriately. Competition among GC B-cells for  
542 antigens and T-cell-help contribute to this flexible dynamic range (Shih, Roederer and  
543 Nussenzweig, 2002). Here, we speculate about a phase-dependent dynamic range in antigen-  
544 affinity discrimination, where BCR-induced apoptosis and NFκB signaling saturation together  
545 tune the dose-dependent synergy and antagonism between BCR and CD40 signals. The  
546 integration of both signals sets a phase-dependent “timer” for B-cell selection. The timer can be  
547 further tuned through BCR-induced NFκB signaling, as previous literature suggested that late  
548 GC B-cells downregulate their BCR-induced NFκB activation compared to early activated B-  
549 cells (Young and Brink, 2021), indicating that remodeling of the BCR signaling network could  
550 also contribute to the phase-dependent dynamic range.

551

552 Consistent with opposing roles of BCR and CD40 signaling, previous work has suggested that  
553 variants that disrupted the signaling of either BCR and CD40 caused an imbalance of positive  
554 and negative selection and lead to immunodeficiency or autoimmune diseases. Specifically,  
555 Yam-Puc et al. found that enhanced BCR signaling through GC-B-cell-specific SHP-1 mutation  
556 led to early GC B-cell death, reducing antibody responses in mice (Yam-Puc *et al.*, 2021). On  
557 the other hand, enhanced CD40 signaling through TRAF3 mutations led to autoimmunity and  
558 increased risk of B-cell malignancy in humans (Rae *et al.*, 2022), while a lack of CD40 signaling  
559 through CD40L mutations led to immunodeficiency in humans (Kroccek *et al.*, 1994).

560

561 In summary, our findings may have implications not only for the maturation of high affinity  
562 antibodies but also for the escape of auto-reactive antibodies from negative selection as in  
563 autoimmunity. We speculate that the opposing roles of BCR and CD40 signals work together in  
564 determining B-cell fates, discriminating highly reactive B-cells as self- versus non-self may  
565 amount to a kinetic proofreading mechanism. Specifically, BCR ligand discrimination is due to  
566 two signaling steps (through BCR and CD40) that reduce the probability of generating unwanted  
567 antibodies. This increased specificity is obtained by introducing cell death, an irreversible step  
568 exiting the pathway that happens faster than the next step in the pathway, when the cell  
569 receives T-cell-help in the form of CD40 stimulation. Furthermore, this delay between ligand  
570 binding and B cell activation consumes free energy due to antigen processing and the activation  
571 of cell-death pathway. Understanding this process in greater detail may enable the design of  
572 vaccination protocols that maximize B-cell activation and proliferation while ensuring temporal  
573 dynamics that selectively induce apoptosis in unwanted B-cell clones, reducing risks of  
574 autoimmunity.

575

576

### 577 **Conflict of Interest Statement**

578 The authors declare no conflict of interest.

579

### 580 **Author Contributions**

581 HH undertook all mathematical modeling work and contributed to experiments. HVN led the  
582 experimental work. All authors contributed to the design of the study. AH secured funding and  
583 provided supervision. HH wrote the manuscript. All authors edited the manuscript.

584

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591

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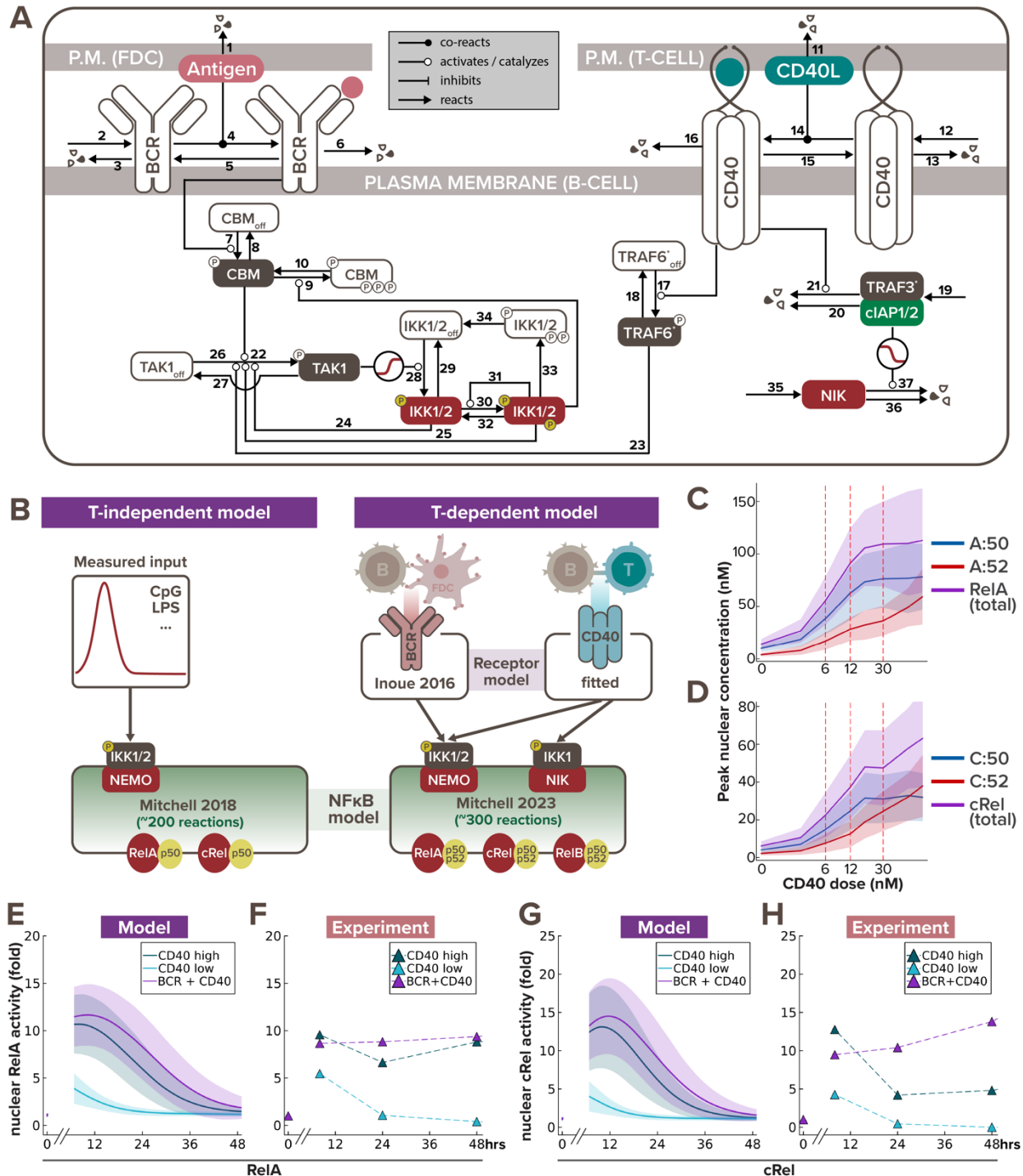
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**Figure 1. Multi-scale model recapitulates B-cell NFκB dynamics in response to T-dependent stimulation.**

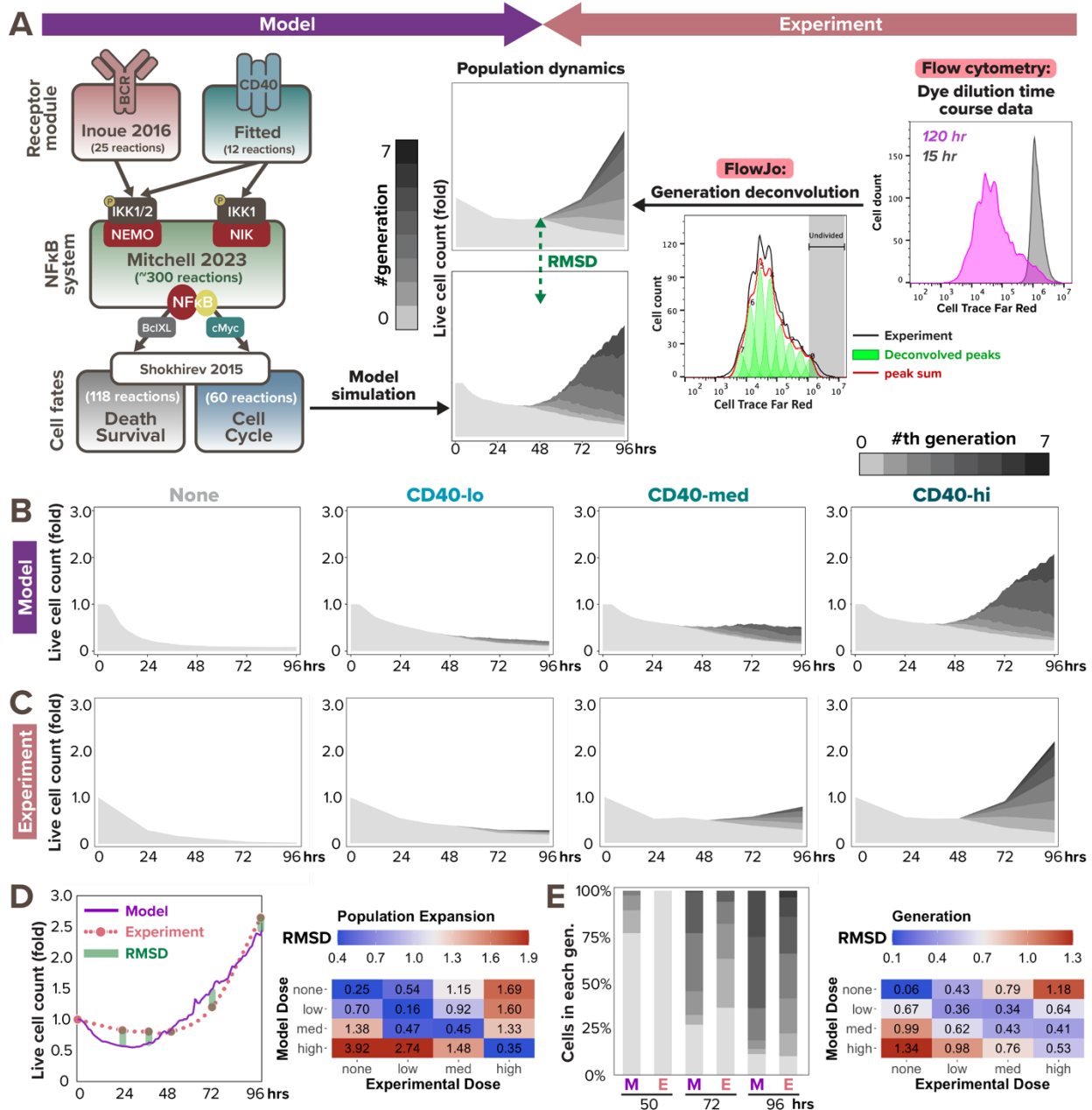
(A) Schematic of BCR-CD40 receptor model to recapitulate T-dependent activation of B-cells. (B) Schematics of existing T-independent (left) and newly integrated T-dependent (right) NFκB signaling modeling frameworks. T-independent stimulation typically only involves a single ligand (e.g. CpG or LPS), while T-dependent stimulation always involve a more complex receptor signaling system of both BCR and CD40 ligands. Low, medium and high dose of CD40 were set to 6nM, 12nM, and 30nM, respectively, to correspond to the three experimental doses we used

747 (1 $\mu$ g/mL, 3.3 $\mu$ g/mL, and 10 $\mu$ g/mL). **(C-D)** Line graph from model simulations show **(C)** peak  
748 nuclear RelA and **(D)** peak nuclear cRel levels in response to increasing CD40 doses, where  
749 the shading represents the sample standard deviation of 1000 cells. X-axes are plotted on a log-  
750 scale to accommodate a wide range of concentrations. **(E)** Line graphs from model simulations  
751 of 1000 virtual cells and **(F)** matching experiments with 600K founder B-cells show temporal  
752 trajectories of nuclear RelA level at 0, 7, 24, and 48hrs following stimulation with low  $\alpha$ -CD40  
753 (1 $\mu$ g/mL), high  $\alpha$ -CD40 (10 $\mu$ g/mL), or costimulation with high  $\alpha$ -CD40 and  $\alpha$ -BCR (10 $\mu$ g/mL  
754 each). Darker colored lines represent the average nuclear RelA level from 1000 cells and the  
755 lighter shading represents the sample standard deviation of the 1000 cells. **(G-H)** Line graphs of  
756 nuclear cRel level in matching stimulation conditions as (E-F).  
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759 **Table 1. Model reactions and parameter values**  
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Module	#	Reactions	Rates	Units	Source
BCR receptor	1	ANTIGEN →	0.05	hr-1	fitted (antibody)
	2	→ BCR	4.93	nM hr-1	fitted
	3	BCR →	1.43	hr-1	fitted
	4	ANTIGEN + BCR → ABCR	66	nM-1 hr-1	fitted
	5	ABCR → ANTIGEN + BCR	1.26	hr-1	fitted
	6	ABCR →	0.35	hr-1	Coulter et al 2018
	7	CBM + ABCR → ACBM + ABCR	6.6	nM-1 hr-1	Inoue et al 2016
	8	ACBM → CBM	0.126	hr-1	Inoue et al 2016
	9	ACBM + IKK → ICBM + IKK	0.181	nM-1 hr-1	Inoue et al 2016
	10	ICBM → CBM	0.068	hr-1	Inoue et al 2016
CD40 receptor	11	CD40L →	0.05	hr-1	fitted (antibody)
	12	→ CD40R	7.672	nM hr-1	fitted
	13	CD40R →	0.05	hr-1	Tucker et al 2008
	14	CD40L + CD40R → CD40LR	0.04	nM-1 hr-1	Ceglia et al 2021
	15	CD40LR → CD40L + CD40R	11.3	hr-1	Ceglia et al 2021
	16	CD40LR →	0.17	hr-1	Tucker et al 2008
	17	CD40LR + TRAF6 <sub>OFF</sub> → CD40LR + TRAF6	0.1	nM-1 hr-1	Cheng et al 2015
	18	TRAF6 → TRAF6 <sub>OFF</sub>	7.5	hr-1	Cheng et al 2015
	19	→ TRAF3	10	nM hr-1	fitted
	20	TRAF3 →	0.5	hr-1	Zhao et al 2016
	21	CD40LR + TRAF3 → CD40LR	10	nM-1 hr-1	fitted
TAK1 dynamics	22	ACBM + TAK1 → ACBM + ATAK1	1050	nM-1 hr-1	Shinohara et al 2014
	23	TRAF6 + TAK1 → TRAF6 + ATAK1	60	nM-1 hr-1	Cheng et al 2015
	24	IKK2 + TAK1 → IKK2 + ATAK1	401.14	nM-1 hr-1	Shinohara et al 2014
	25	IKK3 + TAK1 → IKK3 + ATAK1	1182.86	nM-1 hr-1	Shinohara et al 2014
	26	TAK1 → ATAK1	249	hr-1	Shinohara et al 2014
	27	ATAK1 → TAK1	258600	hr-1	Shinohara et al 2014
IKK dynamics	28	TAK1 + IKK <sub>OFF</sub> → TAK1 + IKK2	80.72	hr-1	Shinohara et al 2016
	29	IKK2 → IKK <sub>OFF</sub>	2175.6	hr-1	Shinohara et al 2016
	30	IKK2 → IKK3	0.009	hr-1	Shinohara et al 2016
	31	IKK3 + IKK2 → IKK3 + IKK3	2094	hr-1	Shinohara et al 2016
	32	IKK3 → IKK2	53844	hr-1	Shinohara et al 2016
	33	IKK3 → IIKK	2528.4	hr-1	Shinohara et al 2016
	34	I IKK → IKK <sub>OFF</sub>	957.6	hr-1	Shinohara et al 2016
NIK dynamics	35	→ NIK	12	nM hr-1	Mitchell et al 2023
	36	NIK →	0.231	hr-1	Mitchell et al 2023
	37	TRAF3 + NIK → TRAF3	2	nM-1 hr-1	Qing et al 2005

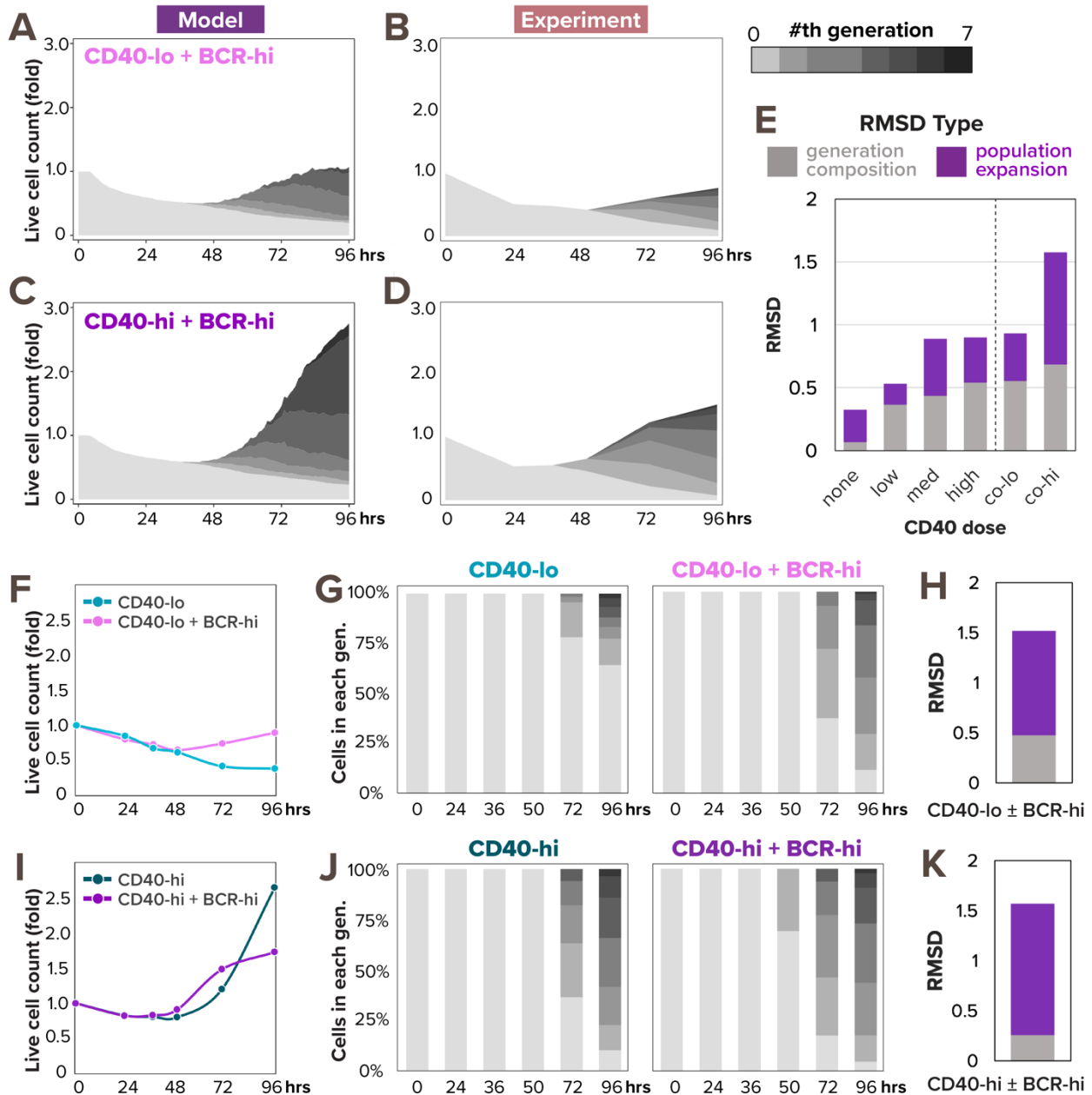
761 (Qing, Qu and Xiao, 2005; Tucker and Schwiebert, 2008; Shinohara *et al.*, 2014, 2016; Cheng *et al.*,  
762 2015; Inoue *et al.*, 2016; Zhao *et al.*, 2016; Coulter *et al.*, 2018; Ceglia *et al.*, 2021; Mitchell *et al.*, 2023)  
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**Figure 2. Multi-scale model recapitulates B-cell population dynamics in response to CD40 stimulation.**

(A) Workflow of fitting model simulations to experimental B-cell population dynamics following stimulation. Left: schematic of full T-dependent modeling framework. Right: experimental workflow with Cell Trace Far Red (CTFR) dye dilution. (B) Stacked area plots from model simulations of 1000 virtual B-cells show their population dynamics in response to stimulation with (from left to right) no (0nM), low (6nM), medium (12nM), and high (30nM) dose of  $\alpha$ -CD40. Each subsequent generation of proliferating cells is indicated with a darker gray. (C) Stacked area plots from matching experiments of 19196 founder B-cells show their population dynamics in response to no (0 $\mu$ g/mL), low (1 $\mu$ g/mL), medium (3.3 $\mu$ g/mL), or high (10 $\mu$ g/mL) dose of  $\alpha$ -CD40. (D-E) Root mean square deviation (RMSD) is calculated between simulated and experimental data, and is composed of 2 scores: RMSD of (D) relative population size expansion and RMSD of (E) generation composition. An example of RMSD between model and

778 experimental data is shown on the left side of (D) and (E), and a heatmap of the RMSD scores  
779 in matching (diagonal) or mismatching (off-diagonal) model-and-experiment pairs is shown on  
780 the right side. Lower RMSD scores correspond to better fit.

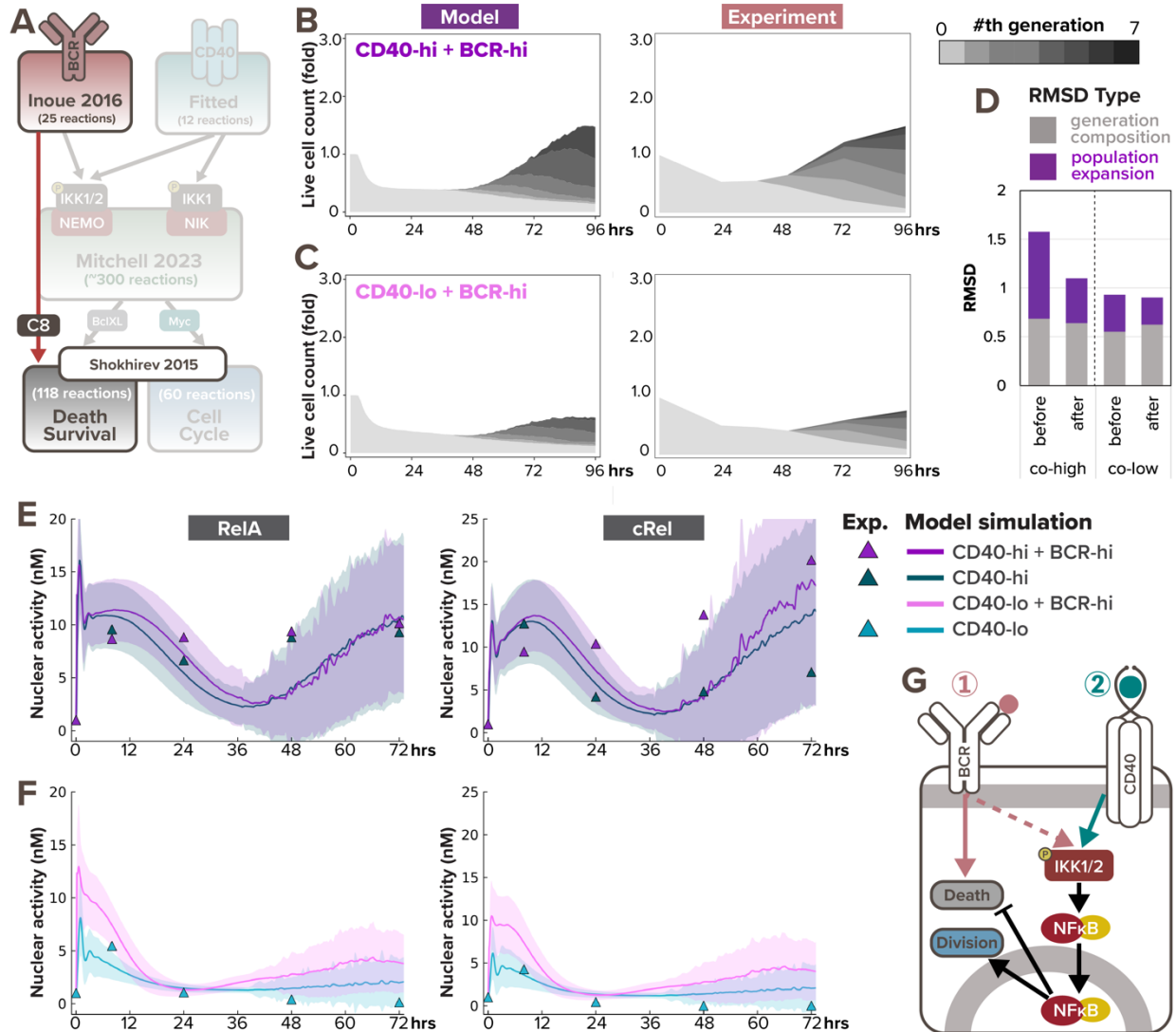


**Figure 3. Model predicts synergistic population expansion in response to BCR and CD40 costimulation, but experiment reveals dose-dependent interaction between the stimuli.**

(A) Stacked area plot from model simulations of 1000 virtual B-cells show their population dynamics in response to costimulation with high  $\alpha$ -IgM (0.25nM) and low  $\alpha$ -CD40 (6nM). Each subsequent generation of proliferating cells is indicated with a darker gray. (B) Stacked area plot from matching experiments of 19196 founder B-cells show their population dynamics in response to high  $\alpha$ -IgM (10 $\mu$ g/mL) and low  $\alpha$ -CD40 (1 $\mu$ g/mL) costimulation. (C) Stacked area plot from model simulations of 1000 virtual B-cells show their population dynamics in response to costimulation with high  $\alpha$ -IgM (0.25nM) and high  $\alpha$ -CD40 (30nM). (D) Stacked area plot from matching experiments of 19196 founder B-cells show their population dynamics in response to high  $\alpha$ -IgM (10 $\mu$ g/mL) and high  $\alpha$ -CD40 (10 $\mu$ g/mL) costimulation. (E) Stacked bar graph shows a breakdown of total RMSD by types in the 2 costimulation conditions compared to the 4 model-and-experiment pairs in Fig. 2B-C which includes no, low, medium, and high dose of CD40. (F)

795 Line graph of experimental population expansion index is higher in response to costimulation  
796 than without  $\alpha$ -IgM. **(G)** Stacked bar graph of experimental generation composition dynamics in  
797 response to low  $\alpha$ -CD40 stimulation with or without high  $\alpha$ -IgM costimulation. **(H)** Stacked bar  
798 graph of RMSD score between the 2 experimental conditions in (G) shows the addition of high  
799  $\alpha$ -IgM changes both population expansion and generation composition. **(I)** Line graph of  
800 experimental population expansion index is lower in response to costimulation than without  $\alpha$ -  
801 IgM. **(J)** Stacked bar graph of experimental generation composition dynamics in response to  
802 high  $\alpha$ -CD40 stimulation with high  $\alpha$ -IgM costimulation. **(K)** Stacked bar graph of RMSD score  
803 between the 2 experimental conditions in (J) shows the addition of high  $\alpha$ -IgM predominantly  
804 affects population expansion.  
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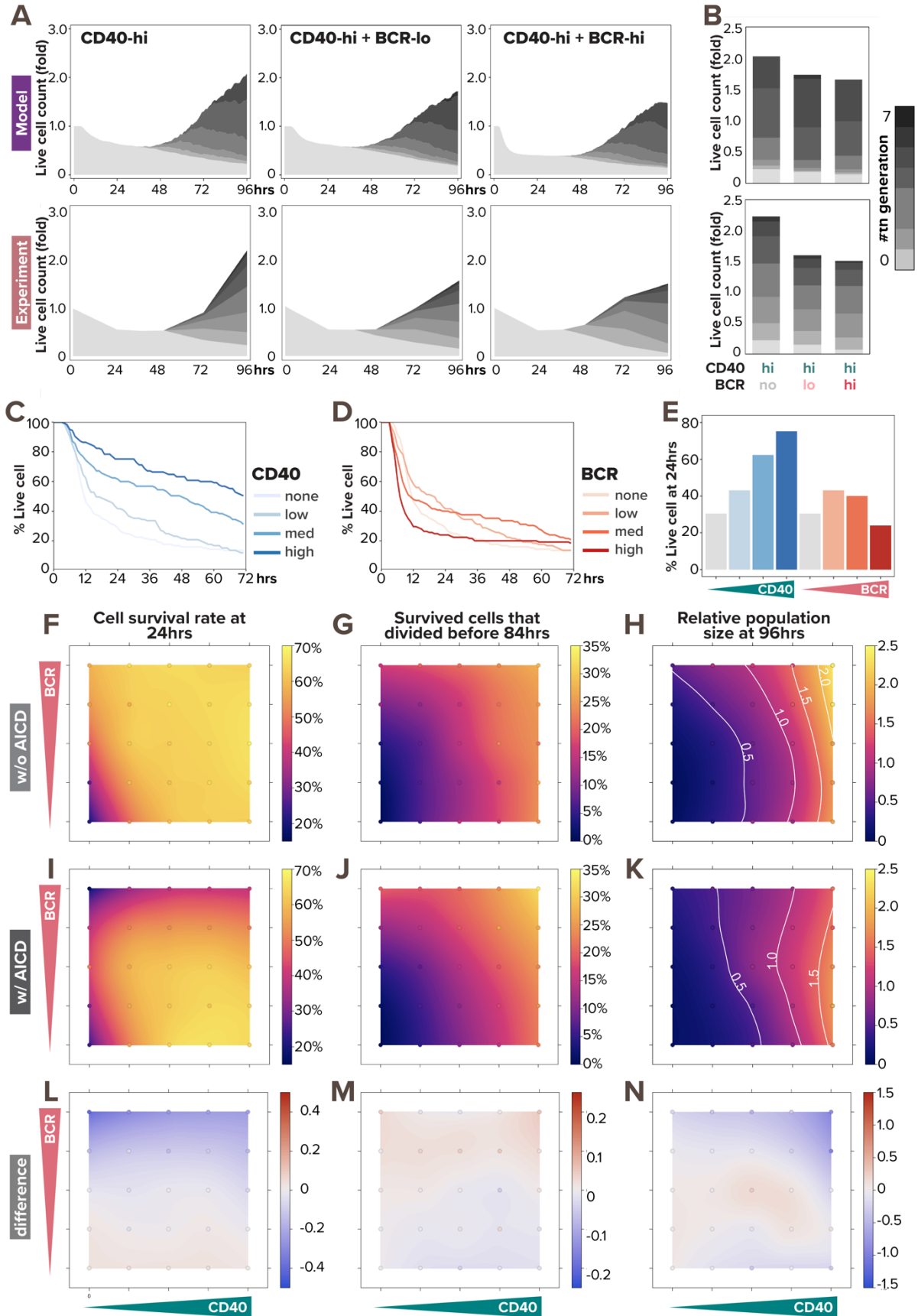




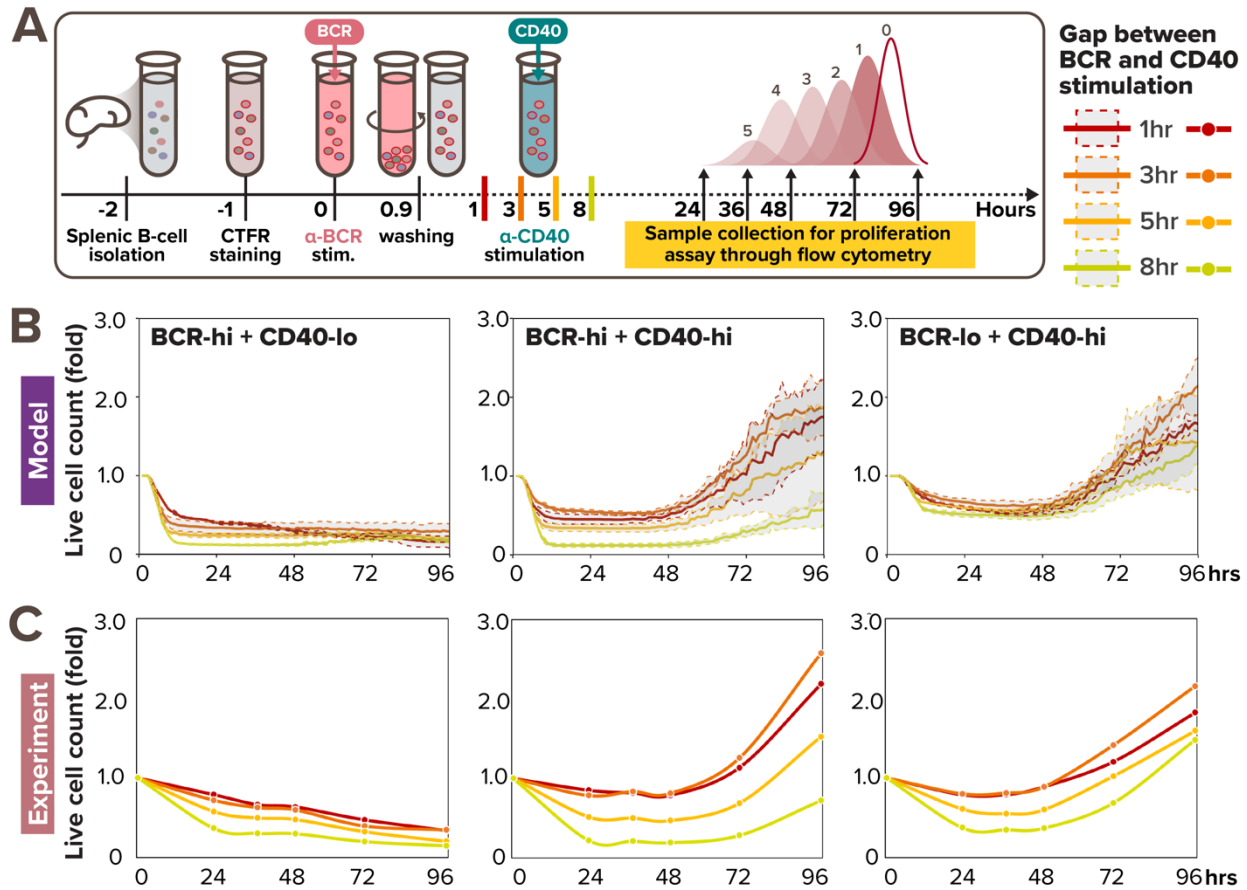
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**Figure 4. BCR-induced caspase-dependent apoptosis and NFκB signaling saturation explains the dose-dependent interaction in costimulation.**

(A) Schematic of updated T-dependent multi-scale B-cell model where activated BCR induces caspase-8 processing. (B) Stacked area plots from model simulation and matching experiment show B-cell population dynamics in response to costimulation with high  $\alpha$ -IgM (0.25nM and 10 $\mu$ g/mL) and high  $\alpha$ -CD40 (30nM and 10 $\mu$ g/mL) with the addition of BCR-induced caspase processing. Each subsequent generation of proliferating cells is indicated with a darker gray. (C) Stacked area plots from model simulation and matching experiment show B-cell population dynamics in response to costimulation with high  $\alpha$ -IgM (0.25nM and 10 $\mu$ g/mL) and low  $\alpha$ -CD40 (6nM and 1 $\mu$ g/mL) with the addition of BCR-induced caspase processing. (D) Bar graph of total RMSDs of the 2 costimulation conditions after the addition of BCR-induced caspase processing compared with before the addition. (E-F) Model simulations (lines) and immunoblot quantification (triangles) show consistent average nuclear RelA and cRel level in naïve B-cells costimulated with (E) high  $\alpha$ -CD40 and high  $\alpha$ -IgM or (F) low  $\alpha$ -CD40 and high  $\alpha$ -IgM at 0, 7, 24, 48hrs, and 72hrs since stimulation onset. Darker colored lines represent average nuclear RelA level from cells that are alive at the timepoint, whereas the shading represents the sample standard deviation of the cells. (G) Schematic of BCR stimulation being pro-proliferative and anti-apoptotic due to NFκB signaling yet pro-apoptotic due to AICD.



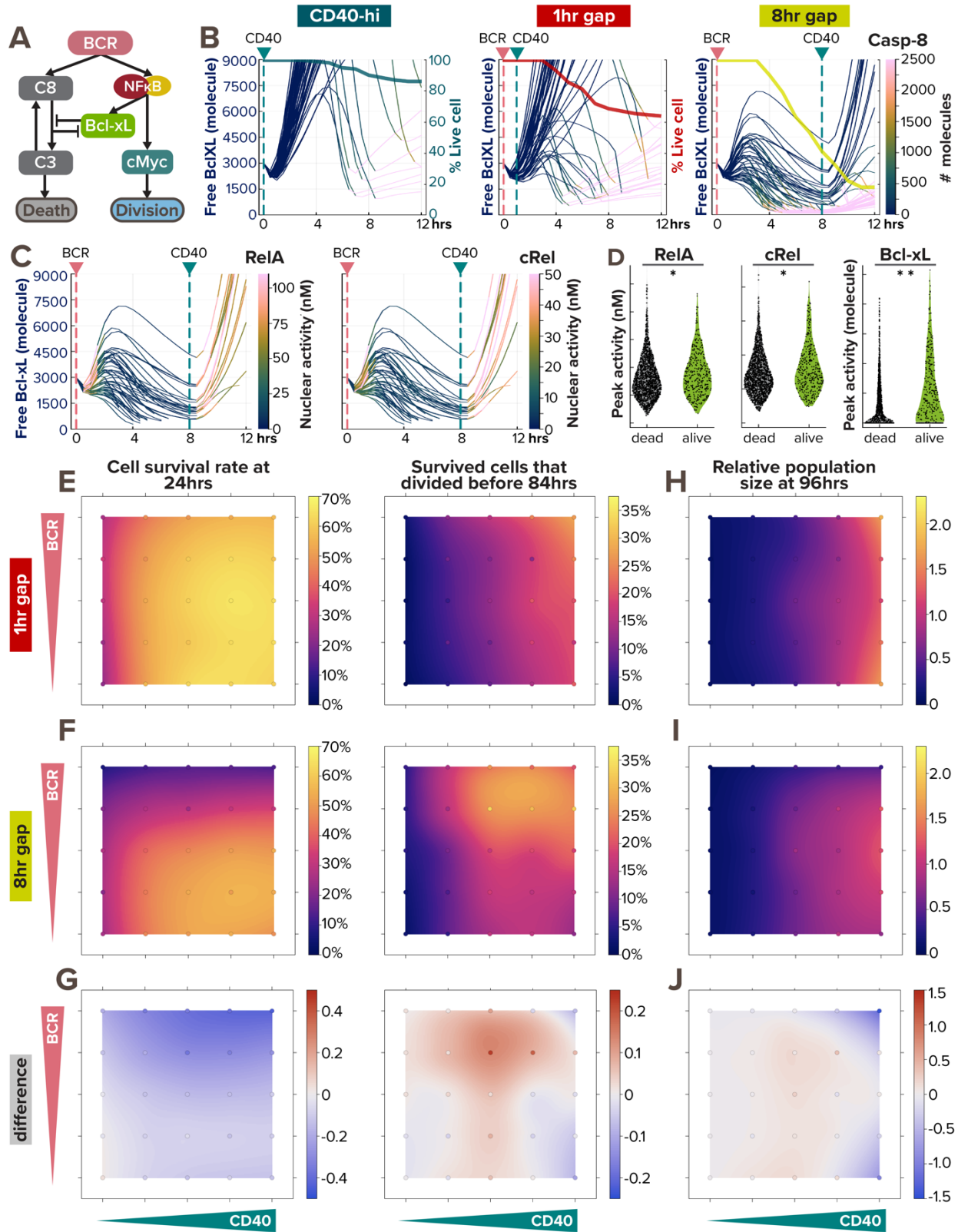
826 **Figure 5. BCR-induced apoptosis prevents BCR stimulation from promoting population**  
827 **growth. (A)** Stacked area plots from model simulations of 1000 virtual B-cells (top) and  
828 matching experiments of 19196 founder B-cells (bottom) show their population dynamics in  
829 response to costimulation with high (30nM and 10 $\mu$ g/mL)  $\alpha$ -CD40 and no (0nM and 0 $\mu$ g/mL),  
830 low (0.005nM and 1 $\mu$ g/mL), or high (0.25nM and 10 $\mu$ g/mL) dose of  $\alpha$ -BCR under the impact of  
831 AICD. Each subsequent generation of proliferating cells is indicated with a darker gray. **(B)**  
832 Stacked bar graph from model simulations (top) and experiments (bottom) show a breakdown of  
833 live B-cells by generation numbers at 96hrs post-stimulation-onset. **(C-D)** Model-simulated  
834 Kaplan-Meier survival curve in response to **(C)**  $\alpha$ -CD40 dose and **(D)**  $\alpha$ -BCR dose shows  
835 distinct pattern regarding monotonicity. **(E)** Bar graph from model simulations show percentage  
836 live B-cells at 24hrs in response to increasing  $\alpha$ -CD40 and  $\alpha$ -BCR doses. **(F-K)** Heatmaps from  
837 model simulations of 1000 virtual B-cells in response to 25 single- or co-stimulation scenarios  
838 (with 5 doses of  $\alpha$ -CD40: 0, 6, 12, 18, and 30nM, and 5 doses of  $\alpha$ -BCR: 0, 0.0005, 0.005, 0.05,  
839 and 0.25nM, combinatorially) show the percentage of survived B-cells at 24 hours under **(F)** no  
840 AICD and **(I)** with AICD, the percentage of proliferative B-cells by 84hrs out of those that  
841 survived **(G)** without AICD and **(J)** with AICD, and the relative population size at 96hrs  
842 (normalized to founder cell population size) **(H)** without AICD and **(K)** with AICD, where white  
843 contour lines represent 0.5-, 1.0-, 1.5-, and 2.0-fold changes. **(L)** Heatmap shows the  
844 differences between (F) and (I). **(M)** Heatmap shows the differences between (G) and (J). **(N)**  
845 Heatmap shows the differences between (H) and (K). In (F-N), the 25 simulated doses are  
846 plotted as colored circles in a scatterplot, whereas the space in between doses is interpolated  
847 with a locally estimated scatterplot smoothing (LOESS) curve.  
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**Figure 6. Sequential BCR-CD40 stimulation reveals a limited window of opportunity to acquire CD40 signal.**

(A) *In vitro* experimental workflow where primary B cells are sequentially stimulated with pulsing  $\alpha$ -BCR, followed by  $\alpha$ -CD40 stimulation 1, 3, 5, or 8hrs later. (B) Line graph from model simulations of 1000 virtual B-cells show their population expansion in response to sequential costimulation with high BCR (0.25nM) and low CD40 (6nM) (left), high BCR (0.25nM) and high CD40 (30nM) (middle), and low BCR (0.005nM) and high CD40 (30nM) (right), colored by the gap between BCR and CD40 stimulation. Each thick colored line represents the average population expansion from 1000 cells, and the shading represents the population standard deviation from the 8 simulations, each with 125 founder cells. (C) Line graph from matching experiments of 19196 founder B-cells show their population expansion in response to sequential costimulation with high BCR (10 $\mu$ g/mL) and low CD40 (1 $\mu$ g/mL) (left), high BCR (10 $\mu$ g/mL) and high CD40 (10 $\mu$ g/mL) (middle), and low BCR (1 $\mu$ g/mL) and high CD40 (10 $\mu$ g/mL) (right), colored by the gap between BCR and CD40 stimulation.

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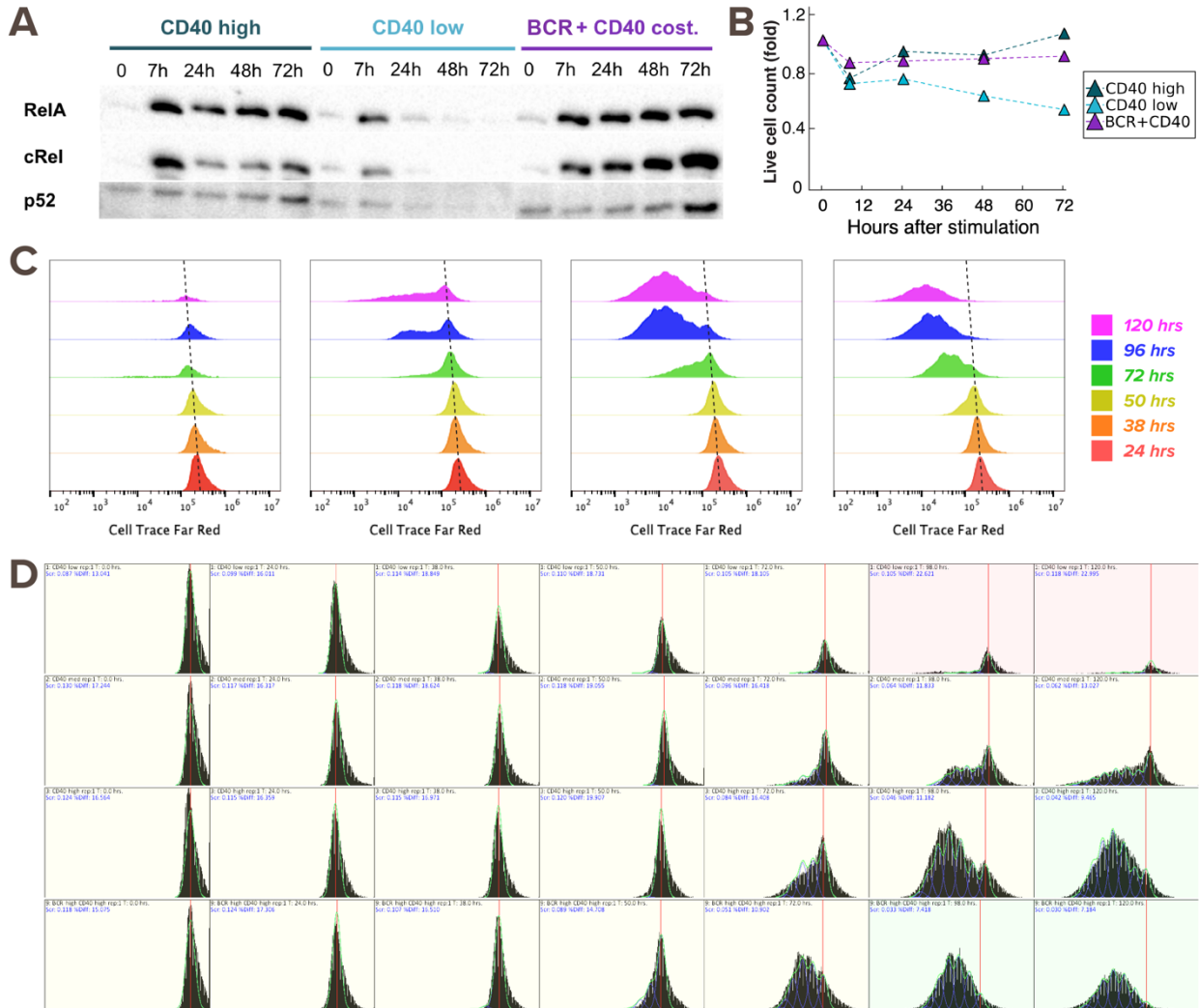
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**Figure 7. BCR-induced anti-apoptotic BclXL protects cells from dying from AICD, as a form of paradoxical signaling.**

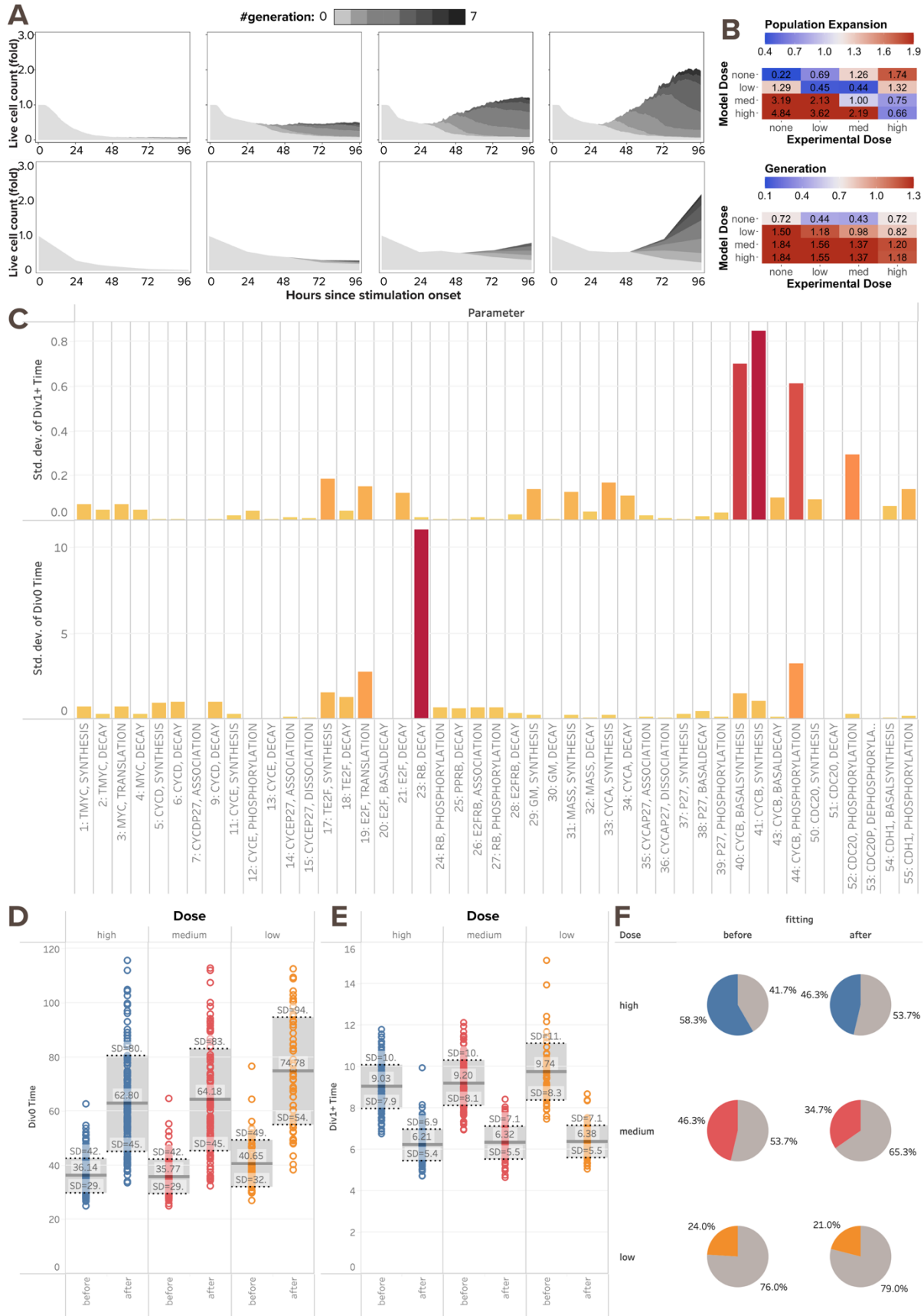
**(A)** Schematic of paradoxical BCR signaling that promotes proliferation, survival, and death of

870 B-cells through cMyc, Bcl-xL, and caspase-8, respectively. **(B)** Line plots of Bcl-xL activity (left  
871 axis) colored by caspase-8 level (color bar) in 50 model-simulated single-cells show the  
872 correlation between Bcl-xL consumption and caspase-8 activity. The thick line overlaid on top is  
873 a Kaplan-Meier survival curve (right axis). The pink and green vertical dashed lines represent  
874 the timing of BCR and CD40 stimulation, respectively. From left to right, the 3 conditions are:  
875 high  $\alpha$ -CD40 costimulation, and sequential  $\alpha$ -BCR and  $\alpha$ -CD40 stimulation with a 1hr and 8hr  
876 gap. When a cell dies, the line continues (and becomes pink due to high caspase-8 level). **(C)**  
877 Line plots of Bcl-xL activity colored by RelA (left) and cRel (right) activity in 50 model-simulated  
878 single-cells demonstrate the correlation between NF $\kappa$ B activation and BclXL upregulation in B-  
879 cells costimulated sequentially with an 8hr gap. When a cell dies, the line discontinues. **(D)**  
880 Violin plot of peak RelA, cRel, and Bcl-xL activity in 2000 model-simulated B-cells in response to  
881 sequential costimulated with an 8hr gap show the differences between cells that died within the  
882 first 12 hours and those that survived. Statistical significance is evaluated using a Mann Whitney  
883 U test, with p-values of 0.0019, 0.0077, and  $<1e-18$ , correspondingly. **(E-J)** Heatmaps from  
884 model simulations of 1000 virtual B-cells in response to 25 single or costimulation scenarios  
885 (with 5 doses of  $\alpha$ -CD40: 0, 6, 12, 18, and 30nM, and 5 doses of  $\alpha$ -BCR: 0, 0.0005, 0.005, 0.05,  
886 and 0.25nM, combinatorially) show the percentage of survived cell at 24 hours after stimulation  
887 onset (left) and the percentage of survived cells that proliferated by 84hrs (right) in **(E)** 1hr  
888 sequential costimulation or **(F)** 8hr sequential costimulation. **(G)** Heatmap highlights the  
889 difference between (E) and (F). **(H-I)** Heatmap of relative population size at 96hrs between **(H)**  
890 1hr sequential stimulation and **(I)** 8hr sequential stimulation shows the biggest difference **(J)** in  
891 the upper right and lower right corners. In (E-J), the 25 simulated doses are plotted as colored  
892 circles in a scatterplot, whereas the space in between doses is interpolated with a locally  
893 estimated scatterplot smoothing (LOESS) curve.  
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895 SUPPLEMENTAL MATERIALS: Supplemental Figures and Materials and Methods  
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897  
898 **Figure S1. Raw experimental data to test the multi-scale B-cell model. (A)** Immunoblot from  
899 experiments with 600K founder B-cells show nuclear RelA, cRel, and p52 levels at 0, 7, 24, 48,  
900 and 72hrs after stimulation with low  $\alpha$ -CD40 (1 $\mu$ g/mL), high  $\alpha$ -CD40 (10 $\mu$ g/mL), or  
901 costimulation with high  $\alpha$ -CD40 and  $\alpha$ -IgM (10 $\mu$ g/mL). **(B)** Line graph of live B-cell count (fold-change)  
902 for each timepoint in (A), to which the samples are adjusted when loading to the gel. The cell count  
903 fluctuation is due to cell death, cell division, and technical error when transferring cells. **(C)** Cell  
904 Trace Far Red (CTFR) dye dilution fluorescence histogram for B-cells stimulated with (from left  
905 to right) low (1 $\mu$ g/mL), medium (3.3 $\mu$ g/mL), and high (10 $\mu$ g/mL) dose of  $\alpha$ -CD40 and  
906 costimulation of high  $\alpha$ -CD40 and  $\alpha$ -IgM (10 $\mu$ g/mL). There is a baseline shift in CTFR  
907 fluorescence by about 2-fold from 24hrs to 120hrs (dotted line), which we adjusted when  
908 deconvolving the cells into each generation. **(D)** Deconvolution of the time courses in (C) into  
909 each generation, where the red line indicates the center of the undivided population of cells, the  
910 blue lines indicate individual proliferation peaks, and the green line represents the model sum.  
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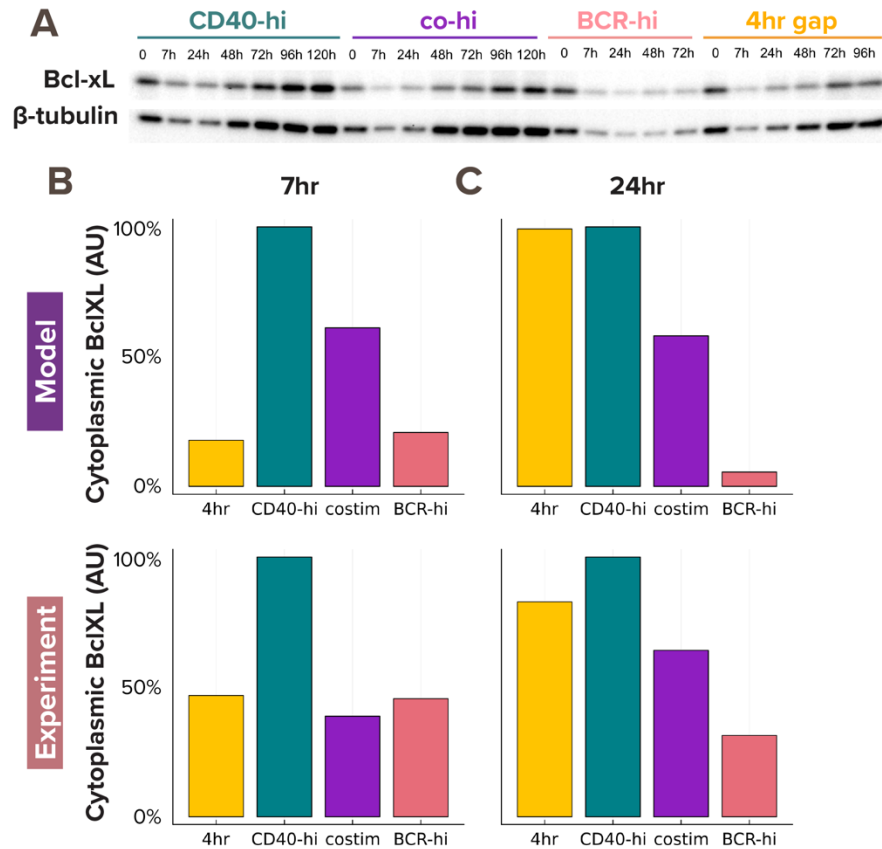




914 **Figure S2. Multi-scale model needs tuning to recapitulate B-cell population dynamics in**  
915 **response to CD40 stimulation.**

916 **(A)** Stacked area plots from model simulations of 1000 virtual B-cells (top) and matching  
917 experiments with 19196 founder B-cells (bottom) show their population dynamics in response to  
918 stimulation with (from left to right) no (0nM and 0 $\mu$ g/mL), low (6nM and 1 $\mu$ g/mL), medium (12nM  
919 and 3.3 $\mu$ g/mL), and high (30nM and 10 $\mu$ g/mL) dose of  $\alpha$ -CD40. Each subsequent generation of  
920 proliferating cells is indicated with a darker gray. **(B)** Heatmap shows RMSD of relative  
921 population size expansion (top) and generation composition (bottom) in matching (diagonal) or  
922 mismatching (off-diagonal) model-and-experiment pairs. Some model doses (medium and low,  
923 respectively), are more deviated from their matching than mismatching experimental doses (high and medium,  
924 respectively), indicating a subpar fit. **(C)** Bar graph from local sensitivity analysis of parameters  
925 in the cell cycle module shows their standard deviations in time to first division (Tdiv0) and time  
926 to later divisions (Tdiv1+). Local sensitivity analysis is achieved by repetitive simulations that  
927 independently scaling each parameter in the cell cycle module by 0.2, 0.33, 0.4, 0.5, 0.66, 1.0,  
928 1.5, 2.0, 2.5, 3.0, or 5.0-fold. 2 out of 55 parameters stand out as the best candidates for tuning  
929 Tdiv0 and Tdiv1+: retinoblastoma (Rb) decay rate and cyclin B (CycB) synthesis rate,  
930 respectively. These parameters were tuned in order to achieve a later and more dose-  
931 responsive Tdiv0, shorter Tdiv1+, and smaller divider percentage. **(D)** Box plots from model  
932 simulations of 300 virtual B-cells show the mean Tdiv0 increases for all doses after parameter  
933 tuning. **(E)** Box plots from model simulations of 300 virtual B-cells show the mean Tdiv1+  
934 decreases for all doses after parameter tuning. **(F)** Pie charts from model simulations of 300  
935 virtual B-cells show the percentage of dividing cells (colored slices) out of all founder cells  
936 decreases for all doses, while maintaining CD40 dose-responsiveness. Grey slices are the non-  
937 dividing founder cells that either die or survive without division.

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**Figure S3. Model-simulated cytoplasmic BclXL level recapitulates experimental results.** (A) Immunoblot from experiments with 600K founder B-cells show cytoplasmic Bcl-xL and  $\beta$ -tubulin levels in response to stimulation with (from left to right) high (10 $\mu$ g/mL) dose of  $\alpha$ -CD40, high  $\alpha$ -CD40 and high  $\alpha$ -BCR, high  $\alpha$ -BCR, and sequential stimulation of high  $\alpha$ -BCR and high  $\alpha$ -BCR with a 4hr delay. (B-C) Bar graphs from model simulations (top) and experiments (bottom) show consistent max-normalized quantification of cytoplasmic Bcl-xL level at (B) 7hrs and (C) 24hrs.

949 **MATERIALS AND METHODS**

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**Key Resources Table**

<b>REAGENT RESOURCE</b>	<b>or</b>	<b>SOURCE</b>	<b>IDENTIFIER</b>
<b>Antibodies</b>			
Rabbit polyclonal anti-RelA		Santa Cruz Biotechnologies	sc-372; RRID: AB_632037
Rabbit polyclonal anti-cRel		Santa Cruz Biotechnologies	sc-71; RRID: AB_2253705
Rabbit polyclonal anti-p50		Santa Cruz Biotechnologies	sc-114; RRID: AB_632034
Mouse monoclonal anti-Bcl-xL		Santa Cruz Biotechnologies	sc-8392; RRID: AB_626739
Rabbit polyclonal anti-p84		Abcam	ab131268
Mouse monoclonal anti- $\beta$ -tubulin		Sigma Aldrich	T5201; RRID: AB_609915
HRP Anti-mouse secondary		Cell Signaling Technology	7076; RRID: AB_330924
HRP Anti-rabbit secondary		Cell Signaling Technology	7074; RRID: AB_2099233
CD40 monoclonal antibody (IC10)		Invitrogen	16-0401-86; RRID: AB_468940
Goat anti-mouse IgM		Jackson ImmunoResearch	115-066-020; RRID: AB_2338579
<b>Chemicals, Peptides, and Recombinant Proteins</b>			
Recombinant murine IL-4		PeproTech	214-14
<b>Critical Commercial Assays</b>			
CellTrace™ Far Red Proliferation Kit		ThermoFisher Scientific	C34564
SuperSignal West		ThermoFisher Scientific	34095, 34580
<b>Experimental Models: Organisms/Strains</b>			
Mouse: C57BL/6		The Jackson Laboratory	JAX: 000664; RRID: IMSR_JAX:000664
<b>Software and Algorithms</b>			
FlowJo V10.8.1		FlowJo LLC	N/A
FlowMax		Shokhirev et al., 2015 (Shokhirev and Hoffmann, 2013)	N/A
Python base:conda v3.7.164-bit		Anaconda v3.0	N/A

REAGENT RESOURCE	or	SOURCE	IDENTIFIER
ImageJ2 v2.9.0			N/A
Julia v1.9.3			N/A
R v4.2.0			N/A

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### Mice

956 Mice were maintained in environmental control facilities at the University of California, Los  
957 Angeles. Female C57BL/6 mice in each replicate experiment were littermates, and were 8-  
958 13 weeks old unless otherwise indicated. Animal work was performed according to University of  
959 California, Los Angeles under approved protocols.

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961

### B cell isolation and culture

962 Spleens were harvested from 8–13-week-old female C57BL/6 mice. Homogenized splenocytes  
963 were incubated with anti-CD43 magnetic beads for 15 min at 4–8°C, washed with MACS buffer  
964 and passed through an LS column (Miltenyi Biotech). The purity of B cells was assessed at >97%  
965 based on B220 staining as described previously (Mitchell et al., 2018). Following isolation, B cells  
966 were stimulated with anti-CD40 (H: 10ug/mL, M: 3.3ug/mL, L: 1ug/mL), IL-4 (H: 20ng/mL, M:  
967 6.6ng/mL, L: 2ng/mL), and anti-IgM (H: 10ug/mL, L: 1ug/mL), unless otherwise specified, and  
968 cultured for 4 days in fresh RPMI-based media at 37°C and 5% CO<sub>2</sub>. All anti-CD40 stimulation  
969 conditions mentioned in the results are stimulated with both anti-CD40 and IL-4 at corresponding  
970 doses.

971

972

### Immunoblot

973 Cells were harvested from culture plates, washed in 1mL PBS and counted on a CytoFlex flow  
974 cytometer (CytoFLEX, Beckman Coulter), prior to preparing lysates for protein content analysis.  
975 Due to varying cell sizes and numbers over time as a result of growth and proliferation, an equal  
976 number of cells (as opposed to equal protein amounts) per sample was analyzed in each  
977 immunoblot. In cases where nuclear fractions were required to be separated, cells were first lysed  
978 in CE buffer on ice, followed by vortexing and centrifugation, and the supernatant containing the  
979 cytoplasmic fraction was removed. Nuclei were then lysed by 3 repeated freeze-thaw cycles  
980 between 37C water and dry ice, followed by centrifugation to clear the lysate of nuclear debris,  
981 after which the supernatant containing the nuclear fraction was harvested.

982 For immunoblotting, lysates were run on 4%–15% Criterion TGX pre-cast polyacrylamide gels  
983 (Bio-Rad), and transferred on to PVDF membranes using wet transfer. The following antibodies  
984 were used to identify the proteins of interest: RelA, cRel, Bcl-xL, p84 (loading control for nuclear  
985 lysates), and b-tubulin (loading control for cytoplasmic and whole cell lysates). Antibody details  
986 are given in the Resources table, and concentrations used were 1:5,000 for RelA and cRel,  
987 1:1,000 for Bcl-xL, 1:10,000 for p84, and 1:10,000 for b-tubulin. Protein bands were detected  
988 using the Bio-Rad ChemiDoc XRS System, with a 10:1 mixture of the SuperSignal West Pico and  
989 Femto Maximum Sensitivity Substrates (Thermo Scientific) applied to detect chemiluminescence  
990 released by HRP-labeled secondary antibodies.

991 RelA, cRel, and Bcl-xL bands were quantified by measuring mean gray value using ImageJ2,  
992 deducting background value per lane (measured by a box of the same size directly below the  
993 target protein band), and normalizing intensities to the 0hr baseline.

994

995

### Media and buffer compositions

996 B cell media: RPMI 1650 (Gibco) supplemented with 100 IU Penicillin, 100 µg/ml Streptomycin, 5  
997 mM L-glutamine, 20 mM HEPES buffer, 1mM MEM non-essential amino acids, 1 mM Sodium  
998 pyruvate, 10% FBS, and 55 µM 2-Mercaptoethanol.

999 MACS buffer: Phosphate buffered saline, (pH 7.4) and 2% bovine serum albumin.

1000 CE Buffer: 50 mM HEPES-KOH pH 7.6, 140 mM NaCl, 1 mM EDTA, 0.5% NP-40, freshly  
1001 supplemented with EDTA-free protease inhibitors (5mM DTT, 1mM PMSF).

1002 NE Buffer: 10 mM Tris-HCl, pH 8.0, 200 mM NaCl, 1 mM EDTA, freshly supplemented with  
1003 EDTA-free protease inhibitors (5mM DTT, 1mM PMSF).

1004

### 1005 **Measurement of generation-specific B cells by CTFR staining**

1006 B cells were stained with Cell Trace Far Red (CTFR) using CellTrace Far Red Cell Proliferation  
1007 Kit (ThermoFisher Scientific, # C34564) as described by the manufacturer protocol. Briefly, 2M  
1008 cells were resuspended in 1mL RT PBS and incubated with 1µL CTFR for 25 min at RT with  
1009 rotation. Cells were washed by centrifugation, resuspension in 1mL RPMI with 10% FBS, and  
1010 incubation for 10 min at RT. The washing steps were repeated 2 more times. CTFR labeled cells  
1011 were treated with anti-CD40, IL-4, and / or anti-IgM for 96hrs as described above. The cells were  
1012 harvested at indicated time points and acquired on the CytoFlex flow cytometer (CytoFLEX,  
1013 Beckman Coulter). The cells were gated based on forward scatter (FSC) and side scatter (SSC)  
1014 to identify live single cells. Doublets were then excluded from the analysis using FSC area and  
1015 height. To deconvolve the cells into different generations based on dilution of CTFR, we used the  
1016 Proliferation Modeling feature on FlowJo V10.8.1. Specifically, generation-0 cells were gated as  
1017 “Undivided” in the APC-A channel according to the unstimulated control and 24hr samples, and  
1018 the number of peaks were set based on visual estimation and then further adjusted based on the  
1019 Peak Ratio and Root Mean Squared outputs to optimize curve fitting.

1020

### 1021 **Computational modeling of the T-dependent receptor signaling pathway**

1022 The mathematical model of T-dependent (TD) B cell stimulation was developed in two parts.  
1023 First, we expanded the previously published BCR-signaling ODE model (Shinohara *et al.*, 2014;  
1024 Inoue *et al.*, 2016) by including the BCR receptor antigen binding (Fig. 1A left side), and scaled  
1025 the parameters to match the units ( $\text{nM}^{-1} \text{hr}^{-1}$ ) in the rest of our model. The Shinohara and Inoue  
1026 models prescribed a signal function for the CBM complex, a downstream adaptor for BCR  
1027 receptor. We bridged the gap between antigen concentration and CBM signaling with a few  
1028 additional ODE equations, and tuned these additional parameters (Table 1) such that the  
1029 signaling dynamics matched the previous version:

$$1030 \quad \frac{d}{dt} [ANTIGEN] = -\varphi_1 * [ANTIGEN] - \varphi_4 [ANTIGEN] * [BCR] * C_{c2m} + \varphi_5 * [ABCR] * C_{c2m}$$

$$1031 \quad \frac{d}{dt} [BCR] = \varphi_2 - \varphi_3 * [BCR] - \varphi_4 * [ANTIGEN] * [BCR] + \varphi_5 * [ABCR]$$

$$1032 \quad \frac{d}{dt} [ABCR] = \varphi_4 [ANTIGEN] * [BCR] - \varphi_5 * [ABCR] - \varphi_6 * [ABCR]$$

1033 where  $[ANTIGEN]$ ,  $[BCR]$ , and  $[ABCR]$  are the concentrations of the antigen, BCR, and their  
1034 complex;  $\varphi_i$ ,  $i = 1,2,3, \dots$ , are the reaction constants (index are listed in Table 1);  $C_{c2m} = 0.01$  is  
1035 a scaling factor for external ligands like ANTIGEN to convert cellular concentration to media  
1036 concentration. In this model,  $[ANTIGEN]$  is the model input corresponds to experimental  
1037 stimulation  $\alpha$ -BCR. As output of the BCR receptor module,  $[ABCR]$  regulates CBM complex  
1038 activation (Fig. 1A left side).

1039

1040 Next, we abstracted the CD40 model from its known signaling pathway (Elgueta *et al.*, 2009;  
1041 Akiyama, Shinzawa and Akiyama, 2012) in a parsimonious way. As mentioned in the  
1042 discussion, to avoid the complexity of combinatorial biochemical reactions among the TRAF  
1043 complexes, we used TRAF3 to represent the TRAF2-TRAF3 complex that constitutively inhibits

1044 the noncanonical NFκB pathway, and TRAF6 to represent the TRAF1-TRAF2, TRAF3-TRAF5,  
1045 and TRAF6-TRAF2 complexes that all activate the canonical NFκB pathway.

$$1046 \quad \frac{d}{dt}[CD40L] = -\varphi_{11} * [CD40L] - \varphi_{14}[CD40L] * [CD40R] * C_{c2m} + \varphi_{15} * [CD40LR] * C_{c2m}$$

$$1047 \quad \frac{d}{dt}[CD40R] = \varphi_{12} - \varphi_{13} * [CD40R] - \varphi_{14} * [CD40L] * [CD40R] + \varphi_{15} * [CD40LR]$$

$$1048 \quad \frac{d}{dt}[CD40LR] = \varphi_{14}[CD40L] * [CD40R] - \varphi_{15} * [CD40LR] - \varphi_{16} * [CD40LR]$$

$$1049 \quad \frac{d}{dt}[TRAF6_{off}] = -\varphi_{17} * [CD40LR] * [TRAF6_{off}] + \varphi_{18} * [TRAF6_{on}]$$

$$1050 \quad \frac{d}{dt}[TRAF6_{on}] = \varphi_{17} * [CD40LR] * [TRAF6_{off}] - \varphi_{18} * [TRAF6_{on}]$$

$$1051 \quad \frac{d}{dt}[TRAF3] = \varphi_{19} * [TRAF3] - \varphi_{20} * [TRAF3] - \varphi_{21} * [CD40LR] * [TRAF3]$$

1052 The subsequent kinases that further relay the receptor signal to NFκB signaling are TAK1 (for  
1053 TRAF6) and NIK (for TRAF3). We used a Hill function for TRAF3-induced degradation of NIK to  
1054 abstract a more complicated complex formation process:

$$1055 \quad \frac{d}{dt}[NIK] = \varphi_{35} - \varphi_{36} * [NIK] - \varphi_{37} * [NIK] * \frac{[TRAF3]^2}{[TRAF3]^2 + 0.5^2}$$

1056  
1057 To generate heterogeneous cell responses, the receptor parameter values were distributed in  
1058 each virtual B-cell the same way as the NFκB signaling module in previous publication, where  
1059 synthesis, degradation, association, dissociation rates were drawn from a normal distribution  
1060 with mean values from the standard parameter set (Table 1) and CV of 11.2% (Mitchell *et al.*,  
1061 2018). These ODEs (with 37 parameters and 12 species) were solved using the Tsit5 solver  
1062 algorithm from the DiffEq.jl package in Julia, with an absolute error tolerance of 1e-5 and  
1063 relative error tolerance of 1e-3. All simulations were carried out on an Ubuntu server with 64  
1064 threads, 2.1 to 3.7 GHz speed, and 384 GB RAM.

## 1065 1066 **Multiscale modeling coupling signaling network and B cell proliferation**

1067 The receptor-extended TD model constructed above was combined with a published MATLAB  
1068 model of B cell proliferation to create a multiscale model capable of simulating the division and  
1069 death of a population of individual B-cells upon TD stimulation. The B cell model integrates a  
1070 biophysically accurate model of canonical NFκB signaling (with about 300 parameters and 61  
1071 species) with models of the cell cycle (with 52 parameters and 23 species) and apoptosis (with  
1072 117 parameters and 59 species) (Shokhirev *et al.*, 2015; Mitchell *et al.*, 2018) to create a  
1073 multiscale model capable of simulating the division and death of a population of individual cells  
1074 upon T-dependent stimulation. Cleaved PARP (cParp) in the apoptosis model and cadherin-1  
1075 (Cdh1) concentration thresholds in the cell cycle model triggered virtual B-cell death and  
1076 division, respectively. We translated the model from MATLAB into Julia 1.9.3 for faster  
1077 execution. All reactions and parameters within the NFκB, apoptosis and cell cycle networks  
1078 were maintained and distributed as described by Mitchell *et al.* (Mitchell *et al.*, 2018), except for  
1079 2 parameters in the cell cycle network that were changed to reduce the discrepancy between  
1080 CD40 and CpG-induced proliferative response (Fig. S2, see more details in “Local sensitivity  
1081 analysis to tune CD40-activated cell fates” section of the Methods).

1082  
1083 Separate modules of the multi-scale model were employed when simulating for different  
1084 purposes. For the NFκB dynamics in figure 1, only the receptor-NFκB model was used for the  
1085 simulation, and the cell fate modules (apoptosis and cell cycle) were excluded. For Fig. S2C-F  
1086 when we tuned the CD40-activated cell fates, the cell death module was excluded to enable  
1087 faster turnaround for parameter tuning in the cell cycle module. For Fig. 5B-D, the cell cycle

1088 module was excluded to isolate the effects of BCR signaling on cell survival. All the other model  
1089 simulation used the full multi-scale model. When we reported the population trajectory of NFκB  
1090 activity in Fig. 1E-H, all 1000 cells contributed to the mean and standard deviation, but in Fig.  
1091 4G-J, only cells that are alive at each timepoint contributed to the mean and standard deviation.

1092  
1093 All of the code to run the model simulations and plot the figures is provided on GitHub  
1094 (<https://github.com/helengracehuang/BCR-CD40-integration>). For each virtual B-cell with its own  
1095 set of parameters, we ran the model in two phases to first identify the steady state, and then  
1096 simulate the dynamic time course upon stimulation, with initial states from this steady state. The  
1097 steady state was solved using Julia's steady state Tsit5 solver with an absolute error tolerance  
1098 of 1e-5 and relative error tolerance of 1e-3. The simulation time for which the given ODE reach  
1099 steady state was limited within 800hrs.

### 1100 1101 **Computational modeling of the BCR-induced cell death pathway**

1102 Since we found α-BCR stimulation had an NFκB-independent anti-survival effect that overrides  
1103 its NFκB-dependent pro-survival effect (Fig. 3), we decided to resolve this difference by  
1104 modifying the multi-scale model. It was reported that ligation of the BCR induces cell death in  
1105 some B cells (Graves, Craxton and Clark, 2004) due to activation of Bcl-2 Interacting Mediator  
1106 of cell death (Bim) (Gao, Kazama and Yonehara, 2012), caspase-2 or -8 (Chen *et al.*, 1999),  
1107 mitochondrial dysfunction (Akkaya *et al.*, 2018) or more. Based on these signaling mechanisms  
1108 that may mediate activation-induced cell death (AICD) in B-cells and the available species in the  
1109 existing cell death module, we revised the cell death module of the T-dependent multi-scale B-  
1110 cell model to include a simplified pathway from activated BCR to caspase-8 processing (Fig.  
1111 4A):

$$1112 \quad \frac{d}{dt}[PC8] = \left( \text{original} \frac{d}{dt}[PC8] \right) - \varphi_{C8,AICD} * [PC8] * [ABCR]$$

$$1113 \quad \frac{d}{dt}[C8] = \left( \text{original} \frac{d}{dt}[C8] \right) + \varphi_{C8,AICD} * [PC8] * [ABCR]$$

1115 where [PC8], [C8], and [ABCR] are the concentrations of the pre-caspase-8, caspase-8, and  
1116 activated BCR; *original*  $\frac{d}{dt}[PC8]$  and *original*  $\frac{d}{dt}[C8]$  are the original differential equations for  
1117 pre-caspase-8 and caspase-8 in Mitchell *et al.*, 2018, abbreviated to highlight the revision we  
1118 made;  $\varphi_{C8,AICD}$  was tuned to be 0.00021 according to experimental data of BCR-CD40  
1119 costimulation versus CD40-only stimulation conditions (Fig. 4B-C).

1120  
1121 Simulations prior to figure 4 and Fig. 5F-H did not include this BCR-induced cell death pathway.  
1122 Fig. 4B-F, Fig. 5A-E, I-K, and figures 6 and 7 were all simulated with the modified caspase-8  
1123 equations.

### 1124 1125 **Model fit evaluation**

1126 Root-mean-squared deviation (RMSD) were calculated on the population dynamics between  
1127 model simulation and experimental results (Fig. 2F, S2B, 3E, 3F, and 4D) and between two  
1128 experimental conditions (Fig. 3H,K) in the same manner. Two RMSD scores, one for population  
1129 expansion index (Fig. 2D), and the other for generational composition (Fig. 2E) between each  
1130 pair of model and experimental outputs at each experimental timepoint (0, 24, 36, 48, 72, and  
1131 96hrs) were calculated.

1132  
1133 For the RMSD on generational composition:

1134

$$RMSD_{gen} = \sqrt{\sum_{i=1}^5 \sum_{j=0}^6 \left( \frac{n_{i,j}}{N_i} - \frac{\hat{n}_{i,j}}{\hat{N}_i} \right)^2}$$

1135 Where  $i$  is the  $i$ -th timepoint of the experimental measurement (i.e.  $i = 1$  corresponds to the  
 1136 measurement at 24 hours, followed by 36, 48, 72, and 96 hours), and  $j$  is the generation  
 1137 number, ranging from generation 0 to 6 corresponding to founder cells to cells that have divided  
 1138 6 times.  $n_{i,j}$  thus means the number of live cells in generation  $j$  and timepoint  $i$  in the  
 1139 experimental data, while  $\hat{n}_{i,j}$  is the corresponding live cell number in generation  $j$  and timepoint  
 1140  $i$  in model simulation. Additionally,  $N_i = \sum_{j=0}^6 n_{i,j}$  represents the total number of live cells at  
 1141 timepoint  $i$  in the experimental data, and  $\hat{N}_i = \sum_{j=0}^6 \hat{n}_{i,j}$  represents the corresponding total live  
 1142 cell number in model simulation.  $\frac{n_{i,j}}{N_i}$  and  $\frac{\hat{n}_{i,j}}{\hat{N}_i}$  are thus the generation decomposition ratios at  
 1143 each time point for experimental data and simulation data, respectively.

1144

1145 For population expansion, the RMSD is composed of two parts, one normalized to population  
 1146 size at 0 hour ( $N_0$ ) and one normalized to the population size at 24 hours ( $N_1$ ) to account for  
 1147 unpredictable mechanical cell death (which typically occur within the first few hours) as a form of  
 1148 technical error in experiments. Both RMSD scores are then normalized to the number of  
 1149 timepoints (5 timepoints for 0hr normalization, and 4 timepoints for 24hr normalization) and the  
 1150 maximum population expansion so that different amount of population expansion at different  
 1151 doses are evaluated on the same scale:

1152

$$RMSD_{pop\_exp} = \sqrt{\frac{\sum_{i=1}^5 \left( \frac{N_t}{N_0} - \frac{\hat{N}_t}{\hat{N}_0} \right)^2}{5 \cdot \max_{i=1,\dots,5} N_i}} + \sqrt{\frac{\sum_{i=2}^5 \left( \frac{N_t}{N_1} - \frac{\hat{N}_t}{\hat{N}_1} \right)^2}{4 \cdot \max_{i=2,\dots,5} N_i}}$$

1153

### 1154 Local sensitivity analysis to tune CD40-activated cell fates

1155 Due to the discrepancy between CD40 and CpG-induced proliferative response, we quantified  
 1156 several key variables in the dye dilution data that determined the population dynamics with  
 1157 FlowMax (Shokhirev and Hoffmann, 2013). After fitting FlowMax model to the experimental  
 1158 data, we quantified the time to first division (Tdiv0), time to later divisions (Tdiv1+), and the  
 1159 fraction of generation 0 cells that respond by dividing (F0) in response to low, medium, and high  
 1160 CD40 doses. In all CD40 doses, the average Tdiv0 is much later and more dose-specific (68.5  
 1161 to 76.9hrs since stimulation onset for high to low dose of CD40) than what the model predicted  
 1162 (36.1 to 40.6hrs). On the other hand, the average Tdiv1+ of the CD40 experimental data were  
 1163 mostly shorter than predicted by the model (Table S1 Exp vs Model(1), Fig. S2A), and the  
 1164 proportion of dividers was lower, indicated by a larger amount of cells in generation 0 at 96hrs in  
 1165 Fig. S2A and a smaller F0 quantified by FlowMax than the model predicted (Table S2 Exp vs  
 1166 Model(1)).

1167

1168 To improve model fit, we identified locally sensitive parameters in the cell cycle module that  
 1169 contribute to Tdiv0 and Tdiv1+ by calculating the standard deviation in division times when  
 1170 scaling each parameter by 0.2, 0.33, 0.4, 0.5, 0.66, 1.0, 1.5, 2.0, 2.5, 3.0, or 5.0-fold. 2 out of 55  
 1171 parameters stood out as the best candidates for tuning Tdiv0 and Tdiv1+: retinoblastoma (Rb)  
 1172 decay rate and cyclin B (CycB) synthesis rate, respectively (Fig. S2C). Rb decay rate was tuned  
 1173 to be 10% of the original value, whereas CycB was tuned to be 1.8-fold the original value to  
 1174 achieve a later and more dose-responsive Tdiv0, shorter Tdiv1+, and smaller divider  
 1175 percentage (Fig. S2D,E,F).



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A simulation of 300 cells with distributed parameters before and after parameter tuning showed that mean Tdiv0 for dividers increased from 36.14 hours to 62.80 hours for high dose of CD40 stimulation, and from 40.65 hours to 74.78 hours for low dose, achieving both a later and more dose-responsive Tdiv0, resulting in much more agreement with the FlowMax output based on experimental data (Table S1, left 3 columns). The mean Tdiv1+ for dividers decreases from around 9 hours to 6 hours for all doses, which is in concordant with high dose of CD40, but in less agreement with medium and low doses (Table S1, right 3 columns). Table S2 also showed the percentage of dividers out of all founder cells decreased for all doses, while maintaining CD40 dose-responsiveness.

**Table S1. Experimental vs. Model proliferation time before (1) & after (2) tuning**

Condition	EXP Tdiv0	MODEL(1) Tdiv0	MODEL(2) Tdiv0	EXP Tdiv1+	MODEL(1) Tdiv1+	MODEL(2) Tdiv1+
CD40 high	68.5	36.1	60.7	6.1	9.0	6.2
CD40 medium	68.6	35.7	66.2	7.8	9.2	6.0
CD40 low	76.9	40.6	79.4	35.2	9.7	6.3

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**Table S2. Experimental vs. Model divider percentage before (1) & after (2) tuning**

Condition	EXP F0	MODEL(1) F0	MODEL(2) F0
CD40 high	46.8%	58.3%	46.3%
CD40 medium	18.4%	46.3%	34.7%
CD40 low	4.4%	24.0%	21.0%

1190