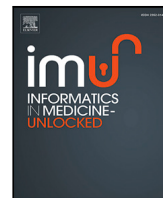**ELSEVIER**

Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Comments on "Optimal control analysis of a COVID-19 and tuberculosis co-dynamics model"

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A B S T R A C T

Study of dynamics of COVID-19 and its co-infection with other diseases through mathematical models is the major focus of recent advancement in mathematical modeling of infectious diseases. There are numerous mathematical models on COVID-19 which describe its dynamics for different geographic regions. However, there are very few research papers dealing with co-infection of COVID-19 and TB. As both TB and COVID-19 are infectious diseases of same nature it becomes very difficult to predict the co-dynamics of these two diseases. The formulation of a correct mathematical model is very important in any kind of modeling and if the mathematical model is not proper then any prediction based on this may not be valid. This letter highlights the important limitations in the proposed mathematical model of co-dynamics of COVID-19 and TB by Goudiaby et al. [1].

A R T I C L E   I N F O

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COVID-19
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1. Introduction

There are two main types of incidences mostly used in the epidemic models that correspond to the movement of individuals from susceptible compartment to either exposed or infected compartment. These are incidences, namely standard incidence and simple mass action type incidence. If force of infection is proportional to the population size, then it is called simple-mass action type incidence and if force of infection is frequency dependent, then one needs to model this situation by considering standard incidence. In the case of both COVID-19 and TB, force of infection is density dependent, so the transmission term should be formulated using simple mass-action type incidence. However, it is noted that there are numerous mathematical models on COVID-19 that are based on standard incidence. But this is not much appropriate considering the nature of spread of COVID-19. The following paper Goudiaby et al. [1] also falls under this category.

M.S. Goudiaby, L.D. Gning, M.L. Diagne, Ben M. Dia, H. Rwezaura, J.M. Tchuenche, Optimal control analysis of a COVID-19 and tuberculosis co-dynamics model, Informatics in Medicine Unlocked 28 (2022) 100849.

However, the usage of simple action over standard incidence considering the nature of TB and COVID-19 diseases is a suggestion; and the focus is on other issues raised. Hence, the important limitations and imperfections related to the basic understanding of the mode of transmission of these diseases imply that the proposed model is restricted in its approach. These are listed as below:

1. Authors have considered exposed compartment as well as infected compartment for both the diseases. Exposed individuals are those individuals who are exposed to infection but not yet infectious. But when we see the transmission term λ_T and λ_C, exposed individuals are also contributing to the transmission of this diseases that is not correct and it needs to be investigated accordingly.

2. Authors have considered the movement of individuals from E_C and E_T to co-infected compartment E_{CT} without interaction from other disease, that is not proper. The individuals from E_C compartment (exposed to COVID-19) is moving to E_{CT} compartment at the rate $\beta_c \lambda_C E_C$. Here λ_C involves E_C and I_{CU} that correspond to exposed to COVID and unreported infected COVID individuals. So there is no TB infection involved in this transmission term, so individuals cannot be exposed to TB without having interaction with individuals infected with TB. Similarly, movement of individuals from E_T compartment to E_{CT} compartment without interaction with COVID infected individuals is not correct as well. So the term $\beta_C \lambda_C E_C$ and $\beta_T \lambda_T E_T$ in the differential equations $\frac{dE_C}{dt}$ and $\frac{dE_T}{dt}$ should be $\beta_T \lambda_T E_C$ and $\beta_C \lambda_C E_T$ respectively, and corresponding changes should be made in the differential equation $\frac{dE_{CT}}{dt}$.

3. Again, movement of individuals from I_{CU} and I_{CR} to I_{CT} are given by γ_{CU} and α_{CR} which are constant rates. Again, this implies movement of COVID-infected individuals (reported or

unreported) to co-infection compartment I_{CT} without having any interaction with TB-infected individuals. So, this assumption too is incorrect. Similar movement is shown from TB-infected compartments I_{TR} and I_{TU} to co-infected compartment without having any interactions.

4. Movement of individuals from I_{CT} to I_{CR} through a rate constant α_C is not proper since α_C represents recovery rate of COVID-19 infected individuals, and movement back to COVID-19 infected class after recovery from COVID-19 is an error which needs to be investigated. Similar changes need to be incorporated when it comes to movement from I_{CT} to I_{TR}.

5. α_C and α_T with red arrows in the flow diagram of the proposed model should be interchanged as it corresponds to recovery from COVID −19 and recovery from TB respectively. So individuals from co-infected compartment I_{CT} will move to I_{CR} compartment (reported COVID-19 compartment) at a rate α_T. Similarly, individuals from co-infected compartment I_{CT} will move to I_{TR} compartment (reported TB compartment) at a rate α_C.

6. α_C and α_T in black arrows in the flow diagram can be assumed different as it corresponds to recovery from single disease from COVID-19 infected or TB-infected compartment alone, whereas earlier it was corresponding to recovery from one disease in the co-infected compartment. So, these rates should not be same as individuals having co-morbidity need more time to recover.

7. There is no recovery from active TB without treatment. So movement of individuals to recovered compartment from unreported TB compartment is not correct.

2. Conclusion

This paper has pointed out some major mistakes in the model formulation due to which the entire dynamical analysis and interpretation can change. The corresponding corrections are clearly presented.

Though a thorough analysis has been carried out in the study, the errors in the very formulation of the mathematical model nullify the correctness of results obtained and interpreted. Since young researchers consider research papers published in their area of work as a source of correct information, mistakes in the formulation of mathematical models need to be addressed precisely. The authors can look into this and present the correct interpretation of results by correcting the model flow.

Although, the mistakes observed by us might have been happened inadvertently, the movement of individuals from a COVID-19 infected class to co-infected class without having any interaction with TB infected patients is untrue and needs to be investigated accordingly. On the similar context, movement of individuals from a TB infected class to co-infected class without having any interaction with COVID-19 infected patients is also untrue and that too needs to be investigated accordingly.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Goudiaby MS, Gning LD, Diagne ML, Dia Ben M, Rwezaura H, Tchuenche JM. Optimal control analysis of a COVID-19 and tuberculosis co-dynamics model. Inf Med Unlocked 2022;28:100849.