



Research article

Dust ion acoustic solitons in a complex dusty plasma system with an adiabatic state

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ABSTRACT

In this theoretical work nonlinear behavior of dust ion acoustic solitary waves (SWs) has been investigated, and then the effect of the adiabatic change on them has been observed. The complex plasma system consists of inertial positive and negative ions, Maxwell's electrons, and positively and negatively charged stationary dust particles. The effects of dust polarity on the dust ion acoustic SWs have also been observed. Using the reductive perturbation method, we first derive K-dV equation which lets to analyze both the positive (bright) and negative (dark) solitons in a very limited region. After that mK-dV equation has been derived, and this let to analyze positive soliton for a large region, but cannot show the negative soliton. Finally, the Gardner equation has been derived employing the same method, through which we were able to analyze both the positive and negative solitons for a large region. It has been found that both the positive and negative solitons significantly depend on the mass number density, ion number density, and dust polarity in the adiabatic and isothermal system.

1. Introduction

For the very first time, Shukla and Silin [1] have theoretically proved the existence of the low-frequency dust-ion-acoustic (DIA) waves in a dusty plasma system. Four years later, Barkan et al. [2] have experimentally verified the existence of dust-ion-acoustic waves differ from usual ion-acoustic waves [3]. The linear properties of the DIA waves in dusty plasma are now well understood [4, 5, 6]. The nonlinear structures associated with the DIA waves are solitary waves [7,8], shock waves [9, 10, 11], double layers [12], envelope solitons [13,14], etc.

In this work particularly DIA SWs have been searched. These waves have received a great deal of interest in understanding the basic properties of localized electrostatic perturbations in space [15,16] and laboratory dusty plasmas [4,5,8]. These DIA solitary waves (SWs) have been investigated by several authors [7, 8, 9, 10, 11, 12] during the last few years. F. Deeba et al. [17] have theoretically shown the existence of dust-ion-acoustic waves in dusty plasma, consisting of inertial ions, Boltzmann electrons, and negatively charged stationary dust, around a critical limit, applying the reductive perturbation method to the Gardner approach. F. Sijo et al. have shown the oblique solitary waves in

five-component complex plasma [19]. Sijo Sebastian et al. in 2014 also worked with a five-component complex plasma system [20].

Generally, in most space and astrophysical plasma systems, dust is ubiquitous. These dust grains often contain colonized charged particles inside that make them charged dust. The polarity of the dust grain significantly modifies the dynamics of the waves by changing the charge density distribution of the system. Because the sizes of the dust grains are much larger than the ions and electrons and they contain more charges, their electrostatic attraction force, and field significantly alter the compressive and rarefactive motions of the ions and the electrons.

Besides the polarity of the constituent, the dynamics of the waves in the plasma system may also depend on the thermal state of the system. Because space is always happening with a lot of events like formations or deformations of stars and galaxies, solar storms, radiations, it is obvious that many spaces go through the adiabatic changes frequently. As the thermal state has a great impact on the motion of the charged particle, it will also change the wave dynamics significantly. Although few studies have been done to analyze the impact of the charged particle and the adiabatic state, to the best of our knowledge no detailed studies have been done so far to analyze the combined effect of the dust polarity and the thermal state and compare the outcomes using different models.

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In this paper, we have analyzed the impacts of the dust polarity and adiabatic changes on the solitary wave profile by deriving and using the solutions of three different model equations: K-dV (Korteweg–de Vries), mK-dV (modified Korteweg–de Vries), and Gardner. As the positive soliton represents an increase in the potential of the medium which is why it is also called a bright soliton. The negative soliton decreases the potential of the medium which often looks like a hole in the space and called dark soliton.

The K-dV equation, which is a famous mathematical model to analyze wave profiles in not only plasma but also other fluid systems like water, lets us analyze both the positive and negative solitons but in a very limited region. The extension of K-dV equation leads us to the mK-dV equation, which lets to analyze positive soliton for a large region but does not show any negative soliton profile. Hence, finally, the Gardner equation has been derived employing the same method, through which lets us analyze both the positive and negative solitons for a large region. The effect of dust polarity, dust number density (μ_d), and the thermal state on the solitons have also been observed. It has been seen that the charge density, the dust polarity, and the thermal state significantly vary the width, amplitude, and the polarity of the solitons. The point of separation for positive and negative K-dV solitons has been observed successfully along with the variation of solitons with a nonlinear coefficient (α).

The manuscript has been organized as follows. At first, the model equations have been provided in Sec. 2. Then the K-dV (Korteweg-de Vries) equation and mK-dV equation have been derived in Sec.3 and Sec. 4 respectively. Finally, the standard Gardner (sG) equation has been derived by using the reductive perturbation method in Sec. 5. A brief discussion has finally been given in Sec. 6.

2. Model equations

The nonlinear propagation of DIA waves has been considered in an unmagnetized collisionless complex dusty plasma system consisting of inertial positive and negative ion, Maxwell's electron, and arbitrarily charged stationary dust. For one dimensional multi-ion dust ion acoustic waves (phase speed is much smaller (larger) than the electron (ion) thermal speed), the nonlinear dynamics are governed by:

$$\partial n_s / \partial t + \partial n_s u_s / \partial x = 0, \tag{1}$$

$$\partial u_i / \partial t + u_i \partial u_i / \partial x = - \partial \psi / \partial x - (\delta_i / n_i) (\partial p_i / \partial x), \tag{2}$$

$$\partial u_n / \partial t + u_n \partial u_n / \partial x = 1 / \mu (\partial \psi / \partial x - \delta_n / n_n \partial p_n / \partial x), \tag{3}$$

$$e \partial \psi / \partial x - 1 / n_e \partial p_e / \partial x = 0, \tag{4}$$

$$\partial p_s / \partial t + u_s \partial p_s / \partial x + \gamma p_s \partial u_s / \partial x = 0, \tag{5}$$

$$\partial^2 \psi / \partial x^2 = [1 - \mu_n + \mu_d] \exp \psi + \mu_n n_n - \mu_d - n_i, \tag{6}$$

where n_s is the number density with $s = n, i, e, d$ of negative ion, positive ion, electron or stationary dust, u_s is the fluid speed of s , m_j is the positive (when $j = i$) or negative (when $j = n$) ion mass, Z_d is the number of electrons residing on the dust grain surface, e is the magnitude of the electron-charge (q), ϕ is the electrostatic wave potential; n_{s0} , n_{j0} , and n_{d0} are the equilibrium values of n_s , n_j , and n_d respectively, i.e., n_s , n_j , and n_d are the number densities normalized by n_{s0} , n_{j0} , and n_{d0} respectively, p_i is the pressure of species i , γ is an adiabatic index, x is the space variable, and t is the time variable. We applied the conditions, $u_s = c_i u_s$, $c_i = (K_B T_{eo} / m_i)^{1/2}$, K_B is Boltzmann constant, T_{eo} normalized electron temperature, m_i the ion mass. $n_s = n_{s0} n_s$, $\phi = (K_B T_{eo} / e) \psi$, $p_s = n_{s0} K_B T_{s0} p_s$, $s = n, i, e$ and d of negative ion, positive ion, electron, stationary dust, $x = \lambda_{Dm} X$, x is the space variable normalized by X . $\lambda_{Dm} = (K_B T_{eo} / 4 \pi n_{i0} e^2)^{1/2}$, $\partial / \partial t = \omega_{pi} \partial / \partial t$, $t = (1 / \omega_{pi}) t$, $\omega_{pi} = (4 \pi n_{i0} e^2 m_i)^{1/2}$, $\partial / \partial x = 1 / \lambda_{Dm} \partial / \partial x$, and $\omega_{pi} \lambda_{Dm} = c_i$, c_i is the ion acoustic speed, ω_{pi} is ion plasma frequency, and λ_{Dm} is the Debye radius.

We have also used the condition; $l = -1$ for negative dust, $l = 1$ for positive dust in Eq. (6). In Eq. (4) mass of electron is considered zero. We choose the ratios are $\delta_i = T_{io} / T_{eo}$, $T_{io}(T_{eo})$ is the normalized temperature of positive (negative) ion, $\delta_n = T_{no} / T_{eo}$, $\mu = m_n / m_i$, m_n (m_i) is the mass of negative ion (positive ion), $\mu_n = n_{no} / n_{io}$, and $\mu_d = Z_d n_{d0} / Z_s n_{s0}$. At equilibrium, $n_n = 1$, $n_i = 1$, $n_e = n_{eo}$, $n_n = n_{no}$, and $n_i = n_{io}$; n_e , n_i , n_n and n_d are the number densities normalized by n_{s0} , where $s = n, i, e, d$ of negative ion, positive ion, electron, and stationary dust respectively.

In Eq. (1) the source and sink terms has been neglected which may arise due to dust-plasma interaction, ionization, recombination, etc., and in Eq. (2) we have also neglected all collisional terms that may arise due to the collisions between plasma and dust particles or vise-versa. These assumptions are valid as long as the collision frequency or the ionization/recombination rates are much less than the frequency of the dust ion acoustic waves under the consideration, and these have been investigated by many authors [1,18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30] to study linear [1] and nonlinear [18, 19, 21, 21, 22, 23, 24, 25,27, 28, 29, 30] dust ion acoustic waves.

It can also be noted that these effects would arise as a source of dissipation and in turn it could lead to the formation of the dust ion acoustic shock waves. However, if dissipation is very small, the shock wave will have an oscillatory profile of which the first few oscillations at the wavefront will be close to solitons [9,18,30]. Many authors showed that there are many space plasma and laboratory dusty plasma situations [8,11,19,24,31,32] in which the effects causing the dissipation can be insignificant compared to that of the dispersion [8] that lead to the formation of dust ion acoustic solitary wave structures.

3. Derivation of K-dV equation

Two stretched coordinates have been considered to obtain K-dV equations. The stretched coordinates are;

$$\zeta = \epsilon^{1/2} (x - V_p t), \text{ and } \tau = \epsilon^{3/2} t, \tag{7}$$

where, V_p is the dust ion acoustic wave phase speed (ω/k), and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). To obtain the dispersion relation, we expand n_s , u_s , p_s , and ϕ with s be the charged species like positive and negative ion, electron in power series of ϵ , to their equilibrium and perturbed parts,

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \epsilon^3 n_s^{(3)} + \dots, \tag{8}$$

$$u_s = 0 + \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \epsilon^3 u_s^{(3)} + \dots, \tag{9}$$

$$p_s = 0 + \epsilon p_s^{(1)} + \epsilon^2 p_s^{(2)} + \epsilon^3 p_s^{(3)} + \dots, \tag{10}$$

$$\psi = 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \epsilon^3 \psi^{(3)} + \dots, \tag{11}$$

where $n_s^{(1)}$, $u_s^{(1)}$, $p_s^{(1)}$, and $\psi^{(1)}$ are the perturbed part of n_s , u_s , p_s , and ψ respectively.

To the lowest order in ϵ , (combining equations(7), (8) (11), and (1, 2, 3, 4, 5, 6)), we get,

$$u_n^{(1)} = \{V_p / [\gamma \delta_n - \mu V_p^2]\} \psi^{(1)}, \tag{12}$$

$$u_e^{(1)} = \{V_p / \gamma\} \psi^{(1)}, \tag{13}$$

$$u_i^{(1)} = \{V_p / [V_p^2 - \gamma \delta_i]\} \psi^{(1)}, \tag{14}$$

$$p_n^{(1)} = \{\gamma / [\gamma \delta_n - \mu V_p^2]\} \psi^{(1)}, \tag{15}$$

$$p_e^{(1)} = \psi^{(1)}, \tag{16}$$

$$p_i^{(1)} = \{1 / [V_p^2 - \gamma \delta_i]\} \psi^{(1)}, \tag{17}$$

$$n_i^{(1)} = \{\gamma / (V_p^2 - \gamma \delta_i)\} \psi^{(1)}, \tag{18}$$

$$n_n^{(1)} = \{1 / (\gamma \delta_n - \mu V_p^2)\} \psi^{(1)}, \tag{19}$$

$$n_e^{(1)} = \{1 / \gamma\} \psi^{(1)}, \tag{20}$$

$$(1 - \mu_n + \mu_d)\psi^{(1)} + \mu_n n_n^{(1)} - n_i^{(1)} = 0. \tag{21}$$

Combining above equations, (12, 13, 14, 15, 16, 17, 18, 19, 20, 21), we get

$$(1 - \mu_n + \mu_d)\psi^{(1)} + (\mu_n/d_1 - 1/d_2)\psi^{(1)} = 0, \tag{22}$$

where

$$d_1 = (\gamma\delta_n - \mu V_p^2)$$

$$d_2 = (V_p^2 - \gamma\delta_i)$$

But $\psi^{(1)} \neq 0$, so that, $(1 - \mu_n + \mu_d) + \mu_n/(\gamma\delta_n - \mu V_p^2) - 1/(V_p^2 - \gamma\delta_i) = 0$,

which gives,

$$V_p = (-b \pm \sqrt{(b^2 - 4ac)/2a})^{(1/2)}, \tag{23}$$

where,

$$\alpha = (1 - \mu_n + \mu_d), a = \mu\alpha, b = \mu_n + \mu - \alpha\gamma\delta_n - \alpha\mu\gamma\delta_i, \text{ and } c = \alpha\gamma^2\delta_n\delta_i - \gamma\delta_i\mu_n$$

Eq. (23) represents the linear dispersion relation for the DIA waves. This clearly indicates that the DIA wave phase speed (V_p) increases with the increase of the dust charge density ($Z_{dn,d0}$). To the next higher order of ϵ , we obtain a set of equations, which, after using (12, 13, 14, 15, 16, 17, 18, 19, 20, 21), can be simplified as

$$\partial n_i^{(2)}/\partial \xi = \{1/v_p(d_2) + v_p(d_2)^2 + \gamma\delta_i/v_p(d_2)^2\} \partial \psi^{(1)}/\partial \tau + \{3/(d_2)^2 + \gamma\delta_i/(d_2)^3 + \gamma^2\delta_i/(d_2)^3\} \psi^{(1)}/\partial \xi + (1/(d_2)) \partial \psi^{(2)}/\partial \xi, \tag{24}$$

$$\partial n_n^{(2)}/\partial \xi = \{1/V_p(d_1) - \mu V_p/(d_1)^2 - \gamma\delta_n/V_p(d_1)^2\} \partial \psi^{(1)}/\partial \tau + \{3/(d_1)^2 - \gamma\delta_n/(d_1)^3 - \gamma^2\delta_n/(d_1)^3\} \psi^{(1)}/\partial \xi + (1/(d_1)) \partial \psi^{(2)}/\partial \xi, \tag{25}$$

$$\partial n_e^{(2)}/\partial \xi = \{2/\gamma^2 + 1/\gamma - 1/\gamma^2 - V_p/\gamma\} \psi^{(1)}/\partial \xi + \{1/\gamma\} \partial \psi^{(2)}/\partial \xi, \tag{26}$$

$$\partial^2 \psi^{(1)}/\partial \xi^2 = (1 - \mu_n + \mu_d)\{\psi^{(2)} + (\psi^{(1)})^2/2\} + \mu_n n_n^{(2)} - n_i^{(2)}. \tag{27}$$

We then, combining Eqn's. (24, 25, 26, 27), and let $\psi^{(1)} = \psi$, obtain an equation of the form:

$$\partial \psi / \partial \tau + A \psi \partial \psi / \partial \xi + \beta \partial^3 \psi / \partial \xi^3 = 0, \tag{28}$$

where,

$$A = Y/X,$$

$$\beta = 1/X,$$

$$X = (2V_p/d_2^2)(2V_p\mu_n/d_1),$$

$$Y = -c_1/d_2^3 + \mu_n c_2/d_1^3 + \mu_n c_3/d_1^3,$$

$$c_1 = V_p^4 + 2 V_p^2 \gamma \delta_i - 3 V_p^2 + \gamma^2 \delta_i^2 - 2 \gamma \delta_i + \gamma^2 \delta_i,$$

$$c_2 = V_p^4 \mu^2 - 2 \mu V_p^2 \gamma \delta_n + 3 \mu V_p^2,$$

$$c_3 = \gamma^2 \delta_n^2 + 2 \gamma \delta_n - \gamma^2 \delta_n.$$

Eq. (28) is known as K-dV (Korteweg-de Vries) equation. The stationary localized solution of (28) can be done by introducing a transformation $\xi = \zeta - U_0 \tau$, and is given by

$$\psi = \psi_m \text{sech}^2[(\zeta - U_0 \tau)/\delta], \tag{29}$$

where the amplitude ψ_m and the width δ are given by $\psi_m = 3U_0/A$ and $\delta = \sqrt{4\beta/U_0}$, respectively. Here, U_0 is the constant speed normalized by c_i . As $U_0 > 0$, (29) clearly indicates that (i) small amplitude solitary waves with $\psi > 0$, i.e. positive soliton (bright soliton) exists if $A > 0$, (ii) small amplitude solitary waves with $\psi < 0$, i.e. negative soliton (dark soliton) exists if $A < 0$, and (iii) no solitons can exist around $A = 0$.

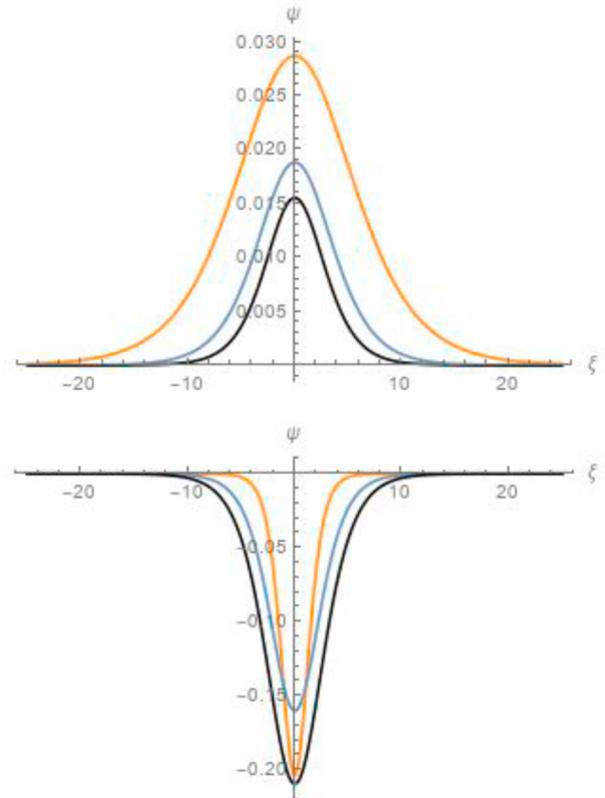


Figure 1. Showing the variations of positive K-dV soliton (up) and negative K-dV solitons (bottom) with mass number density μ (ranges from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the adiabatic system contains positive dust.

Eq. (29) is the solution of K-dV equation. This represents a solitary wave. However, we have numerically analyzed this wave to understand as well as to study its different characteristics.

Figure 1 shows the variation of the positive and negative K-dV solitons with mass number density (μ) when the system contains positively charged dust. Here amplitude increases and width decreases with the increase of ion number density. Whereas, approximately, the same variation for negatively charged dust has shown in Figure 2. Figures 1 and 2 are for the adiabatic system.

In Figures 1 and 2 for the same set of values, we have found a type (positive or negative) soliton in the case of adiabatic system, whereas for the isothermal system (Figure 3 and Figure 4) we have found the opposite type of variation in amplitude and width of the soliton. This is clear in Figure 1, Figure 3 and Figure 2, Figure 4. Again, solitons in the adiabatic system are wider than that in the isothermal system.

The variation of the amplitude and width of the negative K-dV solitons with ion number density (μ_n), for both the adiabatic and isothermal systems containing positive dust, have been observed in Figure 5. Here, it is clear that the width of the negative K-dV solitons for the adiabatic system is wider than that of the isothermal system. The same variations for positive K-dV solitons have been observed.

Figure 6 has shown the variations of the positive K-dV solitons with dust number density μ_d containing positive dust in the adiabatic and isothermal systems, respectively. It is again observed from these figures that solitons for the adiabatic system are wider than that in the isothermal system.

It has observed that ion number density and dust number density for different thermal states must not be the same. Otherwise, no soliton would exist.

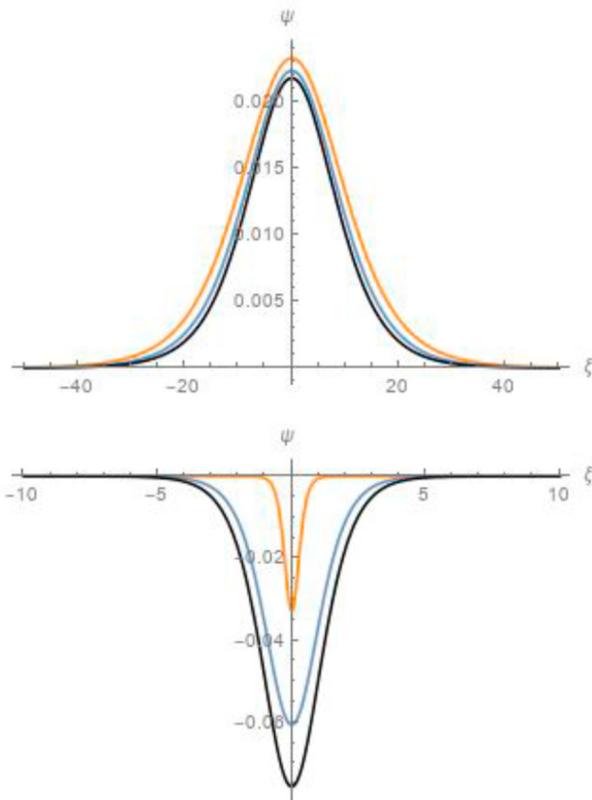


Figure 2. Showing the variation of the positive K-dV soliton (up) and negative K-dV solitons (bottom) with mass number density μ (spaced from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the adiabatic system contains negative dust.

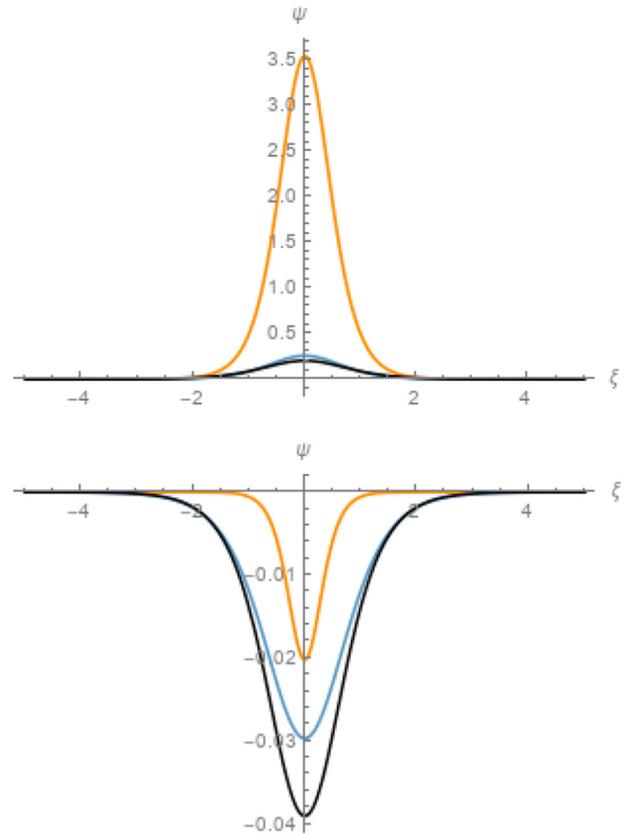


Figure 4. Showing the variations of positive K-dV soliton (up) and negative K-dV solitons (bottom) with mass number density μ (ranges from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the isothermal system contains negative dust.

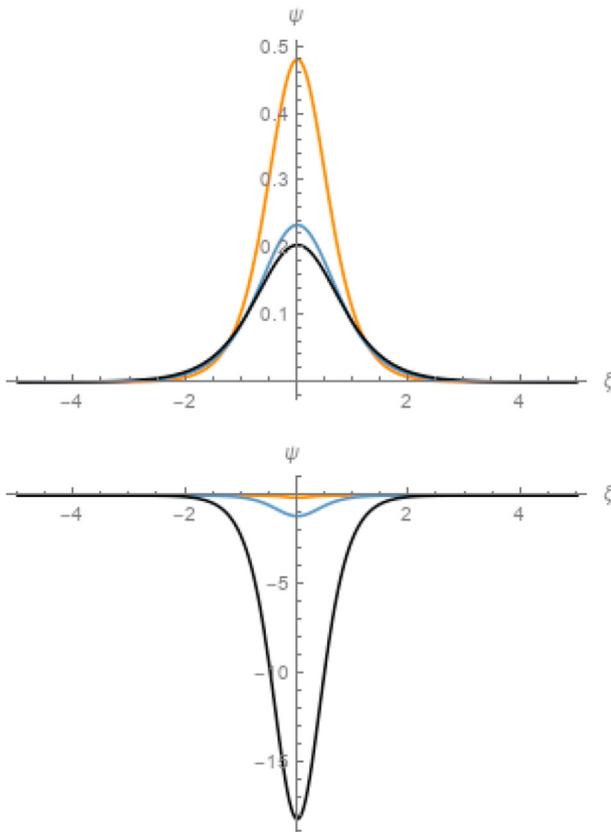


Figure 3. Showing the variations of positive K-dV soliton (up) and negative K-dV solitons (bottom) with mass number density μ (ranges from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the isothermal system contains positive dust.

Point 0.5 in Figure 7 and Figure 8 is the critical point, showed that positive and negative solitons have been observed before and after this point. And no soliton has been obtained at the critical point. At the points 0.475 and 0.53 the soliton goes to infinity, but with opposite polarity. As if, positive soliton ends and negative one begins.

4. Derivation of mK-dV equation

For plasmas with more than two species, there can arise a situation, where A vanishes at the critical region, and K-dV equation fails to describe nonlinear evolution of perturbation. So, higher-order calculation is required in the critical region. We know that the K-dV equation is the result of the second-order calculation of the ϵ . From the third-order calculation, a modified K-dV (mK-dV) equation is obtained to describe the nonlinear evolution near the critical parameter region. So, third-order calculation is needed.

The third-order calculation utilizes a new set of stretched coordinates, $\zeta = \epsilon(x - V_p t)$, $\tau = \epsilon^3 t$, (30)

Using (30) and (8, 9, 10, 11) in (1, 2, 3, 4, 5, 6), we find the same values of $n_i^{(1)}, n_n^{(1)}, n_e^{(1)}, u_i^{(1)}, u_e^{(1)}, u_n^{(1)}, p_i^{(1)}, p_e^{(1)}, p_n^{(1)}$, and V_p as like as that in K-dV.

To the next order approximation of ϵ , we obtain a set of equations, which, after using the values $n_i^{(1)}, n_n^{(1)}, n_e^{(1)}$, and V_p , can be simplified as:

$$n_i^{(2)} = \{3/2(V_p^2 - \gamma\delta_i)^2 + \gamma\delta_i/(V_p^2 - \gamma\delta_i)^3\}\psi^{(1)2} + \psi^{(2)}/(V_p^2 - \gamma\delta_i), \quad (31)$$

$$n_n^{(2)} = \{3/2(\gamma\delta_n - \mu V_p^2)^2 - \gamma\delta_n/(\gamma\delta_n - \mu V_p^2)^3\}\psi^{(1)2} + \psi^{(2)}/(\gamma\delta_n - \mu V_p^2), \quad (32)$$

$$n_e^{(2)} = \psi^{(1)2}/2\gamma^2 + \psi^{(2)}/\gamma, \quad (33)$$

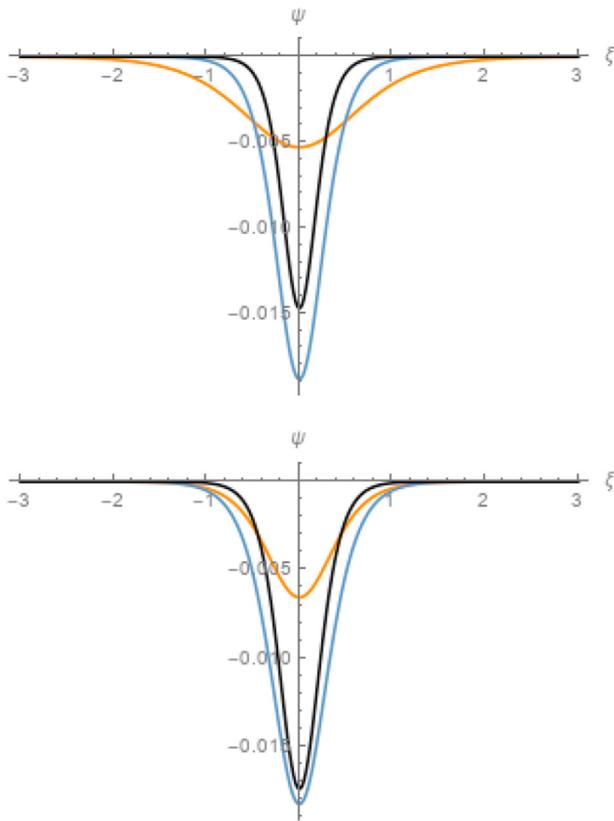


Figure 5. Showing the variations of the negative K-dV soliton with ion number density μ_n (ranges from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the adiabatic (up) isothermal system (bottom) contains positive dust.

$$\partial^2 \psi^{(1)} / \partial \xi^2 - (1 - \mu_n + \mu_d)(\psi^{(2)} + \psi^{(1)2}/2) - \mu_n n_n^{(2)} + n_i^{(2)} = 0. \tag{34}$$

Combining the above Eqs. (31), (32), (33), and (34) and applying the condition, $\psi \neq 0$ (so, its coefficient is zero), we get,

$$[k/2 + 3\mu_n/2a_1^2 - \gamma\delta_n\mu_n/a_1^3 - 3/2a_2^2 - \gamma\delta_i/a_2^3](\psi^{(1)})^2 = 0, \tag{35}$$

where $k = (1 - \mu_n + \mu_d)$. We let,

$$A = k/2 + 3\mu_n/2a_1^2 - \gamma\delta_n\mu_n/a_1^3 - 3/2a_2^2 - \gamma\delta_i/a_2^3 \tag{36}$$

Thus,

$$\frac{1}{2} \{A(\psi^{(1)})^2\} = 0 \tag{37}$$

To the next higher order of ϵ , we obtain a set of equations:

$$\partial n_s^{(1)} / \partial \tau - V_p \partial n_s^{(3)} / \partial \xi + \partial n_s^{(2)} u_s^{(1)} / \partial \xi + \partial n_s^{(1)} u_s^{(2)} / \partial \xi + \partial u_s^{(3)} / \partial \xi = 0, \tag{38}$$

$$\begin{aligned} & \partial u_i^{(1)} / \partial \tau - V_p \partial u_i^{(3)} / \partial \xi - V_p n_i^{(1)} \partial u_i^{(2)} / \partial \xi - V_p n_i^{(2)} \partial u_i^{(1)} / \partial \xi \\ & + u_i^{(1)} \partial u_i^{(2)} / \partial \xi + u_i^{(2)} \partial u_i^{(1)} / \partial \xi + n_i^{(1)} u_i^{(1)} \partial u_i^{(1)} / \partial \xi + \partial \psi^{(3)} / \partial \xi \\ & + n_i^{(1)} \partial \psi^{(2)} / \partial \xi + n_i^{(2)} \partial \psi^{(1)} / \partial \xi + \delta_i \partial p_i^{(3)} / \partial \xi = 0, \end{aligned} \tag{39}$$

$$\begin{aligned} & \partial u_n^{(1)} / \partial \tau - V_p \partial u_n^{(3)} / \partial \xi - V_p n_n^{(1)} \partial u_n^{(2)} / \partial \xi - V_p n_n^{(2)} \partial u_n^{(1)} / \partial \xi + u_n^{(1)} \partial u_n^{(2)} / \partial \xi \\ & + u_n^{(2)} \partial u_n^{(1)} / \partial \xi + u_n^{(1)} \partial u_n^{(1)} / \partial \xi - (1/\mu)(\partial \psi^{(3)} / \partial \xi) + (1/\mu)(n_n^{(1)} \partial \psi^{(2)} / \partial \xi) \\ & - (1/\mu)(n_n^{(2)} \partial \psi^{(1)} / \partial \xi - \delta_n \partial p_n^{(3)} / \partial \xi) = 0, \end{aligned} \tag{40}$$

$$\partial \psi^{(3)} / \partial \xi + n_e^{(1)} \partial \psi^{(2)} / \partial \xi + n_e^{(2)} \partial \psi^{(1)} / \partial \xi - \partial p_e^{(3)} / \partial \xi = 0, \tag{41}$$

$$\begin{aligned} & \partial p_s^{(1)} / \partial \tau - V_p \partial p_s^{(3)} / \partial \xi + \gamma \partial u_s^{(3)} / \partial \xi u_s^{(1)} / \partial \xi + u_s^{(2)} \partial p_s^{(1)} / \partial \xi \\ & + \gamma p_s^{(1)} \partial u_s^{(2)} / \partial \xi + \gamma p_s^{(2)} \partial u_s^{(1)} / \partial \xi = 0, \end{aligned} \tag{42}$$

$$\partial^2 \psi^{(1)} / \partial \xi^2 = (1 - \mu_n + \mu_d) \{ \psi^{(3)} + \psi^{(1)} \psi^{(2)} + (\psi^{(1)3} / 6) + \mu_n n_n^{(3)} - n_i^{(3)} \}. \tag{43}$$

Combining Eqs. (38), (39), (40), (41), (42), and (43), we obtain

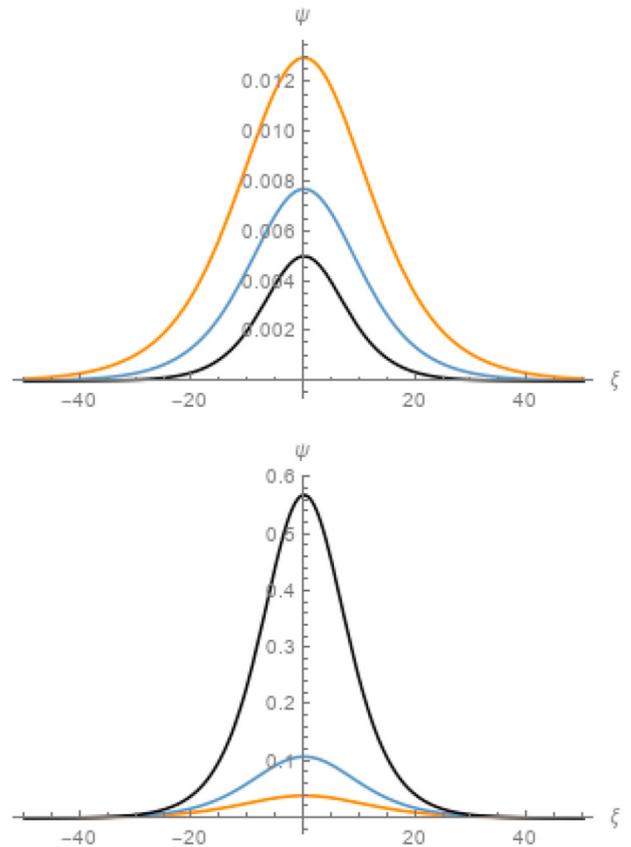


Figure 6. Showing the variations of the positive K-dV soliton with dust number density μ_d (ranges from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the adiabatic (up) and isothermal (down) systems contains positive dust. This soliton in isothermal system is clearly narrower than that in the adiabatic system.

$$\partial \psi / \partial \tau + \alpha \psi^2 \partial \psi / \partial \xi + \beta \partial^3 \psi / \partial \xi^3 = 0, \tag{44}$$

where

$$\alpha = F(-a_1^2 + 15/2 - 21\gamma\delta_n/2a_1 - 5\gamma^2\delta_n/2a_1 - 3\gamma^3\delta_n^2/a_1^2) - F(3\gamma^2\delta_n^2/a_1^2) + G(a_2^2 - 15/2 - 21\gamma\delta_i/2a_2) - G(5\gamma^2\delta_i/2a_2 - 3\gamma^3\delta_i^2/a_2^2 - 3\gamma^2\delta_i^2/a_2^2), \tag{45}$$

$$\beta = V_p a_1^2 a_2^2 / (-2\mu_n a_2^2 V_p^2 - 2\gamma\delta_i a_1^2), \tag{46}$$

Where

$$F = \mu_n/a_1^3, G = 1/a_2^3, a_1 = (\gamma\delta_n - \mu V_p^2), \text{ and}$$

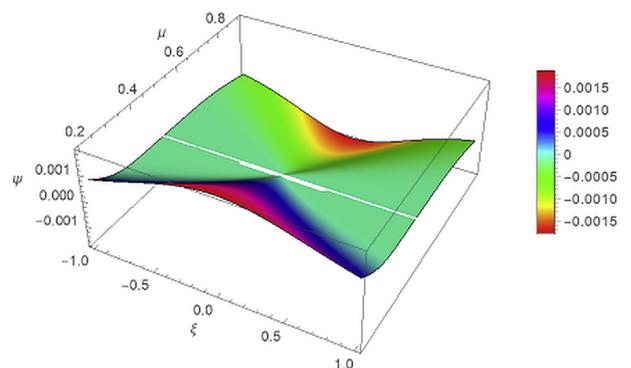


Figure 7. 3D view, showing the point of separation of positive and negative K-dV soliton when the adiabatic system contains positive dust. In the figure white line is indicating the separation.

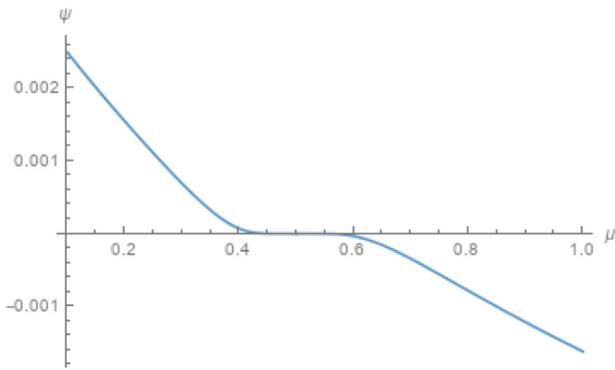


Figure 8. Showing the point of separation of positive and negative K-dV soliton when the adiabatic system contains positive dust.

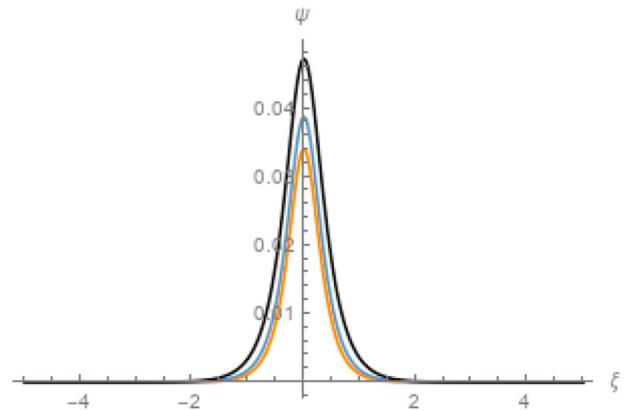
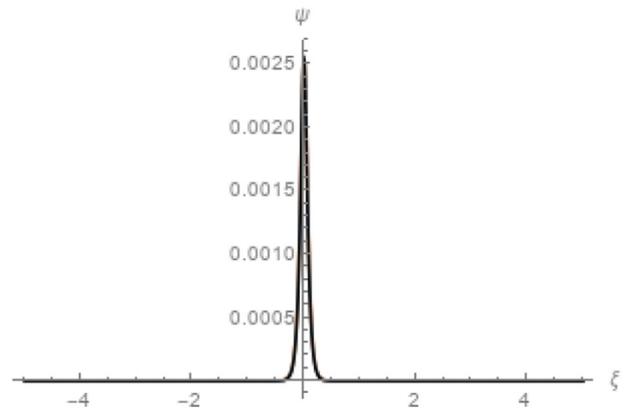


Figure 10. The variation of the mK-dV solitons with dust number density μ_d (spaced from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the adiabatic (up) and isothermal (bottom) system contain negative dust.

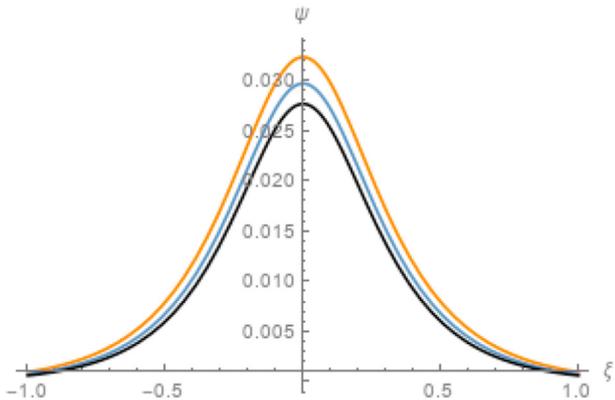
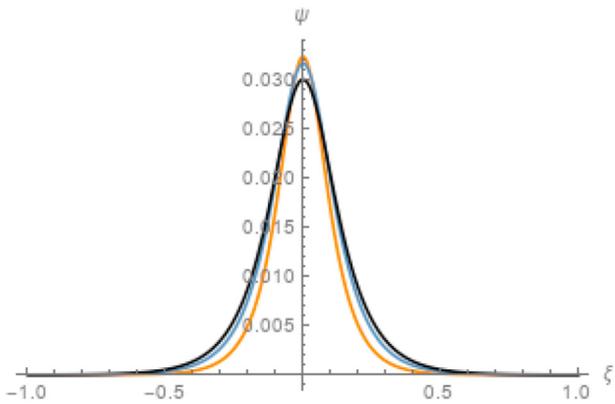


Figure 9. The variation of the mK-dV soliton with dust number density μ_d (spaced from 0.1 (orange) to 0.5 (magenta) to 0.9 (black)) when the adiabatic (up) and isothermal system (bottom) contain positive dust.

$$a_2 = (V_p^2 - \gamma\delta_i)$$

Eq. (44) is known as the mK-dV (modified K-dV) equation. As (44) does not contain any ψ^2 term, it is clear that (44) does not have any DL wave solution. To obtain the stationary localized solution of Eq. (44) a transformation $\xi = \zeta - U_0\tau$ is introduced. The stationary localized solution is obtained as;

$$\psi = \psi_m \text{sech}[\xi/\Delta], \tag{47}$$

where the amplitude ψ_m and the width δ are given by $\psi_m = \sqrt{6U_0/\alpha\beta}$, and $\delta = 1/\psi_m\sqrt{\gamma}$. And the amplitude ψ_m and the width Δ are given by $\psi_m = \sqrt{6U_0/\alpha\beta}$, $\Delta = 1/(\sqrt{\gamma\psi_m})$, and $\gamma = \alpha/6$.

Figure 9 shows the variations of the width and amplitude of mK-dV solitons with the dust number density μ_d for the adiabatic and isothermal systems containing positive dust. In this case, amplitude and width both decrease with the increase of dust number density. However, the width of the soliton for the isothermal system is wider than that for the adiabatic system.

Again, Figure 10 shows the variations of the amplitude and width of the mK-dV soliton with dust number density μ_d where the adiabatic and isothermal systems contain negative dust respectively. For the isothermal system the value of the amplitude and width of the soliton increases with the increase of μ_d , but for the adiabatic system amplitude (width) increases (decreases). Furthermore, the adiabatic index γ has also the opposite effect on the positive and negative dust respectively shown in Figure 11.

5. Derivation of standard Gardner equation

From Eq. (37) we see that $A = 0$ since $\psi \neq 0$. One can find that $A = 0$ at its critical value (μ_c) (which is a solution of $A = 0$). So, for μ around its critical value (μ_c), $A = A_0$ can be expressed as

$$A_0 \simeq s(\partial A/\partial \mu)_{\mu=\mu_c} |\mu - \mu_c| = s\varepsilon, \tag{48}$$

where $|\mu - \mu_c|$ is a small and dimensionless parameter, and can be taken as the expansion parameter ε , i.e. $|\mu - \mu_c| \simeq \varepsilon$, and $s = 1$ for $\mu < \mu_c$ and $s = -1$ for $\mu > \mu_c$. So, $\rho^{(2)}$ can be expressed as

$$\varepsilon^2 \rho^{(2)} \simeq \varepsilon^3 (1/2)s\psi^2, \tag{49}$$

which, therefore, must be included in the third-order Poisson's equation.

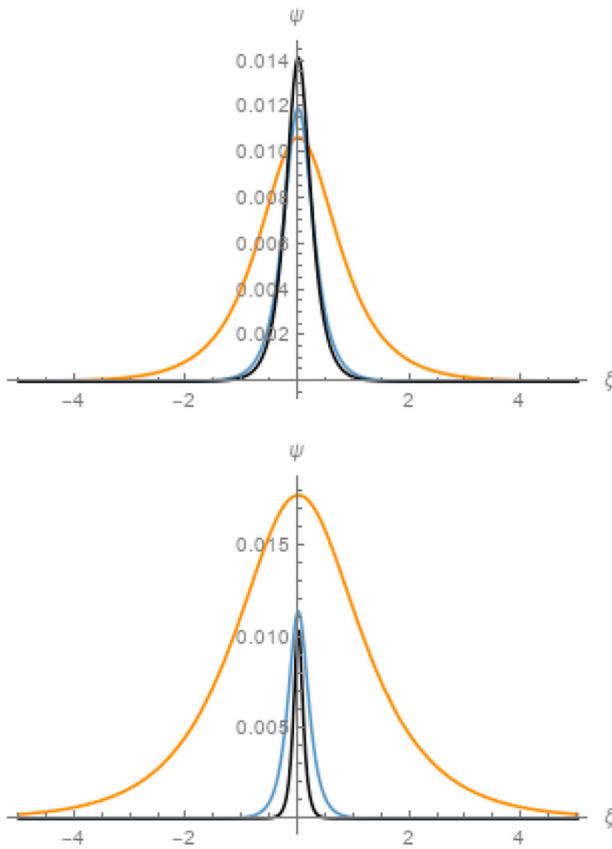


Figure 11. The variation of the mK-dV soliton with the adiabatic index γ (ranges from 0.1 (orange):0.5(magenta): 0.9(black)) for the positive (up) and negative (bottom) dust respectively.

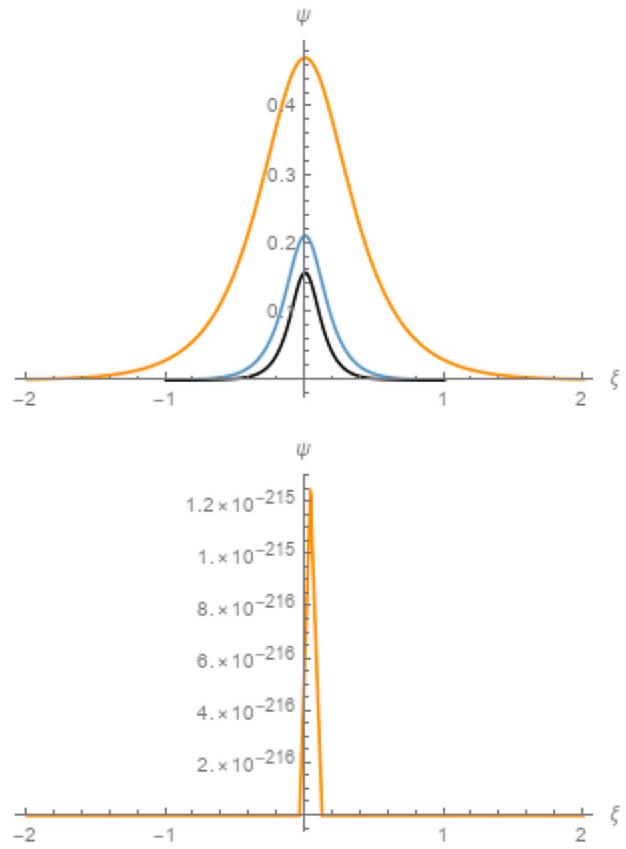


Figure 12. The variation of solitons with nonlinear coefficient α (0.1(orange):0.5(magenta):0.9(black)) when the value of the other nonlinear coefficient β (2.6) is constant (Figure 12 (up)). Figure 13 (bottom) indicates that the values of α and β show a peculiar behavior of shifting of origin and decrease of both the amplitude and width, when one is fixed and other is increased relative to other. The variation of the soliton for the point $\beta = 2.6$ with nonlinear coefficient (α) has been observed (Figure 12(up)) that width and amplitude decrease with the increase of α . After a certain value of β (approximately greater than 273) origin has shifted along with the decrease of both the amplitude and width (Figure 12 (bottom)).

To the next higher order of ϵ , we obtain a set of equations:

$$\partial n_s^{(1)}/\partial \tau - V_p \partial n_s^{(3)}/\partial \xi + \partial n_s^{(2)} u_s^{(1)}/\partial \xi + \partial n_s^{(1)} u_s^{(2)}/\partial \xi + \partial u_s^{(3)}/\partial \xi = 0, \tag{50}$$

$$\begin{aligned} & \partial u_i^{(1)}/\partial \tau - V_p \partial u_i^{(3)}/\partial \xi - V_p n_i^{(1)} \partial u_i^{(2)}/\partial \xi - V_p n_i^{(2)} \partial u_i^{(1)}/\partial \xi + u_i^{(1)} \partial u_i^{(2)}/\partial \xi \\ & + u_i^{(2)} \partial u_i^{(1)}/\partial \xi + n_i^{(1)} u_i^{(1)} \partial u_i^{(1)}/\partial \xi + \partial \psi^{(3)}/\partial \xi + n_i^{(1)} \partial \psi^{(2)}/\partial \xi + n_i^{(2)} \partial \psi^{(1)}/\partial \xi + \\ & \delta_i \partial p_i^{(3)}/\partial \xi = 0, \end{aligned} \tag{51}$$

$$\begin{aligned} & \partial u_n^{(1)}/\partial \tau - V_p \partial u_n^{(3)}/\partial \xi - V_p n_n^{(1)} \partial u_n^{(2)}/\partial \xi - V_p n_n^{(2)} \partial u_n^{(1)}/\partial \xi + u_n^{(1)} \partial u_n^{(2)}/\partial \xi + \\ & u_n^{(2)} \partial u_n^{(1)}/\partial \xi + u_n^{(2)} \partial u_n^{(1)}/\partial \xi - 1/\mu (\partial \psi^{(3)}/\partial \xi) + 1/\mu (n_n^{(1)} \partial \psi^{(2)}/\partial \xi) \\ & - 1/\mu (n_n^{(2)} \partial \psi^{(1)}/\partial \xi - \delta_n \partial p_n^{(3)}/\partial \xi) = 0, \end{aligned} \tag{52}$$

$$\partial \psi^{(3)}/\partial \xi + n_e^{(1)} \partial \psi^{(2)}/\partial \xi + n_e^{(2)} \partial \psi^{(1)}/\partial \xi - \partial p_e^{(3)}/\partial \xi = 0, \tag{53}$$

$$\begin{aligned} & \partial p_s^{(1)}/\partial \tau - V_p \partial p_s^{(3)}/\partial \xi + \gamma \partial u_s^{(3)}/\partial \xi + u_s^{(1)} \partial p_s^{(2)}/\partial \xi + u_s^{(2)} \partial p_s^{(1)}/\partial \xi + \gamma p_s^{(1)} \partial u_s^{(2)}/\partial \xi + \\ & \gamma p_s^{(2)} \partial u_s^{(1)}/\partial \xi = 0, \end{aligned} \tag{54}$$

$$\partial^2 \psi^{(1)}/\partial \xi^2 = (1 - \mu_n + \mu_d)(\psi^{(3)} + \psi^{(1)} \psi^{(2)} + (\psi^{(1)})^3 / 6) + \mu_n N_n^{(3)} - N_i^{(3)} + 1/2 s \psi^2. \tag{55}$$

Now, combining (50)–(55), and let $(\psi^{(1)}) = \psi$, we obtain an equation of the form:

$$\partial \psi / \partial \tau + s \beta \psi \partial \psi / \partial \xi + \alpha \beta \psi^2 \partial \psi / \partial \xi + \beta \partial^3 \psi / \partial \xi^3 = 0, \tag{56}$$

where the coefficients, α and β , has the usual meaning as derived above, (45, 46). Eq. (56) is known as Gardner or often called mixed mK-dV equation because it contains both ψ^2 term of K-dV and ψ^3 term of mK-dV. Eq. (56) is valid for its critical value (μ_c). As (56) contains both ψ^2 and ψ^3 terms, it supports both SWs and DLs solutions.

In Eq. (56) α and β are not any parameters, they are the nonlinear coefficients. And α and β are functions of μ only. If we neglect the ψ^2 term, Eq. (56) reduces to the mK-dV equation which has been derived in Sec. 4,

and to K-dV equation (Sec. 3) by using a lower order stretching viz. $\zeta = \epsilon^{1/2} (x - V_p t)$, $\tau = \epsilon^{3/2} t$. The solitary wave solution [33,34] of standard Gardner equation is, therefore, directly given by

$$\psi = [1/\psi_{m2} - (1/\psi_{m2} - 1/\psi_{m1}) \cosh^2(\xi \delta)]^{-1}, \tag{57}$$

where $\psi_{m1,2}$ are given by

$$\psi_{m1,2} = \psi_m [1 \mp \sqrt{(1 + U_0 V_0)}], \tag{58}$$

where $\psi_m = -s/\alpha$, and $V_0 = s^2 \beta / 6 \alpha$. Soliton width δ is

$$\delta = 2/\sqrt{(-\gamma \psi_{m1} \psi_{m2})} = \sqrt{(\beta/U_0)}, \tag{59}$$

Eq. (57) represents a solitary wave solution of the Gardner equation. From the above equation it is clear that to have GSs we must have $U_0 < V_0$, otherwise $\psi_{m1,2}$ becomes imaginary. When $s = 1$, (57) represents a positive (bright) soliton, whereas when $s = -1$, (57) represents a negative (dark) soliton [35].

Figure 13 shows the variations of the positive and negative Gardner solitons with mass number density μ when the adiabatic system contains positive dust. Whereas, Figure 14 shows the variations of the positive and negative Gardner solitons with mass number density μ when the adiabatic system contains negative dust. It has been observed that the amplitude and width of the Gardner solitons increased with the increase of μ for both positive and negative solitons in the presence of positive and

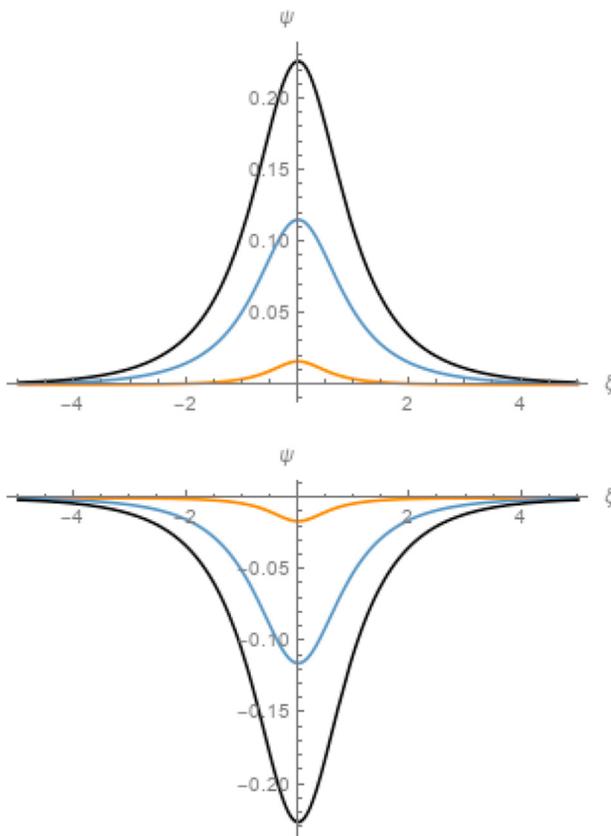


Figure 13. The variation of amplitude and width of positive GSs (up) and negative GSs (bottom) with mass number density μ (0.1(orange):0.5(Magenta) 0.9 (black)) when the adiabatic system contains positively charged dust.

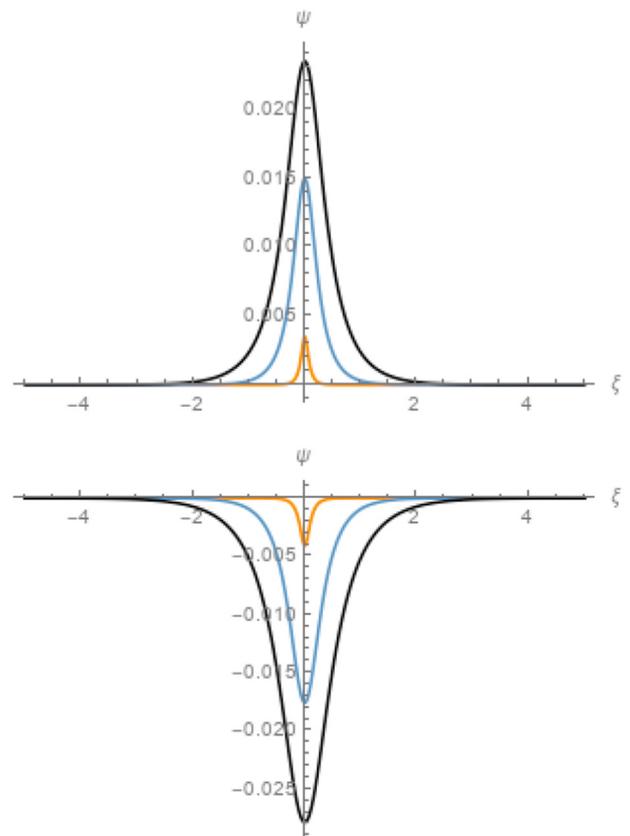


Figure 14. The variation of positive GSs (up) and negative GSs (bottom) with mass number density μ (0.1(orange):0.5(Magenta)0.9 (black)) when the adiabatic system contains negatively charged dust.

negative dust, respectively. We got approximately the same result for the isothermal system.

The variations of positive and negative Gardner solitons with ion number density μ_n for adiabatic and isothermal systems containing positive and negative dust have been observed (Figure 15 and Figure 16). From these figures, it has observed that the amplitude and width decreased (increased) with the increase of μ_n for the positive (negative) solitons when the system contains positive (negative) dust. Again, it is observed that the result is approximately the same for adiabatic and isothermal systems, but the width of the Gardner soliton for the adiabatic system is wider than that of the isothermal system.

6. Results and discussion

Dust ion acoustic K-dV, mK-dV, and Gardner solitons have been theoretically investigated in a complex dusty plasma system consisting of inertial positive and negative ions, Maxwell's electrons, and arbitrarily charged stationary dust. The K-dV solitons are not valid for a parametric regime that vanish the nonlinear coefficients of the K-dV equation. Again, the DIA mK-dV solitons are also not valid for the parametric regime which also vanish the nonlinear coefficient. Finally, the DIA GSs have been investigated in our present work which is valid in the parametric regime. The results have summarized as follows;

1. The existence of positive and negative K-dV soliton is observed.
2. The amplitude and width of the K-dV soliton vary with dust polarity. Amplitude of the positive and negative soliton increases with the increase of mass number density when the system contains positive dust, but it decreases for negative dust.
3. The values of ion number density and dust number density must not be the same; otherwise, no soliton will exist.
4. For the variation of amplitude with ion number density positive K-dV soliton is obtained when the adiabatic system contains positive dust, but if the system is isothermal then the positive soliton changes to negative soliton rapidly. This peculiar result is only obtained for positive dust.
5. K-dV solitons in the adiabatic system are wider than that in the isothermal system.
6. Negative K-dV soliton with negative dust gives comparatively wider soliton than that with positive dust.
7. For the positive K-dV soliton, if dust is positive amplitude increases, and width decreases with the increase of ion number density. However, if dust is negative then amplitude decreases, and width increases with the increase of ion number density. So, ion number density effects on the amplitude and width of the K-dV soliton significantly. For negative K-dV soliton, result is also the same with opposite polarity.
8. The point 0.5, in Figure 8, is the critical point. Positive soliton is obtained before the point and negative soliton is obtained after the point. No soliton exists at the point 0.5. A typical dusty plasma parameter was found by some authors, $\mu = 0.5-1.8$ [36], for the existence of small amplitude solitary waves with negative potential (same as we got), and they also found $\mu = 4-5$ for the existence of small amplitude solitary DIA waves with positive potential. According to our point of view and chosen parameters the critical point, we found, is correct.
9. The existence of positive mK-dV solitons is observed.
10. On the amplitude and width of mK-dV soliton dust number density has just the opposite effect for both the adiabatic and isothermal systems, i.e., the value of the amplitude and the width of the mKdV soliton decreases with the increase of dust number density

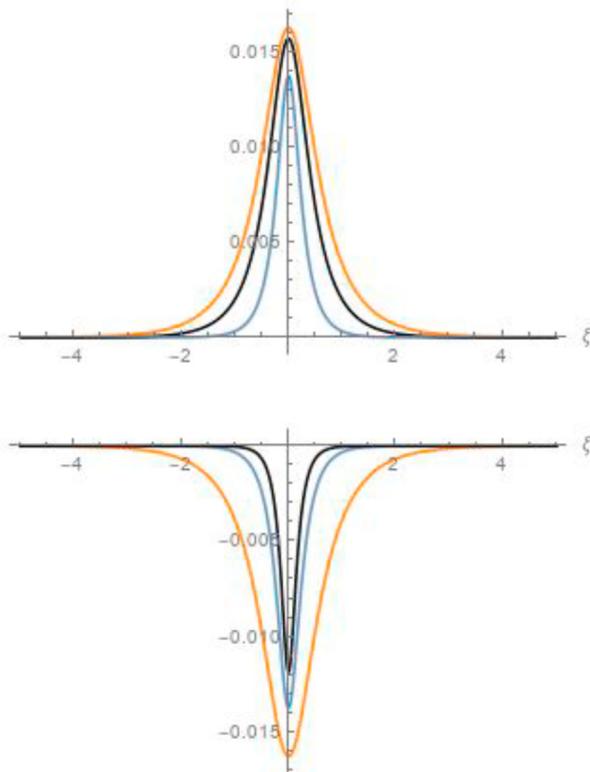


Figure 15. The variation of amplitude of positive (up) and negative GSs (bottom) with ion number density μ_n (0.1(orange):0.5(magenta):0.9(black)) when the adiabatic system contains positive dust.

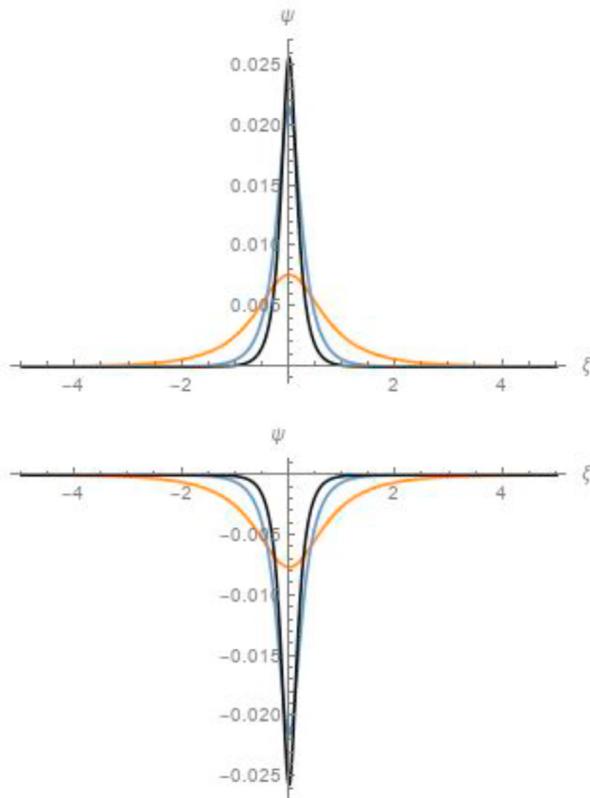


Figure 16. The variation positive (up) and negative (bottom) GSs with ion number density μ_n (0.1(orange):0.5(magenta):0.9(black)) when the adiabatic system contains negatively charged dust.

μ_d for positive dust. But for negative dust, we see different characteristics, i.e., (1) for isothermal system amplitude, and width both increase, (2) for adiabatic system amplitude increases and width decreases.

11. The width of the soliton for the isothermal system is wider than that for the adiabatic system.
12. The adiabatic index γ has an interesting effect on the positive and negative dust (Figure 11), i.e., the amplitude (width) of the positive mK-dV soliton increases (decreases) with adiabatic index γ for positive dust, but both decrease for negative dust.
13. The amplitude and width of the soliton decrease with the increase of nonlinear coefficients α and β .
14. The existence of positive and negative Gardner solitons has been observed.
15. In the adiabatic system, the amplitude and width of the Gardner solitons increase with the increase of mass number density μ for both the positive and negative solitons when the system contains positive and negative dust, respectively. The result is approximately the same for the isothermal system.
16. The amplitude and width decreased (increased) with the increase of μ_n for the positive (negative) solitons when the system contains positive (negative) dust, respectively. It is observed that the result is approximately the same for adiabatic and isothermal systems, but the width of the Gardner soliton for the adiabatic system is wider than that of the isothermal system.

7. Conclusion

Nonlinear waves are the amazing manifestation of nature, arising out properties like dispersion, dissipation, and nonlinearity. Space constitutes a magnificent laboratory to investigate the plasma phenomena and nonlinear wave structures. This theoretical research would offer deep physical insight to uncover the nonlinear phenomena, with the presence of dust particles, happening in space. And of course, this is unique research for dust ion acoustic K-dV, mK-dV, and Gardner solitons in a complex dusty plasma system consisting of inertial positive and negative ions, Maxwell's electrons, and arbitrarily charged stationary dust. The investigation is done to analyze the variations in the wave profiles where space contains plasma with charged dust and undergo through the adiabatic changes frequently, due to the events like formations or deformations of stars and galaxies, solar storms, radiations, etc. The comparisons of outcomes for the isothermal and adiabatic systems, which we believe, make our paper more suitable.

In this theoretical work, Landau damping or effect of phase-mixing is negligible. Generally, ion acoustic waves are subject to ion Landau damping, and is severe for the case of $T_e \sim T_i$, where T_e is electron temperature and T_i is ionization temperature. However, this is not the case, for the dust ion acoustic waves, where wave-particle resonance at $VT_i \sim V_p$ no longer holds since $V_p \gg VT_i$.

Though positive and negative ions, Maxwell's electrons, and arbitrarily charged stationary dust have been considered in this theoretical work, and applicable only for small amplitude waves, the experimental setups of Barkan et al. [5] or Nakamura et al. [8] may be used to observe the solitons, and new experiment based on our results may also be performed.

Declarations

Author contribution statement

Kazi Asraful Islam: Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

F. Deeba, Md. Kamal-Al-Hassan: Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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