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A novel study of spherical fuzzy soft Dombi aggregation operators and their applications to multicriteria decision making

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ABSTRACT

In many decision-making situations, we are not restricted to two kinds of aspects, such as membership degree or nonmembership degree, and sometimes we need to include the abstinence degree (AD). However, many fuzzy set theories fail to cover issues, such as an intuitionistic fuzzy soft set, Pythagorean fuzzy soft set and q-rung orthopair fuzzy soft set. All the above notions can only consider membership degree and a nonmembership degree in their structures. The spherical fuzzy soft set compensates for these drawbacks in its structure. Moreover, the Dombi t-norm and Dombi t-conorm are the fundamental apparatuses to generalize the basic operational laws of sum and product. Therefore, in this article, based on the dominant features of spherical fuzzy soft sets and valuable features of the Dombi t-norm and Dombi t-conorm, we initially developed the basic Dombi operational laws for spherical fuzzy soft numbers. Moreover, based on these newly developed operational laws, we introduced aggregation operators called spherical fuzzy soft Dombi average (weighted, ordered weighted, hybrid) aggregation operators. We discussed the basic properties of these aggregation operators. Additionally, we have developed a multiple criteria decision making (MCDM) approach using an explanatory example via our approach to show its effective utilization. Furthermore, a comparative study of our approach shows the superiority of our introduced notions.

1. Introduction

The concept of the fuzzy set (FS) suggested by Zadeh [1] is a foundation of fuzzy set theory (FST) that generalizes crisp set theory in an improved way. FST is an interesting apparatus to discuss linguistic terms. However, defining a membership degree (MD) for an element of a set is a major difficulty due to the lack of a parameterization tool in the study of FS. To manage the problem, the design of a soft set ($S_{ft}S$) [2] proved to be a very interesting tool by considering the parameterization mechanism. In recent years, $S_{ft}S$ theory has received more attention for solving ambiguous data. Currently, $S_{ft}S$ has an extensive scope, and many researchers have used this structure in several directions [3,4]. The theoretical study of $S_{ft}S$ and its utilization in decision-making (DM) issues are provided in Ref. [5]. After the invention of the soft set, numerous scholars have studied the combined form of the soft set along with the fuzzy set to

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initiate the notion of a fuzzy soft set ($FS_{ft}S$) [6]. The invention of the $FS_{ft}S$ is a hybrid tool that can handle fuzzy information in an improved way; thus, numerous scholars have begun working on this notion, and an algorithm for a $FS_{ft}S$ based on DM for a multi-observer input parameter data set is presented by Alcantud [7]. Jun et al. [8] applied a $FS_{ft}S$ approach to BCK/BCI algebra. The concept of a $FS_{ft}S$ has numerous uses in medical diagnosis [9] and medical science [10]. $FS_{ft}S$ theory has limitations in its structure because human judgment is not only limited to the positive factors but also to the negative aspects. Thus, $FS_{ft}S$ is unidirectional, and to overcome this issue, some extensions have been made in the form of an intuitionistic $FS_{ft}S$ ($IFS_{ft}S$) set established by Maji et al. [11] Evidently, $IFS_{ft}S$ uses the MD and nonmembership degree (NMD) in one frame.

Additionally, $IFS_{ft}Ss$ have been widely utilized in many research directions. Since the entropy of FS shows the fuzziness degree of FSs, entropy based on $IFS_{ft}S$ and interval-valued $IFS_{ft}S$ was initiated by Jiang et al. [12]. Additionally, generalized $IFS_{ft}S$ [13] with executions in DM was proposed. Moreover, Muthukumar and Krishnan [14] established the notion of a similarity measure based on $IFS_{ft}S$ and utilized it to address DM issues. Additionally, IFS_{ft} Bonferroni aggregation operators (AOs) have been established by Garg and Arora [15]. Some robust AOs using the concept of $IFS_{ft}S$ were established by Arora and Garg [16]. Moreover, some new AOs on group-based generalized $IFS_{ft}S$ have been shown by Hayat et al. [17]. Additionally, Khan et al. [18] proposed a generalized $IFS_{ft}S$ and provided its applications to decision support systems. Furthermore, Yaqoob et al. [19] showed the interval-valued intuitionistic (S, T)-fuzzy ideals of ternary semigroups.

The design of $IFS_{ft}S$ was modified into a Pythagorean fuzzy soft set ($PyFS_{ft}S$) by Peng et al. [20]; the various operations of $PyFS_{ft}S$ were deliberated and utilized to profitably address DM issues. Since $PyFS_{ft}S$ uses the constraint that $sum\ (MD^2, NMD^2)$ must belong to [0, 1], $PyFS_{ft}S$ has an extensive scope due to its more generalized structure. Athira et al. [21] established a novel entropy measure based on $PyFS_{ft}S$ and used it in DM scenarios. Naeem et al. [22] initiated $PyFS_{ft}$ TOPSIS and VIKOR methods. Moreover, Zulqarnain et al. [23] established some AOs by using $PyFS_{ft}S$ and developed their applications. Additionally, Shahzadi et al. [24] developed the DM methodology for $PyFS_{ft}$ graphs. Hussain et al. [25] noted that $PyFS_{ft}S$ is a limited structure and presented the notion of the q-rung orthopair fuzzy soft set ($q-ROFS_{ft}S$). Additionally, based on $q-ROFS_{ft}S$ and the Dombi t-norm and Dombi t-conorm, some Dombi AOs were established by Ref. [26]. Moreover, Chinram et al. [27] initiated the idea of $q-ROFS_{ft}$ geometric AOs. Furthermore, Hayat et al. [28] presented the concept of group generalized $q-ROFS_{ft}$ and proposed new AOs and their applications.

Note that $FS_{ft}S, IFS_{ft}S, PyFS_{ft}S$ and $q-ROFS_{ft}S$ can either discuss MD or both MD and NMD in their structures, and they have no concept to discuss AD in their structure. Thus, if the given data involves AD, then these above structures have no concept to tackle this type of data. To resolve this issue, the notion of picture fuzzy soft set ($PFS_{ft}S$) [29] has been presented. $PFS_{ft}S$ can use the setting that $sum(MD, AD, NMD) \in [0,1]$. Recently, many researchers have started working on this idea, and some PFS_{ft} robust VIKOR methods and their utilization are provided by Khan et al. [30]. Additionally, Jan et al. [31] invented the concept of multivalued $PFS_{ft}Ss$ and their execution to group DM scenarios. Moreover, a hybrid notion of picture fuzzy N-soft set ($PFN-S_{ft}S$) [32] has been introduced, and they have presented their applications to DM problems. Moreover, Dhurmas and Bajaj [33] introduced modified EDAS techniques based on PFS_{ft} Dombi aggregation operators utilized these notions in robotic agri-farming.

With time, researchers have noted the limitation of $PFS_{ft}S$ in its constraint. They note that when decision-makers provide 0.5 as the MD, 0.3 as the NMD and 0.6 as the AD, then the necessary condition for $PFS_{ft}S$ fails to cope with this type of data; thus, the notion of a spherical fuzzy soft set ($SFS_{ft}S$) [34] was introduced to address this difficulty. Moreover, dealing with uncertainty and ambiguous data is a very difficult task in many real-life problems, and this has become a popular topic for researchers. To address imprecise and complex data, $SFS_{ft}S$ plays a very important role in this regard. When there are several human opinions such as "Yes," "abstain" and "refusal," then the $SFS_{ft}S$ can effectively model that situation. For instance, examine the case of voting. In this situation, we notice that a person can vote in support of someone, vote against someone, abstain from voting or refuse to vote. In this situation, we can say that all the above theories of $FS_{ft}S, IFS_{ft}S, PyFS_{ft}S$ *and* $q-ROFS_{ft}S$ fail to handle this situation, while $PFS_{ft}S$ *and* $SFS_{ft}S$ can handle that situation. Therefore, all the above theories have some drawbacks in their structures. Additionally, $SFS_{ft}S$ has an advantage on $PFS_{ft}S$ in terms of its constraint that $sum(MD^2, AD^2, NMD^2) \in [0,1]$. Thus, $SFS_{ft}S$ provides more space for decision-makers. Therefore, we can say that $SFS_{ft}S$ is more dominant in all the above existing theories.

As $SFS_{ft}S$ is a more general tool to deal with uncertain and ambiguous data, AOs are very valuable mechanisms to change the input data into a single number. Therefore, Ahmmad et al. [35] proposed some average AOs based on $SFS_{ft}Ns$. Moreover, Guner and Aygun [36] introduced the theory and AOs based on $SFS_{ft}S$.

$SFS_{ft}S$ is a more general notion because of the following:

1. When a power 1 is used in the basic condition for $SFS_{ft}Ss$, then the notion of $SFS_{ft}S$ degenerates into a $PFS_{ft}S$.
2. Additionally, by disregarding the abstinence degree in the basic notion of the spherical fuzzy soft set, we can obtain the notion of $PyFS_{ft}S$.
3. Moreover, by disregarding the abstinence degree and using power 1 in the main condition for $SFS_{ft}S$, $SFS_{ft}S$ is reduced to $IFS_{ft}S$.
4. By using one parameter in $SFS_{ft}S$, $SFS_{ft}S$ degenerates into a spherical fuzzy set.
5. By using one parameter and power 1 in the basic definition of $SFS_{ft}S$, $SFS_{ft}S$ degenerates into a picture fuzzy set.

Therefore, based on the dominant feature for $SFS_{ft}S$ and the Dombi t-norm and Dombi t-conorm, in this article, we initially established Dombi operational laws for $SFS_{ft}Ns$ and then effectively applied this theory to initiate some new aggregation operators, such as SFS_{ft} Dombi weighted average, SFS_{ft} Dombi ordered weighted average and SFS_{ft} Dombi hybrid average AOs. Moreover, some basic results for these newly developed AOs were introduced, and supportive examples were provided to define the reliability of our initiated work. Additionally, an algorithm along with an example was established, and a comparative study of the established algorithm was carried out to show the primacy of the introduced work.

The article's remaining sections are organized as follows: Section 2 covers the basic definition of a soft set, $SFS_{ft}S$ and their basic operating rules; in Section 3, we have initiated the basic Dombi operating rules for $SFS_{ft}Ns$; Section 4 elaborates on the SFS_{ft} Dombi weighted average AOs; Section 5 provides the MCDM problem to support our work; Section 6 carries out a comparative study of the invented work along with some existing theories; and Section 7 provides the concluding remarks.

2. Preliminaries

We addressed the key concepts of soft sets and $SFS_{ft}Ss$ in this section of the article. Additionally, the basic operating rules, score function (SF) and accuracy function (AF) of these concepts are discussed.

Definition 1. [2]: Let U be a general set and E be a set of parameters. The soft set is a pair (f, M) and $M \subseteq E$, where f is the map given by $f : M \rightarrow P(U)$, and $P(U)$ denotes the power set of U.

Definition 2. [34]: A $SFS_{ft}S$ on a universal set U is the pair of the form (q, N) where $N \subseteq E$ and $q : N \rightarrow SFS^U$ defined as follows:

$$\S_{\hat{e}}(l_i) = \{\langle l_\ell, \epsilon_\ell(l_\ell), \wp_\ell(l_\ell), \mathscr{L}_\ell(l_\ell)\rangle | l_\ell \in U\}$$

where SFS^U present a collection of spherical fuzzy sets. Here $\epsilon_\ell(l_\ell), \wp_\ell(l_\ell), and\ \mathscr{L}_\ell(l_\ell)$ denote the MD, AD and NMD, respectively with $0 \leq (\epsilon_\ell(l_\ell))^2 + (\wp_\ell(l_\ell))^2 + (\mathscr{L}_\ell(l_\ell))^2 \leq 1$. For clarity, the triplet $\{\langle \epsilon_\ell(l_\ell), \wp_\ell(l_\ell), \mathscr{L}_\ell(l_\ell)\rangle\}$ is called $SFS_{ft}N$. Moreover, a refusal degree is given as follows: $Ref_{q_{\hat{e}_{\ell\ell}}} = \sqrt{1 - ((\epsilon_\ell(l_\ell))^2 + (\wp_\ell(l_\ell))^2 + (\mathscr{L}_\ell(l_\ell))^2)}$.

Definition 3. [35]: Let $\S_{\hat{e}_{\ell\ell}} = (\epsilon_{\ell\ell}, \wp_{\ell\ell}, \mathscr{L}_{\ell\ell})$, $\S_{\hat{e}'_{\ell\ell}} = (\epsilon'_{\ell\ell} \wp'_{\ell\ell} \mathscr{L}'_{\ell\ell})$ be two $SFS_{ft}Ns$ and $\flat > 0$. The fundamental laws are given as follows:

1) $\S_{\hat{e}_{\ell\ell}} \subseteq \S_{\hat{e}'_{\ell\ell}}$ iff $\epsilon_{\ell\ell} \leq \epsilon'_{\ell\ell} \wp_{\ell\ell} \leq \wp'_{\ell\ell}$ and $\mathscr{L}_{\ell\ell} \geq \mathscr{L}'_{\ell\ell}$.
2) $\S_{\hat{e}_{\ell\ell}} = \S_{\hat{e}'_{\ell\ell}}$ iff $\S_{\hat{e}_{\ell\ell}} \subseteq \S_{\hat{e}'_{\ell\ell}}$ and $\S_{\hat{e}'_{\ell\ell}} \subseteq \S_{\hat{e}_{\ell\ell}}$.
3) $\S_{\hat{e}_{\ell\ell}} \cup \S_{\hat{e}'_{\ell\ell}} = \langle \max(\epsilon_{\ell\ell}, \epsilon'_{\ell\ell}), \min(\wp_{\ell\ell}, \wp'_{\ell\ell}), \min(\mathscr{L}_{\ell\ell}, \mathscr{L}'_{\ell\ell})\rangle$.
4) $\S_{\hat{e}_{\ell\ell}} \bigcap \S_{\hat{e}'_{\ell\ell}} = \langle \min(\epsilon_{\ell\ell}, \epsilon'_{\ell\ell}), \min(\wp_{\ell\ell}, \wp'_{\ell\ell}), \max(\mathscr{L}_{\ell\ell}, \mathscr{L}'_{\ell\ell})\rangle$.
5) $\S_{\hat{e}_{\ell\ell}}{}^c = (\mathscr{L}_{\ell\ell}, \wp_{\ell\ell}, \epsilon_{\ell\ell})$.
6) $\S_{\hat{e}_{\ell\ell}} \oplus \S_{\hat{e}'_{\ell\ell}} = (\sqrt{(\epsilon_{\ell\ell})^2 + (\epsilon'_{\ell\ell})^2 - (\epsilon_{\ell\ell})^2(\epsilon'_{\ell\ell})^2}, \wp_{\ell\ell} \wp'_{\ell\ell} \mathscr{L}_{\ell\ell} \mathscr{L}'_{\ell\ell})$.
7) $\S_{\hat{e}_{\ell\ell}} \bigotimes \S_{\hat{e}'_{\ell\ell}} = (\epsilon_{\ell\ell} \epsilon'_{\ell\ell} \wp_{\ell\ell} \wp'_{\ell\ell} \sqrt{(\mathscr{L}_{\ell\ell})^2 + (\mathscr{L}'_{\ell\ell})^2 - (\mathscr{L}_{\ell\ell})^2(\mathscr{L}'_{\ell\ell})^2})$.
8) $\flat \S_{\hat{e}_{\ell\ell}} = (\sqrt{1 - (1 - \epsilon_{\ell\ell}{}^2)^\flat}, \wp_{\ell\ell}{}^\flat, \mathscr{L}_{\ell\ell}{}^\flat)$..
9) $\S_{\hat{e}_{\ell\ell}}{}^\flat = (\epsilon_{\ell\ell}{}^\flat, \wp_{\ell\ell}{}^\flat, \sqrt{1 - (1 - \mathscr{L}_{\ell\ell}{}^2)^\flat})$.

Definition 4. [35]: For a family of $S_{ft}Ns$ $\S_{\hat{e}_{\ell\ell}} = (\epsilon_{\ell\ell}, \wp_{\ell\ell}, \mathscr{L}_{\ell\ell})$, the SF and AF are given by the following:

$Sc(\S_{\hat{e}_{\ell\ell}}) = \frac{(2 + \epsilon_{\ell\ell} - \wp_{\ell\ell} - \mathscr{L}_{\ell\ell})}{3}$ and

$$Ac(\S_{\hat{e}_{\ell\ell}}) = \epsilon_{\ell\ell} - \mathscr{L}_{\ell\ell}.$$

Note that $Sc(\S_{\hat{e}_{\ell\ell}}) \in [-1, 1]$.

3. Dombi operations for spherical fuzzy soft numbers

In this section, the fundamental laws for $SFS_{ft}S$ based on the Dombi t-norm and Dombi t-conorm suggested by Dombi [37] are discussed.

Definition 5. [37]: For two arbitrary real numbers f, g and $\propto \geq 1$., Dombi operations for real numbers are given by the following:

$$\tilde{O}_{Đ}(f, g) = \frac{1}{1 + \left\{\left(\frac{1-f}{f}\right)^\propto + \left(\frac{1-g}{g}\right)^\propto\right\}^{\frac{1}{\propto}}}$$

$$\tilde{O}'_{Đ}(f, g) = 1 - \frac{1}{1 + \left\{\left(\frac{f}{1-f}\right)^\propto + \left(\frac{g}{1-g}\right)^\propto\right\}^{\frac{1}{\propto}}}$$

Now, fundamental laws based on the above-given definition are defined.

Definition 6. Suppose $\S_{\hat{e}_{11}} = (\text{€}_{11}, \wp_{11}, \mathscr{L}_{11})$ and $\S_{\hat{e}_{12}} = (\text{€}_{12}, \wp_{12}, \mathscr{L}_{12})$ are two arbitrary *SFS* $SFS_{ft}Ns$, $\propto \geq 1$ and $\flat > 0$. Then, using Dombi t-norms and Dombi t-conorm, the Dombi operations for $SFS_{ft}Ns$ are defined by the following:

1) $$\S_{\hat{e}_{11}} \bigoplus \S_{\hat{e}_{12}} = \left(\sqrt{1 - \frac{1}{1+\left\{\left(\frac{\text{€}_{11}^2}{1-\text{€}_{11}^2}\right)^{\propto} + \left(\frac{\text{€}_{12}^2}{1-\text{€}_{12}^2}\right)^{\propto}\right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{\left(\frac{1-\wp_{11}}{\wp_{11}}\right)^{\propto} + \left(\frac{1-\wp_{12}}{\wp_{12}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{\left(\frac{1-\mathscr{L}_{11}}{\mathscr{L}_{11}}\right)^{\propto} + \left(\frac{1-\mathscr{L}_{12}}{\mathscr{L}_{12}}\right)^{\propto}\right\}^{\frac{1}{\propto}}} \right)$$

2) $$\S_{\hat{e}_{11}} \bigotimes \S_{\hat{e}_{12}} = \left(\frac{1}{1+\left\{\left(\frac{1-\text{€}_{11}}{\text{€}_{11}}\right)^{\propto} + \left(\frac{1-\text{€}_{12}}{\text{€}_{12}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}, \sqrt{1 - \frac{1}{1+\left\{\left(\frac{\wp_{11}^2}{1-\wp_{11}^2}\right)^{\propto} + \left(\frac{\wp_{12}^2}{1-\wp_{12}^2}\right)^{\propto}\right\}^{\frac{1}{\propto}}}}, \sqrt{1 - \frac{1}{1+\left\{\left(\frac{\mathscr{L}_{11}^2}{1-\mathscr{L}_{11}^2}\right)^{\propto} + \left(\frac{\mathscr{L}_{12}^2}{1-\mathscr{L}_{12}^2}\right)^{\propto}\right\}^{\frac{1}{\propto}}}} \right)$$

3) $$\flat \S_{\hat{e}_{11}} = \left(\sqrt{1 - \frac{1}{1+\left\{\flat\left(\frac{\text{€}_{11}^2}{1-\text{€}_{11}^2}\right)^{\propto}\right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{\flat\left(\frac{1-\wp_{11}}{\wp_{11}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{\flat\left(\frac{1-\mathscr{L}_{11}}{\mathscr{L}_{11}}\right)^{\propto}\right\}^{\frac{1}{\propto}}} \right)$$

4) $$\S_{\hat{e}_{11}}{}^{\flat} = \left(\frac{1}{1+\left\{\flat\left(\frac{1-\text{€}_{11}}{\text{€}_{11}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}, \sqrt{1 - \frac{1}{1+\left\{\flat\left(\frac{\wp_{11}^2}{1-\wp_{11}^2}\right)^{\propto}\right\}^{\frac{1}{\propto}}}}, \sqrt{1 - \frac{1}{1+\left\{\flat\left(\frac{\mathscr{L}_{11}^2}{1-\mathscr{L}_{11}^2}\right)^{\propto}\right\}^{\frac{1}{\propto}}}} \right)$$

Theorem 1. *For any two arbitrary $SFS_{ft}Ns$ $\S_{\hat{e}_{11}} = (\text{€}_{11}, \wp_{11}, \mathscr{L}_{11})$ and $\S_{\hat{e}_{12}} = (\text{€}_{12}, \wp_{12}, \mathscr{L}_{12})$ and $\flat, \flat_1, \flat_2 > 0$, the following results hold:*

1) $\S_{\hat{e}_{11}} \bigoplus \S_{\hat{e}_{12}} = \S_{\hat{e}_{12}} \bigoplus \S_{\hat{e}_{11}}$
2) $\S_{\hat{e}_{11}} \bigotimes \S_{\hat{e}_{12}} = \S_{\hat{e}_{12}} \bigotimes \S_{\hat{e}_{11}}$
3) $\flat(\S_{\hat{e}_{11}} \bigoplus \S_{\hat{e}_{12}}) = (\flat \S_{\hat{e}_{11}} \bigoplus \flat \S_{\hat{e}_{12}})$
4) $\flat_1 \S_{\hat{e}_{11}} \bigoplus \flat_2 \S_{\hat{e}_{11}} = (\flat_1 + \flat_2) \S_{\hat{e}_{11}}$
5) $\S_{\hat{e}_{11}}{}^{\flat} \bigotimes \S_{\hat{e}_{12}}{}^{\flat} = (\S_{\hat{e}_{11}} \bigotimes \S_{\hat{e}_{12}})^{\flat}$
6) $\S_{\hat{e}_{11}}{}^{\flat_1} \bigotimes \S_{\hat{e}_{11}}{}^{\flat_2} = \S_{\hat{e}_{11}}{}^{(\flat_1 + \flat_2)}$

Proof: The proofs are straightforward.

4. Spherical fuzzy soft Dombi average aggregation operators

In this part, some Dombi average AOs based on $SFS_{ft}Ns$ known as $SFS_{ft}DWA$, $SFS_{ft}DOWA$, $SFS_{ft}HA$ are debated. Furthermore, the basic characteristics of these AOs are elaborated.

4.1. Spherical fuzzy soft Dombi weighted average operators

Definition 7. Consider the family of $SFS_{ft}Ns$, i.e., $\S_{\hat{e}_{\ell\mathscr{k}}} = (\text{€}_{\ell\mathscr{k}}, \wp_{\ell\mathscr{k}}, \mathscr{L}_{\ell\mathscr{k}})$ for $\ell = 1, 2, \ldots, \mathfrak{m}$ and $\mathscr{k} = 1, 2, \ldots, \tilde{n}$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathfrak{m}})$ denote the weight vector (WV) for experts $ð_{\ell}$ and $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ denote the WVs for $\hat{e}_{\mathscr{k}}$ parameters with the condition that $\grave{a}_{\ell}, \mathscr{o}_{\mathscr{k}} \in [0, 1]$ and $\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_{\ell} = 1, \sum_{\mathscr{k}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{k}} = 1$. Now, the $SFS_{ft}DWA$ operator can be defined as $SFS_{ft}DWA : ©^{\tilde{n}} \rightarrow ©$, where © represents the family of $SFS_{ft}Ns$, as follows:

$$SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{mn}}) = \bigoplus_{\mathscr{k}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{k}} \left(\bigoplus_{\ell=1}^{\mathfrak{m}} \grave{a}_{\ell} \S_{\hat{e}_{\ell\mathscr{k}}} \right) \tag{1}$$

Now, using equation (1), we can define $SFS_{ft}DWA$ AOs, as follows:

Theorem 2. *Suppose the family of $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\mathscr{k}}} = (\text{€}_{\ell\mathscr{k}}, \wp_{\ell\mathscr{k}}, \mathscr{L}_{\ell\mathscr{k}})$ where $\ell = 1, 2, \ldots, \mathfrak{m}$ and $\mathscr{k} = 1, 2, \ldots, \tilde{n}$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathfrak{m}})$ present the WV of experts $ð_{\ell}$ and $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ denote the WVs of $\hat{e}_{\mathscr{k}}$ parameters such that $\grave{a}_{\ell}, \mathscr{o}_{\mathscr{k}} \in [0, 1]$ and $\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_{\ell} = 1, \sum_{\mathscr{k}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{k}} = 1$. Then, the aggregated outcome for the $SFS_{ft}DWA$ operator is defined as follows:*

$$SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{mn}}) = \bigoplus_{\mathscr{k}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{k}} \left(\bigoplus_{\ell=1}^{\mathfrak{m}} \grave{a}_{\ell} \S_{\hat{e}_{\ell\mathscr{k}}} \right)$$

$$= \left(\sqrt{1 - \frac{1}{1+\left\{ \sum_{\ell=1}^{\tilde{n}} \wp_\ell \left(\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell \left(\frac{\epsilon_{\ell\ell}^2}{1-\epsilon_{\ell\ell}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \sum_{\ell=1}^{\tilde{n}} \wp_\ell \left(\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell \left(\frac{1-\wp_{\ell\ell}}{\wp_{\ell\ell}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \sum_{\ell=1}^{\tilde{n}} \wp_\ell \left(\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell\ell}}{\mathscr{L}_{\ell\ell}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right) \quad (2)$$

***Proof*: Here, the mathematical induction method is used, as follows**:

$$\S_{\hat{e}_{11}} \bigoplus \S_{\hat{e}_{12}} = \left(\sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\epsilon_{11}^2}{1-\epsilon_{11}^2} \right)^{\propto} + \left(\frac{\epsilon_{12}^2}{1-\epsilon_{12}^2} \right)^{\propto} \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \left(\frac{1-\wp_{11}}{\wp_{11}} \right)^{\propto} + \left(\frac{1-\wp_{12}}{\wp_{12}} \right)^{\propto} \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \left(\frac{1-\mathscr{L}_{11}}{\mathscr{L}_{11}} \right)^{\propto} + \left(\frac{1-\mathscr{L}_{12}}{\mathscr{L}_{12}} \right)^{\propto} \right\}^{\frac{1}{\propto}}} \right)$$

And

$$\beta \S_{\hat{e}} = \left(\sqrt{1 - \frac{1}{1+\left\{ \beta \left(\frac{\epsilon_{\S_{\hat{e}}}^2}{1-\epsilon_{\S_{\hat{e}}}^2} \right)^{\propto} \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \beta \left(\frac{1-\wp_{\S_{\hat{e}}}}{\wp_{\S_{\hat{e}}}} \right)^{\propto} \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \beta \left(\frac{1-\mathscr{L}_{\S_{\hat{e}}}}{\mathscr{L}_{\S_{\hat{e}}}} \right)^{\propto} \right\}^{\frac{1}{\propto}}} \right)$$

First, we prove that the results are valid for $\mathbb{m} = 2$ *and* $\tilde{n} = 2$, *as follows*:

$$SFS_{fi}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}) = \bigoplus_{\ell=1}^{2} \wp_\ell \left(\bigoplus_{\ell=1}^{2} \grave{a}_\ell \S_{\hat{e}_{\ell\ell}} \right) = \wp_1 (\grave{a}_1 \S_{\hat{e}_{11}} \bigoplus \grave{a}_2 \S_{\hat{e}_{21}}) \bigoplus \wp_2 (\grave{a}_1 \S_{\hat{e}_{12}} \bigoplus \grave{a}_2 \S_{\hat{e}_{22}})$$

$$= \wp_1 \left(\sqrt{1 - \frac{1}{1+\left\{ \grave{a}_1 \left(\frac{\epsilon_{11}^2}{1-\epsilon_{11}^2} \right)^{\propto} + \grave{a}_2 \left(\frac{\epsilon_{21}^2}{1-\epsilon_{21}^2} \right)^{\propto} \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \grave{a}_1 \left(\frac{1-\wp_{11}}{\wp_{11}} \right)^{\propto} + \grave{a}_2 \left(\frac{1-\wp_{21}}{\wp_{21}} \right)^{\propto} \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \grave{a}_1 \left(\frac{1-\mathscr{L}_{11}}{\mathscr{L}_{11}} \right)^{\propto} + \grave{a}_2 \left(\frac{1-\mathscr{L}_{21}}{\mathscr{L}_{21}} \right)^{\propto} \right\}^{\frac{1}{\propto}}} \right) \bigoplus$$

$$\wp_2 \left(\sqrt{1 - \frac{1}{1+\left\{ \grave{a}_1 \left(\frac{\epsilon_{12}^2}{1-\epsilon_{12}^2} \right)^{\propto} + \grave{a}_2 \left(\frac{\epsilon_{22}^2}{1-\epsilon_{22}^2} \right)^{\propto} \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \grave{a}_1 \left(\frac{1-\wp_{12}}{\wp_{12}} \right)^{\propto} + \grave{a}_2 \left(\frac{1-\wp_{22}}{\wp_{22}} \right)^{\propto} \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \grave{a}_1 \left(\frac{1-\mathscr{L}_{12}}{\mathscr{L}_{12}} \right)^{\propto} + \grave{a}_2 \left(\frac{1-\mathscr{L}_{22}}{\mathscr{L}_{22}} \right)^{\propto} \right\}^{\frac{1}{\propto}}} \right)$$

$$= \wp_1 \left(\sqrt{1 - \frac{1}{1+\left\{ \sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{\epsilon_{\ell 1}^2}{1-\epsilon_{\ell 1}^2} \right)^{\propto} \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\wp_{\ell 1}}{\wp_{\ell 1}} \right)^{\propto} \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell 1}}{\mathscr{L}_{\ell 1}} \right)^{\propto} \right\}^{\frac{1}{\propto}}} \right) \bigoplus$$

$$\wp_2 \left(\sqrt{1 - \frac{1}{1+\left\{ \sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{\epsilon_{\ell 2}^2}{1-\epsilon_{\ell 2}^2} \right)^{\propto} \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\wp_{\ell 2}}{\wp_{\ell 2}} \right)^{\propto} \right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{ \sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell 2}}{\mathscr{L}_{\ell 2}} \right)^{\propto} \right\}^{\frac{1}{\propto}}} \right)$$

$$= \left(\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{ \wp_1 \left(\sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{\epsilon_{\ell 1}^2}{1-\epsilon_{\ell 1}^2} \right)^{\propto} \right) + \wp_2 \left(\sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{\epsilon_{\ell 2}^2}{1-\epsilon_{\ell 2}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{ \wp_1 \left(\sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\wp_{\ell 1}}{\wp_{\ell 1}} \right)^{\propto} \right) + \wp_2 \left(\sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\wp_{\ell 2}}{\wp_{\ell 2}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \\ \frac{1}{1+\left\{ \wp_1 \left(\sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell 1}}{\mathscr{L}_{\ell 1}} \right)^{\propto} \right) + \wp_2 \left(\sum_{\ell=1}^{2} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell 2}}{\mathscr{L}_{\ell 2}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \end{array} \right)$$

$$= \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{2} \grave{a}_{\ell} \left(\frac{{€_{\ell\mathscr{b}}}^2}{1 - {€_{\ell\mathscr{b}}}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{2} \grave{a}_{\ell} \left(\frac{1 - \wp_{\ell\mathscr{b}}}{\wp_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{2} \grave{a}_{\ell} \left(\frac{1 - \mathscr{L}_{\ell\mathscr{b}}}{\mathscr{L}_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right)$$

Therefore, this outcome is true for $\mathbb{m} = 2$ and $\tilde{n} = 2$.

Now, we assume that the outcome is valid for $\mathbb{m} = \mathfrak{R}_1$ and $\tilde{n} = \mathfrak{R}_2$, as follows:

$$SFS_{ft}DWA\left(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{\mathfrak{R}_1\mathfrak{R}_2}}\right) = \bigoplus_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\bigoplus_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \S_{\hat{e}_{\ell\mathscr{b}}}\right)$$

$$= \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{{€_{\ell\mathscr{b}}}^2}{1-{€_{\ell\mathscr{b}}}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{1-\wp_{\ell\mathscr{b}}}{\wp_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{1-\mathscr{L}_{\ell\mathscr{b}}}{\mathscr{L}_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right)$$

Next, we prove that the outcome is true for $\mathbb{m} = \mathfrak{R}_1 + 1$ and $\tilde{n} = \mathfrak{R}_2 + 1$, as follows:

$$SFS_{ft}DWA\left(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{\mathfrak{R}_1\mathfrak{R}_2}}, \S_{\hat{e}_{(\mathfrak{R}_{1+1})(\mathfrak{R}_{2+1})}}\right) = \left(\bigoplus_{\ell=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\bigoplus_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \S_{\hat{e}_{\ell\mathscr{b}}}\right)\right) \bigoplus \left(\mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \S_{\hat{e}_{(\mathfrak{R}_2+1)(\mathfrak{R}_1+1)}}\right)\right)$$

$$= \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{{€_{\ell\mathscr{b}}}^2}{1-{€_{\ell\mathscr{b}}}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{1-\wp_{\ell\mathscr{b}}}{\wp_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{1-\mathscr{L}_{\ell\mathscr{b}}}{\mathscr{L}_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right) \bigoplus$$

$$\left(\sqrt{1 - \frac{1}{1 + \left\{ \mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \left(\frac{{€_{\ell\mathscr{b}}}^2}{1-{€_{\ell\mathscr{b}}}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1 + \left\{ \mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \left(\frac{1-\wp_{\ell\mathscr{b}}}{\wp_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1 + \left\{ \mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \left(\frac{1-\mathscr{L}_{\ell\mathscr{b}}}{\mathscr{L}_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right)$$

$$= \left(\begin{array}{c} \sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{{€_{\ell\mathscr{b}}}^2}{1 - {€_{\ell\mathscr{b}}}^2} \right)^{\propto} \right) + \mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \left(\frac{{€_{\ell\mathscr{b}}}^2}{1 - {€_{\ell\mathscr{b}}}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \\ \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{1 - \wp_{\ell\mathscr{b}}}{\wp_{\ell\mathscr{b}}} \right)^{\propto} \right) + \mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \left(\frac{1 - \wp_{\ell\mathscr{b}}}{\wp_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \\ \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\mathfrak{R}_2} \mathscr{e}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{R}_1} \grave{a}_{\ell} \left(\frac{1 - \mathscr{L}_{\ell\mathscr{b}}}{\mathscr{L}_{\ell\mathscr{b}}} \right)^{\propto} \right) + \mathscr{e}_{\mathfrak{R}_2+1} \left(\grave{a}_{\mathfrak{R}_1+1} \left(\frac{1 - \mathscr{L}_{\ell\mathscr{b}}}{\mathscr{L}_{\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \end{array} \right)$$

Table 1
Spherical fuzzy soft data

Đ	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$
$ð_1$	(0.7, 0.3, 0.3)	(0.7, 0.3, 0.4)	(0.8, 0.3, 0.3)
$ð_2$	(0.8, 0.3, 0.3)	(0.6, 0.2, 0.3)	(0.8, 0.4, 0.3)
$ð_3$	(0.7, 0.4, 0.3)	(0.7, 0.5, 0.4)	(0.7, 0.4, 0.3)
$ð_4$	(0.6, 0.4, 0.3)	(0.7, 0.4, 0.4)	(0.6, 0.3, 0.4)

$$= \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\ell=1}^{\Re_2+1} \varkappa_\ell \left(\sum_{\ell=1}^{\Re_1+1} \grave{a}_\ell \left(\frac{\epsilon_{\ell\ell}^2}{1-\epsilon_{\ell\ell}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1 + \left\{ \sum_{\ell=1}^{\Re_2+1} \varkappa_\ell \left(\sum_{\ell=1}^{\Re_1+1} \grave{a}_\ell \left(\frac{1-\wp_{\ell\ell}}{\wp_{\ell\ell}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1 + \left\{ \sum_{\ell=1}^{\Re_2+1} \varkappa_\ell \left(\sum_{\ell=1}^{\Re_1+1} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell\ell}}{\mathscr{L}_{\ell\ell}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right)$$

Therefore, the outcome is valid for $\mathbb{m} = \Re_1 + 1$ and $\tilde{n} = \Re_2 + 1$. Hence, the result is true for all $\mathbb{m}, \tilde{n} \geq 1$..

It is clear from the above expression that the aggregated results established for $SFS_{ft}DWA$ operators are again $SFS_{ft}N$..

Example 1. Let $\eth = \{\eth_1, \eth_2, \eth_3, \eth_4\}$ be the family of experts who evaluate the progress of football player $'\mathscr{F}'$ using the parameter set given *as* $\mathbb{F} = \{\hat{e}_1 = \textit{Technique}, \hat{e}_2 = \textit{Fitness}, \hat{e}_3 = \textit{Mindset}\}$. *Let* $\grave{a} = (0.22, 0.31, 0.24, 0.23)$ be the WV for experts $\eth_\ell$ and $\varkappa = \{0.36, 0.27, 0.37\}$ denote the WVs for $\hat{e}_\ell$ parameters and $\propto = 2$. Suppose decision-makers provide their assessment information as *$SFS_{ft}Ns$* given in Table 1. Now, we use the established operator to evaluate the result by using equation (2), and we have the following:

$$SFS_{ft}DWA\left(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{43}}\right)$$

$$= \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\ell=1}^{3} \varkappa_\ell \left(\sum_{\ell=1}^{4} \grave{a}_\ell \left(\frac{\epsilon_{\ell\ell}^2}{1-\epsilon_{\ell\ell}^2} \right)^{2} \right) \right\}^{\frac{1}{2}}}}, \frac{1}{1 + \left\{ \sum_{\ell=1}^{3} \varkappa_\ell \left(\sum_{\ell=1}^{4} \grave{a}_\ell \left(\frac{1-\wp_{\ell\ell}}{\wp_{\ell\ell}} \right)^{2} \right) \right\}^{\frac{1}{2}}}, \frac{1}{1 + \left\{ \sum_{\ell=1}^{3} \varkappa_\ell \left(\sum_{\ell=1}^{4} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\ell\ell}}{\mathscr{L}_{\ell\ell}} \right)^{2} \right) \right\}^{\frac{1}{2}}} \right)$$

Therefore, the following is obtained:

$$SFS_{ft}DWA\left(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{43}}\right) = (0.4628, 0.0441, 0.3970)$$

Remark 1. 1) If we change the power 2 by 1 in *$SFS_{ft}DWA$* AOs, then the proposed *$SFS_{ft}DWA$* AOs are converted into a *PFS_{ft}* Dombi weighted average (*$PFS_{ft}DWA$*) operator.

2) If we use only one parameter $\hat{e}_1$ ($\mathbb{m} = 1$), then the presented *$SFS_{ft}DWA$* operator degenerates into spherical fuzzy Dombi weighted average (SFDWA) AOs.

Theorem 3. Let $\S_{\hat{e}_{\ell\ell}} = (\epsilon_{\ell\ell}, \wp_{\ell\ell}, \mathscr{L}_{\ell\ell})$ be the family of *$SFS_{ft}Ns$*, where $\ell = 1, 2, \ldots, \mathbb{m}$ and $\ell = 1, 2, \ldots, \tilde{n}$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathbb{m}})$ and $\varkappa = \{\varkappa_1, \varkappa_2, \ldots, \varkappa_{\tilde{n}}\}$ represent the WV of $\eth_\ell$ experts $\hat{e}_\ell$ parameters, respectively such that $\grave{a}_\ell, \varkappa_\ell \in [0, 1]$ and $\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell = 1, \sum_{\ell=1}^{\tilde{n}} \varkappa_\ell = 1$. Then, *$SFS_{ft}DWA$* AO satisfies the following properties:

1) (*Idempotency*): *Let* $\S_{\hat{e}_{\ell\ell}} = \mathscr{Q}_{\hat{e}}$ for all ($\ell = 1, 2, \ldots, \mathbb{m}$) *and* $\ell = 1, 2, \ldots, \tilde{n}$ *where* $\mathscr{Q}_{\hat{e}} = (¢, \varrho, \hat{u})$. Then, the following is obtained:

$$SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) = \mathscr{Q}_{\hat{e}}.$$

2) (Boundedness): Let $\S^{-}_{\hat{e}_{\ell\ell}} = \left(\underset{\ell}{\min}\underset{\ell}{\min}(\epsilon_{\ell\ell}), \underset{\ell}{\max}\underset{\ell}{\max}(\wp_{\ell\ell}), \underset{\ell}{\max}\underset{\ell}{\max}(\mathscr{L}_{\ell\ell}) \right)$ and $\S^{+}_{\hat{e}_{\ell\ell}} = \left(\underset{\ell}{\max}\underset{\ell}{\max}(\epsilon_{\ell\ell}), \underset{\ell}{\min}\underset{\ell}{\min}(\wp_{\ell\ell}), \underset{\ell}{\min}\underset{\ell}{\min}(\mathscr{L}_{\ell\ell}) \right)$, then, the following is obtained:

$$\S^{-}_{\hat{e}_{\ell\ell}} \leq SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) \leq \S^{+}_{\hat{e}_{\ell\ell}}.$$

3) (Monotonicity): Suppose another collection of *$SFS_{ft}Ns$* $\mathscr{Q}_{\hat{e}_{\ell\ell}} = (¢_{\ell\ell}, \varrho_{\ell\ell}, \hat{u}_{\ell\ell})$ *for* ($\ell = 1.2, \ldots, \mathbb{m}$) *and* $\ell = 1, 2, \ldots, \tilde{n}$ *such that* $\epsilon_{\ell\ell} \leq ¢_{\ell\ell}, \wp_{\ell\ell} \geq \varrho_{\ell\ell}$ *and* $\mathscr{L}_{\ell\ell} \geq \hat{u}_{\ell\ell}$. Then, the following is obtained:

$$SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) \leq SFS_{ft}DWA\left(\mathscr{Q}_{\hat{e}_{11}}, \mathscr{Q}_{\hat{e}_{12}}, \ldots, \mathscr{Q}_{\mathbb{m}n}\right).$$

4) (Shift Invariance): Let $\mathscr{Q}_{\hat{e}} = (¢, \varrho, \hat{u})$ be a *$SFS_{ft}Ns$*. Then, the following is obtained:

$$SFS_{ft}DWA(\S_{\hat{e}_{11}} \bigoplus \mathscr{Q}_{\hat{e}}, \S_{\hat{e}_{12}} \bigoplus \mathscr{Q}_{\hat{e}}, \ldots, \S_{\mathbb{m}n} \bigoplus \mathscr{Q}_{\hat{e}}) = SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) \bigoplus \mathscr{Q}_{\hat{e}}.$$

5) (Homogeneity): For any $\flat > 0$, the following applies:

$$SFS_{ft}DWA(\flat\S_{\hat{e}_{11}}, \flat\S_{\hat{e}_{12}}, \ldots, \flat\S_{\mathbb{m}n}) = \flat SFS_{ft}DWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}).$$

Proof: Straightforward

4.2. Spherical fuzzy soft Dombi ordered weighted average operators

This section is committed to the construction of SFS_{ft} Dombi-ordered weighted average ($SFS_{ft}DOWA$) AOs. Furthermore, we analyze the basic properties related to $SFS_{ft}DOWA$ AOs.

Definition 8. Consider the family of $SFS_{ft}Ns$, i.e., $\S_{\hat{e}_{\ell\mathscr{b}}} = (€_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ for $\ell = 1, 2, \ldots, \mathfrak{m}$ and $\mathscr{b} = 1, 2, \ldots, \tilde{n}$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathfrak{m}})$ denote the WV for experts $\eth_\ell$ and $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ denote the WVs for $\hat{e}_{\mathscr{b}}$ parameters such that $\grave{a}_\ell, \mathscr{o}_{\mathscr{b}} \in [0,1]$ and $\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell = 1$, $\sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} = 1$. Now, $SFS_{ft}DOWA$ AO can be defined as $SFS_{ft}DOWA : ©^{\tilde{n}} \rightarrow ©$, where © represents the family of $SFS_{ft}Ns$ such that the following applies:

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{mn}}) = \bigoplus_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} \left(\bigoplus_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell \S_{\hat{e}_{\eqsim\ell\mathscr{b}}} \right) \tag{3}$$

where $\S_{\hat{e}_{\eqsim\ell\mathscr{b}}} = (€_{\eqsim\ell\mathscr{b}}, \wp_{\eqsim\ell\mathscr{b}}, \mathscr{L}_{\eqsim\ell\mathscr{b}})$ is the permutation of the ℓth row and $\mathscr{b}th$ largest elements of the collection for $\ell \times \mathscr{b}$ $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\mathscr{b}}} = (€_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ for $\ell = 1, 2, \ldots, \mathfrak{m}$ and $\mathscr{b} = 1, 2, \ldots, \tilde{n}$.

Now, using equation (3), we can define $SFS_{ft}DOWA$ AOs, as follows:

Theorem 4. *Suppose the set of* $\mathrm{SFS_{ft}Ns}$ $\S_{\hat{e}_{\ell\mathscr{b}}} = (€_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ where $\ell = 1, 2, \ldots, \mathfrak{m}$ *and* $\mathscr{b} = 1, 2, \ldots, \tilde{n}$. *Let* $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathfrak{m}})$ represent the WV of experts $\eth_\ell$ and $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ denote the WVs of $\hat{e}_{\mathscr{b}}$ parameters such that $\grave{a}_\ell, \mathscr{o}_{\mathscr{b}} \in [0,1]$ and $\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell = 1$, $\sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} = 1$. Then, the aggregated outcome for the $SFS_{ft}DOWA$ operator is defined as follows:

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{mn}}) = \bigoplus_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} \left(\bigoplus_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell \S_{\hat{e}_{\eqsim\ell\mathscr{b}}} \right)$$

$$= \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell \left(\frac{€_{\eqsim\ell\mathscr{b}}^2}{1-€_{\eqsim\ell\mathscr{b}}^2} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell \left(\frac{1-\wp_{\eqsim\ell\mathscr{b}}}{\wp_{\eqsim\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} \left(\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\eqsim\ell\mathscr{b}}}{\mathscr{L}_{\eqsim\ell\mathscr{b}}} \right)^{\propto} \right) \right\}^{\frac{1}{\propto}}} \right).$$

where $\S_{\hat{e}_{\eqsim\ell\mathscr{b}}} = (€_{\eqsim\ell\mathscr{b}}, \wp_{\eqsim\ell\mathscr{b}}, \mathscr{L}_{\eqsim\ell\mathscr{b}})$ *is the permutation of the* ℓth *row and* $\mathscr{b}th$ *largest* elements of the collection for $\ell \times \mathscr{b}$ $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\mathscr{b}}} = (€_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ *for* $\ell = 1, 2, \ldots, \mathfrak{m}$ *and* $\mathscr{b} = 1, 2, \ldots, \tilde{n}$. **Proof:** The proof of the Theorem is straightforward, similar to Theorem 2.

Example 2:. Let $\mathrm{SFS_{ft}Ns}$ $\S_{\hat{e}_{\ell\mathscr{b}}} = (€_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ for $\ell = 1, 2, \ldots, 4$ and $\mathscr{b} = 1, 2, 3$ be the family of $SFS_{ft}Ns$ as given in Table 1 of example 1. Now, by definition (4) of the SF for $SFS_{ft}Ns$, the tabular representation of $\S_{\hat{e}_{\eqsim\ell\mathscr{b}}} = (€_{\eqsim\ell\mathscr{b}}, \wp_{\eqsim\ell\mathscr{b}}, \mathscr{L}_{\eqsim\ell\mathscr{b}})$ is provided in Table 2.

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{43}}) = \left(\sqrt{1 - \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{3} \mathscr{o}_{\mathscr{b}} \left(\sum_{\ell=1}^{4} \grave{a}_\ell \left(\frac{€_{\eqsim\ell\mathscr{b}}^2}{1-€_{\eqsim\ell\mathscr{b}}^2} \right)^2 \right) \right\}^{\frac{1}{2}}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{3} \mathscr{o}_{\mathscr{b}} \left(\sum_{\ell=1}^{4} \grave{a}_\ell \left(\frac{1-\wp_{\eqsim\ell\mathscr{b}}}{\wp_{\eqsim\ell\mathscr{b}}} \right)^2 \right) \right\}^{\frac{1}{2}}}, \frac{1}{1 + \left\{ \sum_{\mathscr{b}=1}^{3} \mathscr{o}_{\mathscr{b}} \left(\sum_{\ell=1}^{4} \grave{a}_\ell \left(\frac{1-\mathscr{L}_{\eqsim\ell\mathscr{b}}}{\mathscr{L}_{\eqsim\ell\mathscr{b}}} \right)^2 \right) \right\}^{\frac{1}{2}}} \right)$$

Therefore, the following is obtained;

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{43}}) = (0.4389, 0.04916, 0.3963)$$

Remarks 2:

1) If we change power 2 by 1 in $SFS_{ft}DOWA$ AOs, then the proposed $SFS_{ft}DOWA$ operator is reduced to PFS_{ft} Dombi ordered weighted average ($PFS_{ft}DOWA$) AO.

Table 2
Tabular representation of $\S_{\hat{e}_{\eqsim\ell\mathscr{b}}} = (€_{\eqsim\ell\mathscr{b}}, \wp_{\eqsim\ell\mathscr{b}}, \mathscr{L}_{\eqsim\ell\mathscr{b}})$ for $\propto = 2$.

Ð	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$
$\eth_1$	(0.7, 0.3, 0.3)	(0.7, 0.3, 0.4)	(0.8, 0.3, 0.3)
$\eth_2$	(0.6, 0.4, 0.3)	(0.7, 0.5, 0.4)	(0.7, 0.4, 0.3)
$\eth_3$	(0.8, 0.3, 0.3)	(0.7, 0.4, 0.4)	(0.8, 0.4, 0.3)
$\eth_4$	(0.7, 0.4, 0.3)	(0.6, 0.2, 0.3)	(0.6, 0.3, 0.4)

2) If we use only one parameter $\hat{e}_1$ ($\mathbb{m} = 1$), then the presented *$SFS_{ft}DOWA$* operator degenerates into an SF Dombi-ordered weighted average (*SFDOWA*) operator.

Theorem 5. *Let* $\S_{\hat{e}_{\ell\mathscr{b}}} = (\text{€}_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ be the family of *$SFS_{ft}Ns$*, where $\ell = 1, 2, \ldots, \mathbb{m}$ and $\mathscr{b} = 1, 2, \ldots, \tilde{n}$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathbb{m}})$ and $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ *represent the* WV *of* $ð_\ell$ *experts* $\hat{e}_{\mathscr{b}}$ *parameters such that* $\grave{a}_\ell, \mathscr{o}_{\mathscr{b}} \in [0,1]$ *and* $\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell = 1, \sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} = 1$. *Then, $SFS_{ft}DOWA$* AOs satisfy the following properties:

1) (*Idempotency*): *Let* $\S_{\hat{e}_{\ell\mathscr{b}}} = \mathscr{Q}_{\hat{e}}$ *for all* ($\ell = 1.2, \ldots, \mathbb{m}$) *and* $\mathscr{b} = 1, 2, \ldots, \tilde{n}$ *where* $\mathscr{Q}_{\hat{e}} = (¢, ϱ, \hat{u})$. *Then, the following is obtained*:

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) = \mathscr{Q}_{\hat{e}}.$$

2) (*Boundedness*): *Let* $\S^-_{\hat{e}_{\ell\mathscr{b}}} = \left(\min_{\mathscr{b}}\min_{\ell}(\text{€}_{\ell\mathscr{b}}), \max_{\mathscr{b}}\max_{\ell}(\wp_{\ell\mathscr{b}}), \max_{\mathscr{b}}\max_{\ell}(\mathscr{L}_{\ell\mathscr{b}})\right)$ *and* $\S^+_{\hat{e}_{\ell\mathscr{b}}} = \left(\max_{\mathscr{b}}\max_{\ell}(\text{€}_{\ell\mathscr{b}}), \min_{\mathscr{b}}\min_{\ell}(\wp_{\ell\mathscr{b}}), \min_{\mathscr{b}}\min_{\ell}(\mathscr{L}_{\ell\mathscr{b}})\right)$, *then, the following is obtained*:

$$\S^-_{\hat{e}_{\ell\mathscr{b}}} \le SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) \le \S^+_{\hat{e}_{\ell\mathscr{b}}}.$$

3) (*Monotonicity*): *Suppose another collection of $SFS_{ft}Ns$* $\mathscr{Q}_{\hat{e}_{\ell\mathscr{b}}} = (¢_{\ell\mathscr{b}}, ϱ_{\ell\mathscr{b}}, \hat{u}_{\ell\mathscr{b}})$ *for* ($\ell = 1.2, \ldots, \mathbb{m}$) *and* $\mathscr{b} = 1, 2, \ldots, \tilde{n}$ *such that* $\text{€}_{\ell\mathscr{b}} \le ¢_{\ell\mathscr{b}}$, $\wp_{\ell\mathscr{b}} \ge ϱ_{\ell\mathscr{b}}$ *and* $\mathscr{L}_{\ell\mathscr{b}} \ge \hat{u}_{\ell\mathscr{b}}$. *Then, the following is obtained*:

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) \le SFS_{ft}DOWA\left(\mathscr{Q}_{\hat{e}_{11}}, \mathscr{Q}_{\hat{e}_{12}}, \ldots, \mathscr{Q}_{\mathbb{m}n}\right).$$

4) (*Shift Invariance*): *Let* $\mathscr{Q}_{\hat{e}} = (¢, ϱ, \hat{u})$ *be a $SFS_{ft}Ns$. Then, the following is obtained*:

$$SFS_{ft}DOWA(\S_{\hat{e}_{11}} \bigoplus \mathscr{Q}_{\hat{e}}, \S_{\hat{e}_{12}} \bigoplus \mathscr{Q}_{\hat{e}}, \ldots, \S_{\mathbb{m}n} \bigoplus \mathscr{Q}_{\hat{e}}) = SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}) \bigoplus \mathscr{Q}_{\hat{e}}.$$

5) (*Homogeneity*): *For any* $\not{p} > 0$, *then, the following applies*:

$$SFS_{ft}DOWA(\not{p}\S_{\hat{e}_{11}}, \not{p}\S_{\hat{e}_{12}}, \ldots, \not{p}\S_{\mathbb{m}n}) = \not{p}SFS_{ft}DOWA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathbb{m}n}).$$

4.3. Spherical fuzzy soft Dombi hybrid average aggregation operators

In this subsection, we introduce *SFS_{ft}* Dombi hybrid average (*$SFS_{ft}DHA$*) AOs and their basic properties.

Definition 9. Consider the family of *$SFS_{ft}Ns$*, i.e., $\S_{\hat{e}_{\ell\mathscr{b}}} = (\text{€}_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ for $\ell = 1, 2, \ldots, \mathbb{m}$ and $\mathscr{b} = 1, 2, \ldots, \tilde{n}$, and let $\tau = \{\tau_1, \tau_2, \ldots, \tau_{\mathbb{m}}\}, \partial = \{\partial_1, \partial_2, \ldots, \partial_{\tilde{n}}\}$ be the WVs of $\S_{\hat{e}_{\ell\mathscr{b}}} = (\text{€}_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ using the condition that $\tau_\ell, \partial_{\mathscr{b}} \in [0,1]$ with $\sum_{\ell=1}^{\mathbb{m}} \tau_\ell = 1$ and $\sum_{\mathscr{b}=1}^{\tilde{n}} \partial_{\mathscr{b}} = 1$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathbb{m}})$ denote the WV for experts $ð_\ell$ and $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ denote the WVs for $\hat{e}_{\mathscr{b}}$ parameters such that $\grave{a}_\ell, \mathscr{o}_{\mathscr{b}} \in [0,1]$ and $\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell = 1, \sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} = 1$. Now, the *$SFS_{ft}DHA$* operator can be defined as *$SFS_{ft}DHA$* : $©^{\tilde{n}} \to ©$, where © represents the family of *$SFS_{ft}Ns$* as follows:

$$SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{\mathbb{m}n}}) = \bigoplus_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} \left(\bigoplus_{\ell=1}^{\mathbb{m}} \grave{a}_\ell \overline{\S}_{\hat{e}_{\sigma\ell\mathscr{b}}}\right) \tag{4}$$

where $\overline{\S}_{\hat{e}_{\sigma\ell\mathscr{b}}} = n\tau_\ell\partial_{\mathscr{b}}\S_{\hat{e}_{\ell\mathscr{b}}}$ is the permutation of the ℓth row and $\mathscr{b} th$ largest elements of the collection for $\ell \times \mathscr{b}$ *$SFS_{ft}Ns$* $\S_{\hat{e}_{\ell\mathscr{b}}} = (\text{€}_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ and '*n*' is the balancing coefficient. Now, using equation (4), we can define the *$SFS_{ft}DHA$* operator given as follows:

Theorem 6. *Suppose the family of $SFS_{ft}Ns$* $\S_{\hat{e}_{\ell\mathscr{b}}} = (\text{€}_{\ell\mathscr{b}}, \wp_{\ell\mathscr{b}}, \mathscr{L}_{\ell\mathscr{b}})$ *where* $\ell = 1, 2, \ldots, \mathbb{m}$ *and* $\mathscr{b} = 1, 2, \ldots, \tilde{n}$. *Let* $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathbb{m}})$ *denote the WV of experts* $ð_\ell$ *and* $\mathscr{o} = \{\mathscr{o}_1, \mathscr{o}_2, \ldots, \mathscr{o}_{\tilde{n}}\}$ *denote the WVs of* $\hat{e}_{\mathscr{b}}$ *parameters such that* $\grave{a}_\ell, \mathscr{o}_{\mathscr{b}} \in [0,1]$ *and* $\sum_{\ell=1}^{\mathbb{m}} \grave{a}_\ell = 1, \sum_{\mathscr{b}=1}^{\tilde{n}} \mathscr{o}_{\mathscr{b}} = 1$. *Then, the aggregated result for $SFS_{ft}DHA$ AOs is defined as follows*:

Table 3
Tabular description of $\overline{\S}_{\hat{e}_{\sigma\ell\mathscr{b}}} = n\tau_\ell\partial_{\mathscr{b}}\S_{\hat{e}_{\ell\mathscr{b}}}$ for $\propto = 2$.

Ð	$\hat{e}_{\sigma_1}$	$\hat{e}_{\sigma_2}$	$\hat{e}_{\sigma_3}$
$ð_1$	(0.5107, 0.5384, 0.5882)	(0.5107, 0.4829, 0.5922)	(0.6285, 0.4929, 0.4929)
$ð_2$	(0.6700, 0.4832, 0.5338)	(0.4527, 0.3040, 0.4281)	(0.6700, 0.5479, 0.4379)
$ð_3$	(0.5420, 0.6062, 0.5479)	(0.5420, 0.6490, 0.5521)	(0.5420, 0.5619, 0.4519)
$ð_4$	(0.4558, 0.5883, 0.5295)	(0.5562, 0.5336, 0.5336)	(0.4558, 0.4336, 0.5436)

Table 4
Tabular description of $\overline{\S}_{\hat{e}_{\ell\hbar}} = n\tau_{\ell}\partial_{\hbar}\S_{\hat{e}_{\ell\hbar}}$ for $\propto = 2$.

Ð	$\hat{e}_{1}$	$\hat{e}_{2}$	$\hat{e}_{3}$
ð$_1$	$(0.6700, 0.4832, 0.5338)$	$(0.4527, 0.3040, 0.4281)$	$(0.6700, 0.5479, 0.4379)$
ð$_2$	$(0.5420, 0.6062, 0.5479)$	$(0.5562, 0.5336, 0.5336)$	$(0.6285, 0.4929, 0.4929)$
ð$_3$	$(0.5107, 0.5384, 0.5882)$	$(0.5107, 0.4829, 0.5922)$	$(0.5420, 0.5619, 0.4519)$
ð$_4$	$(0.4558, 0.5883, 0.5295)$	$(0.5420, 0.6490, 0.5521)$	$(0.4558, 0.4336, 0.5436)$

$$SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{mn}}) = \bigoplus_{\hbar=1}^{\tilde{n}} \varrho_{\hbar}\left(\bigoplus_{\ell=1}^{m} \grave{a}_{\ell}\overline{\S}_{\hat{e}_{\ell\hbar}}\right)$$

$$= \left(\sqrt{1-\frac{1}{1+\left\{\sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar}\left(\sum_{\ell=1}^{m}\grave{a}_{\ell}\left(\frac{\overline{\epsilon}_{\ell\hbar}^{\,2}}{1-\overline{\epsilon}_{\ell\hbar}^{\,2}}\right)^{\propto}\right)\right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{\sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar}\left(\sum_{\ell=1}^{m}\grave{a}_{\ell}\left(\frac{1-\overline{\wp}_{\ell\hbar}}{\overline{\wp}_{\ell\hbar}}\right)^{\propto}\right)\right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{\sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar}\left(\sum_{\ell=1}^{m}\grave{a}_{\ell}\left(\frac{1-\overline{\mathscr{L}}_{\ell\hbar}}{\overline{\mathscr{L}}_{\ell\hbar}}\right)^{\propto}\right)\right\}^{\frac{1}{\propto}}}\right).$$

where $\overline{\S}_{\hat{e}_{\ell\hbar}} = n\tau_{\ell}\partial_{\hbar}\S_{\hat{e}_{\ell\hbar}}$ *is the permutation of the* ℓ*th row and* $\hbar$*th largest elements of the collection for* $\ell \times \hbar$ *SFS$_{ft}$Ns* $\S_{\hat{e}_{\ell\hbar}} = (\epsilon_{\ell\hbar}, \wp_{\ell\hbar}, \mathscr{L}_{\ell\hbar})$ *with* $\tau = \{\tau_1, \tau_2, \ldots, \tau_m\}, \partial = \{\partial_1, \partial_2, \ldots, \partial_{\tilde{n}}\}$ *being the WVs and 'n' is the balancing coefficient.*

Proof: *The proof of the* Theorem *is straightforward, similar to* Theorem 2.

Example 3. *Let SFS$_{ft}$Ns* $\S_{\hat{e}_{\ell\hbar}} = (\epsilon_{\ell\hbar}, \wp_{\ell\hbar}, \mathscr{L}_{\ell\hbar})$ *for* $\ell = 1, 2, \ldots, 4$ *and* $\hbar = 1, 2, 3$ *be the family of SFS$_{ft}$Ns as provided in* Table 1 *of example* 1. *Let* $\tau = \{0.18, 0.28, 0.25, 0.29\}, \partial = \{0.25, 0.39, 0.36\}$ *be the WVs of* ð$_{\ell}$ *experts and parameters* $\hat{e}_{\hbar}$. *Let their corresponding associated WVs* $\grave{a} = (0.22, 0.31, 0.24, 0.23)$ *for* ð$_{\ell}$ experts and $\varrho = \{0.36, 0.27, 0.37\}$ for parameters $\hat{e}_{\hbar}$. *Now, we use* equation (5), *and their results are listed in* Table 3. *Additionally, by using definition* (4) *of SF, we obtain a new ordering of* $\overline{\S}_{\hat{e}_{\ell\hbar}} = n\tau_{\ell}\partial_{\hbar}\S_{\hat{e}_{\ell\hbar}}$ *given in* Table 4.

$$\not{p}\S_{\hat{e}} = \left(\sqrt{1-\frac{1}{1+\left\{\not{p}\left(\frac{\epsilon_{\S_{\hat{e}}}^{\,2}}{1-\epsilon_{\S_{\hat{e}}}^{\,2}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}}, \frac{1}{1+\left\{\not{p}\left(\frac{1-\wp_{\S_{\hat{e}}}}{\wp_{\S_{\hat{e}}}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}, \frac{1}{1+\left\{\not{p}\left(\frac{1-\mathscr{L}_{\S_{\hat{e}}}}{\mathscr{L}_{\S_{\hat{e}}}}\right)^{\propto}\right\}^{\frac{1}{\propto}}}\right) \tag{5}$$

Now, we use the following:

$$SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{43}}) = \bigoplus_{\hbar=1}^{3} \varrho_{\hbar}\left(\bigoplus_{\ell=1}^{4} \grave{a}_{\ell}\overline{\S}_{\hat{e}_{\ell\hbar}}\right)$$

$$= \left(\sqrt{1-\frac{1}{1+\left\{\sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar}\left(\sum_{\ell=1}^{m}\grave{a}_{\ell}\left(\frac{\overline{\epsilon}_{\ell\hbar}^{\,2}}{1-\overline{\epsilon}_{\ell\hbar}^{\,2}}\right)\right)^{2}\right\}^{\frac{1}{2}}}}, \frac{1}{1+\left\{\sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar}\left(\sum_{\ell=1}^{m}\grave{a}_{\ell}\left(\frac{1-\overline{\wp}_{\ell\hbar}}{\overline{\wp}_{\ell\hbar}}\right)\right)^{2}\right\}^{\frac{1}{2}}}, \frac{1}{1+\left\{\sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar}\left(\sum_{\ell=1}^{m}\grave{a}_{\ell}\left(\frac{1-\overline{\mathscr{L}}_{\ell\hbar}}{\overline{\mathscr{L}}_{\ell\hbar}}\right)\right)^{2}\right\}^{\frac{1}{2}}}\right)$$

$$SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\hat{e}_{43}}) = (0.003565, 0.4590, 0.4829)$$

Remark 3:

1) *If we change the power* 2 by 1 *in SFS$_{ft}$DHA AOs, then the proposed SFS$_{ft}$DHA operator is reduced to the PFS$_{ft}$ Dombi hybrid average* (*PFS$_{ft}$DHA*) *operator.*
2) *If we use only one parameter* $\hat{e}_1$ $(m = 1)$, *then the presented SFS$_{ft}$DHA operator degenerates into a spherical fuzzy Dombi hybrid average* (*SFDHA*) *operator.*

Theorem 7. *Let* $\S_{\hat{e}_{\ell\hbar}} = (\epsilon_{\ell\hbar}, \wp_{\ell\hbar}, \mathscr{L}_{\ell\hbar})$ *be the family of SFS$_{ft}$Ns, where* $\ell = 1, 2, \ldots, m$ *and* $\hbar = 1, 2, \ldots, \tilde{n}$ *with* $\tau = \{\tau_1, \tau_2, \ldots, \tau_m\}, \partial = \{\partial_1, \partial_2, \ldots, \partial_{\tilde{n}}\}$ *be the WVs of* $\S_{\hat{e}_{\ell\hbar}} = (\epsilon_{\ell\hbar}, \wp_{\ell\hbar}, \mathscr{L}_{\ell\hbar})$ *such that* $\tau_{\ell}, \partial_{\hbar} \in [0, 1]$ *and* $\sum_{\ell}^{m}\tau_{\ell} = 1$ *and* $\sum_{\hbar}^{\tilde{n}}\partial_{\hbar} = 1$. *Let* $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_m)$ *and* $\varrho = \{\varrho_1, \varrho_2, \ldots, \varrho_{\tilde{n}}\}$ *present the WV of* ð$_{\ell}$ *experts* $\hat{e}_{\hbar}$ *parameters such that* $\grave{a}_{\ell}, \varrho_{\hbar} \in [0, 1]$ *and* $\sum_{\ell=1}^{m}\grave{a}_{\ell} = 1, \sum_{\hbar=1}^{\tilde{n}}\varrho_{\hbar} = 1$. *Then, SFS$_{ft}$DHA AOs satisfy the following properties*:

1) (*Idempotency*): *Let* $\S_{\hat{e}_{\ell\hbar}} = \mathscr{Q}_{\hat{e}}$ *for all* $(\ell = 1.2, \ldots, m)$ *and* $\hbar = 1, 2, \ldots, \tilde{n}$ *where* $\mathscr{Q}_{\hat{e}} = (¢, \varrho, \hat{u})$. *Then, the following is obtained*:

$$SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{mn}) = \mathscr{Q}_{\hat{e}}.$$

2) (*Boundedness*): *Let* $\S^{-}_{\hat{e}_{\ell\ell}} = \left(\underset{\ell}{\min}\underset{\ell}{\min}(\text{€}_{\ell\ell}), \underset{\ell}{\max}\underset{\ell}{\max}(\wp_{\ell\ell}), \underset{\ell}{\max}\underset{\ell}{\max}(\mathscr{L}_{\ell\ell})\right)$ *and* $\S^{+}_{\hat{e}_{\ell\ell}} = \left(\underset{\ell}{\max}\underset{\ell}{\max}(\text{€}_{\ell\ell}), \underset{\ell}{\min}\underset{\ell}{\min}(\wp_{\ell\ell}), \underset{\ell}{\min}\underset{\ell}{\min}(\mathscr{L}_{\ell\ell})\right)$, *then, the following is obtained*:

$$\S^{-}_{\hat{e}_{\ell\ell}} \leq SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathfrak{m}n}) \leq \S^{+}_{\hat{e}_{\ell\ell}}.$$

3) (*Monotonicity*): *Suppose another collection of* $SFS_{ft}Ns$ $\mathscr{Q}_{\hat{e}_{\ell\ell}} = (\text{¢}_{\ell\ell}, \varrho_{\ell\ell}, \hat{u}_{\ell\ell})$ *for* $(\ell = 1.2, \ldots, \mathfrak{m})$ *and* $\ell = 1, 2, \ldots, \tilde{n}$ *such that* $\text{€}_{\ell\ell} \leq \text{¢}_{\ell\ell}$, $\wp_{\ell\ell} \geq \varrho_{\ell\ell}$ *and* $\mathscr{L}_{\ell\ell} \geq \hat{u}_{\ell\ell}$. *Then, the following is obtained*:

$$SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathfrak{m}n}) \leq SFS_{ft}DHA\left(\mathscr{Q}_{\hat{e}_{11}}, \mathscr{Q}_{\hat{e}_{12}}, \ldots, \mathscr{Q}_{\mathfrak{m}n}\right).$$

4) (*Shift Invariance*): *Let* $\mathscr{Q}_{\hat{e}} = (\text{¢}, \varrho, \hat{u})$ *be a* $SFS_{ft}Ns$. *Then, the following is obtained*:

$$SFS_{ft}DHA(\S_{\hat{e}_{11}} \bigoplus \mathscr{Q}_{\hat{e}}, \S_{\hat{e}_{12}} \bigoplus \mathscr{Q}_{\hat{e}}, \ldots, \S_{\mathfrak{m}n} \bigoplus \mathscr{Q}_{\hat{e}}) = SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathfrak{m}n}) \bigoplus \mathscr{Q}_{\hat{e}}.$$

5) (*Homogeneity*): *For any* $\beta > 0$, *the following applies*:

$$SFS_{ft}DHA(\beta\S_{\hat{e}_{11}}, \beta\S_{\hat{e}_{12}}, \ldots, \beta\S_{\mathfrak{m}n}) = \beta SFS_{ft}DHA(\S_{\hat{e}_{11}}, \S_{\hat{e}_{12}}, \ldots, \S_{\mathfrak{m}n}).$$

5. MCDM approach using the spherical fuzzy soft Dombi aggregation operators

This section is devoted to determining the MCDM technique by using the introduced operators. Additionally, the stepwise general algorithm is given as follows:

Let $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_l\}$ denote the family of alternatives. The target is to evaluate the best alternative by the family of decision-making experts $\varkappa = \{\varkappa_1, \varkappa_2, \ldots, \varkappa_{\mathfrak{m}}\}$ corresponding to their parameter set given as $E = \{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_{\tilde{n}}\}$. Let $\grave{a} = (\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_{\mathfrak{m}})$ and $\wp = \{\wp_1, \wp_2, \ldots, \wp_{\tilde{n}}\}$ presents the WV of $\eth_\ell$ experts $\hat{e}_\ell$ parameters such that $\grave{a}_\ell, \wp_\ell \in [0, 1]$ and $\sum_{\ell=1}^{\mathfrak{m}} \grave{a}_\ell = 1, \sum_{\ell=1}^{\tilde{n}} \wp_\ell = 1$. The team of $\mathfrak{m}$ experts assesses each object Ω_l corresponding to their parameters $\hat{e}_{\tilde{n}}$. The experts provide their assessment in the form of $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\ell}} = (\text{€}_{\ell\ell}, \wp_{\ell\ell}, \mathscr{L}_{\ell\ell})$. The overall evaluation of the experts is provided in a decision matrix $\mathfrak{M} = [\S_{\hat{e}_{\ell\ell}}]_{\mathfrak{m}\times\tilde{n}}$. Using the preferences values of senior experts, the aggregated results $\mathscr{G}_{\mathscr{J}}$ for alternatives $\Omega_{\mathscr{J}}$ $(\mathscr{J} = 1, 2, \ldots, l)$ is $\mathscr{G}_{\mathscr{J}} = (\text{€}_{\mathscr{J}}, \wp_{\mathscr{J}}, \mathscr{L}_{\mathscr{J}})$ by using the $SFS_{ft}DW$ averaging operators that are given in equations (2), (4) and (6). Finally, definition 4 is applied to each aggregated result $\mathscr{G}_{\mathscr{J}}$, and $\mathscr{G}_{\mathscr{J}}$ values are ranked to obtain the optimal choice.

5.1. Algorithm

The algorithm for our established work to solve MCDM problems is summarized by the following:

Step 1. The overall data of senior experts proposed for each alternative corresponding to their parameter as $\mathfrak{M} = [\S_{\hat{e}_{\ell\ell}}]_{\mathfrak{m}\times\tilde{n}}$ are collected, as follows:

$$\mathfrak{M} = \begin{bmatrix} (\text{€}_{11}, \wp_{11}, \mathscr{L}_{11}) & (\text{€}_{12}, \wp_{12}, \mathscr{L}_{12}) & \ldots & (\text{€}_{1\tilde{n}}, \wp_{1\tilde{n}}, \mathscr{L}_{1\tilde{n}}) \\ (\text{€}_{21}, \wp_{21}, \mathscr{L}_{21}) & (\text{€}_{22}, \wp_{22}, \mathscr{L}_{22}) & \ldots & (\text{€}_{2\tilde{n}}, \wp_{2\tilde{n}}, \mathscr{L}_{2\tilde{n}}) \\ \vdots & \vdots & \ldots & \vdots \\ (\text{€}_{\mathfrak{m}1}, \wp_{\mathfrak{m}1}, \mathscr{L}_{\mathfrak{m}1}) & (\text{€}_{\mathfrak{m}2}, \wp_{\mathfrak{m}2}, \mathscr{L}_{\mathfrak{m}2}) & \ldots & (\text{€}_{\mathfrak{m}n}, \wp_{\mathfrak{m}n}, \mathscr{L}_{\mathfrak{m}n}) \end{bmatrix}$$

Step 2. $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\ell}}$ is aggregated for alternative $\Omega_{\mathscr{J}}$ $(\mathscr{J} = 1, 2, \ldots, l)$ into an overall decision matrix $\mathscr{G}_{\mathscr{J}} = (\text{€}_{\mathscr{J}}, \wp_{\mathscr{J}}, \mathscr{L}_{\mathscr{J}})$ by using the $SFS_{ft}DWA$ AOs.

Step 3. The score values for each $\mathscr{G}_{\mathscr{J}} = (\text{€}_{\mathscr{J}}, \wp_{\mathscr{J}}, \mathscr{L}_{\mathscr{J}})$ are evaluated for each alternative $\Omega_{\mathscr{J}}(\mathscr{J} = 1, 2, \ldots, l)$.

Table 5
Tabular representation of $SFS_{ft}Ns$ for $\propto = 2$.

Ω	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$
Ω_1	(0.91, 0.3, 0.5)	(0.3, 0.9, 0.3)	(0.2, 0.8, 0.6)
Ω_2	(0.2, 0.4, 0.6)	(0.6, 0.6, 0.4)	(0.3, 0.9, 0.4)
Ω_3	(0.5, 0.5, 0.5)	(0.3, 0.8, 0.5)	(0.5, 0.7, 0.3)
Ω_4	(0.6, 0.4, 0.5)	(0.2, 0.9, 0.42)	(0.4, 0.5, 0.4)

Table 6
Tabular representation of $SFS_{ft}Ns$ for $\propto\ = 2$.

Ω	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$
Ω_1	$(0.3, 0.3, 0.3)$	$(0.5, 0.7, 0.3)$	$(0.91, 0.3, 0.5)$
Ω_2	$(0.6, 0.6, 0.4)$	$(0.4, 0.5, 0.4)$	$(0.2, 0.4, 0.6)$
Ω_3	$(0.3, 0.4, 0.5)$	$(0.2, 0.8, 0.6)$	$(0.5, 0.5, 0.5)$
Ω_4	$(0.2, 0.3, 0.4)$	$(0.3, 0.9, 0.4)$	$(0.6, 0.4, 0.5)$

Table 7
Tabular representation of $SFS_{ft}Ns$ for $\propto\ = 2$.

Ω	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$
Ω_1	$(0.6, 0.6, 0.4)$	$(0.5, 0.7, 0.3)$	$(0.2, 0.8, 0.6)$
Ω_2	$(0.4, 0.5, 0.4)$	$(0.91, 0.3, 0.5)$	$(0.5, 0.5, 0.5)$
Ω_3	$(0.3, 0.4, 0.6)$	$(0.3, 0.5, 0.4)$	$(0.6, 0.4, 0.5)$
Ω_4	$(0.3, 0.4, 0.3)$	$(0.2, 0.8, 0.6)$	$(0.6, 0.6, 0.4)$

Table 8
Tabular representation of $SFS_{ft}Ns$ for $\propto\ = 2$.

Ω	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$
Ω_1	$(0.2, 0.8, 0.6)$	$(0.3, 0.9, 0.3)$	$(0.3, 0.4, 0.6)$
Ω_2	$(0.6, 0.4, 0.5)$	$(0.6, 0.6, 0.4)$	$(0.6, 0.6, 0.4)$
Ω_3	$(0.91, 0.3, 0.5)$	$(0.2, 0.8, 0.6)$	$(0.4, 0.5, 0.4)$
Ω_4	$(0.5, 0.5, 0.5)$	$(0.5, 0.7, 0.3)$	$(0.3, 0.9, 0.4)$

Step 4. If the score values obtained from the above step are the same, then the formula of the accuracy function as given in definition 4 is applied to evaluate score values.

Step 5. Finally, the score values are ranked to select the optimal choice.

5.2. Numerical example

In this section, we will verify the validity and applicability of the introduced work. To do this, we have provided the numerical example of introduced work.

Let X be a company that wants to invest its money in the best alternative from a set of four alternatives $\Omega = \{\Omega_1 = \textit{Car company}, \Omega_2 = \textit{computer company}, \Omega_3 = \textit{TV company} \text{ and } \Omega_4 = \textit{Food company}\}$. Let $\varkappa = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$ be the set of experts with WVs $\grave{a} = \{0.21, 0.12, 0.27, 0.20, 0.17\}$ and $E = \{\hat{e}_1 = \textit{Strong leadership}, \hat{e}_2 = \textit{Encourage a healthy work environment}, \hat{e}_3 = \textit{Gaols and strategies}, \hat{e}_4 = \textit{Clear and defined purpose}\}$ be the set of parameters with WVs $\wp = \{0.28, 0.21, 0.27, 0.24\}$. The experts provide their opinion for each alternative corresponding to their parameters in the form of $SFS_{ft}Ns$. Now following steps are used for the selection of the most suitable company for investment.

By using the $\boldsymbol{SFS_{ft}DWA}$ operator

Step 1. The overall expert data based on $SFS_{ft}Ns$ are provided in Tables 5–8.

Step 2. The $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\ell}}$ is aggregated for alternatives $\Omega_{\mathscr{J}}$ $(\mathscr{J} = 1, 2, \ldots, l)$ by using the introduced $SFS_{ft}DW$ A AOs for $\propto = 2$ are given as follows:

$$\mathscr{G}_1 = (0.6179, 0.4499, 0.4348),\ \mathscr{G}_2 = (0.63330, 0.1703, 0.4191)$$

$$\mathscr{G}_3 = (0.6521, 0.3314, 0.4176),\ \mathscr{G}_4 = (0.6702, 0.4152, 0.4064)$$

Step 3. The score values are determined for each $\mathscr{G}_{\mathscr{J}} = (€_{\mathscr{J}}, \wp_{\mathscr{J}}, \mathscr{L}_{\mathscr{J}})$ for each alternative $\Omega_{\mathscr{J}}(\mathscr{J} = 1, 2, \ldots, l)$, as follows:

$$Sc(\mathscr{G}_1) = 0.5762, Sc(\mathscr{G}_2) = 0.6812,$$

$$Sc(\mathscr{G}_3) = 0.6342, Sc(\mathscr{G}_4) = 0.6161$$

Step 4. Since all score values in step 3 are different, there is no need to apply the formula of the accuracy function.

Step 5. The prime choice is selected by ranking the score values, as follows:

$$Sc(\mathscr{G}_2) > Sc(\mathscr{G}_3) > Sc(\mathscr{G}_4) > Sc(\mathscr{G}_1)$$

Hence, it is clear that $''\Omega_2''$ is the optimal result.
By using the $\boldsymbol{SFS_{ft}DOWA}$ operator

Step 1. Same as above.

Step 2. The $SFS_{ft}Ns$ $\S_{\hat{e}_{\ell\ell}}$ is aggregated for alternatives $\Omega_{\mathscr{J}}$ $(\mathscr{J} = 1, 2, \ldots, l)$ by using the introduced $SFS_{ft}DOWA$ operators for $\propto = 2$, as follows:

$$\mathscr{G}_1 = (0.6182, 0.4421, 0.4458),$$

$$\mathscr{G}_2 = (0.6357, 0.1703, 0.4359),$$

$$\mathscr{G}_3 = (0.4758, 0.4101, 0.4189),$$

$$\mathscr{G}_4 = (0.6251, 0.4012, 0.4623)$$

Step 3. The score values are calculated for each $\mathscr{G}_{\mathscr{J}} = (€_{\mathscr{J}}, \wp_{\mathscr{J}}, \mathscr{L}_{\mathscr{J}})$ for each alternative $\Omega_{\mathscr{J}}(\mathscr{J} = 1, 2, \ldots, l)$, as follows:

$$Sc(\mathscr{G}_1) = 0.5767, Sc(\mathscr{G}_2) = 0.67464,$$

$$Sc(\mathscr{G}_3) = 0.5489, Sc(\mathscr{G}_4) = 0.5868$$

Step 4. As all score values in step 3 are different, there is no need to apply the formula of the accuracy function.

Step 5. The prime choice is selected by ranking the score values, as follows:

$$Sc(\mathscr{G}_2) > Sc(\mathscr{G}_4) > Sc(\mathscr{G}_1) > Sc(\mathscr{G}_3)$$

Therefore, $''\Omega_2''$ is the optimal outcome.

6. Comparative study of the introduced approach

In this section, we initiate a comparative assessment of the given operators with some available theories to show the effectiveness and authenticity of our initiated methods.

We differentiate our methods from the Jana et al. [38] method, Aydemir and Gunduz [39] method, Sheikh and Mandal [40] method, Zhang et al. [41] method, Ashraf et al. [42] method and Hussian et al. [26] method. For collective information, different parameters of $SFS_{ft}Ns$ are aggregated by using the weighted average operators corresponding to $\grave{a} = \{0.22, 0.33, 0.17, 0.28\}$ as WVs of experts and $\wp = \{0.20, 0.35, 0.19, 0.26\}$ as WVs of parameters to determine the aggregated SFS_{ft} decision matrix for different alternatives $\Omega_{\ell} = \{\ell = 1, \ldots, 4\}$ and obtain the collective decision matrix in Table 9. The results are provided in Table 10 by comparing our initiated method with some other existing methods to show the effectiveness and primacy of our initiated work. From the analysis of information presented in Table 10, although the results are marginally different, the overall optimal alternative is the same, that is, Ω_1.

1. Note that the Sheikh and Mandal [40] method, Jana et al. [38] method and Aydemir and Gunduz [39] method are based on intuitionistic fuzzy data, Pythagorean fuzzy data and q-rung orthopair fuzzy data, respectively. Moreover, all the above existing methods can only discover the MD and NMD in their structures. The data given in Table 9 consist of spherical fuzzy soft information that contains not only the MD and NMD but also the AD in its structure. Thus, all the above existing methods cannot deal with that kind of information due to their limited structures. Therefore, these existing theories fail to cover the data provided in Table 9, which is why no result is obtained in these cases.
2. Additionally the Sheikh and Mandal [40] method, Jana et al. [38] method and Aydemir and Gunduz [39] method cannot discuss the parameterization tool, while our developed notions use the parameterization tool. Therefore, the extra feature of the developed notions causes the introduced notions to be superior to the existing notions.

Table 9
Tabular representation of $SFS_{ft}Ns$ for $\propto = 2$.

Z	Ω_1	Ω_2	Ω_3	Ω_4
z_1	$(0.4994, 0.4333, 0.6012)$	$(0.5043, 0.5972, 0.5209)$	$(0.3469, 0.4658, 0.5760)$	$(0.2676, 0.6020, 0.6141)$
z_2	$(0.3248, 0.5554, 0.6289)$	$(0.3455, 0.5794, 0.6276)$	$(0.4819, 0.5897, 0.6380)$	$(0.3987, 0.6690, 0.6145)$
z_3	$(0.3419, 0.7115, 0.5693)$	$(0.3069, 0.6597, 0.6981)$	$(0.3365, 0.6548, 0.6670)$	$(0.5031, 0.5584, 0.6515)$
z_4	$(0.3417, 0.6243, 0.6208)$	$(0.32670, 0.5909, 0.6137)$	$(0.3290, 0.6797, 0.5540)$	$(0.3358, 0.5802, 0.5670)$

Table 10
Tabular representation of $SFS_{ft}Ns$ for $\propto = 2$.

Methods	Score values	Ranking results
(Jana et al. [38] method	Failed	Failed
Aydemir and Gunduz [39] method	Failed	Failed
Sheikh and Mandal [40] method	Failed	Failed
(Hussian et al. [26] method	Failed	Failed
Zhang et al. [41] method	Failed	Failed
Ashraf et al. [42] method	$Sc(\Omega_1) = 0.2569$ $Sc(\Omega_2) = 0.1954$ $Sc(\Omega_3) = 0.1793$ $Sc(\Omega_4) = 0.1689$	$\Omega_1 > \Omega_2 > \Omega_3 > \Omega_4$
$SFS_{ft}DWA$ operator proposed work	$Sc(\Omega_1) = 0.2306$ $Sc(\Omega_2) = 0.1904$ $Sc(\Omega_3) = 0.1753$ $Sc(\Omega_4) = 0.1907$	$\Omega_1 > \Omega_4 > \Omega_2 > \Omega_3$
$SFS_{ft}DOWA$ operators proposed work	$Sc(\Omega_1) = 0.2367$ $Sc(\Omega_2) = 0.1974$ $Sc(\Omega_3) = 0.1830$ $Sc(\Omega_4) = 0.1668$	$\Omega_1 > \Omega_2 > \Omega_3 > \Omega_4$
$SFS_{ft}DHA$ operators proposed work	$Sc(\Omega_1) = 0.2206$ $Sc(\Omega_2) = 0.1874$ $Sc(\Omega_3) = 0.1923$ $Sc(\Omega_4) = 0.1701$	$\Omega_1 > \Omega_3 > \Omega_2 > \Omega_4$

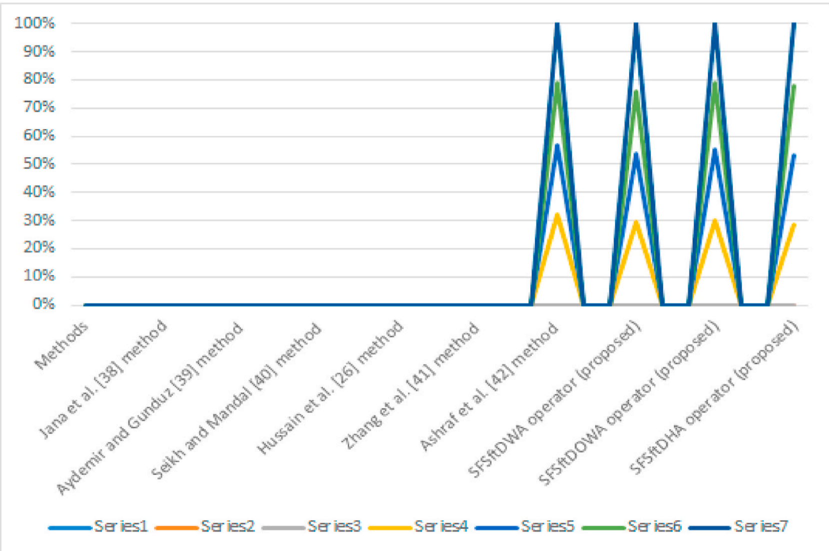

Fig. 1. A pictorial presentation of data provided in Table 10.

Table 11
Characteristic assessment of our developed approach with existing notions.

Methods	Consider the fuzzy information	Consider the parameterization tool
Jana et al. [38] method	Yes	No
Aydemir and Gunduz [39] method	Yes	No
Sheikh and Mandal [40] method	Yes	No
(Hussian et al. [26] method	Yes	Yes
Zhang et al. [41] method	Yes	No
Ashraf et al. [42] method	Yes	No
$SFS_{ft}DWA$ operator proposed work	Yes	Yes
$SFS_{ft}DOWA$ operators proposed work	Yes	Yes
$SFS_{ft}DHA$ operators proposed work	Yes	Yes

3. Note that the method given by Hussian et al. [26] is based on $q-ROFS_{ft}$ information and uses the parameterization tool as well. However, $q-ROFS_{ft}$ data cannot discuss the AD, while our developed approach can discuss the AD in its structure; thus, in this regard, our introduced work is superior to the existing notion.
4. When we compare our work with the Zhang et al. [41] method, the Zhang et al. [41] method consists of picture fuzzy data and uses the condition that $sum(MD, AD, NMD) \in [0,1]$, while the existing notions use the condition that $sum(MD^2, AD^2, NMD^2) \in [0,1]$. From Tables 9 and if we use the term in the third row and first column that is $(0.3419, 0.7115, 0.5693)$, then the Zhang et al. [41] method fails to handle this information because $sum(0.3419, 0.7115, 0.5693) \notin [0,1]$. Thus, the Zhang et al. [41] method fails to handle these data, and no result can be found in this case. Moreover, the Zhang et al. [41] method cannot discuss the parameterization tool, while our existing notions can. Therefore, the developed approach is more dominant than the existing notions.
5. The structure provided in the Ashraf et al. [42] method consists of T-spherical fuzzy information and can handle the data given in Table 9. The results are provided in Table 10. From Tables 10 and in all cases the optimal alternative is the same, showing the reliability of the developed approach.

Additionally, the pictorial presentation of the data given in Table 10 is presented in Fig. 1.
Moreover, the characteristic analysis of our introduced work with some existing notions is provided in Table 11.

7. Conclusion

A spherical fuzzy soft set is stronger than other fuzzy structures for handling the uncertainty of data. It can provide more space for decision-makers to handle fuzzy information. By being able to discuss the MD, NMD and AD in one structure, $SFS_{ft}S$ has the ability to show all three aspects in one structure. Moreover, the Dombi t-norm and t-conorm are two fuzzy logic operators and are great substitutes for the sum and product. Therefore, in this article, based on the dominant features of $SFS_{ft}S$ and valuable features of the Dombi t-norm and Dombi t-conorm, we have established the Dombi operational laws for SFS_{ft} numbers. After, we introduced SFS_{ft} Dombi average AOs called $SFS_{ft}DWA, SFS_{ft}DOWA,$ and $SFS_{ft}DHA$ AOs. Moreover, the basic properties of these introduced AOs were established. Additionally, an algorithm along with a numerical example was explored to show the advantages of our developed work. A comparative assessment of the initiated work along with literature was provided to produce the benefit and significance of our initiated work.

From the complexity of our introduced notions, our study is also limited because if the decision makers use 0.7 as MD, 0.8 as AD and 0.5 as NMD, then our proposed approach cannot handle that kind of information because the $sum\ (0.7^2, 0.8^2, 0.5^2) \notin [0,1]$. Therefore, our developed approach is limited.

In the future, this work can be extended to spherical fuzzy sets [43], complex picture fuzzy N-soft sets [44], bipolar soft sets [45] and rough fuzzy bipolar soft sets [46]. Moreover, this concept can be extended to other concepts provided in Refs. [47,48]. In addition, we can extend these notions to the 3,4-quasiring fuzzy sets [49] and quasiring orthopair fuzzy set theory [50]. Based on the Frank t-norm and t-conorm, we can develop some Frank aggregation operators as provided in Refs. [51,52]. We can extend these notions to the T-spherical fuzzy set theory introduced by Guleria and Bajaj [53]. Additionally, we can define some correlation measures based on the developed approach provided in Ref. [54].

Ethics declaration statement

The authors state that this is their original work and it is neither submitted nor under consideration in any other journal simultaneously.

Author contribution statement

Xiaopeng Yang: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data.
Tahir Mahmood: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.
Jabbar Ahmmad: Contributed reagents, materials, analysis tools or data; Wrote the paper.
Khizar Hayat: Contributed reagents, materials, analysis tools or data.

Data availability statement

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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