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Linguistic Multi-Attribute Group Decision Making with Risk Preferences and Its Use in Low-Carbon Tourism Destination Selection

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Abstract: Low-carbon tourism plays an important role in carbon emission reduction and environmental protection. Low-carbon tourism destination selection often involves multiple conflicting and incommensurate attributes or criteria and can be modelled as a multi-attribute decision-making problem. This paper develops a framework to solve multi-attribute group decision-making problems, where attribute evaluation values are provided as linguistic terms and the attribute weight information is incomplete. In order to obtain a group risk preference captured by a linguistic term set with triangular fuzzy semantic information, a nonlinear programming model is established on the basis of individual risk preferences. We first convert individual linguistic-term-based decision matrices to their respective triangular fuzzy decision matrices, which are then aggregated into a group triangular fuzzy decision matrix. Based on this group decision matrix and the incomplete attribute weight information, a linear program is developed to find an optimal attribute weight vector. A detailed procedure is devised for tackling linguistic multi-attribute group decision making problems. A low-carbon tourism destination selection case study is offered to illustrate how to use the developed group decision-making model in practice.

Keywords: low-carbon tourism destination selection; linguistic multi-attribute group decision making; risk preference; incomplete weight information; linear program

1. Introduction

Climate change caused by carbon emissions has resulted in global warming and created an increasing threat to the environment and survival of all living things on earth. To cope with this challenge, the Chinese government declared at the 2009 United Nations Climate Change Conference its goal to reduce the intensity of carbon emission by 40–45% below 2005 levels, by 2020. A low-carbon economy has been considered to be an effective development framework for carbon reduction and environmental protection without affecting economic enhancement [1,2]. As a significant part of economic development, the tourism industry is encouraging low-carbon tourism and developing low-carbon tourism destinations (LCTDs) [3,4]. Meanwhile, more and more tourists are paying attention to carbon reduction and environmental protection, and thus select low-carbon tourism destinations to relieve the mental pressure caused by their work. Therefore, in order to obtain a high-quality travel experience, it is important for tourists to select the best option(s) from multiple low-carbon tourism destinations based on multi-attributes while considering carbon reduction, lower energy consumption and environmental protection. Generally speaking, tourism destination selection often involves multiple tourists. Each tourist may have his/her own demands and may approach the selection process with different expectations, but all of these tourists have a mutual interest in reaching final agreement on selecting the best travel destination(s). On the other hand, it is difficult

for the tourist group to create a ranking order of all possible tourism destinations due to the fact that the multiple attributes or criteria are frequently conflicting. To address such problems in selecting tourism destinations, this paper develops an approach to solve group decision-making problems, where evaluations of all of the alternatives with respect to each attribute are provided as linguistic terms, and the attribute weights are partly known.

Linguistic multi-attribute group decision making (MAGDM) uses linguistic terms in a linguistic term set (LTS) to express decision makers' evaluations of alternatives with respect to each attribute. In order to aggregate such evaluation values into an overall value in the linguistic environment, Herrera and Martínez [5] developed a 2-tuple linguistic representation model, in which a linguistic term in a balanced LTS is expressed by a linguistic 2-tuple. Wang and Hao [6] devised another 2-tuple linguistic representation model, where trapezoidal fuzzy numbers are used to characterize semantic information of linguistic terms in an unbalanced LTS. Based on these two representation models, a number of aggregation operators have been developed to solve linguistic MAGDM problems, such as the weighted averaging operator [5], the linguistic power aggregation operator [7], the linguistic hybrid harmonic operator [8] and the linguistic Choquet aggregation operator [9]. However, attribute weight information may be incomplete because of the complexity and indeterminacy of MAGDM problems [10,11]. Some researchers have focused their attention on linguistic MAGDM with incomplete weight information. For instance, Wei [12] developed another method to solve linguistic MAGDM problems, where linguistic terms are transformed into 2-tuples and TOPSIS (technique for order performance by similarity to ideal solution [13]) is used to devise an optimization model for determining attribute weights. Wei [14] proposed an approach to MAGDM, in which a maximizing-deviation-based optimization model is established to obtain attribute weights. Zhang and Guo [15] put forward an approach to linguistic MAGDM with multi-granularity and incomplete attribute weight information. By using the positive and negative ideal solutions, Ju [16] presented a method for solving linguistic MAGDM problems with incomplete linguistic weight information. In recent years, MAGDM with linguistic information has been widely used in many different areas, such as company performance assessment [17], recommender systems [18] and supplier selection [19].

Decision-making methods have been applied in low-carbon economy development. Tong and Wang [20] proposed a group decision-making framework with intuitionistic fuzzy preference relations and applied it to low-carbon supplier selection. Cho et al. [3] adopted the fuzzy analytic hierarchy process to construct evaluation indicators of Taiwan's low-carbon tourism development. Cheng [3] used the Delphi method and the analytic hierarchy process to establish evaluation indicators of low-carbon tourist attractions. Zhang [21] employed the analytic network process to evaluate regional low-carbon tourism strategies.

In real-life linguistic MAGDM problems, different decision makers often have various expectations and considerations for semantic scale values of linguistic terms, which can be characterized by their risk preferences [22–24]. For example, if linguistic terms are used to describe the evaluation of a tourism destination's low-carbon facilities, then different tourists may have various expectative scale values for the linguistic term "Good". Zhou and Xu [23] introduced two parameters reflecting risk preferences to extend the sigmoid function and proposed the notion of generalized linguistic term sets (GLTSs). Lin and Wang [24] developed GLTSs with triangular fuzzy semantic information and put forward an approach to solve qualitative decision-making problems. In this paper, we establish an optimization model to obtain an optimal group GLTS based on individual decision maker's risk preferences. By using the triangular fuzzy weighted average based aggregation method, individual linguistic evaluations are fused into group triangular fuzzy evaluations. Based on the group evaluation information, a linear program is established to obtain optimal attribute weights. A detailed procedure is developed to solve linguistic MAGDM problems with risk preferences and incomplete weight information.

The remaining contents of this article are organized as follows. The next section gives preliminaries on LTSs, GLTSs and the Euclidean distance of any two positive triangular fuzzy numbers. Section 3 describes linguistic MAGDM problems and establishes a nonlinear programming model to obtain a

group GLTS. A linear program and a procedure are developed for solving linguistic MAGDM problems with risk preferences and incomplete weight information in Section 4. Section 5 provides a case study of a low-carbon tourism destination selection problem in order to examine the proposed decision models. Finally, Section 6 offers concluding remarks.

2. Preliminaries

This section offers preliminaries on LTSs, GLTSs and the Euclidean distance between two positive triangular fuzzy numbers.

Let $S = \{s_{-\tau_1}, s_{-\tau_1+1}, \dots, s_0, \dots, s_{\tau_2-1}, s_{\tau_2}\}$ be an LTS, where τ_1 and τ_2 are two positive integers, $\tau_1 + \tau_2 + 1$ is the granularity of S , and s_0 is the neutral linguistic term in S , such as “middle”, “fair” and “indifference”. If $\tau_1 = \tau_2$, then S satisfies the following characteristics [25]:

- (i) The set S is ordered, i.e., $s_i > s_j$ if and only if $i > j$;
- (ii) A negation operator can be defined as $Neg(s_i) = s_{-i}$, where $Neg(s_0) = s_0$.

A LTS S is called a balanced LTS if $\tau_1 = \tau_2$ and the distribution of its semantic information is uniform and symmetrical; otherwise, S is an unbalanced LTS. For example, an LTS including seven linguistic terms ($\tau_1 = \tau_2 = 3$) is expressed as:

$$S = \left\{ \begin{array}{l} s_{-3} = \text{very poor}(VP), s_{-2} = \text{poor}(P), s_{-1} = \text{slightly poor}(SP), s_0 = \text{medium}(M), \\ s_1 = \text{slightly good}(SG), s_2 = \text{good}(G), s_3 = \text{very good}(VG) \end{array} \right\} \quad (1)$$

In order to characterize semantic information with risk preferences for linguistic terms in a LTS, Lin and Wang [24] introduced the following notion of a GLTS.

Definition 1 [24]. For an LTS $S = \{s_{-\tau_1}, s_{-\tau_1+1}, \dots, s_0, \dots, s_{\tau_2-1}, s_{\tau_2}\}$, a GLTS is defined as

$$\tilde{S} = \left\{ (s_i, \tilde{v}_i) \mid i = -\tau_1, \dots, 0, \dots, \tau_2, \tilde{v}_i = (v_i^L, v_i^M, v_i^U) \right\} \quad (2)$$

where \tilde{v}_i is a triangular fuzzy number with two parameters θ_1 and θ_2 ($\theta_1, \theta_2 > 0$) and indicates a fuzzy semantic value of s_i for each $i = -\tau_1, \dots, 0, \dots, \tau_2$, and v_i^L, v_i^M and v_i^U are given as

$$v_i^L = \begin{cases} (1 + e^{-\theta_2 i})^{-1}, & i = -\tau_1 \\ (1 + e^{-\theta_2(i-1)})^{-1}, & i = -\tau_1 + 1, \dots, -1, 0 \\ (1 + e^{-\theta_1(i-1)})^{-1}, & i = 1, 2, \dots, \tau_2 \end{cases} \quad (3)$$

$$v_i^M = \begin{cases} (1 + e^{-\theta_2 i})^{-1}, & i = -\tau_1, \dots, -1 \\ 0.5, & i = 0 \\ (1 + e^{-\theta_1 i})^{-1}, & i = 1, 2, \dots, \tau_2 \end{cases} \quad (4)$$

$$v_i^U = \begin{cases} (1 + e^{-\theta_2(i+1)})^{-1}, & i = -\tau_1, \dots, -1 \\ (1 + e^{-\theta_1(i+1)})^{-1}, & i = 0, 1, \dots, \tau_2 - 1 \\ (1 + e^{-\theta_1 i})^{-1}, & i = \tau_2 \end{cases} \quad (5)$$

It is obvious that $v_{-1}^U = v_0^M = v_1^L = 0.5$ and $0 < v_i^L, v_i^M, v_i^U < 1$ for $i = -\tau_1, \dots, 0, \dots, \tau_2$. If $\theta_1 > \theta_2$, then the risk preferences of semantic values of linguistic terms in \tilde{S} are radical. If $\theta_1 = \theta_2$, then the risk preferences of semantic values of linguistic terms in \tilde{S} are neutral. If $\theta_1 < \theta_2$, then the risk preferences of semantic values of linguistic terms in \tilde{S} are aversive.

For any two positive triangular fuzzy numbers $\tilde{v}_\alpha = (v_\alpha^L, v_\alpha^M, v_\alpha^U)$ and $\tilde{v}_\beta = (v_\beta^L, v_\beta^M, v_\beta^U)$, their Euclidean distance is given as [26]

$$d(\tilde{v}_\alpha, \tilde{v}_\beta) = \sqrt{\frac{1}{3} \left((v_\alpha^L - v_\beta^L)^2 + (v_\alpha^M - v_\beta^M)^2 + (v_\alpha^U - v_\beta^U)^2 \right)} \tag{6}$$

It is obvious that $0 \leq d(\tilde{v}_\alpha, \tilde{v}_\beta) < 1$ if $0 < v_\alpha^L \leq v_\alpha^M \leq v_\alpha^U \leq 1$ and $0 < v_\beta^L \leq v_\beta^M \leq v_\beta^U \leq 1$. According to (2)–(6), we have $d(\tilde{v}_{-i}, \tilde{v}_0) = d(\tilde{v}_i, \tilde{v}_0)$ ($i = 1, 2, \dots, \tau_2$) if $\tau_1 = \tau_2 > 1$ and $\theta_1 = \theta_2$. This implies that the distribution of semantic values of linguistic terms in \tilde{S} is symmetrical. In this situation, if $d(\tilde{v}_i, \tilde{v}_{i+1})$ is approximately equal to $d(\tilde{v}_{i-1}, \tilde{v}_i)$, i.e., $d(\tilde{v}_i, \tilde{v}_{i+1}) \approx d(\tilde{v}_{i-1}, \tilde{v}_i)$ for $i = -\tau_1 + 1, \dots, -1, 0, 1, \dots, \tau_2 - 1$, then \tilde{S} is said to have symmetry and approximate uniformity; otherwise, \tilde{S} is symmetrical and non-uniform.

If $\tau_1 = \tau_2 > 1$ and $\theta_1 \neq \theta_2$, then one has $d(\tilde{v}_{-i}, \tilde{v}_0) \neq d(\tilde{v}_i, \tilde{v}_0)$ ($i = 1, 2, \dots, \tau_2$) and there exists $i \in \{-\tau_1 + 1, \dots, -1, 0, 1, \dots, \tau_2 - 1\}$ satisfying $d(\tilde{v}_i, \tilde{v}_{i+1}) \neq d(\tilde{v}_{i-1}, \tilde{v}_i)$. This indicates that the distribution of semantic values of linguistic terms in \tilde{S} is asymmetrical and non-uniform.

In order to compare and rank triangular fuzzy numbers, the following formula is used to obtain the score of a triangular fuzzy number $\tilde{v} = (v^L, v^M, v^U)$ [27].

$$S(\tilde{v}) = \frac{v^L + 2v^M + v^U}{4} \tag{7}$$

3. An Optimization Model for Determining a Group Generalized Linguistic Term Set

This section describes an MAGDM problem and establishes an optimization model to obtain an optimal group GLTS.

Given n feasible alternatives x_i ($i = 1, 2, \dots, n$) and m qualitative attributes a_j ($j = 1, 2, \dots, m$). Let $X = \{x_1, x_2, \dots, x_n\}$ be the alternative set and $A = \{a_1, a_2, \dots, a_m\}$ be the attribute set, then an MAGDM problem is to determine a ranking of all alternatives or find the best alternative(s) from feasible alternatives in X according to the evaluation information offered by a group of experts or decision makers denoted by $E = \{e_1, e_2, \dots, e_q\}$.

Assume that the important weight vector of the experts is $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_q)^T$, where $0 \leq \lambda_k \leq 1$ for each $k = 1, 2, \dots, q$, and the weight vector is normalized to one, i.e., $\sum_{k=1}^q \lambda_k = 1$. In linguistic MAGDM, each expert $e_k \in E$ uses linguistic terms in S to evaluate the alternatives in X with respect to the attributes in A and provides a decision matrix as $R_k = (s_{rijk})_{n \times m}$, where s_{rijk} is a linguistic term in S , i.e., $s_{rijk} \in S$ for $i = 1, 2, \dots, n, j = 1, 2, \dots, m, k = 1, 2, \dots, q$.

In linguistic MAGDM with risk preferences, different experts have various expectations and considerations for the semantic scale value of a linguistic term. In other words, different decision makers have various risk preferences on providing their evaluation information. In order to aggregate individual evaluations into a group evaluation, it is necessary to determine a group GLTS based on the expected triangular fuzzy semantic values given by the decision maker $e_k \in E, k = 1, 2, \dots, q$ for linguistic terms in S .

Let $\tilde{V}^{(k)} = \{\tilde{v}_{t_\alpha}^{(k)}, \tilde{v}_{t_\beta}^{(k)}, \dots, \tilde{v}_{t_\gamma}^{(k)}\}$ be the set of the expected triangular fuzzy semantic information provided by the decision maker $e_k \in E$, then the following optimization model is devised to determine an optimal group GLTS.

$$\begin{aligned} \min \quad & J = \sum_{k=1}^q \lambda_k \left(d(\tilde{v}_{t_\alpha}^{(k)}, \tilde{v}_{t_\alpha}^{(k)}) + d(\tilde{v}_{t_\beta}^{(k)}, \tilde{v}_{t_\beta}^{(k)}) + \dots + d(\tilde{v}_{t_\gamma}^{(k)}, \tilde{v}_{t_\gamma}^{(k)}) \right) \\ \text{s.t.} \quad & \theta_1 > 0, \theta_2 > 0 \end{aligned} \tag{8}$$

where $d(\cdot, \cdot)$ is the Euclidean distance defined by (6) and θ_1, θ_2 are decision variables.

Solving the above nonlinear programming model yields an optimal solution denoted by θ_1^* and θ_2^* . By plugging θ_1^* and θ_2^* into (3)–(5), we obtain an optimal group GLTS as

$$\tilde{S}^* = \left\{ \langle s_i, \tilde{v}_i^* \rangle \mid i = -\tau_1, \dots, 0, \dots, \tau_2, \tilde{v}_i^* = (v_i^{*L}, v_i^{*M}, v_i^{*U}) \right\} \tag{9}$$

where

$$v_i^{*L} = \begin{cases} (1 + e^{-\theta_2^* i})^{-1}, & i = -\tau_1 \\ (1 + e^{-\theta_2^*(i-1)})^{-1}, & i = -\tau_1 + 1, \dots, -1, 0 \\ (1 + e^{-\theta_1^*(i-1)})^{-1}, & i = 1, 2, \dots, \tau_2 \end{cases} \tag{10}$$

$$v_i^{*M} = \begin{cases} (1 + e^{-\theta_2^* i})^{-1}, & i = -\tau_1, \dots, -1 \\ 0.5, & i = 0 \\ (1 + e^{-\theta_1^* i})^{-1}, & i = 1, 2, \dots, \tau_2 \end{cases} \tag{11}$$

$$v_i^{*U} = \begin{cases} (1 + e^{-\theta_2^*(i+1)})^{-1}, & i = -\tau_1, \dots, -1 \\ (1 + e^{-\theta_1^*(i+1)})^{-1}, & i = 0, 1, \dots, \tau_2 - 1 \\ (1 + e^{-\theta_1^* i})^{-1}, & i = \tau_2 \end{cases} \tag{12}$$

We can see from (9)–(12) that the optimal group GLTS \tilde{S}^* captures and synthesizes individual decision makers’ risk preferences. If $\theta_1^* > \theta_2^*$, then the expert group prefers to make a risk-seeking decision. If $\theta_1^* = \theta_2^*$, then the expert group prefers to obtain a neutral-risk decision result. If $\theta_1^* < \theta_2^*$, then the expert group prefers to make a risk-aversion decision.

Example 1. Consider the LTS S given by (1). Three decision makers e_1, e_2 and e_3 provide their expected triangular fuzzy semantic information as follows.

$$\begin{aligned} \tilde{V}^{(1)} &= \left\{ \tilde{v}_{-2}^{(1)} = (0.2, 0.3, 0.4), \tilde{v}_0^{(1)} = (0.6, 0.65, 0.75), \tilde{v}_1^{(1)} = (0.65, 0.75, 0.8) \right\}, \\ \tilde{V}^{(2)} &= \left\{ \tilde{v}_{-3}^{(2)} = (0.25, 0.25, 0.35), \tilde{v}_1^{(2)} = (0.6, 0.7, 0.75), \tilde{v}_2^{(2)} = (0.7, 0.75, 0.85) \right\}, \\ \tilde{V}^{(3)} &= \left\{ \begin{aligned} \tilde{v}_{-3}^{(3)} &= (0.18, 0.18, 0.28), \tilde{v}_{-2}^{(3)} = (0.18, 0.28, 0.38), \tilde{v}_{-1}^{(3)} = (0.28, 0.38, 0.48), \\ \tilde{v}_0^{(3)} &= (0.38, 0.48, 0.58), \tilde{v}_1^{(3)} = (0.48, 0.58, 0.68), \tilde{v}_2^{(3)} = (0.58, 0.68, 0.78), \\ \tilde{v}_3^{(3)} &= (0.68, 0.78, 0.78) \end{aligned} \right\}. \end{aligned}$$

Assume that the important weights of the three decision makers are 0.4, 0.4 and 0.2 respectively. By solving the optimization model (8), we obtain an optimal solution of $\theta_1^* = 0.5114$ and $\theta_2^* = 0.4397$. Thus, as per (9)–(12), an optimal group GLTS is determined as

$$\tilde{S}_1^* = \left\{ \begin{aligned} &\langle s_{-3}, (0.2110, 0.2110, 0.2933) \rangle, \langle s_{-2}, (0.2110, 0.2933, 0.3918) \rangle, \\ &\langle s_{-1}, (0.2933, 0.3918, 0.5000) \rangle, \langle s_0, (0.3918, 0.5000, 0.6251) \rangle, \\ &\langle s_1, (0.5000, 0.6251, 0.7355) \rangle, \langle s_2, (0.6251, 0.7355, 0.8226) \rangle, \\ &\langle s_3, (0.7355, 0.8226, 0.8226) \rangle \end{aligned} \right\} \tag{13}$$

Obviously, $\theta_1^* > \theta_2^*$, indicating that \tilde{S}^* is a risk-seeking GLTS. The distribution of the semantic values of \tilde{S}^* is shown in Figure 1, where VP, P, SP, M, SG, G and VG are defined in (1). It is easy to verify that $d(\tilde{v}_{-i}, \tilde{v}_0) \neq d(\tilde{v}_i^*, \tilde{v}_0^*) (i = 1, 2, 3)$ and there exists $i \in \{-2, -1, 0, 1, 2\}$ satisfying $d(\tilde{v}_i^*, \tilde{v}_{i+1}^*) \neq d(\tilde{v}_{i-1}^*, \tilde{v}_i^*)$. Hence, the distribution of the semantic values of \tilde{S}^* is asymmetrical and non-uniform.

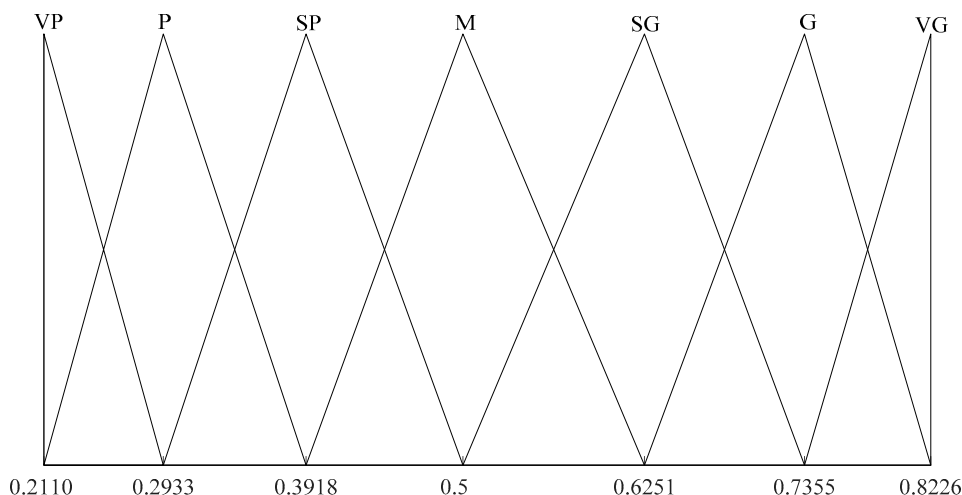


Figure 1. Distribution of the semantic values of the generalized linguistic term sets (GLTS) \tilde{S}_1^* .

4. An Approach to Linguistic MAGDM with Risk Preferences and Incomplete Weight Information

This section uses the triangular fuzzy weighted average based aggregation method to fuse individual linguistic evaluations into a group triangular fuzzy evaluation and develops a linear program to obtain optimal attribute weights. A procedure is also devised for solving linguistic MAGDM problems with risk preferences and incomplete weight information.

Once a group GLTS \tilde{S}^* is determined, each linguistic-term-based decision matrix $R_k = (s_{rijk})_{n \times m}$ ($k = 1, 2, \dots, q$) can be transformed into a triangular fuzzy decision matrix denoted by

$$\tilde{D}_k = (\tilde{d}_{ijk})_{n \times m} = ((d_{ijk}^L, d_{ijk}^M, d_{ijk}^U))_{n \times m}, k = 1, 2, \dots, q \tag{14}$$

where

$$\tilde{d}_{ijk} = \tilde{v}_{rijk}^* = (v_{rijk}^{*L}, v_{rijk}^{*M}, v_{rijk}^{*U}), i = 1, 2, \dots, n, j = 1, 2, \dots, m, k = 1, 2, \dots, q \tag{15}$$

Based on the triangular fuzzy decision matrices \tilde{D}_k ($k = 1, 2, \dots, q$), using the triangular fuzzy weighted average operator together with the decision makers' weight vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_q)^T$ yields a group triangular fuzzy decision matrix as

$$\tilde{G} = (\tilde{g}_{ij})_{n \times m} = ((g_{ij}^L, g_{ij}^M, g_{ij}^U))_{n \times m} \tag{16}$$

where

$$\tilde{g}_{ij} = \sum_{k=1}^q \lambda_k \tilde{d}_{ijk} = \left(\sum_{k=1}^q \lambda_k v_{rijk}^{*L}, \sum_{k=1}^q \lambda_k v_{rijk}^{*M}, \sum_{k=1}^q \lambda_k v_{rijk}^{*U} \right) \tag{17}$$

In MAGDM problems, it is clear that different attribute weights reflect their importance in selecting the best alternative and ranking alternatives. Let $w = (w_1, w_2, \dots, w_m)^T$ be the attribute crisp weight vector, where $\sum_{j=1}^m w_j = 1$ and $w_j \geq 0, j = 1, 2, \dots, m$. If the attribute weights in w are completely known, then from the group decision matrix \tilde{G} , a group overall evaluation value of alternative x_i is determined as

$$\begin{aligned} \tilde{d}_i^{(G)} &= \sum_{j=1}^m w_j \tilde{g}_{ij} = \left(\sum_{j=1}^m g_{ij}^L w_j, \sum_{j=1}^m g_{ij}^M w_j, \sum_{j=1}^m g_{ij}^U w_j \right) \\ &= \left(\sum_{j=1}^m \left(w_j \sum_{k=1}^q \lambda_k v_{r_{ijk}}^{*L} \right), \sum_{j=1}^m \left(w_j \sum_{k=1}^q \lambda_k v_{r_{ijk}}^{*M} \right), \sum_{j=1}^m \left(w_j \sum_{k=1}^q \lambda_k v_{r_{ijk}}^{*U} \right) \right) \end{aligned} \tag{18}$$

for $i = 1, 2, \dots, n$.

In reality, it is often difficult for decision makers to offer exact values for attribute weights due to the complexity of practical decision cases and the limitation of the decision makers' knowledge. Thus, the attribute weight information may be incomplete or partially known, which can be characterized by a nonempty subset Ω of all combinations of the following five forms.

- (i) A weak ranking: $\{w_i \geq w_j\}, i \neq j$;
- (ii) A strict ranking: $\{w_i - w_j \geq \varepsilon_{ij}\}, i \neq j$, where $\varepsilon_{ij} > 0$;
- (iii) An interval form: $\{\alpha_j \leq w_j \leq \alpha_j + \varepsilon_j\}$, where $0 \leq \alpha_j < \alpha_j + \varepsilon_j \leq 1$;
- (iv) A ranking with multiples: $\{w_i \geq \beta_{ij} w_j\}$, where $0 \leq \beta_{ij} \leq 1, i \neq j$; and
- (v) A ranking of deviations: $\{w_i - w_j \geq w_k - w_l\}$, where $i \neq j \neq k \neq l$.

Since the value $\tilde{d}_i^{(G)}$ defined by (18) indicates the overall evaluation of alternative x_i , the larger the triangular fuzzy number $\tilde{d}_i^{(G)}$, the better the alternative x_i is. Thus, as per the score of a triangular fuzzy number given by (7), we should find a weight vector $w = (w_1, w_2, \dots, w_m)^T$ such that $S(\tilde{d}_i^{(G)})$ is maximized for all $i = 1, 2, \dots, n$. Therefore, the following multi-objective optimization model is established to determine attribute weights.

$$\begin{aligned} \max_{J_{x_i}} &= S(\tilde{d}_i^{(G)}) && i = 1, 2, \dots, n \\ \text{s.t.} &\begin{cases} 0 \leq w_j \leq 1, & j = 1, 2, \dots, m \\ \sum_{j=1}^m w_j = 1, \\ w = (w_1, w_2, \dots, w_m)^T \in \Omega. \end{cases} \end{aligned} \tag{19}$$

Since each alternative $x_i \in X$ is a feasible and non-inferior alternative and the maximization problems $\max_{J_{x_i}} = S(\tilde{d}_i^{(G)}) (i = 1, 2, \dots, n)$ have the same constraint conditions, the multi-objective optimization model (19) can be converted into the following aggregated optimization model by setting the same important weight for each goal $J_{x_i} (i = 1, 2, \dots, n)$.

$$\begin{aligned} \max J &= \frac{1}{n} \sum_{i=1}^n S(\tilde{d}_i^{(G)}) \\ \text{s.t.} &\begin{cases} 0 \leq w_j \leq 1, & j = 1, 2, \dots, m \\ \sum_{j=1}^m w_j = 1, \\ w = (w_1, w_2, \dots, w_m)^T \in \Omega. \end{cases} \end{aligned} \tag{20}$$

As per (7) and (18), the optimization model (20) can be equivalently rewritten as the following linear program.

$$\begin{aligned} \max J &= \frac{1}{4n} \sum_{i=1}^n \sum_{j=1}^m (g_{ij}^L + 2g_{ij}^M + g_{ij}^U) w_j \\ \text{s.t.} &\begin{cases} 0 \leq w_j \leq 1, & j = 1, 2, \dots, m \\ \sum_{j=1}^m w_j = 1, \\ w = (w_1, w_2, \dots, w_m)^T \in \Omega. \end{cases} \end{aligned} \tag{21}$$

where w_j is a decision variable for all $j = 1, 2, \dots, m$.

By solving the linear program (21), we obtain an optimal attribute weight vector denoted by $w^* = (w_1^*, w_2^*, \dots, w_m^*)^T$.

Substituting w^* into (18) yields an optimal group overall evaluation value of alternative x_i as

$$\begin{aligned} \tilde{d}_i^{(*G)} &= \left(\sum_{j=1}^m g_{ij}^L w_j^*, \sum_{j=1}^m g_{ij}^M w_j^*, \sum_{j=1}^m g_{ij}^U w_j^* \right) \\ &= \left(\sum_{j=1}^m \left(w_j^* \sum_{k=1}^q \lambda_k v_{r_{ijk}}^{*L} \right), \sum_{j=1}^m \left(w_j^* \sum_{k=1}^q \lambda_k v_{r_{ijk}}^{*M} \right), \sum_{j=1}^m \left(w_j^* \sum_{k=1}^q \lambda_k v_{r_{ijk}}^{*U} \right) \right) \end{aligned} \tag{22}$$

Based on the aforementioned analysis, a procedure is now developed for linguistic MAGDM with risk preferences and incomplete attribute weigh information.

Procedure

Step 1: Each decision maker $e_k \in E$ ($k = 1, 2, \dots, q$) adopts linguistic terms in S to evaluate alternatives in X with respect to each attribute in A , which are given by a decision matrix $R_k = (s_{r_{ijk}})_{n \times m}$, and provides his/her risk preference information $\tilde{V}^{(k)}$.

Step 2: Obtain an optimal group GLTS \tilde{S}^* by solving the optimization model (8) and using (9)–(12).

Step 3: Transform the linguistic-term-based decision matrix R_k into a triangular fuzzy decision matrix \tilde{D}_k as per (14) and (15) for each $k = 1, 2, \dots, q$.

Step 4: Aggregate individual decision matrix \tilde{D}_k ($k = 1, 2, \dots, q$) into a group triangular fuzzy decision matrix $\tilde{G} = (\tilde{g}_{ij})_{n \times m} = (g_{ij}^L, g_{ij}^M, g_{ij}^U)_{n \times m}$ according to (17).

Step 5: Determine optimal attribute weights by solving the linear program (21).

Step 6: Employ (22) to obtain optimal group overall evaluation values $\tilde{d}_i^{(*G)}$ ($i = 1, 2, \dots, n$) for all alternatives in X .

Step 7: Use (7) to calculate scores $S(\tilde{d}_i^{(*G)})(i = 1, 2, \dots, n)$.

Step 8: Obtain a ranking order of all alternatives in terms of the decreasing order of the scores $S(\tilde{d}_i^{(*G)})(i = 1, 2, \dots, n)$, and $x_i \succ x_j$ is employed to express that alternative x_i is preferred to x_j .

5. A Case Study of the Low-Carbon Tourism Destination Selection Problem

This section applies the proposed linguistic MAGDM model to examine a low-carbon tourism destination selection problem.

With the continuing advocacy and promotion of the Chinese government, many tourism destinations have been developed to reduce carbon emissions and save energy. Moreover, many tourists have recognized the importance of low-carbon tourism for environmental protection. In order to find a good balance between the enjoyment of a trip and carbon emission reduction, it is crucial for tourists to compare and evaluate some known low-carbon tourism destinations, and then choose the best one(s) from these options. Generally speaking, this evaluation and selection process is based on several criteria or attributes. In this case study, the attributes consist of the following five aspects:

- (i) a_1 : Low-carbon transportation, low-energy consumption vehicles and pick-up and drop-off services as reflected in connecting different scenic sites and reaching the destination.
- (ii) a_2 : Food service including green food, a low-carbon environment and low-energy waste handling mechanisms.
- (iii) a_3 : Hotels and accommodation, as reflected in green-material labels, low-carbon facilities and a low-carbon environment and education management.
- (iv) a_4 : Consumption satisfaction, as reflected in the service cost of travel agencies, ticket prices for scenic sites and the cost of accommodation.
- (v) a_5 : Attraction and impact of scenic sites, including low-carbon customer service and low-carbon management and control.

Without loss of generality, assume that three tourists (i.e., decision makers) e_1, e_2 and e_3 want to go on a low-carbon trip and their importance weights are 0.4, 0.4 and 0.2, respectively, i.e., $\lambda = (0.4, 0.4, 0.2)^T$. After preliminary screening there are four low-carbon tourism destinations x_1, x_2, x_3 and x_4 as the alternatives. Based on the LTS S given by (1), the three tourists provide their linguistic evaluations for the four tourism destinations with respect to each attribute a_j ($j = 1, 2, \dots, n$). The three tourists' linguistic evaluations are shown in Tables 1–3, respectively.

Table 1. Linguistic-term-based decision matrix $R_1 = (s_{r_{ij1}})_{4 \times 5}$ given by e_1 .

Alternative	a_1	a_2	a_3	a_4	a_5
x_1	P	G	VG	M	SG
x_2	SP	M	G	P	G
x_3	VG	G	SP	SP	M
x_4	SG	SG	M	G	M

Table 2. Linguistic-term-based decision matrix $R_2 = (s_{r_{ij2}})_{4 \times 5}$ given by e_2 .

Alternative	a_1	a_2	a_3	a_4	a_5
x_1	SP	VG	SG	G	G
x_2	SG	VG	VG	VP	VG
x_3	G	VG	M	G	SG
x_4	SG	G	SP	SG	SG

Table 3. Linguistic-term-based decision matrix $R_3 = (s_{r_{ij3}})_{4 \times 5}$ given by e_3 .

Alternative	a_1	a_2	a_3	a_4	a_5
x_1	M	SP	G	SG	VG
x_2	G	SG	SG	M	G
x_3	G	SG	G	G	SP
x_4	M	VG	SG	VG	G

Based on the expectations of semantic scale values of linguistic terms in S , the expected triangular fuzzy semantic values for the three tourists are as follows:

$$\begin{aligned} \tilde{V}^{(1)} &= \left\{ \tilde{v}_{-2}^{(1)} = (0.15, 0.25, 0.35), \tilde{v}_1^{(1)} = (0.55, 0.65, 0.75), \tilde{v}_2^{(1)} = (0.65, 0.75, 0.85) \right\} \\ \tilde{V}^{(2)} &= \left\{ \tilde{v}_{-2}^{(2)} = (0.25, 0.35, 0.45), \tilde{v}_1^{(2)} = (0.6, 0.7, 0.8), \tilde{v}_2^{(2)} = (0.7, 0.8, 0.9) \right\}, \\ \tilde{V}^{(3)} &= \left\{ \begin{aligned} &\tilde{v}_{-3}^{(3)} = (0.2, 0.2, 0.3), \tilde{v}_{-2}^{(3)} = (0.2, 0.3, 0.4), \tilde{v}_{-1}^{(3)} = (0.3, 0.4, 0.5), \tilde{v}_0^{(3)} = (0.4, 0.5, 0.6), \\ &\tilde{v}_1^{(3)} = (0.5, 0.6, 0.7), \tilde{v}_2^{(3)} = (0.6, 0.7, 0.8), \tilde{v}_3^{(3)} = (0.7, 0.8, 0.8) \end{aligned} \right\} \end{aligned}$$

Solving the nonlinear programming model (8) yields an optimal solution of $\theta_1^* = 0.5668$, $\theta_2^* = 0.4417$. By (9)–(12), an optimal group GLTS is obtained as

$$\tilde{S}_2^* = \left\{ \begin{aligned} &\langle s_{-3}, (0.2100, 0.2100, 0.2925) \rangle, \langle s_{-2}, (0.2100, 0.2925, 0.3913) \rangle, \\ &\langle s_{-1}, (0.2925, 0.3913, 0.5000) \rangle, \langle s_0, (0.3913, 0.5000, 0.6380) \rangle, \\ &\langle s_1, (0.5000, 0.6380, 0.7565) \rangle, \langle s_2, (0.6380, 0.7565, 0.8456) \rangle, \\ &\langle s_3, (0.7565, 0.8456, 0.8456) \rangle \end{aligned} \right\}$$

According to (14) and (15), the decision matrices R_k ($k = 1, 2, 3$) are converted to triangular fuzzy decision matrices \tilde{D}_k ($k = 1, 2, 3$), which are shown in Tables 4–6, respectively. As per (17), a group triangular fuzzy decision matrix \tilde{G} is determined as listed in Table 7.

Assume further that the three tourists provide their incomplete attribute weight information as

$$\Omega = \{w_3 - w_1 \geq w_2 - w_4, w_1 \geq 0.8w_4, 0.1 \leq w_3 \leq 0.2, 0.15 \leq w_5 \leq 0.25\}$$

Thus, based on (21), a linear program is established as

$$\begin{aligned} \max J &= 0.5813w_1 + 0.71w_2 + 0.6214w_3 + 0.5666w_4 + 0.6594w_5 \\ \text{s.t.} &\begin{cases} 0 \leq w_1 \leq 1, 0 \leq w_2 \leq 1, 0 \leq w_3 \leq 1, 0 \leq w_4 \leq 1, 0 \leq w_5 \leq 1, \\ \sum_{j=1}^5 w_j = 1, \\ w_3 - w_1 \geq w_2 - w_4, w_1 \geq 0.8w_4, 0.1 \leq w_3 \leq 0.2, 0.15 \leq w_5 \leq 0.25. \end{cases} \end{aligned} \tag{23}$$

Solving (23) yields an optimal attribute weight vector as $w = (0.1400, 0.2350, 0.2, 0.1750, 0.2500)^T$.

As per (22), the optimal group overall evaluation values are determined as follows.

$$\begin{aligned} \tilde{d}_1^{(*G)} &= (0.5511, 0.6622, 0.7451), \tilde{d}_2^{(*G)} = (0.5396, 0.6397, 0.7162), \\ \tilde{d}_3^{(*G)} &= (0.5220, 0.6332, 0.7260), \tilde{d}_4^{(*G)} = (0.5114, 0.6319, 0.7377). \end{aligned}$$

Using (7), we obtain $S(\tilde{d}_1^{(*G)}) = 0.6451$, $S(\tilde{d}_2^{(*G)}) = 0.6338$, $S(\tilde{d}_3^{(*G)}) = 0.6286$ and $S(\tilde{d}_4^{(*G)}) = 0.6282$.

Since $S(\tilde{d}_1^{(*G)}) > S(\tilde{d}_2^{(*G)}) > S(\tilde{d}_3^{(*G)}) > S(\tilde{d}_4^{(*G)})$, the four low-carbon tourism destinations are ranked as $x_1 \succ x_2 \succ x_3 \succ x_4$ and thus, x_1 is the best low-carbon tourism destination.

Next, a study is made to compare the attribute weight vector and the ranking order obtained from the proposed model herein with the results derived from the 2-tuple linguistic based approaches by Wei [12,14] and Ju [16].

Wei [12] first converted individual linguistic-term-based decision matrices to 2-tuple linguistic decision matrices, which are then aggregated into a group decision matrix. Based on the TOPSIS method, Wei [12] developed an optimization model to obtain an optimal attribute weight vector. For this case study, using this optimization model yields the optimal attribute weight vector as $w = (0.3333, 0, 0.1, 0.4167, 0.15)^T$ and a ranking order of the four low-carbon tourism destinations is determined as $x_3 \succ x_4 \succ x_1 \succ x_2$. The results are shown in the second row in Table 8.

Wei [14] used the maximizing deviation method to establish an optimization model for determining an optimal attribute weight vector and employed grey relational analysis to obtain a ranking order of all alternatives. Using this maximizing deviation based model generates the optimal attribute weight vector as $w = (0.425, 0, 0.2, 0.225, 0.15)^T$ and thus, a ranking order of the four low-carbon tourism destinations is obtained as $x_3 \succ x_2 \succ x_4 \succ x_1$. The results are listed in the third row in Table 8.

Ju [16] aggregated individual 2-tuple linguistic decision matrices into a group decision matrix whose symbolic translation values belong to $\left[-\frac{1}{2(\tau_1+\tau_2)}, \frac{1}{2(\tau_1+\tau_2)}\right)$ and constructed a TOPSIS-based optimization model to obtain an optimal attribute weight vector. By using Ju’s approach [16], we obtain the optimal attribute weight vector as $w = (0.425, 0, 0.1, 0.325, 0.15)^T$ and the ranking order of the four low-carbon tourism destinations as $x_3 \succ x_4 \succ x_1 \succ x_2$, which are shown in the penultimate row in Table 8.

Table 4. Triangular fuzzy decision matrix $\tilde{D}_1 = (\tilde{d}_{ij1})_{4 \times 5}$.

Alternative	a_1	a_2	a_3	a_4	a_5
x_1	(0.2100, 0.2925, 0.3913)	(0.6380, 0.7565, 0.8456)	(0.7565, 0.8456, 0.8456)	(0.3913, 0.5000, 0.6380)	(0.5000, 0.6380, 0.7565)
x_2	(0.2925, 0.3913, 0.5000)	(0.3913, 0.5000, 0.6380)	(0.6380, 0.7565, 0.8456)	(0.2100, 0.2925, 0.3913)	(0.6380, 0.7565, 0.8456)
x_3	(0.7565, 0.8456, 0.8456)	(0.6380, 0.7565, 0.8456)	(0.2925, 0.3913, 0.5000)	(0.2925, 0.3913, 0.5000)	(0.3913, 0.5000, 0.6380)
x_4	(0.5000, 0.6380, 0.7565)	(0.5000, 0.6380, 0.7565)	(0.3913, 0.5000, 0.6380)	(0.6380, 0.7565, 0.8456)	(0.3913, 0.5000, 0.6380)

Table 5. Triangular fuzzy decision matrix $\tilde{D}_2 = (\tilde{d}_{ij2})_{4 \times 5}$.

Alternative	a_1	a_2	a_3	a_4	a_5
x_1	(0.2925, 0.3913, 0.5000)	(0.7565, 0.8456, 0.8456)	(0.5000, 0.6380, 0.7565)	(0.6380, 0.7565, 0.8456)	(0.6380, 0.7565, 0.8456)
x_2	(0.5000, 0.6380, 0.7565)	(0.7565, 0.8456, 0.8456)	(0.7565, 0.8456, 0.8456)	(0.2100, 0.2100, 0.2925)	(0.7565, 0.8456, 0.8456)
x_3	(0.6380, 0.7565, 0.8456)	(0.7565, 0.8456, 0.8456)	(0.3913, 0.5000, 0.6380)	(0.6380, 0.7565, 0.8456)	(0.5000, 0.6380, 0.7565)
x_4	(0.5000, 0.6380, 0.7565)	(0.6380, 0.7565, 0.8456)	(0.2925, 0.3913, 0.5000)	(0.5000, 0.6380, 0.7565)	(0.5000, 0.6380, 0.7565)

Table 6. Triangular fuzzy decision matrix $\tilde{D}_3 = (\tilde{d}_{ij3})_{4 \times 5}$.

Alternative	a_1	a_2	a_3	a_4	a_5
x_1	(0.3913, 0.5000, 0.6380)	(0.2925, 0.3913, 0.5000)	(0.6380, 0.7565, 0.8456)	(0.5000, 0.6380, 0.7565)	(0.7565, 0.8456, 0.8456)
x_2	(0.6380, 0.7565, 0.8456)	(0.5000, 0.6380, 0.7565)	(0.5000, 0.6380, 0.7565)	(0.3913, 0.5000, 0.6380)	(0.6380, 0.7565, 0.8456)
x_3	(0.6380, 0.7565, 0.8456)	(0.5000, 0.6380, 0.7565)	(0.6380, 0.7565, 0.8456)	(0.6380, 0.7565, 0.8456)	(0.2925, 0.3913, 0.5000)
x_4	(0.3913, 0.5000, 0.6380)	(0.7565, 0.8456, 0.8456)	(0.5000, 0.6380, 0.7565)	(0.7565, 0.8456, 0.8456)	(0.6380, 0.7565, 0.8456)

Table 7. Group triangular fuzzy decision matrix $\tilde{G} = (\tilde{g}_{ij})_{4 \times 5} = ((g_{ij}^L, g_{ij}^M, g_{ij}^U))_{4 \times 5}$.

Alternative	A ₁	A ₂	A ₃	A ₄	A ₅
X ₁	(0.2793, 0.3735, 0.4841)	(0.6163, 0.7191, 0.7765)	(0.6302, 0.7447, 0.8100)	(0.5117, 0.6302, 0.7447)	(0.6065, 0.7269, 0.8100)
X ₂	(0.4446, 0.5630, 0.6717)	(0.5591, 0.6658, 0.7447)	(0.6578, 0.7684, 0.8278)	(0.2463, 0.3010, 0.4011)	(0.6854, 0.7921, 0.8456)
X ₃	(0.6854, 0.7921, 0.8456)	(0.6578, 0.7684, 0.8278)	(0.4011, 0.5078, 0.6243)	(0.4998, 0.6104, 0.7074)	(0.4150, 0.5335, 0.6578)
X ₄	(0.4783, 0.6104, 0.7328)	(0.6065, 0.7269, 0.8100)	(0.3735, 0.4841, 0.6065)	(0.6065, 0.7269, 0.8100)	(0.4841, 0.6065, 0.7269)

Table 8. A comparative study for attribute weight vectors and ranking results obtained from different models.

Model	Ref.	Attribute Weight Vector	Ranking Result
(M-3) and (20)	Wei [12]	$w = (0.3333, 0, 0.1, 0.4167, 0.15)^T$	$x_3 \succ x_4 \succ x_1 \succ x_2$
(M-2) and (11)–(19)	Wei [14]	$w = (0.425, 0, 0.2, 0.225, 0.15)^T$	$x_3 \succ x_2 \succ x_4 \succ x_1$
(M-5) and (8)	Ju [16]	$w = (0.425, 0, 0.1, 0.325, 0.15)^T$	$x_3 \succ x_4 \succ x_1 \succ x_2$
(21) and (22)	This paper	$w = (0.14, 0.235, 0.2, 0.175, 0.25)^T$	$x_1 \succ x_2 \succ x_3 \succ x_4$

Table 8 reveals that the ranking order obtained by the proposed model in this paper differs from the results derived by the 2-tuple linguistic based approaches in [12,14,16]. This difference is mainly due to the fact that the 2-tuple linguistic based approaches [12,14,16] adopt symbolic translation models to obtain a group decision matrix and do not consider decision makers' risk preferences for semantic scales of linguistic terms. As a result of this treatment, the importance weight of attribute a_2 is determined to be 0. In other words, when the linguistic MAGDM methods [12,14,16] are applied in solving this low-carbon tourism destination selection problem, the evaluation criterion a_2 is excluded from the consideration. It can be seen from Table 8 that by our proposed model, a_2 is determined to be a pivotal criterion for the low-carbon tourism destination selection.

6. Conclusions

In this paper, a nonlinear programming model has been established to obtain an optimal group GLTS based on individual risk preferences. An aggregation method has been presented to fuse individual linguistic-term-based evaluation values into a group evaluation with triangular fuzzy information. By maximizing the score of the group overall evaluation value for each alternative, a multi-objective optimization model has been devised and converted into a linear program for determining an optimal attribute weight vector. An approach has been developed for linguistic MAGDM with risk preferences and incomplete weight information. A low-carbon tourism destination selection case study has been provided to demonstrate the use of the proposed group decision-making model.

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