

BinomiRare: A robust test for association of a rare genetic variant with a binary outcome for mixed models and any case-control proportion

Tamar Sofer,^{1,2,3,*} Jiwon Lee,³ Nuzulul Kurniansyah,³ Deepti Jain,⁴ Cecelia A. Laurie,⁴ Stephanie M. Gogarten,⁴ Matthew P. Conomos,⁴ Ben Heavner,⁴ Yao Hu,⁵ Charles Kooperberg,⁵ Jeffrey Haessler,⁵ Ramachandran S. Vasani,^{6,7} L. Adrienne Cupples,^{7,8} Brandon J. Coombes,⁹ Amanda Seyerle,¹⁰ Sina A. Gharib,¹¹ Han Chen,^{12,13} Jeffrey R. O'Connell,¹⁴ Man Zhang,¹⁵ Daniel J. Gottlieb,³ Bruce M. Psaty,^{16,17} W.T. Longstreth, Jr.,¹⁷ Jerome I. Rotter,¹⁹ Kent D. Taylor,¹⁹ Stephen S. Rich,²⁰ Xiuqing Guo,¹⁹ Eric Boerwinkle,^{12,21} Alanna C. Morrison,¹² James S. Pankow,²² Andrew D. Johnson,^{7,23} Nathan Pankratz,²⁴ NHLBI Trans-Omics for Precision Medicine (TOPMed) Consortium, Alex P. Reiner,⁵ Susan Redline,^{1,3} Nicholas L. Smith,^{18,25,26} Kenneth M. Rice,⁴ and Elizabeth D. Schifano²⁷

Summary

Whole-genome sequencing (WGS) and whole-exome sequencing studies have become increasingly available and are being used to identify rare genetic variants associated with health and disease outcomes. Investigators routinely use mixed models to account for genetic relatedness or other clustering variables (e.g., family or household) when testing genetic associations. However, no existing tests of the association of a rare variant with a binary outcome in the presence of correlated data control the type 1 error where there are (1) few individuals harboring the rare allele, (2) a small proportion of cases relative to controls, and (3) covariates to adjust for. Here, we address all three issues in developing a framework for testing rare variant association with a binary trait in individuals harboring at least one risk allele. In this framework, we estimate outcome probabilities under the null hypothesis and then use them, within the individuals with at least one risk allele, to test variant associations. We extend the BinomiRare test, which was previously proposed for independent observations, and develop the Conway-Maxwell-Poisson (CMP) test and study their properties in simulations. We show that the BinomiRare test always controls the type 1 error, while the CMP test sometimes does not. We then use the BinomiRare test to test the association of rare genetic variants in target genes with small-vessel disease (SVD) stroke, short sleep, and venous thromboembolism (VTE), in whole-genome sequence data from the Trans-Omics for Precision Medicine (TOPMed) program.

Introduction

Whole-genome sequencing (WGS) and whole-exome sequencing studies are becoming increasingly available to public health researchers, for example, from the National Heart, Lung, and Blood Institute (NHLBI) Trans-Omics for Precision Medicine (TOPMed) program,¹ National Hu-

man Genome Research Institute (NHGRI) Centers for Common Disease Genetics (CCDG), and the UK Biobank.² As most variants in sequencing datasets are rare, researchers may be interested in using such datasets for detecting rare variant associations, genome-wide or in a genomic region of interest. They may also seek to confirm suggested associations from other studies or populations or

¹Department of Medicine, Harvard Medical School, Boston, MA, USA; ²Department of Biostatistics, Harvard T.H. Chan School of Public Health, Boston, MA, USA; ³Division of Sleep and Circadian Disorders, Brigham and Women's Hospital, Boston, MA, USA; ⁴Department of Biostatistics, University of Washington, Seattle, WA, USA; ⁵Division of Public Health Sciences, Fred Hutchinson Cancer Research Center, Seattle, WA, USA; ⁶Departments of Medicine and Epidemiology, Boston University Schools of Medicine and Public Health, Boston, MA, USA; ⁷Framingham Heart Study, Framingham, MA, USA; ⁸Department of Biostatistics, Boston University, Boston, MA, USA; ⁹Department of Health Sciences Research, Mayo Clinic, Rochester, MN, USA; ¹⁰Division of Pharmaceutical Outcomes and Policy, Gillings School of Global Public Health, University of North Carolina at Chapel Hill, Chapel Hill, NC, USA; ¹¹Computational Medicine Core, Center for Lung Biology, Department of Medicine, University of Washington, Seattle, WA, USA; ¹²Human Genetics Center, Department of Epidemiology, Human Genetics, and Environmental Sciences, School of Public Health, The University of Texas Health Science Center at Houston, Houston, TX, USA; ¹³Center for Precision Health, School of Biomedical Informatics, The University of Texas Health Science Center at Houston, Houston, TX, USA; ¹⁴Department of Medicine, Division of Endocrinology, Diabetes, and Nutrition, University of Maryland School of Medicine, Baltimore, MD, USA; ¹⁵Division of Endocrinology, Diabetes, and Nutrition, Department of Medicine, University of Maryland School of Medicine, Baltimore, MD, USA; ¹⁶Cardiovascular Health Research Unit, Departments of Medicine, Epidemiology, and Health Services, University of Washington, Seattle, WA, USA; ¹⁷Departments of Neurology and Epidemiology, University of Washington, Seattle, WA, USA; ¹⁸Kaiser Permanente Washington Health Research Institute, Seattle, WA, USA; ¹⁹The Institute for Translational Genomics and Population Sciences, Department of Pediatrics, The Lundquist Institute for Biomedical Innovation at Harbor-UCLA Medical Center, Torrance, CA, USA; ²⁰Center for Public Health Genomics, University of Virginia School of Medicine, Charlottesville, VA, USA; ²¹Human Genome Sequencing Center, Baylor College of Medicine, Houston, TX, USA; ²²Division of Epidemiology and Community Health, University of Minnesota, Minneapolis, MN, USA; ²³Population Sciences Branch, Division of Intramural Research, National Heart, Lung and Blood Institute, Framingham, MA, USA; ²⁴Department of Laboratory Medicine and Pathology, University of Minnesota, Minneapolis, MN, USA; ²⁵Department of Epidemiology, University of Washington, Seattle, WA, USA; ²⁶Seattle Epidemiologic Research and Information Center, Department of Veterans Affairs Office of Research and Development, Seattle, WA, USA; ²⁷Department of Statistics, University of Connecticut, Storrs, CT, USA

*Correspondence: tsofer@bwh.harvard.edu

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to assess pathogenicity in large population-based studies of rare variant alleles reported from small family-based studies. For example, Amininejad et al.³ studied the association of genetic variants within genes associated with monogenic immunodeficiency disorders with Crohn disease. Wright et al.⁴ assessed the pathogenicity and penetrance of rare variants identified in clinical studies in the population-based UK Biobank. Tuijnenburg et al.⁵ studied rare genetic variants within *NGKBI* for association with primary immunodeficiency disease. Do et al.⁶ studied risk of myocardial infarction in individuals with rare *LDLR* and *APOA5* alleles. Kendall et al.⁷ studied cognitive outcomes in individuals with rare copy-number variants. These studies demonstrate that there is an interest in testing single rare genetic variant associations with a wide range of health outcomes, including binary outcomes such as disease or affection status.

Testing rare variant associations with binary traits is challenging. It was previously shown that likelihood-based tests such as the Wald, Score, and likelihood ratio tests poorly control the type 1 error when testing for rare variant associations with a binary trait.^{8,9} The Score test performance depends on the case-control ratio, and for rare variants, even a small imbalance causes “inflation” (i.e., too many false-positive results). A few approaches have been used previously to study rare variant associations in a set of unrelated individuals. Amininejad et al.³ used a permutation approach to test for association of rare genetic variants with Crohn disease. Wright et al.⁴ used Fisher’s exact test. While it is possible to adjust for covariates in the permutation approach and when using Fisher’s exact test to some extent through stratification,¹⁰ they do not have the full flexibility of covariate adjustment of a generalized linear model (i.e., they still require the identification of distinct groups in which no additional adjustment is required). Further, permutation tests may also be computationally intensive if low p values are desired, because the number of required permutations may be large, although there are ways to reduce this computational burden.¹⁰ Alternatively, Tuijnenburg et al.⁵ used a method called BeviMed,¹¹ implementing a Bayesian model to estimate posterior disease probabilities. The BinomiRare test has also been proposed as a powerful method to test for rare variant associations that can account for covariates.⁹ The BinomiRare test uses standard methods to compute the disease probabilities in the entire dataset, under the null hypothesis of no association between a specific genetic variant and the binary outcome. Then, for each specific genetic variant, it uses the estimated probabilities in individuals harboring at least one copy of the rare variant to test the hypothesis that the disease probabilities under the null are the true outcome probabilities in these individuals. The null hypothesis is rejected if the number of individuals with both the rare variant and the outcome is inconsistent with their outcome probabilities. However, the previously published version of this method assumed the sample contains only unrelated individuals. Currently, there is no sin-

gle-variant test that is generally appropriate for testing rare variants when individuals are correlated (e.g., due to known or cryptic genetic relatedness). Notably, the saddle point approximation to compute p values (henceforth SPA¹²) was first developed to improve the calibration of the Score test when there is case-control imbalance and was then extended in the SAIGE framework for the settings where related individuals are used.¹³ However, it does not reliably control the type I error rate when the number of individuals harboring the rare variant is very small (i.e., tens of individuals¹⁴). Therefore, there is a need for a statistical test that is well-calibrated when the number of individuals with the rare allele is low, individuals are potentially related, and there is case-control imbalance.

The previously published version of the BinomiRare test¹⁴ is useful in the presence of case-control imbalance, allows for covariate adjustment, controls the type I error rate for any number of individuals with the rare allele, and can also be used when combining heterogeneous studies, and here we expand its framework for testing rare variant associations when study individuals are correlated. We developed two tests: first, we extended the BinomiRare test to the mixed models setting by applying it on conditional probabilities computed with a mixed model, rather than on marginal probabilities. Second, we developed the Conway-Maxwell-Poisson (CMP) test, which follows the same framework by using estimated (conditional) disease probabilities like the BinomiRare. For a given rare variant it uses the estimated disease probabilities in individuals with the rare allele to fit the parameters of the CMP distribution, under the null. It then tests whether the observed number of individuals with both one or more copies of the rare allele and the outcome is consistent with this distribution. We study these tests using synthetic simulations with varying outcome probabilities, variant allele frequencies, and strengths of correlation between individuals due to genetic relatedness. We apply the BinomiRare test to test rare variant associations in known disease-causing genes for specific disorders: the *NOTCH3* gene and small vessel disease (SVD) ischemic stroke, the *DEC2* (also known as *BHLHE41*) gene and short sleep, and the *F5* gene and venous thromboembolism (VTE).

Material and methods

Statistical approach

Let D_i be an indicator of the disease, or another binary outcome, of participant i , with value 1 if the person is affected and 0 otherwise, where $i = 1, \dots, n$ and the n individuals may be correlated. Let \mathbf{x}_i be a $p \times 1$ vector of covariate values for the i th participant, and g_i be their count of minor alleles for a specified genetic variant. Under the logistic disease model for correlated data:

$$\text{logit}(p_i) = \mathbf{x}_i^T \boldsymbol{\alpha} + g_i \beta + b_i, \quad i = 1, \dots, n$$

with $p_i = \Pr(D_i = 1 | \mathbf{x}_i, g_i, b_i)$ being the conditional outcome probability in the sample (regardless of the population probability),

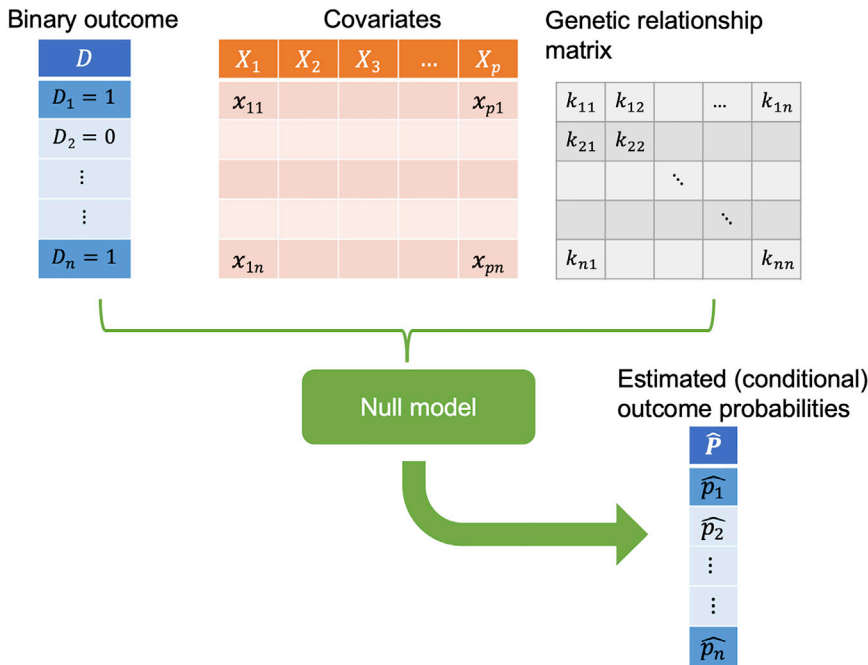


Figure 1. Step 1 of testing genetic association using the proposed framework
A null model of association between the binary outcomes and covariates of interest is fitted, accounting for genetic relationship. Then, estimated conditional outcome probabilities are extracted to be used in the testing step.

and b_i is the i th entry of the vector $\mathbf{b} = (b_1, \dots, b_n)^T \sim N(0, \sum_{k=1}^K \sigma_k^2 V_k)$ of correlated random effects with possibly K variance components $\sigma_k^2, k = 1, \dots, K$ and V_k modeling the correlation structure corresponding to a particular source of correlation. While the methods proposed here can be applied for an arbitrary $K \geq 1$, we simplify presentation by focusing on the scenario of a single correlation matrix modeling genetic relatedness, possibly cryptic, so that $\mathbf{b} \sim N(0, \sigma_g^2 \mathbf{G})$, with \mathbf{G} being any genetic relationship matrix (GRM), or possibly kinship matrix, and σ_g^2 is the corresponding variance component.

We assume that the genetic variant is rare, so that the minor allele frequency (MAF) is low and that individuals harboring the minor allele are overwhelmingly heterozygotes. While having homozygotes does not invalidate our approach, it also does not increase statistical power. Our approach first estimates a disease probability for each individual in the sample under the null hypothesis of no association between the genetic variant and disease status, (i.e., under the assumption that $\beta = 0$) by not including any variant of interest in the regression (step 1, demonstrated in Figure 1), and then considers individuals with at least one copy of the rare variant (in short, with the rare allele), testing whether the number of diseased individuals with the rare allele is consistent with their estimated disease probabilities (step 2, demonstrated in Figure 2).

Step 1: Estimating disease probabilities under the null hypothesis

At step 1, we fit a null model under the assumption $\beta = 0$, using the existing penalized quasi-likelihood algorithm for logistic mixed models.¹⁵ This approach is implemented in multiple software, including the GENESIS R package,¹⁶ GMMAT,¹⁷ and SAIGE.¹³ In both GENESIS and GMMAT, the vector of fixed effects α and the variance component σ_g^2 are estimated using an implementation of an AI-REML (average information restricted maximum likelihood) algorithm on top of the penalized quasi-likelihood (PQL) approach,¹⁷ but the proposed tests do not depend on the specific algorithm used for estimating the outcome

probabilities. From the fitted null model, we obtain estimates $\hat{\alpha}, \hat{\mathbf{b}}$ and an estimated disease probability vector by plugging them in to obtain $\hat{p}_i = \text{expit}(\mathbf{x}_i^T \hat{\alpha} + \hat{b}_i)$, $i = 1, \dots, n$, where expit is the inverse of the logit function. If the variance component σ_g^2 is estimated as 0, so is $\hat{\mathbf{b}} = 0$, and the analysis reverts to the independent individual settings.

Step 2: Testing the association between a genetic variant and disease status

Suppose that we obtained disease probability estimates $\hat{p}_i, i = 1, \dots, n$, under the null as described above. Denote n_c as the number of individuals harboring at least one copy of the rare variant (i.e., those with $g > 0$), so that $\sum_{i=1}^n 1(g_i > 0) = n_c$. Without loss of generality, assume that participants $i = 1, \dots, n_c$ have the rare allele. Let n_d be the number of diseased individuals with rare allele:

$$n_d = \sum_{i=1}^{n_c} 1(d_i = 1) = \sum_{i=1}^{n_c} 1(d_i = 1, g_i > 0).$$

Let $\hat{\mathbf{p}}_{n_c} = (\hat{p}_1, \dots, \hat{p}_{n_c})^T$ denote the vector of estimated disease probabilities for individuals with the rare variant. Despite $\hat{\mathbf{p}}_{n_c}$ being estimated, we treat it as fixed. For testing, we assess the goodness-of-fit of the estimated model to the observed disease status in the individuals with the rare allele by testing the null hypothesis:

$$H_0 : \mathbf{p}_{n_c} = \hat{\mathbf{p}}_{n_c},$$

where \mathbf{p}_{n_c} is the true, unknown, vector of outcome probabilities among those with the rare allele.

The p value for testing the null hypothesis of no variant-disease association is given by:

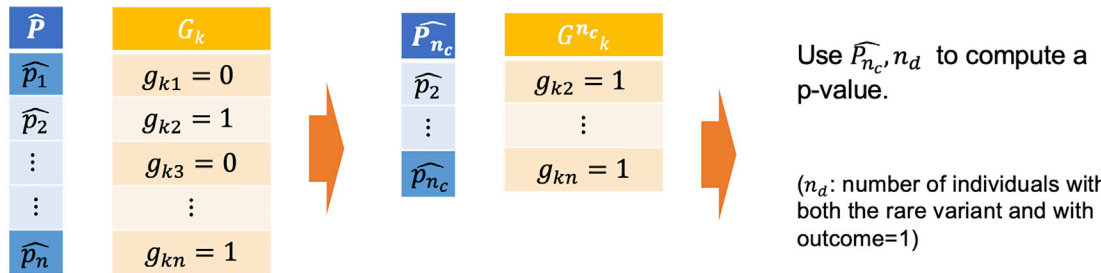
$$p\text{-value} = \Pr \left\{ n_d \text{ diseased individuals among those with the rare allele or more extreme} \mid \hat{\mathbf{p}}_{n_c} \right\} \tag{Equation 1}$$

This is a two-tailed p value, because n_d can appear to be lower or higher than expected. When only a single person carries the rare variant (i.e., $n_c = 1$), the calculation is trivial; Equation 1 reduces to the single individual fitted probability \hat{p}_i if they are an affected individual, and $1 - \hat{p}_i$ if they are a control individual. When $n_c > 1$, there are two special cases that are already developed. If \hat{p}_i for all individuals with the rare allele are equal, and outcomes for all those individuals are independent, then $n_d \sim \text{Binomial}(n, \hat{p}_i)$, and the p value is the tail area (possibly two tails) of the standard binomial distribution (i.e., a binomial exact test). If the \hat{p}_i for

Associations are tested for each variant at a time

\widehat{P}	G_1	G_2	G_3	...	G_q
\widehat{p}_1	g_{11}				g_{q1}
\widehat{p}_2					
\vdots					
\widehat{p}_n	g_{1n}				g_{qn}

For a given variant G_k :



1. Identify individuals with the rare allele j: $g_{kj} > 0$
2. Filter to these individuals
3. Test association

Figure 2. Step 2 of testing genetic associations using the proposed framework

Based on estimated outcome probabilities, variants are inspected one at a time. For a given variant, individuals harboring the rare allele are identified, and a test of the null hypothesis $H_0 : P_{n_c} = \widehat{p}_{n_c}$ is performed testing whether n_d is consistent with the outcome probabilities within individuals with the rare allele, based on the null model.

the individuals with the rare allele differ but independence still holds, the distribution is the Poisson-binomial distribution, and the test is the previously proposed BinomiRare test for independent data.⁹ In the general case, an arbitrary sum of binomial variables, possibly correlated, has the CMP-binomial distribution, which can be approximated by the CMP distribution^{18,19} when the number of individuals with the rare allele is “large enough” (see Appendix A).

In addition to the p value above, we also study the mid-p value, which was previously shown to improve properties of discrete tests²⁰ and to be less conservative. The mid-p value is always smaller than the p value, because when summing the tail area probabilities, it accounts for only half of the probability of the observed event n_d , whereas the p value uses it as it is, without dividing in half.

BinomiRare and CMP tests using conditional probabilities

In Appendix A, we show that the distribution of n_d in the general case can be approximated by the CMP distribution and develop the CMP test. However, because approximations may not work well in practice for low n_c , we also attempt a different approach. Note that for two individuals i and j , we have that D_i and D_j were independent if the true conditional disease probabilities were known. In other words, given conditional disease probabilities, knowing the disease status of individual i does not inform of the disease status of individual j . Therefore, we consider using the BinomiRare test, which was developed for independent data—with the conditional probabilities. We note that this inde-

pendence may not hold when probabilities are estimated, and therefore it is not trivially true that the BinomiRare is appropriate in this setting. Both the CMP and the BinomiRare tests for correlated data are available in the GENESIS R package for genetic association analysis.²¹

Simulation study: Testing rare variant associations using BinomiRare and CMP in a sample of trios

We carried out a simulation study to evaluate the performance of BinomiRare and CMP tests in samples of correlated individuals. In each simulation, we generated 3,000 individuals as 1,000 trios (two parents and one offspring), as follows. For 1,000 pairs of parents, and each of two chromosomal copies, we generated 20 independent “non-causal” genetic variants by first sampling MAFs from a uniform $U[0.05, 0.5]$ distribution and setting $MAF \in \{0.05, 0.02, 0.01, 0.001\}$ for one “causal” variant, followed by sampling of genetic variants using a binary distribution based on these MAFs. For each parent, allele count was the sum of the two sampled alleles. For each variant independently, an offspring inherited one allele from each of the parents. The parental allele was sampled at random with equal probabilities from the two alleles. We used the 21 (1 causal and 20 non-causal) simulated genotypes to generate a variable mimicking a principal component (PC), as a weighted sum of all allele counts, with weights sampled from a standard normal distribution $N(0, 1)$. Next, we simulated probability of disease using a mixed logistic model:

$$\text{logit}[p(D_i = 1)] = \beta_0 + PC_i \times \beta_{pc} + g_i \beta_g + b_i, \quad i = 1, \dots, n.$$

Here, $\exp(\beta_0) \in \{0.01, 0.05, 0.5\}$ is the probability of disease in individuals without the rare allele ($g_i = 0$) with genetic PC and b_i equal to zero. β_{pc} models the association of the PC with disease probability, β_g is the effect of the (causal) variant of interest, and $\mathbf{b} = (b_1, \dots, b_n)^T$, representing the correlation across individuals, is sampled from a multivariate normal distribution $\mathbf{b} \sim MVT - N(0, \sigma_g^2 \mathbf{K})$, with the correlation matrix \mathbf{K} being a block diagonal kinship matrix, having twice the kinship coefficient between a child and each of their parents (i.e., 0.5). We set $\sigma_g^2 \in \{0.06, 0.6\}$. In all simulations we had $\beta_{pc} = 0.1$. The variant effect was varied from zero when evaluating type 1 error rate to $\beta_g = \log(\text{Odds Ratio}) \in \{\log(2), \log(3), \log(4)\}$ when evaluating power. We then sampled disease status for each individual from a binary distribution with the computed disease probability. Finally, we applied the BinomiRare and CMP tests and computed p values and mid-p values. We performed 1×10^7 replicates to estimate type 1 error rate and 1×10^5 replicates to estimate power. We estimated type 1 error rate and power for p value threshold for declaring significance $\{1 \times 10^{-2}, 1 \times 10^{-3}, 1 \times 10^{-4}\}$. For tests that did not, empirically, control the type I error rate for a given p value threshold (i.e., the proportion of simulations passing the threshold was higher than the threshold), we computed a calibrated threshold, defined as a value for which the proportion of simulations with p value less than this value was the desired threshold. We then used this calibrated threshold to estimate power, specifically power at an “honest alpha” (Supplemental material and methods). Our main results are those focused on simulations in which the variance component had a non-zero estimate, but we analyzed all simulations.

The TOPMed whole-genome sequencing study

Whole-genome sequencing was performed via TOPMed and the NHGRI's CCDG programs, using DNA from blood at multiple sequencing centers using Illumina X10 technology at an average sequencing depth of $>30\times$. Studies and samples were sequenced in multiple phases. Periodically, the TOPMed Informatics Research Center (IRC) performed variant calling on the combined TOPMed and CCDG samples, resulting in multiple releases of data “freezes.” Details regarding sequencing methods and quality control are provided elsewhere²² and in the TOPMed website.

We used three TOPMed multi-ethnic datasets: a dataset of SVD stroke in the Women's Health Initiative (WHI), a study of short sleep, and a study of VTE, with the latter two comprised of individuals from multiple TOPMed cohorts. We performed data analysis to demonstrate the BinomiRare test. The approaches for data analysis were similar. GRMs were constructed based on the analytic datasets of each of the analyses, using all genetic variants with minor allele frequency ≥ 0.001 . Logistic mixed models under the null were fit and adjusted for age, sex, and self-reported race/ethnic group, and, for short sleep, also for parent study/cohort. SVD stroke and short sleep analyses used TOPMed freeze 5b release, while the VTE analysis used TOPMed freeze 8 genotype release. All participants provided written informed consent at their recruitment centers.

The TOPMed WHI stroke dataset

The WHI is a long-term health study following postmenopausal women aged 50–79 years who were recruited from 1993 through 1998 from 40 clinical centers throughout the United States.²³ In the present analysis, we focus on a subset of 5,358 WHI participants who were sequenced through TOPMed with data available

via freeze 5b and had SVD stroke case-control classification, according to the following methodology: stroke diagnosis requiring and/or occurring during hospitalization was based on the rapid onset of a neurological deficit attributable to an obstruction or rupture of an arterial vessel system. Hospitalized incident stroke events were identified by semiannual questionnaires and adjudicated following medical record review, which occurred both locally (at individual study sites) and centrally. Ischemic strokes were further classified by the central neurologist adjudicators into cardio-embolic stroke, larger artery stroke, and SVD stroke according to the Trial of Org 10172 Acute Stroke Trial (TOAST) criteria.²⁴ The TOAST classification focuses on the presumed underlying stroke mechanism and requires detailed investigations (such as brain computed tomography, magnetic resonance imaging, angiography, carotid ultrasound, and echocardiography). Baseline stroke cases were excluded from the analysis, and VTE cases were excluded from the control samples. Further, participants who had non-SVD stroke were excluded.

The TOPMed short sleep dataset

We used sleep duration data from multiple TOPMed cohorts, as described in the Supplemental information detailing phenotype harmonization for short sleep analysis. Short sleep was defined as self-reported sleep duration during weekday, or usual sleep (if sleep duration during the weekdays was not available), being 5 h or less. Otherwise, if self-reported sleep duration was 6 h or longer and less than 9 h, sleep was “normal.” Individuals with self-reported sleep duration longer than 5 h and shorter than 6 h were excluded to minimize risk of misclassification. Because of a well-known U-shaped relationship between sleep duration and cardiovascular disease,²⁵ suggesting that potential non-linearity in genetic associations may exist as well, we also excluded “long sleepers” reporting usual sleep of 9 h or longer.

The TOPMed VTE dataset

The TOPMed VTE dataset includes TOPMed participants from six studies, combining prospective cohort and case-only studies. Individuals were matched across groups defined to be homogeneous with respect to race/ethnicity and sex, and strata defined by age at event (determined according to cases). The matching strategy resulted in a sample set mimicking a case-control study, with 11,627 individuals, of whom 3,793 are cases and 7,834 are controls.

Association testing of rare coding variants within known disease-causing genes

For each of the SVD stroke, short sleep, and VTE datasets, we considered a known gene associated with the disorder. For stroke, we focused on the *NOTCH3* gene, in which mutations may cause cerebral autosomal dominant arteriopathy with subcortical infarcts and leukoencephalopathy (CADASIL), which causes ischemic stroke.²⁶ For short sleep, we focused on the gene *DEC2* (also known as *BHLHE41*), a transcription inhibitor of orexin, a neuropeptide that regulates wakefulness.^{27,28} For VTE, we focused on the coagulation factor V gene, *F5*,^{29,30} which has a known common variant highly associated with VTE, factor V Leiden (rs6025). We performed single-variant analysis within the candidate genes, as follows. We selected a subset of rare variants within the genes based on functional annotations, with the goal of increasing power by focusing on variants that are more likely to be functional compared to others. In detail, the filter based on functional

annotation included the selection of variants that were: (1) high-confidence loss-of-function variants according to the Ensembl Variant Effect Predictor;³¹ (2) missense variants if they are predicted deleterious by either SIFT 4G,³² PolyPhen2-HDIV,³³ PolyPhen2-HVAR,³³ or LRT-pred;³⁴ (3) inframe insertions or deletions (indels) with FATHMM-XF coding score > 0.5 ;³⁵ or (4) variants that are synonymous according to the Ensembl Variant Effect Predictor and have FATHMM-XF coding score > 0.5 . The annotation-based variant filtering was performed using the annotation explorer application on the NHLBI's BioData Catalyst.³⁶ We further filtered variants to those that passed the TOPMed quality control (QC) filter²² and had at least 3 and no more than 300 individuals with the rare allele. This upper threshold was defined because we were interested specifically in rare variants and because it was previously shown that properties of statistical tests of rare variant associations depend on the number of individuals with the rare allele, rather than on allele frequency.¹⁴ Finally, we further restricted the set of variants to those that had reasonable statistical power according to a power analysis performed as follows. We arbitrarily assumed an odds ratio (OR) of 2 for a causal variant, and for each variant we computed power based on a function developed for the BinomiRare test. The function uses the estimated outcome probabilities in the sample, an OR, the number of individuals harboring the rare variant, and p value threshold to compute power. To increase accuracy, for each variant we specifically used the estimated disease probabilities among the individuals with the rare allele.

Although the proposed testing approach is developed primarily for studies of candidate gene or association regions, some investigators may be interested in applying it genome-wide. Therefore, in the [Supplemental information](#) we provide Manhattan and QQ-plots and report computation times from applying BinomiRare, CMP, and SPA tests genome-wide for the three outcomes of interest. When applied genome-wide, we used the Score test and recomputed p values (mid-p value for BinomiRare and p value for CMP) whenever the Score test p value was < 0.05 . Results are reported for all variants with at least 3 individuals with the rare allele and $MAF \leq 0.01$.

Results

Simulation studies

We studied the performance of the tests in simulations of 1,000 trios. In the setting where $\sigma_g^2 = 0.06$, about half of the simulations estimated the variance component to be zero. When $\sigma_g^2 = 0.6$, this happened in about a third of the simulations. The number of individuals with the simulated rare variant allele was in the range [0, 27] when $MAF = 0.001$, [17, 119] when $MAF = 0.01$, [58, 203] when $MAF = 0.02$, and [195, 401] when $MAF = 0.05$. [Table 1](#) provides estimated type 1 error rates in the simulations, restricted to those simulations in which the estimated variance component was $\hat{\sigma}_g^2 > 0$. For BinomiRare, we only provide results for the mid-p value, because in our simulations it always controlled the type 1 error rate, while the usual p value controlled it as well while being more conservative. For CMP, we only provide results for the usual p value, because it sometimes did not control the type 1 error, and the lack of control was worse with the mid-p value. The CMP test

usually did not control the type 1 error when the variant was very rare ($MAF = 0.001$) and when the case proportion was low ($\exp(\beta_0) = 0.05$). Its performance improved as the MAF increased. In [Tables S3–S5](#), we provide complete simulation results, including both mid-p value and the usual p value for both the CMP and BinomiRare tests and results computed over all simulations and computed over the simulations in which $\hat{\sigma}_g^2 = 0$.

In power analysis, after appropriately calibrating the p value threshold for the CMP test, CMP was either equally powerful as BinomiRare or more powerful ([Figures S1–S3](#)). The patterns were similar across p value thresholds used and across the two variance component parameters used in the simulations. Notably, when the disease was common ($\exp(\beta_0) = 0.5$), the power was lower when the variance component was high ($\sigma_g^2 = 0.6$) compared to when it was low ($\sigma_g^2 = 0.06$). When the disease was rare ($\exp(\beta_0) = 0.05$), the power was essentially the same with both values of variance components.

Data analysis: TOPMed datasets

For each of the three TOPMed datasets that we considered, [Table 2](#) provides the sample sizes, gene of interest, and number of variants according to sequential filtering: the number of available (non-monomorphic) variants in the sample that passed the functional filters described in the Material and methods section, number of variants after applying quality filters, number of variants after further restricting to variants with at least 3 and no more than 300 individuals with the rare allele, and the number of variants with at least 50% power to reject the null hypothesis at the 0.05 level under the assumption of $OR = 2$. There were 3 such variants in the *NOTCH3*-SVD stroke analysis, 1 variant in the *DEC2*-short sleep analysis, and 4 variants in the *F5*-VTE analysis.

[Table 3](#) provides the results from testing each of the variants passing this estimated power filter. Of the three tested *NOTCH3* variants, rs115582213 had p value = 0.03. For short sleep, only a single *DEC2* variant was tested; it had BinomiRare mid-p value = 0.03, suggesting association with short sleep. For the *F5* gene and VTE, none of the four tested variants showed evidence of association.

[Figures S4–S6](#) provide Manhattan and QQ-plots from genome-wide analyses of the three phenotypes, and [Table S6](#) provides computation times when testing associations in a single segment of 1×10^7 base pairs.

Discussion

We extended the BinomiRare test and studied the CMP test for testing the association of a rare genetic variant with a binary outcome in the mixed-model framework. These tests were specifically developed to handle variants with very low minor allele counts (tens of individuals harboring the rare allele), because it was previously shown that other tests that allow for covariate adjustment, such as the naive

Table 1. Estimated type 1 error rates of BinomiRare and CMP tests in simulations with related individuals

		Estimated type 1 error by p value threshold					
		$\sigma_g^2 = 0.06$			$\sigma_g^2 = 0.6$		
MAF	$\exp(\beta_0)$	10^{-2}	10^{-3}	10^{-4}	10^{-2}	10^{-3}	10^{-4}
BinomiRare (mid-p value)							
0.001	0.01	3.38E-03	2.08E-04	1.25E-05	3.78E-03	2.55E-04	1.88E-05
0.001	0.05	5.61E-03	4.45E-04	2.76E-05	5.63E-03	4.4E-04	3.24E-05
0.001	0.5	5.78E-03	3.22E-04	1.57E-05	5.44E-03	2.94E-04	1.26E-05
0.01	0.01	6.22E-03	4.62E-04	3.25E-05	6.36E-03	4.64E-04	3.35E-05
0.01	0.05	7.70E-03	6.58E-04	4.42E-05	7.83E-03	6.58E-04	5.1E-05
0.01	0.5	8.68E-03	8.28E-04	6.67E-05	8.21E-03	7.3E-04	6.56E-05
0.02	0.01	6.17E-03	4.27E-04	2.73E-05	6.41E-03	4.71E-04	3.38E-05
0.02	0.05	7.53E-03	6.27E-04	5.34E-05	7.52E-03	6.15E-04	5.27E-05
0.02	0.5	7.90E-03	6.87E-04	6.51E-05	7.56E-03	6.34E-04	5.55E-05
0.05	0.01	4.99E-03	2.88E-04	1.84E-05	5.21E-03	2.97E-04	1.48E-05
0.05	0.05	5.94E-03	4.49E-04	3.10E-05	5.93E-03	4.26E-04	2.73E-05
0.05	0.5	6.02E-03	4.69E-04	3.91E-05	5.67E-03	4.16E-04	3.34E-05
CMP (usual p value)							
0.001	0.01	5.75E-02	6.59E-03	4.18E-04	6.01E-02	6.54E-03	4.41E-04
0.001	0.05	4.50E-02*	4.44E-03*	2.83E-04*	4.22E-02*	3.89E-03*	2.26E-04*
0.001	0.5	3.44E-02*	9.29E-04	6.28E-06	3.34E-02*	7.78E-04	8.21E-06
0.01	0.01	2.34E-02	2.19E-03	1.70E-04	2.25E-02	2.09E-03	1.68E-04
0.01	0.05	1.62E-02*	1.55E-03*	1.37E-04*	1.53E-02*	1.44E-03*	1.20E-04*
0.01	0.5	9.30E-03	7.53E-04	4.41E-05	8.71E-03	6.74E-04	4.65E-05
0.02	0.01	1.76E-02	1.50E-03	1.11E-04	1.69E-02	1.44E-03	1.16E-04
0.02	0.05	1.11E-02*	1.10E-03*	1.02E-04	1.02E-02	9.77E-04	8.58E-05
0.02	0.5	7.86E-03	6.30E-04	5.49E-05	7.45E-03	5.76E-04	4.36E-05
0.05	0.01	1.06E-02	7.29E-04	4.14E-05	1.01E-02	7.19E-04	4.04E-05
0.05	0.05	6.53E-03	4.94E-04	3.44E-05	6.38E-03	4.52E-04	3.38E-05
0.05	0.5	5.80E-03	4.37E-04	3.30E-05	5.46E-03	3.83E-04	2.99E-05

*Settings in which the type 1 error was not controlled, defined according to type 1 error rate being larger than the highest value in a 95% confidence interval around the expected type 1 error rate, based on binomial distribution with parameters being the p value threshold and number of simulations used.

Score test and the SPA test, do not always control the type 1 error in the very low count settings.¹⁴ Both BinomiRare and CMP tests first estimate the outcome probabilities for each person in a dataset, while accounting for covariates and for genetic relatedness (and possibly other covariance matrices) via a mixed model, and then use the estimated conditional disease probabilities. For a single variant, individuals with the rare alleles are identified, and based on their disease probabilities and the observed number of cases, a p value is computed, as the probability of observing the given number of cases or more extreme given the estimated outcome probabilities. The BinomiRare test assumes a Poisson binomial distribution on the number of cases, and the CMP test assumes a CMP distribution. The

BinomiRare test using estimated conditional outcome probabilities assuming that the individuals with the rare allele are independent performed well, while, surprisingly, the CMP test, which was constructed specifically for correlated data, did not control the type 1 error rate for settings with a low number of individuals harboring the rare allele. This was likely because the approximations on which it relies are asymptotic in its non-centrality parameter λ , which is related to the number of individuals with the rare allele.

We demonstrated the application of the BinomiRare test using three TOPMed studies: SVD stroke, short sleep, and VTE. Due to the low power for testing low-count variants, we filtered variants according to functional annotation and according to computed statistical power. The

Table 2. Characteristics of the TOPMed datasets and variants considered for association testing

	SVD stroke	Short sleep	VTE
No. of individuals in the analysis	5,358	20,021	11,627
No. of cases	692 (12.9%)	2,408 (12%)	3,793 (32.6%)
No. of controls	4,666 (87.1%)	17,613 (88%)	7,834 (67.4%)
Gene of interest	<i>NOTCH3</i>	<i>DEC2/BHLHE41</i>	<i>F5</i>
No. of potentially functional non-monomorphic variants identified	122	58	142
No. of variants further passing TOPMed quality filters	117	49	132
No. of variants further having $2 < \text{individuals with the rare allele} < 300$	20	9	25
No. of variants with estimated power > 0.5 at the 0.05α level	3	1	4

limitations of this approach are that (1) the deleteriousness predicting annotations used and the filters applied to them may not have captured the true functional variant set, and (2) the power analysis was based on an arbitrarily selected OR parameters. In this study, we chose $OR = 2$ and only considered the handful of variants that had estimated power > 0.5 for testing, while requiring p value (α level) < 0.05 . We recognize that many rare variants have larger effect sizes. However, if we specified a larger OR parameter, and thus included more variants in our analysis, a more stringent α level would be needed. Thus, the resulting list of variants to test may have been similar. More work is needed developing strategies for identifying single rare variant associations.

For each of the phenotypes, SVD stroke, VTE, and short sleep, we searched for rare variants within genes with known trait associations. For SVD stroke, we considered *NOTCH3*, because some *NOTCH3* variants have been reported in individuals with CADASIL, which poses a risk

for stroke. Most *NOTCH3* mutations reported as associated with CADASIL are those involving loss or gain of a cysteine residue, leading to unpaired cysteine.³⁷ Single-nucleotide variants in *NOTCH3* have not yet consistently been identified as associated with SVD stroke in population-based studies. Here, we identified the rare variant rs115582213 (BinomiRare mid-p value = 0.03). This variant was rare, with 87 out of 5,358 individuals in the dataset harboring the rare allele. Of these, 17 individuals had SVD stroke. More work is needed to study the association of rs115582213 with SVD stroke, as, after accounting for multiple testing, its association is not statistically significant.

For VTE, we considered the *F5* gene. The *F5* gene harbors the strongest known, relatively common, genetic risk factor for VTE, the rs6025 variant.^{38,39} This motivated the search for rare variants in this gene. We did not identify any variant associated with VTE at the p value < 0.05 level. We did not consider rs6025 as part of our testing strategy because it was common with $MAF = 0.04$, and 839

Table 3. Results from association analysis of rare genetic variants within monogenic disease genes of interest

rsID	Variant	BinomiRare p value	BinomiRare mid-p value	n_c	n_d	Estimated power (OR = 2)	ClinVar interpretation	CADD PHRED	FATHMM-XF coding
SVD stroke: <i>NOTCH3</i> gene									
rs115582213	chr-19-15162524-C-T	0.04	0.03	87	17	0.7	benign/likely benign	25.4	0.66
rs112197217	chr-19-15179425-G-T	0.53	0.49	166	23	0.91	benign/likely benign	21	0.42
rs11670799	chr-19-15188240-G-A	0.81	0.77	180	23	0.94	benign/likely benign	28.8	0.68
Short sleep: <i>DEC2</i> gene									
rs121912617	chr-12-26122364-G-T	0.04	0.03	127	38	0.98	not available	27.5	0.66
VTE: <i>F5</i> gene									
rs6026	chr-1-169528054-C-T	0.37	0.34	115	31	0.94	benign/likely benign	25.7	0.75
rs6034	chr-1-169529782-G-C	1.00	0.94	46	16	0.57	conflicting interpretations	21.3	0.56
rs78958618	chr-1-169542985-G-A	0.67	0.63	130	32	0.94	benign	15.18	0.11
rs9332485	chr-1-169586344-C-T	0.37	0.34	222	55	1	benign/likely benign	22.5	0.23

Genetic variants presented are those that passed functional annotation and statistical power filters. For each variant we provide its BinomiRare p value and mid-p value, the number of individuals with the rare allele n_c , the number of individuals with both the rare allele and the outcome n_d , the estimated power computed while assuming effect size $OR = 2$ and p value threshold = 0.05, pathogenicity interpretation from ClinVar, CADD score, and FATHMM-XF coding score.

individuals had the rare allele, a setting in which other tests such as the SPA should be able to control the type 1 error well and also be more powerful. Still, as a positive control we tested its association with VTE using BinomiRare, and the p value was 1.5×10^{-14} .

Short sleep has been consistently associated with cardiovascular and cardiometabolic disease.^{40,41} Genetic determinant of short sleep may help elucidate this connection.⁴² We considered the *DEC2/BHLHE4* gene, which has a mutation with a known familial aggregation associated with short sleep. Our filtering strategy resulted in a single variant considered for testing: rs121912617, the known short sleep mutation.²⁷ In our data, it was associated with short sleep with BinomiRare mid-p value = 0.03. rs121912617 is substantially more common (yet is still rare) in African Americans compared to European Americans (0.01 MAF in African Americans from the TOPMed short sleep datasets, compared to MAF < 0.001 in European Americans from the same dataset), allowing for observing this association in a population-based, rather than a family-based, study.

Here, we demonstrated the BinomiRare test for testing single-variant associations in data with known or cryptic relatedness. It can also be used to test sets of rare variants, by focusing on individuals with at least one rare allele in the variant set. It is a topic of future research to extend this framework to use the counts of the rare variant allele and increase power.

Appendix A

The CMP test

Let $W = \sum_{i=1}^{n_c} D_i$, for $D_i \sim \text{Binom}(p_i, 1)$, $i = 1, \dots, n_c$ be a random variable with the CMP-binomial probability function. When m increases, this distribution is approximated by the CMP distribution (Theorem 4.1. in Daly and Gaunt⁴³) so that $W \sim \text{CMP}(\lambda, \nu)$. Consider proposition 2 in Kadane¹⁹ stating:

Proposition 2 (Kadane¹⁹): Suppose D_1, \dots, D_{n_c} take values on $\{0, 1\}$. Let $P(W = k) = \tilde{p}_k \geq 0$, where $\sum_{k=0}^{n_c} \tilde{p}_k = 1$. Then there exists a unique distribution on D_1, \dots, D_{n_c} such that D_1, \dots, D_{n_c} are exchangeable of order n_c , and $\sum_{i=1}^{n_c} D_i$ has the same distribution as W , where we made a small change in the statement of the proposition compared to Kadane¹⁹ so it is clear that D_1, \dots, D_{n_c} could have different means without the context provided in Kadane.¹⁹ According to this proposition, an arbitrary sum of binary variables is distributed as a sum of exchangeable binary variables, where the exchangeable variables are such that there is a unique combination of probability parameter $p =$

$\Pr(D_{i_1} = 1) = \dots = \Pr(D_{i_{n_c}} = 1)$ and a parameter ρ modeling the dependency between each pair D_{i_1}, D_{i_2} , $i_1 \neq i_2$. Therefore, two parameters suffice to characterize the distribution of an arbitrary sum of binary variables. Specifically, for a given set of individuals with the rare genetic variant, the sum of their disease statuses,

$$W = \sum_{i=1}^{n_c} D_i,$$

is distributed like a unique sum of exchangeable binary variables. Based on the estimated disease probabilities, we estimate the two parameters (different than the probability and dependency parameters p and ρ above) of the CMP distribution to obtain an estimated probability function in a variation of a method-of-moment approach that is based on estimated probabilities, rather than on the observed data. Daly and Gaunt⁴³ provided an approximation to the moments of the CMP distribution:

Proposition 2.3. (Daly and Gaunt⁴³): Let $W \sim \text{CMP}(\lambda, \nu)$. Then, for $k \in \mathbb{N}$,

$$E[W^k] \sim \lambda^{\frac{k}{\nu}} \left[1 + O\left(\lambda^{-\frac{1}{\nu}}\right) \right],$$

as $\lambda \rightarrow \infty$.

Assuming that $\lambda^{-1/\nu}$ is small (which, as we shall see, is true when λ is very large, because ν tends to be well bounded), we get that, approximately:

$$E[W] \approx \lambda^{1/\nu}. \quad (\text{Equation A1})$$

Daly and Gaunt also showed, in their Equation 2.4 and based on the result in Shmueli et al.,¹⁸ that:

$$\text{Var}(W) \approx \frac{1}{\nu} \lambda^{\frac{1}{\nu}} + O(1), \text{ as } \lambda \rightarrow \infty. \quad (\text{Equation A2})$$

Therefore, noting that $\text{Var}(W) = E[W^2] - (E[W])^2$, once we estimate $E[W]$ and $E[W^2]$, we use Equations A1 and A2 to obtain estimators of λ and ν by:

$$\hat{\nu} = \frac{\widehat{E}[W]}{E[\widehat{W}^2] - \left(\widehat{E}[W]\right)^2} = \frac{\widehat{E}[W]}{\widehat{\text{Var}}(W)}, \quad (\text{Equation A3})$$

$$\hat{\lambda} = \left(\widehat{E}[W]\right)^{\hat{\nu}} \quad (\text{Equation A4})$$

Estimating parameters of the CMP distribution from estimated diseased probabilities

We consider two approaches to estimate components of $\hat{\lambda}$ and $\hat{\nu}$, i.e., $E[W]$, $E[W^2]$, and $\text{Var}(W)$: an analytic approach and a sampling-based approach. In the analytic approach, we compute $\widehat{E}[W] = \sum_{i=1}^{n_c} \hat{p}_i$ and $\widehat{\text{Var}}[W] = \sum_{i=1}^{n_c} \hat{p}_i(1 - \hat{p}_i)$. In the sampling-based approach, we generate random

variables \tilde{W} with the same distribution as W_{n_c} (the sum of disease statuses among the n_c individuals with the rare genetic variant) and treat them as observed data to estimate the desired quantities. More specifically, let $\tilde{D}_{i,s} \sim \text{Binom}(\hat{p}_i)$ be the sampled disease status of the i th individual in the $s = 1, \dots, S$ sample. Then:

$$\tilde{W}_s = \sum_{i=1}^{n_c} \tilde{D}_{i,s}, \quad (\text{Equation A5})$$

and we estimate:

$$E[\widehat{W}] = \frac{1}{S} \sum_{s=1}^S \tilde{W}_s$$

$$E[\widehat{W}^2] = \frac{1}{S} \sum_{s=1}^S \tilde{W}_s^2$$

To summarize, to calculate the p value and the mid-p value, formally given by:

$$p\text{-value} = \widehat{\Pr}(W = n_d) + \sum_{k=1}^{n_c} \widehat{\Pr}(W = k) \times 1 \left[\widehat{\Pr}(W = k) < \widehat{\Pr}(W = n_d) \right] \quad (\text{Equation A6})$$

$$\text{mid-p-value} = \frac{\widehat{\Pr}(W = n_d)}{2} + \sum_{k=1}^{n_c} \widehat{\Pr}(W = k) \times 1 \left[\widehat{\Pr}(W = k) < \widehat{\Pr}(W = n_d) \right], \quad (\text{Equation A7})$$

we estimate probabilities for each potential number of individuals who have both the rare allele and the disease, in the following process:

1. Obtain individual disease probability estimates $\hat{p}_1, \dots, \hat{p}_{n_c}$ via standard approaches (e.g., logistic mixed model).
2. Compute estimates $\widehat{E}[W]$ and $\widehat{Var}[W]$ in the analytic approach, or compute $\widehat{E}[W]$ and $\widehat{E}[W^2]$ in the sampling approach.
3. Compute estimates $\hat{\lambda}, \hat{\nu}$ using Equations A3 and A4.
4. Compute $\widehat{\Pr}(W = k)$ for $k = 1, \dots, n_c$ using the R package COMPOISSONREG.⁴⁴

Data and code availability

The code generated during this study for the BinomiRare and CMP tests is provided in the publicly available GENESIS R/Bioconductor package.²¹ A script for computing power for a single variant test is provided using the BinomiRare test. Source TOPMed whole-genome sequencing and phenotype data for this paper are available by applica-

tion to dbGaP according to the study specific accessions: Amish: phs000956.v5.p1, ARIC: phs001416.v2.p1, CHS: phs001368.v2.p1, CFS: phs000954.v3.p2, FHS: phs000974.v4.p3, HVH: phs000993.v4.p2, JHS: phs000964.v1.p1, Mayo VTE: phs001402.v2.p1, MESA: phs001211.v3.p2, WHI: phs001237.v2.p1. Additional study phenotypes are available by application to dbGaP via parent studies accession: ARIC: phs000090.v7.p1, CHS: phs000287.v7.p1, CFS: phs000284.v2.p1, FHS: phs000007.v32.p13, JHS: phs000286.v6.p2, MESA: phs000209.v13.p3, WHI: phs000200.v12.p3.

Supplemental information

Supplemental information can be found online at <https://doi.org/10.1016/j.xhgg.2021.100040>.

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Declaration of interests

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Web resources

Power computation function for the BinomiRare test, https://github.com/tamartsi/Binary_combine/blob/master/compute_power.R

GENESIS R/Bioconductor package, <https://bioconductor.org/packages/release/bioc/html/GENESIS.html>

dbGaP, <https://www.ncbi.nlm.nih.gov/gap/>

TOPMed datasets, <https://www.nhlbiwgs.org/data-sets>

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