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Product quality evaluation by confidence intervals of process yield index

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Statistical techniques have a beneficial effect on measuring process variability, analyzing the variability concerning product requirements, and eliminating the variability in product manufacturing. Process capability indices (PCIs) are not only easy to understand but also able to be directly employed by the manufacturing industry. The process yield index offers accurate measurement of the process yield, and it is a function of two unilateral six sigma quality indices. This paper initiates to develop the confidence intervals of the process yield index by using joint confidence regions of two unilateral six sigma quality indices for all quality characteristics of a product. Then integrate these joint confidence regions to find the confidence intervals of the product yield index. All manufacturing industries can use these confidence intervals to make statistical inferences to assess whether the process capability of the product and all quality characteristics has reached the required level, and to grasp the opportunities for improvement. An illustrated example on driver integrated circuit of micro hard disk is provided.

Process capability indices are commonly employed to assess whether the product quality can meet specifications defined in the manufacturing industry¹⁻³. At the same time, the process capability indices are also commonly used in the industry, and many studies have invested in related discussions⁴⁻⁶. Based on some studies, a product usually contains multi-quality characteristics, including smaller-the-better (STB), larger-the-better (LTB), and nominal-the-best (NTB) at the same time^{7,8}. Each quality characteristic needs to meet the required quality level, so that the quality of the final product can be guaranteed⁹. Moreover, numerous statisticians and quality engineers have studied process capability indices, aiming to come up with more effective methods to evaluate process potential and performance¹⁰⁻¹². The six-sigma method is also a commonly used in the industry. Some studies are discussing the relationship between the six-sigma method quality level and the process capability index¹³, and then study and propose some six-sigma quality indices¹⁴. Two well-known unilateral six sigma quality indices, Q_{pu} and Q_{pl} proposed by Chang et al.¹⁴, are used to measure the STB and LTB quality characteristics as follows:

$$Q_{pu} = \frac{USL - \mu}{\sigma}, \quad (1)$$

and

$$Q_{pl} = \frac{\mu - LSL}{\sigma}, \quad (2)$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ refers to the process standard deviation. In normal condition, the process yield (% Yield) and unilateral six sigma quality indices Q_{pu} and Q_{pl} have close relations displayed below:

$$STB : \%Yield = p\{X \leq USL\} = p\left\{Z \leq \frac{USL - \mu}{\sigma}\right\} = \Phi(Q_{pu}) \quad (3)$$

and

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$$LTB : \%Yield = p\{X \geq LSL\} = p\left\{Z \leq \frac{\mu - LSL}{\sigma}\right\} = \Phi(Q_{pl}), \quad (4)$$

where Φ is a standard function of the normal cumulative distribution. The process yield and the unilateral six sigma quality indices own a one-to-one relationship in mathematics. Therefore, some studies use these two unilateral six sigma quality indices to develop fuzzy quality evaluation model^{15,16} and fuzzy supplier selection model¹⁷.

For bilateral process capability index, Kane⁵ proposed a yield built on capability index C_{pk} below:

$$\begin{aligned} C_{pk} &= \frac{1}{3} \text{Min}\{Q_{pu}, Q_{pl}\} \\ &= \frac{1}{3} \text{Min}\left\{\frac{USL - \mu}{\sigma}, \frac{\mu - LSL}{\sigma}\right\}. \end{aligned} \quad (5)$$

Given that C_{pk} is a function of μ and σ , it simultaneously depends on μ and σ . Hence, a confidence interval for C_{pk} can be obtained using a joint confidence region for these two parameters. The lower confidence bound is the minimum value of C_{pk} over the region. The approximate confidence bounds of C_{pk} can then be obtained¹⁸. Hence, the process evaluation of C_{pk} cannot provide exact process capability measurement and process yield.

Therefore, Boyles¹⁹ has put forward a new bilateral process capability index which develops a one-to-one relation with the process yield as follows:

$$\begin{aligned} S_{pk} &= \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi \left(\frac{USL - \mu}{\sigma} \right) + \frac{1}{2} \Phi \left(\frac{\mu - LSL}{\sigma} \right) \right\} \\ &= \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi(Q_{pu}) + \frac{1}{2} \Phi(Q_{pl}) \right\}. \end{aligned} \quad (6)$$

Then, process yield index S_{pk} offers accurate measurement of the process yield. When $S_{pk} = c$, then the process yield is $Yield = 2\Phi(3c) - 1$. For processes of the normal distribution, the number of non-conformities is 2700 ppm, corresponding to a capable process with $S_{pk} = 1.0$.

Index S_{pk} has been widely used to judge whether the process quality can meet specifications and exactly measure the process yield. For example, Lee et al.²⁰ have proposed an asymptotic distribution for an estimator \hat{S}_{pk} . The asymptotic distribution of \hat{S}_{pk} is functional in statistical inferences for S_{pk} . Huang et al.⁸ have applied process yield index S_{pk} to assess the product quality of a backlight module with multiple process characteristics. Whereas, first, the integrated product capability (S_{pk}^T) of the backlight module is defined and, second, the individual process quality for each quality characteristic is determined. In addition, Chen et al.⁹ have considered generalizing the process yield index S_{pk} targeted at processes with multiple quality characteristics. Wang and Du²¹ used index S_{pk} to assess the performance of the supply chain. According to index S_{pk} , Wang et al.²² developed a new index to assess the measurement of the yield for a multiple-stream process. Lin and Pearn²³ identified the problem of process selection with S_{pk} so as to compare two processes as well as choose the one with a better production yield.

Unfortunately, the sampling distribution of yield index S_{pk} is pretty complicate, and it is not easy to derive the confidence interval of the process yield index S_{pk} . To overcome this difficulty, we have adopted several existing techniques to construct the confidence bounds for S_{pk} ²⁴⁻³¹. For example, Chen²⁴ used the bootstrap simulation technique to figure out four approximate lower confidence limits of the yield index S_{pk} . Shu and Wu²⁸ developed a useful method to gain the fuzzy estimate of the process yield index S_{pk} for measuring the manufacturing process yield. Wu et al.³⁰ proposed a generalized confidence intervals for S_{pk} to assess the process yield. However, most of the confidence bound studies for S_{pk} are complicated and approximate estimation.

The yield index S_{pk} is a function of indices Q_{pu} and Q_{pl} . Based on the Kushler and Hurley¹⁸ method constructing the minimum of C_{pk} over the region of the lower confidence bound, this paper initiates to develop the confidence regions of yield index S_{pk} by using the joint confidence regions of the two indices Q_{pu} and Q_{pl} . This research aims to appraise the individual process quality of a multiple-process product according to the confidence intervals of the process yield index S_{pk} . The method suggested by this study is a simple and intuitive tool. Then quality engineer can evaluate the process capability of the product and all quality characteristics, and decide whether to carry out process improvement. Therefore, the quality evaluation model in this paper can help the industry to improve.

Confidence intervals

Many studies have suggested that companies use control charts to perform process control. If the process is under statistical process control, then the process capability will be evaluated^{32,33}. It is assumed that each subsample contains n observations on quality characteristics, and there are m subsamples available.

In each subsample, we let \bar{X}_i be the sample mean and S_i be the sample variance of the i -th subsample, as displayed below:

$$\bar{X}_i = \frac{1}{n} \sum_{j=1}^n X_{ij} \quad (7)$$

and

$$S_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (X_{ij} - \bar{X}_i)^2}. \quad (8)$$

We define the overall sample mean and the pooled sample variance as follows:

$$\bar{X} = \frac{1}{m} \sum_{i=1}^m \bar{X}_i \tag{9}$$

and

$$\bar{S} = \frac{1}{m} \sum_{i=1}^m S_i. \tag{10}$$

The estimator \hat{Q}_{pi} of index Q_{pi} is displayed as follows:

$$\hat{Q}_{pi} = \frac{USL - \bar{X}}{\bar{S}} \text{ or } \frac{\bar{X} - LSL}{\bar{S}} \tag{11}$$

For unilateral six sigma quality index Q_{pi} , the $100 \times (1 - \alpha)\%$ lower and upper confidence limits LQ_{pi} and UQ_{pi} for Q_{pi} satisfy

$$P(LQ_{pi} \leq Q_{pi} \leq UQ_{pi}) = 1 - \alpha, \text{ where } i = u \text{ or } l. \tag{12}$$

Based on Choi and Owen³⁴, $\sqrt{mn} \times \hat{Q}_{pi}$ follows a non-central t distribution with $m(n - 1)$ degrees of freedom, where n is the subsample size and m is the number of sample groups. The non-central parameter of $\delta = \sqrt{mn} \times Q_{pi}$ is expressed as $T'_{n-1}(\delta = \sqrt{mn} \times Q_{pi})$. Then, the above two equations can be rewritten as

$$P\left[T'_{n-1}(\delta = \sqrt{mn} \times Q_{Li}) \leq \sqrt{mn} \times \hat{Q}_{pi}\right] = 1 - \frac{\alpha'}{2} \tag{13}$$

and

$$P\left[T'_{n-1}(\delta = \sqrt{mn} \times Q_{Ui}) \leq \sqrt{mn} \times \hat{Q}_{pi}\right] = \frac{\alpha'}{2}, \tag{14}$$

where $i = u$ or l and $\alpha' = \alpha/q$; q is the total number of quality characteristics for a multi-process product, m is the number of sample groups, and n is the sample size for each sub-sample.

Online Appendix S1 displays the lower limit (LQ_{pi}) and the upper limits (UQ_{pi}) of the 95% confidence intervals for different Q_{pi} values with $n = 11$, $m = 30$, and $q = 6(1)10$. For instance, given sub-sample size $n = 11$, $m = 30$, $q = 6$, $\alpha = 0.05$, when $Q_{pi} = 3.60$, from SAS program results, the LQ_{pi} and UQ_{pi} values are 3.222 and 3.981. The confidence intervals for different Q_{pi} values can be obtained in Online Appendix S1. Online Appendix S1 only provides the numerical values which are used in this study. Different sample size, different number of sample groups or different number of quality characteristics could possibly occur in the practical application. For easier explanation, we only perform the selected sample size and selected number of sample groups and the results are listed in Online Appendix S1. The numbers of quality characteristics are computed from 6 to 10 in Online Appendix S1.

Assume there are q quality characteristics in a product, the process capability index for the j th characteristic will become

$$S_{pkj} = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi(Q_{puj}) + \frac{1}{2} \Phi(Q_{plj}) \right\} \tag{15}$$

Let the confidence intervals of indices Q_{puj} and Q_{plj} are denoted by (LQ_{puj}, UQ_{puj}) and (LQ_{plj}, UQ_{plj}) , respectively. And index S_{pkj} is a function of indices Q_{puj} and Q_{plj} , the confidence intervals for index S_{pkj} can be described as follows.

Lower confidence interval for the j th characteristic (LS_{pkj}):

$$LS_{pkj} = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi(LQ_{puj}) + \frac{1}{2} \Phi(LQ_{plj}) \right\} \tag{16}$$

Upper confidence interval for the j th characteristic (US_{pkj}):

$$US_{pkj} = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi(UQ_{puj}) + \frac{1}{2} \Phi(UQ_{plj}) \right\} \tag{17}$$

Thus, the integrated process capability index for the entire product is

$$S_{pk}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \left(\prod_{j=1}^q [2\Phi(3S_{pkj}) - 1] + 1 \right) \right\} \tag{18}$$

Let the confidence intervals of indices S_{pk}^T are denoted by (LS_{pk}^T, US_{pk}^T) , and the confidence intervals for index S_{pk}^T can be described as follows.

Lower confidence interval for the integrated product (LS_{pk}^T):

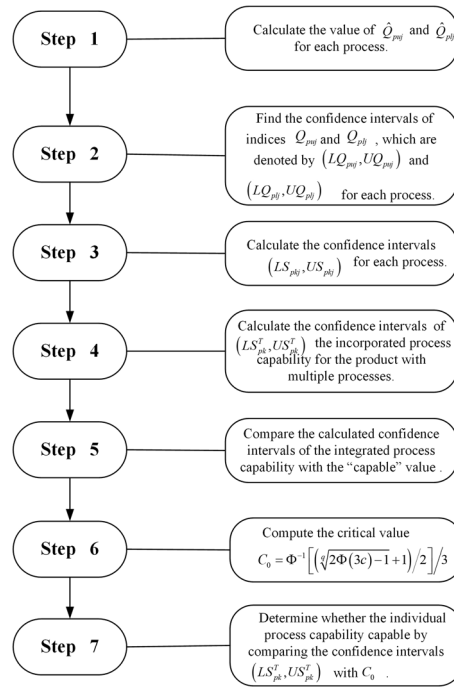


Figure 1. Evaluation procedure steps chart.

$$LS_{pk}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \left(\prod_{j=1}^q [2\Phi(3LS_{pkj}) - 1] + 1 \right) \right\} \tag{19}$$

Upper confidence interval for the integrated product (US_{pk}^T):

$$US_{pk}^T = \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \left(\prod_{j=1}^q [2\Phi(3US_{pkj}) - 1] + 1 \right) \right\} \tag{20}$$

As previously mentioned and based on Eqs. (16, 17, 19, 20), these confidence intervals allow statistical inferences to be made to assess whether the product’s process capability and all quality characteristics are at the required level.

Application procedures of confidence intervals

Products are commonly designed with many quality characteristics. Pearn et al.²⁵ has proposed how to determine the incorporated process of a product which contains multiple processes using index S_{pk} . What we emphasize here is the construction and the application through the confidence intervals of the process yield index. The previous section illustrates how to calculate the confidence intervals for the incorporated process capability of a product which contains multiple processes on the basis of the process yield index S_{pk} .

We assume that q quality characteristics in a product, m sample groups, and sub-sample sizes n are collected for each process. Since process yield index S_{pkj} is a function of unilateral indices Q_{puj} and Q_{plj} , the estimates of Q_{puj} and Q_{plj} for each process should be calculated. Then the confidence intervals (LQ_{puj}, UQ_{puj}) and (LQ_{plj}, UQ_{plj}) for the estimates of Q_{puj} and Q_{plj} are determined according to the formulas in “Application procedures of confidence intervals”. Or confidence intervals can also be found in Online Appendix S1. According to the confidence intervals for the estimates of Q_{puj} and Q_{plj} , the confidence intervals (LS_{pkj}, US_{pkj}) for each process is calculated. Finally, the confidence intervals for the incorporated process capability of the product with multiple processes are determined according to the formula of LS_{pk}^T and US_{pk}^T . For practical and easier application, the Fig. 1 evaluation procedure steps chart is shown as below.

The above evaluation steps are provided for manufacturers to follow and calculate the index confidence interval of the product and all quality characteristics, thereby evaluating whether the process capability of the product and all quality characteristics meets the quality level requirements. Next, this paper will use an example to illustrate the application of the above evaluation steps.

Mold thickness		0.25 mm	
Item	Layers	μm	Tolerance (μm)
A	Top Space	95	10
B	Top Loop	55	15
C	Die	70	5
D	Film	30	5
E	Mold thickness	250	50
F	Substrate thickness	110	25

Table 1. The quality characteristics and specifications for driver IC.

Item	USL _j	LSL _j	$\bar{\bar{X}}_j$	\bar{s}_j	\hat{Q}_{puj}	LQ _{puj}	UQ _{puj}	\hat{C}_{plj}	LQ _{plj}	UQ _{plj}	LS _{pkj}	US _{pkj}
A*	105	85	97.0	8.8	2.73	2.43	3.033	4.08	3.66	4.503	0.889	1.079
B	70	40	52.7	12.4	4.17	3.741	4.602	3.06	2.73	3.39	0.981	1.191
C*	75	65	69.1	4.4	4.02	3.606	4.44	2.79	2.484	3.096	0.905	1.098
D*	35	25	29.4	4.8	3.51	3.141	3.882	2.76	2.457	3.063	0.886	1.084
E*	300	200	253	51	2.76	2.457	3.063	3.12	2.787	3.456	0.863	1.067
F	135	85	108	22	3.69	3.306	4.077	3.15	2.814	3.489	0.992	1.216
Lower confidence interval for the integrated product									$LS_{pk}^T = 0.9626$			
Upper confidence interval for the integrated product									$US_{pk}^T = 0.9977$			

Table 2. The analytical results of a driver IC process.

An illustrative example

As the market demand on DVD player, MP3 and PDA rapidly grows, the demand on gigabyte capacity of multimedia data storage also increases. Nowadays, micro Hard Disk Driver (micro HDD) and Flash Memory card are the major products in terms of multimedia data storage. The storage capacity of a Flash Memory card ranges from 16 to 128 GB, while the storage capacity of micro HDD ranges from 500 GB to 5 TB. Driver IC is the key electronic component of micro HDD, functioning as anti-mechanical shock and read head control. In addition, it is responsible for the performance and effectiveness of micro HDD. The hard disk adopts the PCMCIA interface to connect to other equipment. To fit the design of thinner, smaller and low profile of micro HDD, the vertical dimension of driver IC gets thinner and thinner. There are six essential quality dimensions including top space, top loop, die, film, mold thickness and substrate thickness for a driver IC. The abovementioned evaluation method is adopted to assess the incorporated capability for a multi-process product, driver IC, for 1.8" HDD with mold thickness of 0.25 mm. Table 1 shows brief information about this production line. We illustrate the above procedure step by step with a numerical example as follows:

- Step 1 Calculate the estimates \hat{Q}_{puj} and \hat{Q}_{plj} of indices Q_{puj} and Q_{plj} for each process in Table 2. For example, the \hat{Q}_{puA} and \hat{Q}_{plA} for process A are 2.73 and 4.08, respectively.
- Step 2 Find the confidence intervals of indices Q_{puj} and Q_{plj} , which are denoted by (LQ_{puj}, UQ_{puj}) and (LQ_{plj}, UQ_{plj}) for each process in Table 2. For example, the (LQ_{puA}, UQ_{puA}) and (LQ_{plA}, UQ_{plA}) for process A are (2.430, 3.033) and (3.660, 4.503), respectively.
- Step 3 Calculate the confidence intervals (LS_{pkj}, US_{pkj}) for each process in Table 2. For example, the confidence intervals (LS_{pkA}, US_{pkA}) for process A are (2.667, 3.237).
- Step 4 Calculate the confidence intervals for the integrated process capability for the 6-process driver IC. The confidence interval for the product is (2.888, 2.993).
- Step 5 Compare the confidence intervals of driver IC with capable value, $c = 1.0$. Since the capable value does not range in the confidence intervals, the integrated process capability for the driver IC is not capable.
- Step 6 There are six essential dimensions for this product. Thus from the computation of the formula $C_0 = \Phi^{-1}[(\sqrt{2}\Phi(3c) - 1) / 2] / 3$, the critical value for individual process capability is 1.170 ($C_0 = 1.170$).
- Step 7 From Table 2, compare the required value C_0 with the confidence interval for each process and make a decision to determine which quality characteristic need to improvement. If C_0 ranges LS_{pk}^T and US_{pk}^T , then it is concluded that the individual process capability can meet the preset target; otherwise, the conclusion will be reverse. The processes marked with an "*" indicate that the process capabilities are not capable.

According to the above evaluation steps, the quality engineer can complete the process capability evaluation of the product and all quality characteristics, and decide whether to carry out process improvement. From Table 2, items B and F are capable processes. Quality engineers need to launch quality enhancement projects on incapable items A, C, D and E for process capability improvement. Obviously, through the evaluation steps in this

article, process engineers can simultaneously master and improve four quality characteristics with insufficient process capabilities. When the process capabilities of all quality characteristics meet the quality requirements, the product's process capabilities will meet the quality requirements.

Conclusions

PCIs are widely employed by the manufacturing industry to evaluate whether the process capability can meet the specifications. Regarding the product with multiple processes, customers concern the incorporated capability of the product. Based on the yield index S_{pkj} for quality characteristic j , this paper discussed the incorporated process capability of a product with multiple processes in terms of calculating the confidence intervals of the yield index S_{pkj} . Whereas the yield index S_{pkj} is a function of indices Q_{puj} and Q_{plj} , the confidence intervals for individual process of indices Q_{puj} and Q_{plj} are computed to attain the confidence intervals of S_{pkj} . Then integrate these confidence intervals to derive the confidence intervals of the entire product yield index S_{pk}^T . Evaluation procedures are presented in steps to assist practical application. The quality engineer can follow the evaluation procedures to complete the process capability evaluation of the product and all quality characteristics, and decide whether to carry out process improvement. The above research is based on the premise of the normal process. When the process distribution is non-normal, the method in this paper will have a large error, so it can be the focus of future research.

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Author contributions

Conceptualization, K.-S.C. and C.-H.H.; methodology, K.-S.C. and C.-H.H.; software, K.-C.C.; validation, K.-C.C.; formal analysis, K.-S.C. and C.-H.H.; data curation, K.-C. C.; writing—original draft preparation, K.-S.C., C.-H.H. and K.-C. C.; writing—review and editing, K.-S.C. and C.-H.H.; visualization, K.-C. C.; supervision, K.-S.C.; project administration, C.-H.H. All authors reviewed the manuscript.

Competing interests

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Additional information

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