Heliyon 8 (2022) e11036

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Data-driven reduced order model and simplicial homology global optimization for reliability analysis and application

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ARTICLE INFO

Keywords:
Geomechanics
Reduced-order model
First-order reliability method
Monte Carlo simulation
Uncertainty

ABSTRACT

A novel framework of reliability analysis was developed in this study to consider the uncertainty of geomaterials and geological conditions by combining the reduced-order model (ROM), reliability analysis, and numerical model. The reliability method was used to determine the reliability index using the simplicial homology global optimization (SHGO) based on the ROM. The developed method was verified and illustrated using three numerical examples and a simple slope. The limit state curve in all three numerical examples was in excellent agreement with the actual curve. The reliability index and failure probability were also in excellent agreement with those of the actual limit state function using the first-order reliability method (FORM) and Monte Carlo simulation, respectively, indicating that the ROM method can present the limit state function well. The results showed that the developed method is feasible and effective for reliability analysis of geotechnical and geological engineering problems with a complex, nonlinear, and implicit limit state function. Furthermore, the developed method is effective, efficient, and accurate for reliability analysis. It provides an excellent way to approximate the limit state function to avoid the time-consuming numerical model in a practical engineering system.

1. Introduction

Uncertainty is one of the most important factors affecting design, stability analysis, and the failure of the engineering system. The probabilistic approach to uncertainty analysis is one of the most reliable and widely used approaches in engineering systems (Panthi and Nilsen, 2007). In the probabilistic approach, the reliability analysis method has become a significant research field in past decades. It has broadly been used to measure the safety of the engineering system in uncertain conditions. Reliability analysis aims to compute the reliability index and estimate the failure probability of the engineering system. Geomechanics and geotechnical engineering are more complex than other engineering systems because geomaterials, such as rock and soil, are the natural geology media formed by the geological processes. Uncertainty is the intrinsic characteristic of geomaterials. Reliability analysis has attracted more and more attention in geomechanics and geotechnical engineering in the last decades (Hoek, 1998; Deng et al., 2005; Li and Low, 2010; Mollon et al., 2009; Tsompanakis et al., 2010; Lv and Low, 2011; Zhao et al., 2014; Li et al., 2015a,b; Zhang and Goh, 2018; Leon and Garduno, 2020; Zhao, 2022).

In order to calculate the reliability index, various reliability analyses have been developed for the engineering system. In the early stage of reliability analysis, moment-based methods, such as the first-order reliability method (FORM) (Ji et al., 2018) and the high-order moment method (Zhao and Lu, 2007), were developed to estimate the reliability index. Recently, a hybrid iterative conjugate approach was developed for fuzzy reliability analysis of stiffened panels by combining the first-order reliability method and adaptive dynamical harmony search optimization (Zhu et al., 2020a). A simpler and more efficient approach was developed to evaluate the most probable point using the cumulative distribution function of basic random variables for reliability problems with non-normal random variables (Zhu et al., 2020b). These methods can obtain the reliability of the engineering system with less computing cost, but in these methods, derivative information of the limit state function must be known. In this regard, the major challenge is that there is a large error in the complex and nonlinear limit state function. This is because it is challenging

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https://doi.org/10.1016/j.heliyon.2022.e11036
Received 15 February 2022; Received in revised form 29 April 2022; Accepted 6 October 2022

to get the derivative information special to the implicit limit state function such as that for geotechnical and geological engineering problems. Simulation-based methods, such as Monte Carlo Simulation (MCS) (Ditlevsen and Madsen, 1996), importance sampling (IS) (Au and Beck, 1999), and subset simulation (SS) (Hsu and Ching, 2010), are broadly utilized to calculate the failure probability of an engineering system because of their great accuracy and their avoidance to compute the derivative information. Luo et al. (2022) developed a hybrid enhanced MCS approach to accurately approximate failure probability with high-efficiency computations based on the machine learning method. However, the disadvantage of simulation-based methods is that they generally need a high computational cost. Thus, it is challenging to implement a large-scale practical project and limits the application of these methods in engineering. The surrogate model received more attention in practical engineering applications to improve the efficiency and accuracy of reliability analyses. There are various surrogate models, such as the polynomial response surface method (RSM) (Cheng and Lu, 2018), Artificial Neural Network (ANN) (Karunanithi, 1992), Support Vector Machine (SVM) (Bourinet et al., 2011), and Polynomial Chaos Expansion (PCE) (Lophaven et al., 2002). In geotechnical and geological engineering, it is not easy to obtain an analytical solution, and therefore, the numerical solution is widely adopted. The surrogate model provides an excellent way to approximate the solution of geotechnical and geological engineering problems using a numerical model (Li et al., 2016).

The response surface method is commonly used to construct a surrogate model in reliability analysis. Various response surface methods have been developed for the reliability analysis of geotechnical and geological engineering problems. Quadratic polynomials without cross-terms were adopted in the reliability analysis of geotechnical engineering problems based on the RSM (Xu and Low, 2006; Zhang et al., 2011; Ji and Low, 2012; Tan et al., 2013; Li et al., 2015a,b). Lv and Low (2011) performed the probabilistic analysis of underground rock excavation using the RSM and the second-order reliability method (SORM) (Lv and Low, 2011). The Kriging-based response surface was adopted to simulate performance functions. This response surface was successfully applied to solve several geotechnical reliability problems (Zhang et al., 2011). The high dimensional model representation (HDMR) was used to approximate the limit state function to establish the RSM for geotechnical and geological engineering applications (Chowdhury and Rao, 2010). However, it is difficult to approximate the complex nonlinear function using polynomial RSM in practical geotechnical and geological engineering problems. With the development of computational intelligence, artificial intelligence was employed to approximate the limit state function for reliability analysis of geotechnical and geological engineering problems to enhance the efficiency and performance of the RSM. Deng et al. (2005) proposed an ANN-based second-order reliability method and an ANN-based Monte Carlo simulation method (Deng et al., 2005). Cho developed an ANN-based RSM for slope reliability analysis (Cho, 2009). Zhao proposed the reliability analysis of the tunnel by combing the least square support vector machine and FORM (Zhao et al., 2014). An artificial bee colony algorithm was adopted to determine the SVM model to enhance the performance of SVM and then the SVM model was utilized to build the RSM for reliability analysis of slope (Kang and Li, 2016). To reduce the number of evaluations of the actual performance function, Zhao et al. (2017) developed an adaptive sampling method and combined it with the least square support vector machine to build the RSM to improve the efficiency of reliability analysis for geotechnical and geological engineering applications. Tan et al. (2011) built the ANN-based RSM and SVM-based RSM for geotechnical and geological engineering applications, respectively, and compared their similarities and differences. However, artificial intelligence (including ANN, SVM, etc.) has some inherent drawbacks, such as their slow convergence, a less generalized performance, arriving at a local minimum, and over-fitting problems. These limits will be detrimental to the engineering application of artificial intelligence, especially in the case of practical, complex geotechnical and geological engineering.

Determining the surrogate model is an important step in the reliability analysis of geotechnical and geological engineering problems. The development of the surrogate model enhanced the efficiency of the reliability analysis of large-scale practical geotechnical and geological engineering problems. Li et al. (2022) reviewed the recent reliability analysis advances of engineering application in aeroengine rotor systems and highlighted the importance of surrogate models in practical engineering applications. However, the traditional surrogate model, such as RSM, does not reflect the physical mechanism of geotechnical and geological engineering problems. Meanwhile, not only the reliability index and probability of failure have to be determined, but also the fields of stress and deformation are important for the design and construction in geotechnical and geological engineering. The traditional surrogate model only obtains information about the discrete stress or deformation on several key points. Therefore, it is difficult to obtain the total stress and deformation fields using this classical approach. The development of the data sciences provides a good way to reveal the mechanism behind the data (LeCun et al., 2015). Data sciences have been successfully applied in the field of medicine (Veer and Bernards, 2008), energy (Severson et al., 2019), biology (Alipanahi et al., 2015; Huys et al., 2016), and geoscience (Markus et al., 2019). The physics-based reduced-order model containing some information about geotechnical and geological engineering challenges was developed recently (Zhao and Chen, 2021).

The proper orthogonal decomposition is a popular physics-based reduced-order model that has been successfully used in many engineering fields (Fic et al., 2006; Cizmas et al., 2008; Freno and Cizmas, 2014). In this study, simplicial homology global optimization (SHGO) was chosen as an optimal method and was employed to determine the reliability index based on the FORM. The proper orthogonal decomposition was adopted to construct the reduced-order model (ROM) to generate the surrogate model. Then, a novel reliability analysis framework was developed by combining the reliability analysis method, SHGO, and ROM. The developed framework make full of the merits of SHGO and ROM and significantly enhances the efficiency, accuracy, and feasibility of reliability analysis. The snapshots (samples) were generated based on numerical simulation and Latin hypercube sampling, which was employed to construct a set of design variables. The proper orthogonal decomposition was adopted to determine the ROM. Then, reliability analysis was implemented according to the ROM. The remainder of this paper is organized as follows: First, the reliability analysis method is introduced in detail in Section 2. Second, the idea and theory of the ROM method are described briefly in Section 3. Then the proposed reliability analysis framework and procedure are presented in Section 4. In Section 5, the proposed method is verified using three numerical examples. Then, the proposed method is applied to geotechnical and geological engineering using two slopes in Section 6. Finally, some conclusions are drawn in Section 7.

2. Reliability analysis method

2.1. First order second moment (FOSM)

First-order second moment (FOSM) is a reliability analysis method based on the moment of the limit state function. The limit state function of geotechnical and geological engineering problems may be established using Eq. (1) as follows (Zhao, 2008):

$$Z = g\left(X_1, X_1, \ldots, X_n\right) = F_s\left(X_1, X_1, \ldots, X_n\right) - F_{lim} \tag{1}$$

where n is the number of random variables; X_i $(i = 1, 2, \ldots, n)$ are the random variables in the geotechnical and geological engineering problems; $g(X_1, X_2, X_3, \ldots, X_n)$ is the limit state function; Z > 0 indicates that the engineering structure is safe, Z < 0 indicates that it is failed, and Z = 0 means that boundary is fluctuating between safety and unsafety. $F_s(X_1, X_2, X_3, \ldots, X_n)$ is the response to geotechnical and geological engineering problems such as safety factor, displacement, strain, rock bolt bearing capacity, etc. F_{lim} is the limit value of geotechnical and geological engineering safety. According to the FOSM method, if the limit state function is linear and the random variables are normally distributed, the reliability index β can be calculated by the following equation (Eq. (2)):

$$\beta = \frac{\mu_z}{\sigma_z} \tag{2}$$

where μ_z and σ_z are the mean value and the standard deviation of the limit state function Z, respectively. To the nonlinear limit state function, it can be approximated by a first-order polynomial using Taylor's series expansion with the maximum probabilistic point. An iteration procedure could be used to determine the reliability index based on Taylor's series expansion. The detailed procedure can refer to the relevant literature (Wu et al., 2013).

In FOSM, the most important step is to calculate the statistical moments of the limit state function Z (i.e., the mean value μ_z and the standard deviation σ_z). However, it is difficult to obtain them in practical geotechnical and geological engineering problems because the response of surrounding rock mass and support structure will be calculated by a numerical model and cannot be represented as an explicit function of the random variables for geotechnical and geological engineering problems. To overcome this issue, the RSM can be used to estimate the response of geotechnical and geological structures by replacing the numerical model, and then the limit state function is determined. The statistics moment is calculated based on the RSM. Once the reliability index was obtained, the probability of failure can be obtained based on Eq. (3):

$$p_f \approx 1 - \Phi(\beta) \tag{3}$$

where Φ is the cumulative distribution function of the standard normal variable.

2.2. FORM

The first-order reliability method (FORM) is a popular reliability analysis method that is broadly adopted in geotechnical and geological engineering. Low (2004) developed a practical FORM procedure based on the Hasofer–Lind index for geotechnical and geological engineering applications (Low, 2004). In their method, the computation of the reliability index was transferred to a constrained optimization based on the perspective of an expanding ellipsoid in the original space of the basic random variables. Meanwhile, they proposed a new efficient algorithm for the FORM by varying the dimensionless number n_i to avoid the computation of equivalent normal means and equivalent normal standard deviations in Eq. (4) (Low and Tang, 2007).

$$\beta = \min_{X \in F_z} \sqrt{[\mathbf{n}]^T [R]^{-1} [\mathbf{n}]} \tag{4}$$

where R is the correlation matrix, X is a vector representing the set of random variables x_i, F_z is the failure domain, and $\mathbf{n}$ is a column vector of n_i. When the value of n_i changes during strained optimization, the corresponding value of random variables x_i is calculated as follows (Eq. (5)):

$$x_i = F^{-1}\left[\phi(n_i)\right] \tag{5}$$

Where $F^{-1}()$ is the inverse of the original non-normal cumulative distribution function, $\phi()$ is the standard normal cumulative distribution function.

In this study, the reliability index was determined by solving the optimization problem in Eq. (4). SHGO was chosen as an optimization algorithm based on the Python Scipy optimization package. The procedure of determining the reliability index is explained in Python.

2.3. Monte Carlo Simulation (MCS)

MCS is a simple and directed reliability analysis method through computing the failure probability based on a large volume of repeated simulations of geotechnical and geological engineering problems. In this research, the ROM method was employed to replace the numerical model in MCS to avoid a large volume of repeated numerical simulation.

A reliability problem is usually formulated by a limit state function (Eq. (1)). In order to calculate the failure probability, an adequate number of n independent random samples are produced based on the statistical feature for each random variable. The Monte Carlo method allows the determination of an estimate of the probability of failure according to the following Eq. (6) (Li et al., 2013):

$$N_f = \sum_{i=1}^{N} I(X_1, X_2, \ldots, X_n) \tag{6}$$

where N is the total times of MCS computation of a geotechnical and geological engineering problem, N_f is the failure time in N MCS computations, and $I(X_1; X_2; \ldots; X_n)$ is a function defined in the following Eq. (7):

$$I\left(X_1, X_2, \ldots, X_n\right) = \begin{cases} 1 & if\ g(X_1, X_2, \ldots, X_n) \le 0 \\ 0 & if\ g\left(X_1, X_2, \ldots, X_n\right) > 0 \end{cases} \tag{7}$$

According to Eq. (6), N independent sets of values $X_1, X_2, \ldots, X_n$ are obtained based on the statistical feature for each random variable, and the limit state function is calculated for each sample. The failure probability of geotechnical and geological engineering problems can be calculated based on MCS using the following equation (Eq. (8)):

$$p_f = \frac{N_f}{N} \tag{8}$$

2.4. Response surface

In a complex engineering system such as geotechnical and geological engineering systems, the analytical solution of the response is generally unavailable. Although reliability analysis of geotechnical and geological engineering systems can be carried out by FORM and Monte Carlo simulations, the large number of required numerical simulations results in prohibitively high computational costs. Using polynomial approximations of actual limit states function in the reliability analysis reduces the number of required analyses. Such approximations are referred to as response surfaces. The response surface method (RSM) is a simple mathematical form or regression model that avoids lengthy computations in the probabilistic analysis of complex systems. Furthermore, the RSM is an important technique to find out how response variables are related to variations in the experimental conditions.

In geotechnical and geological engineering, lots of variables influence the response of the surrounding rock mass and support structure. Suppose the response variable of engineering system y depends on the input variables (x_1, x_2, x_3,...). Experiments are conducted with design variables (x_1, x_2, x_3,...) for a sufficient number of times to define the response surface to the level of accuracy desired. The basic procedure of the RSM is to approximate response by an nth order polynomial with undetermined coefficients. The RSM involves the generation of a polynomial equation using regression analysis and an approximate linear or non-linear functional relationship between dependent output y and input variables (x_1, x_2, x_3,...) as follows (Eq. (9)):

$$y = f\left(x_1, x_2, x_3, \dots\right) + e \tag{9}$$

The following model (Eq. (10)) is a second-order regression model without cross-terms containing two input variables x_1, x_2.

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1^2 + a_4 x_2^2 + e \tag{10}$$

where a_0, a_1, a_2, … are regression coefficients and 'e' represents the error involved in neglecting other sources of uncertainties. To obtain these constants in the RSM, $(2n+1)$ experiments have to be implemented, n is the number of input variables. Recently, some intelligent regression models, including ANN and SVM, were used to obtain the response surface. To enhance the efficiency of the RSM, Zhao et al. (2017) developed an adaptive sample constructing method for reliability analysis. In this research, the ROM was adopted to approximate the response of geotechnical and geological engineering systems based on the RSM.

3. Reduced-order model

3.1. Constructing the ROM

ROM are simplifications of high-fidelity, complex system models and captures the behavior of the complex system. So, the researcher can quickly study a system's dominant effects using minimal computational costs. The ROM method was used to generate a low-order surrogate model for the numerical model of geotechnical and geological engineering systems based on the proper orthogonal decomposition. The proper orthogonal decomposition originates from the field of turbulence. It is a technique used to decompose a matrix and characterize it by its principal components, called proper orthogonal modes of the function. According to the proper orthogonal decomposition algorithm, we can obtain the following equation for any x_i, $i = 1, 2, \dots, I$, and θ_j, $\mathrm{j} = 1, 2, \dots, J$ (Audouze et al., 2009).

$$\tilde{u}^h\left(x_i, \theta_j\right) = \sum_{k=1}^{K} \beta_k(\theta_j)\varphi^k(x_i) + \tilde{g}(x_i, \theta_j) \tag{11}$$

The above equations (Eq. (11)) can be written as follows (Eq. (12)):

$$\tilde{u}^h = \varphi\beta + \tilde{g} \tag{12}$$

where $\tilde{u}^h$ is the solution of field variables for geotechnical and geological engineering systems, x_i and θ_j are the design and parameter variables of a numerical model, φ and β need to be determined, and $\tilde{g}(x, \theta)$ is an extension of the boundary conditions in the whole domain based on Eq. (13).

$$\tilde{g}\,(x, \theta) = \begin{cases} g\,(x, \theta) & on\ \partial\Omega \\ 0 & elsewhere \end{cases} \tag{13}$$

In order to determine the unknown coefficient φ, Latin hypercube sampling (LHS) was utilized to generate the set of design variables θ_j, $j = 1, 2, \dots, J$. And then, a set of discrete solutions (snapshots) of the numerical model, $w_j = \tilde{u}^h\left(\theta_j\right) - \tilde{g}\left(\theta_j\right)$, $j = 1, 2, \dots, J$, were obtained based on numerical models such as finite element. Let us denote the spatial Gram matrix by $\mathbf{M}^{x}$ using Eq. (14):

$$M_{ij}^{x} = \left(w_i \cdot w_j\right),\ i, j = 1, 2, \dots, J \tag{14}$$

where $\left(w_i \cdot w_j\right)$ is the scalar product between w_i and w_j.

The positive eigenvalues of $\mathbf{M}^{x}$ are arranged in descending order according to Eq. (15).

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_J \geq 0 \tag{15}$$

The K first eigenfunctions $\varphi^k(x)$, $k = 1, 2, ..K$ associated with the K first eigenvalues provide the orthogonal principal direction of snapshots. If $r^k = (r_j^k)_{j=i,i,\dots J}$ is the kth eigenvector of $\mathbf{M}^{x}$, then its dual kth eigenfunctions $\varphi^k(x)$ are obtained by Eq. (16):

$$\varphi^k(x) = \sum_{j=1}^{K} r_j^k w_j(x) \tag{16}$$

where K is the dimension of the proper orthogonal decomposition basis and can be determined in Eq. (17):

$$\frac{\sum_{i=1}^{K}\lambda_i}{\sum_{i=1}^{J}\lambda_i} > k \tag{17}$$

where k is the user-specified tolerance, in this study, k was 0.9999.

The unknown coefficient β can be obtained by solving the following penalized minimization problem based on Eq. (18):

$$\min_{\beta_j \in R^K} \left\| u^{h,j} - \varphi\beta_j - \tilde{g}_j \right\|^2 + \mu \left\| \beta_j \right\|^2 \tag{18}$$

β_j can be acquired by solving the following normal equation (Eq. (19)):

$$\left(\varphi^T\varphi + \mu I_K\right)\beta_j = \varphi^T\left(u^{h,j} - \tilde{g}_j\right),\ j = 1, 2, \ldots, J \tag{19}$$

where μ is a small regularization parameter.

3.2. Predicting the field variables

In order to determine the field variables for unknown design variables θ and space variables x, the coefficient $\beta_k(\theta)$ is expanded using the radial basis function (RBF) as follows:

$$\beta_k(\theta) = \sum_{j=1}^{J}\alpha_{jk}\psi\left(\frac{\left|\theta - \theta_j\right|}{\sigma}\right) \tag{20}$$

For any $\theta_{j'}$, $j' = 1, 2, \ldots, J$, the following equation (Eq. (21)) will be obtained based on Eq. (20):

$$\sum_{j=1}^{J}\alpha_{jk}\psi\left(\frac{\left|\theta_{j'} - \theta_j\right|}{\sigma}\right) = \beta_{kj'} \tag{21}$$

where $\beta_{kj'}$ are obtained using Eq. (19). The above equations can be written in the following Eq. (22):

$$A\alpha_k = \beta_k \tag{22}$$

The unknown coefficient α_k can be obtained by solving the following Eq. (23):

$$\left(A^T A + \mu I_J\right)\alpha_k = A^T\beta_k,\ k = 1, 2, \ldots, K \tag{23}$$

3.3. Procedure of the ROM method

This study developed the ROM based on the numerical model and the proper orthogonal decomposition. LHS was used to generate the set of design variables for the snapshot. Then the numerical model was adopted to obtain the solution corresponding to each design variable in the above set. Based on the above snapshots, the proper orthogonal decomposition basis vector and its coefficient were determined based on the proper orthogonal decomposition algorithm for geotechnical and geological engineering systems. In order to obtain the unknown field of the new design variable, RBF functions were employed to expand the coefficient of the proper orthogonal decomposition basis and then obtain the coefficients of the orthogonal decomposition ROM. The unknown field variables corresponding to the new design were calculated based on the ROM. Fig. 1 shows the main flowchart and steps of the ROM method. In what follows, the procedure of the ROM is presented in detail.

Step 1: Collect the geotechnical and geological engineering data, including in-situ stress, geomaterial mechanical parameters, and boundary conditions.
Step 2: Build the numerical model (FEM) based on the above engineering information.
Step 3: Generate the set of design variables $\boldsymbol{\theta}$ for the numerical model using LHS.
Step 4: Compute the field variables w_i (displacement or stress field) at space domain $\mathbf{X}$ using a numerical model for each design variable. Collect all the field variables and obtain the snapshots.
Step 5: Construct the spatial Gram matrix $\mathbf{M}^x$ based on the above snapshots.
Step 6: Compute the eigenvalues λ and eigenvectors $\boldsymbol{r}$ based on the spatial Gram matrix.
Step 7: Determine the rank number K of $\mathbf{M}^x$ and K first eigenfunction vector $\boldsymbol{\varphi}$.
Step 8: Compute the undetermined coefficient β based on eigenfunction vector $\boldsymbol{\varphi}$ and snapshots.
Step 9: To find a new design variable θ, construct element ϕ based on the design variables $\boldsymbol{\theta}$ generated by LHS using the RBF function.
Step 10: Compute the interpolation matrix $\mathbf{A}$ of elements ϕ.
Step 11: Compute the vector of element $\boldsymbol{\alpha}$ using the penalized linear systems.
Step 12: Determine the coefficients $\beta(\theta)$ based on the RBF function.
Step 13: Predict the unknown field variables $\tilde{u}^h(\theta)$ based on coefficients $\beta(\theta)$ and eigenfunction vector $\boldsymbol{\varphi}$ using the ROM method.

4. Reliability analysis based on the ROM

In this study, the ROM was adopted to replace the numerical model to approximate the physical field of geotechnical and geological engineering problems. The statistical moment of the safety factor, displacement, and stress can be calculated based on the ROM in the reliability analysis. The reliability index and failure probability were obtained based on the reliability analysis method.

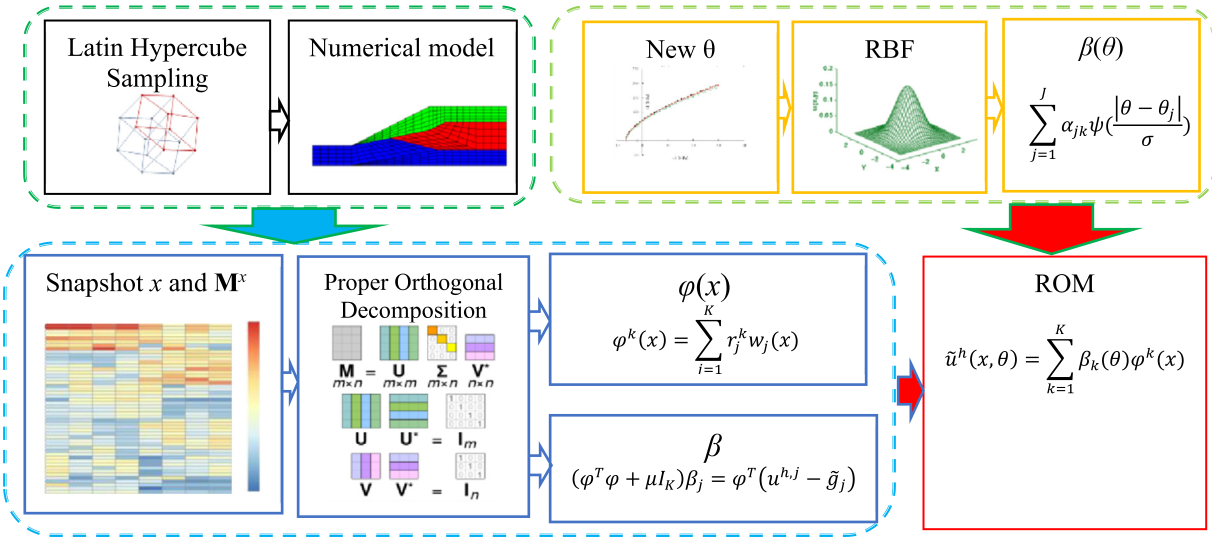


Fig. 1. The main part of the ROM method.

4.1. Simplicial homology global optimization (SHGO)

The simplicial homology global optimization (SHGO) algorithm is a general-purpose global optimization algorithm based on applications of simplicial integral homology and combinatorial topology. SHGO approximates the homology groups of a complex that is built on a hypersurface, which is homeomorphic to a complex on the objective function. The SHGO algorithms only make use of function evaluations without requiring the derivatives of objective functions. This makes them applicable to black-box global optimization problems, such as the optimal method of FORM in geotechnical and geological engineering. The algorithm itself consists of four steps: 1) generation of uniform sampling vertices in the search space, 2) construction of the directed simplicial complex by triangulation of the vertices, 3) construction of the minimizer pool using Sperner's lemma (Sperner, 1928), and 4) local minimization using the starting points. In this study, the Scipy optimization package of Python was chosen to solve the optimization method in Eq. (4). Endres et al. (2018) described the idea and algorithm briefly (Endres et al., 2018).

4.2. Response surface based on ROM

For geotechnical and geological engineering problems with different mechanical parameters, boundary conditions, and in-situ stresses, a numerical model can be used to calculate the response of engineering structures, including displacement, stress, strain, and safety factor. In geotechnical and geological engineering, it is difficult to obtain an analytical solution, and the numerical solution is widely used in practical engineering for design, stability analysis, and construction. In reliability analysis, the numerical model needs to be called hundreds and thousands of times for computing the reliability index. This procedure is costly and time-consuming and even is impossible to implement in a practical large-scale project. In this study, the ROM was adopted to replace the numerical model to approximate the limit state function in reliability analysis. The limit state function was obtained based on the ROM. In terms of the random variables, the ROM (Eq. (11)) was adopted to calculate the total physical field of geotechnical and geological engineering problems. The displacement, stress, and strain information at some key points were extracted from the calculated field by the ROM. The relationship between random variables and the response of geotechnical and geological engineering systems was presented by the ROM. It can be presented as follows (Eqs. (24)&(25)):

$$ROM(\boldsymbol{X}):\ \ R^N \rightarrow R \tag{24}$$

$$y = ROM(\boldsymbol{X}) \tag{25}$$

where $\boldsymbol{X} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N)$, $\boldsymbol{x}_i$ $(i = 1, 2, \ldots, N)$ is a vector of geomaterial mechanical parameters (such as Young's modulus, friction angle, or in-situ stress) and $\boldsymbol{y}$ is the response of the tunnel such as displacement, stress, and plastic zone. To obtain ROM($\boldsymbol{X}$), the ROM was implemented in Python 3.0 according to the procedure in Section 3.3.

4.3. Determination of the limit state function

The safety factor and displacement are selected as the stability index of geotechnical and geological engineering problems in the reliability analysis of geotechnical and geological engineering problems, respectively. The limit state function is essential for the reliability analysis. For the slope reliability analysis, the limit state function is expressed as follows (Eq. (26)):

$$Z = FOS\left(X_1, X_1, \ldots, X_n\right) - FOS_{lim} \tag{26}$$

where $FOS\left(X_1, X_1, \ldots, X_n\right)$ is the value of the safety factor for the slope, which is predicted by the ROM and FOS_{lim} is the permissible value of the safety factor for the slope (FOS_{lim} was 1.0 in this study).

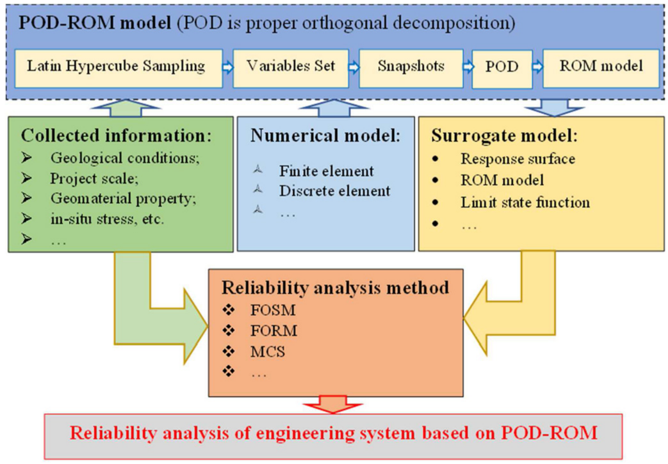


Fig. 2. The flowchart of the reliability analysis according to the ROM.

Table 1. Limit state functions and random variables used in the three numerical examples.

Example	Limit state function	Random variables				
		Random variable distribution	Mean value		standard deviation	
			x_1	x_2	x_1	x_2
1	$g = 0.01846154 - 74.76923x_1/x_2^3$	Normal	1000	250	200	37.5
2	$g = \exp[0.4(x_1+2)+6.2] - \exp(0.3x_2+5) - 200$	Normal	0	0	1	1
3	$g = 2.5 - 0.2357(x_1 - x_2) + 0.00463(x_1 + x_2 - 20)^4$	Normal	10	10	3	3

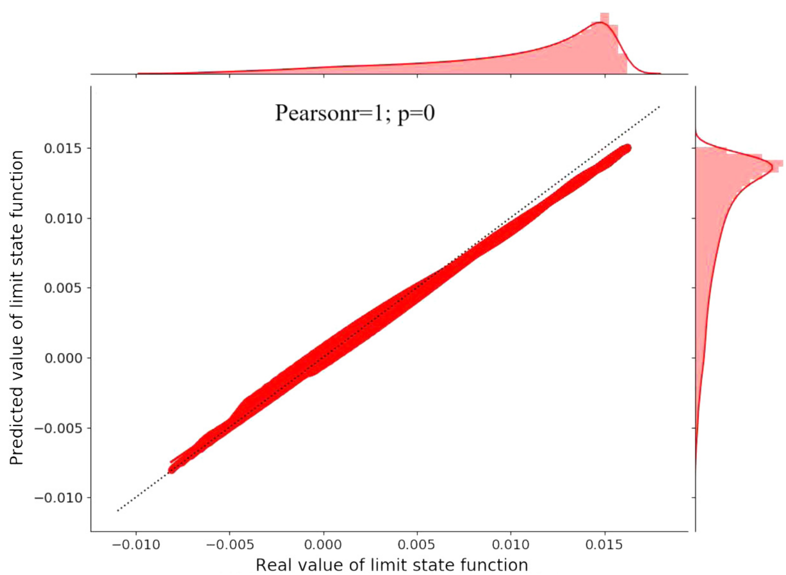


Fig. 3. The comparison between the real and predicted values of the limit state function for example 1.

4.4. Determination of the reliability index and failure probability

The reliability index and failure probability were calculated based on the ROM in different reliability analysis methods. In the FOSM approach, the statistical moment was calculated based on the ROM, and then Eq. (2) and Eq. (3) were used to determine the reliability index and failure probability. SHGO was utilized to solve the optimal problem (Eq. (4)) based on the ROM in the Python Scipy optimization package. According to the MCS method, the failure probability was determined by repeatedly calling the ROM.

4.5. Procedure of the reliability analysis

The ROM was adopted to replace the numerical model in the reliability analysis. First, the ROM was built by combining the numerical model and the ROM algorithm. Based on the ROM, the response of geotechnical and geological engineering problems, such as safety factors and displacement, was obtained. Then the reliability index and failure probability were calculated by combining the reliability analysis method and the ROM. The ROM was adopted to approximate the response of geotechnical and geological engineering systems instead of the numerical method. SHGO was chosen as the optimal method in the Python Scipy package. The procedure is shown in Fig. 2 and is elaborated in the following.

Step 1: Collect the engineering information, including geological conditions, project scale, rock mechanical parameters, and in-situ stress.

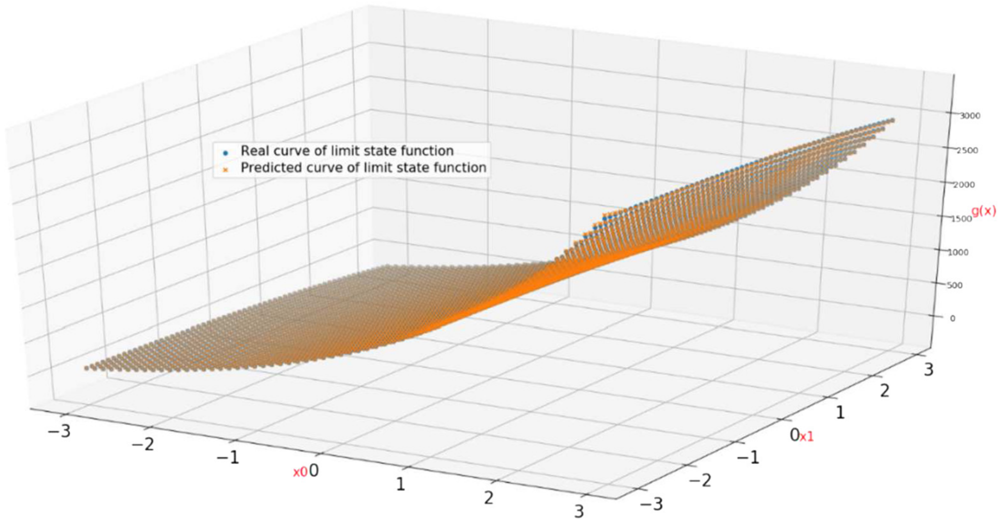


Fig. 4. The curve comparison of the limit state function for example 2.

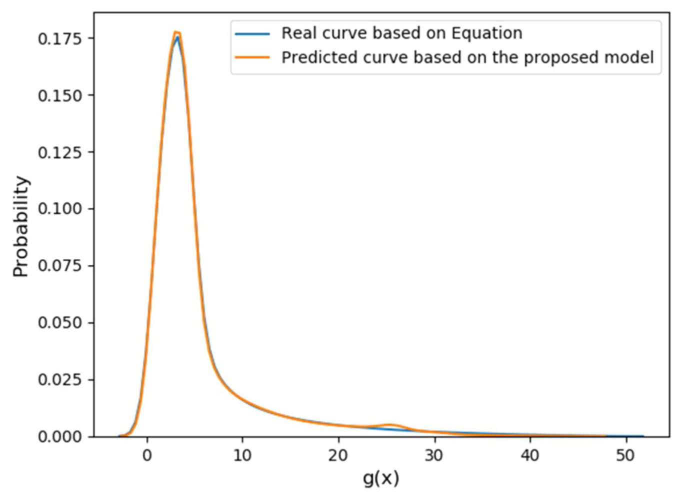


Fig. 5. The distribution comparison of the limit state function for example 3.

Step 2: Build a numerical model using continuous or discontinuous finite element/finite difference software, such as Phase 2 and Fast Lagrangian Analysis of Continua (FLAC) or the Discrete Element Method (DEM).

Step 3: According to the information collected in Step 1, determine the random variables and their statistical properties.

Step 4: Generate the snapshots and construct the ROM according to the procedure in Section 3.3.

Step 5: Extract the response information of the geotechnical and geological engineering system, such as stress, displacement, and safety factor, from the ROM.

Step 6: Calculate the reliability index and failure probability using the reliability analysis method.

Step 7: Evaluate the reliability of the geotechnical and geological engineering system.

5. Numerical examples

Based on Low and Tang's method, FORM was used to compute the reference reliability index to verify the proposed method (Low and Tang, 2007). In order to demonstrate the effectiveness of the proposed method, three classic examples from the literature were investigated (Table 1) (Zhao et al., 2017). All examples contained two independent and normally distributed random variables. The mean values and standard deviations of the random variables are presented in Table 1. The ROM was built based on the actual equation in Table 1 according to the procedure of ROM in Section 3.3. Fig. 3 shows the comparison of the limit state function between the predicted value by the ROM and the computed value by the real equation for Example 1. Fig. 4 shows the surface of the limit state function using the ROM and real equation, respectively, for Example 2. Fig. 5 exhibits the distribution of limit state function value using the ROM and real equation for Example 3. It is clear that the ROM could approximate the limit state function with great accuracy.

The reliability of the three numerical examples was calculated based on the ROM using FORM and MCS, respectively. The reliability index and failure probability are presented in Table 2. From Table 2, we can observe that the reliability index based on the ROM was in excellent agreement with that based on the actual equation in Table 1. The maximum relative error was less than 0.65% of the reliability index. The failure probability of examples 1 and 2 were also in excellent agreement with the results based on the actual equation in Table 1. However, in example 3, the failure probability base on the ROM was 0.35, far larger than the 0.3 obtained based on the actual equation. The relative error was about 17%. Fig. 6 displays the limit state function, limit state curve, and design points, which were determined by SHGO, based on the ROM and actual equation,

Table 2. Results of the three numerical examples.

	Reliability index by FORM			Failure probability by MCS		
	Equation	ROM	Relative error (%)	Equation	ROM	Relative error (%)
Example 1	2.33	2.32	−0.65	0.94	0.95	1.06
Example 2	2.71	2.71	0.06	0.37	0.37	0.00
Example 3	2.50	2.50	−0.03	0.3	0.35	16.67

Table 3. The statistical parameters of the random variables.

Random variable	Distribution	Mean	Standard deviation
c'	Normal	10	3
tan(φ')	Normal	0.5774	0.1732

Table 4. The results of the infinite slope frictional/cohesive soil.

Method	FORM			MCS	
	Excel Solver	Scipy SHGO	SHGO with ROM	Analytical solution	ROM
β	0.8743	0.8743	0.8763	-	–
Probability of failure (%)	-	-	-	19.08	17.75
Relation error (%)	-	0.00	0.23	-	−6.96

Table 5. The statistical parameters of the random variables.

Random variable	Distribution	Mean	Standard deviation
tanβ	Normal	0.3250	0.0325
tanφ	Normal	0.5770	0.1732
γ	Normal	18.0000	0.5000
u	Normal	12.0000	1.2000

Table 6. The results of the infinite slope frictional/cohesive soil.

Method	FORM			MCS	
	Excel Solver	Scipy SHGO	SHGO with ROM	Analytical solution	ROM
β	1.1014	1.1037	1.1542	-	–
Probability of failure (%)	-	-	-	13.53	11.93
Relation error (%)	-	0.20	4.79	-	−11.83

respectively. In examples 1 and 2, the design points based on the ROM were also in excellent agreement with those by the actual equation using SHGO and Excel Solver (Figs. 6a-b). In example 3, the design point was very close to the position obtained based on the actual equation as a reliability index, as shown in Table 2. There are some errors in example 3 because of the high nonlinearity of the limit state function (Fig. 6c). However, it can meet the requirement of the reliability analysis for geotechnical and geological engineering problems. From the three numerical examples, we can conclude that the reliability index can be calculated with great accuracy using the proposed method. The design point obtained using the ROM was very closer to the actual design point, indicating that in reliability analysis, it is feasible to replace the limit state function using the ROM.

6. Application

6.1. Infinite slope

The proposed method was applied to an infinite soil slope with an analytical solution (Griffiths et al., 2011). A typical slice of the infinite slope is shown in Fig. 7. The safety factor of the infinite slope can be calculated using the following equation (Eq. (27)):

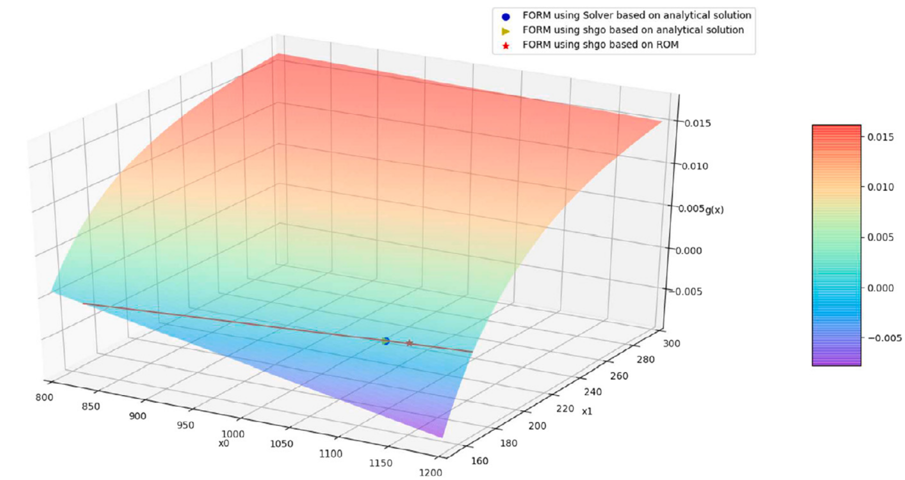
$$\mathrm{FOS} = \frac{\left(H\gamma cos^2\beta - u\right) tan\varphi' + c'}{H\gamma sin\beta cos\beta} \tag{27}$$

where FOS is the safety factor of the slope, H is the depth of the soil layer to the potential failure surface, β is the slope inclination, γ is the total unit weight of soil, u is the pore pressure at the base of the slice, φ' is the effective soil friction angle at the base of the slice, and c' is the effective cohesion at the base of the slice.

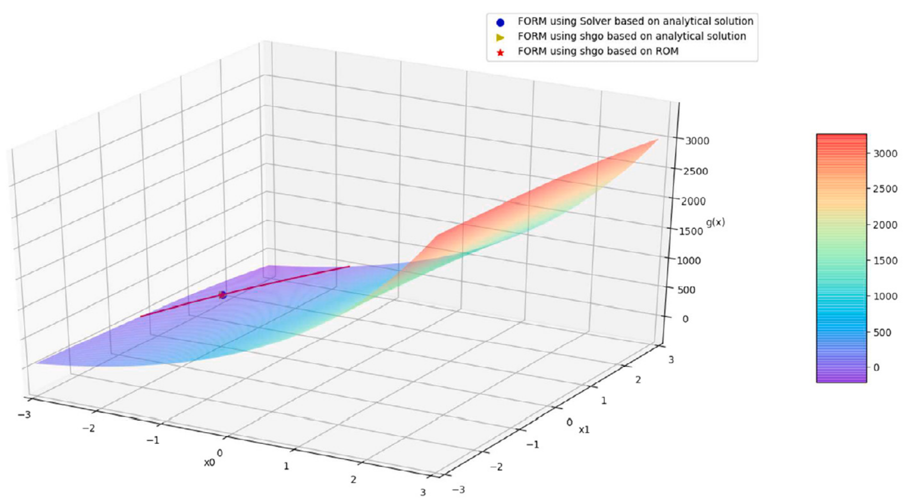
The limited equilibrium method was employed to compute the safety factor. The reliability index and failure probability were calculated using FORM, ROM response surface-based FORM, and MCS. The results were compared with the results of MCS. In the infinite slope, there were six influencing factors for the safety factor of the slope. To illustrate and verify the proposed method, two cases of slope stability were analyzed: 1) frictional/cohesive soil (two random variables) and 2) frictional soil with pore pressure (four random variables).

6.1.1. Frictional/cohesive soil

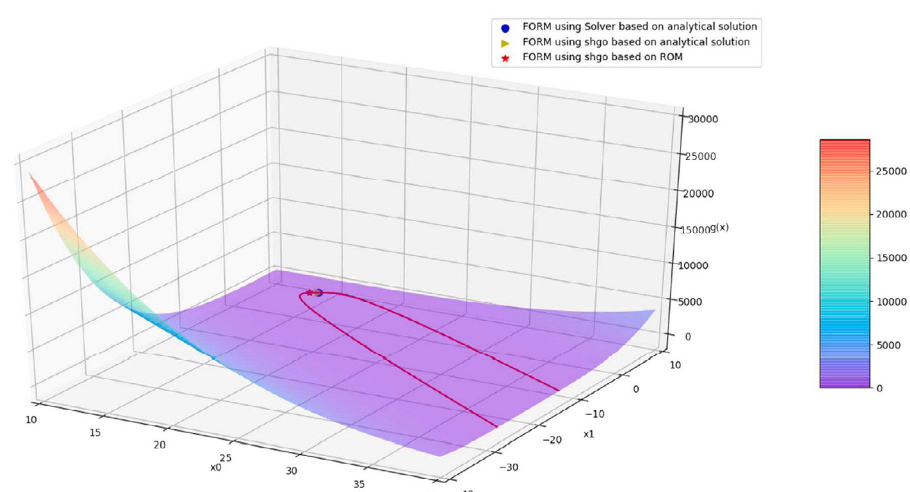
For frictional/cohesive soil, we considered an effective stress analysis of the infinite slope with shear strength parameters (c' and $\tan\varphi'$), while no pore pressure was taken into account. In this case, the safety factor of the slope was calculated using Eq. (28). Soil strength parameters (c$'$ and tanφ') were considered random variables with a normal distribution. The statistical parameters of the random variables are presented in Table 3. The other parameters were $H = 5$ m, $\beta = 30$, and $\gamma = 17$ kN/m^3. The random variables were regarded as the input of ROM, and the slope safety factor was computed using Eq. (28). The snapshots were generated according to the procedure in Section 3. To verify the proposed method, the ROM, which was generated based on the snapshots and captured the relation between slope safety factor and random variables, was used to approximate the analytical solution. Fig. 8 compares the safety factors of the slope obtained by the ROM and the analytical solution. The predicted values of



(a) Example 1



(b) Example 2



(c) Example 3

Fig. 6. Limit state function, design point, and limit state curve for three examples using different methods.

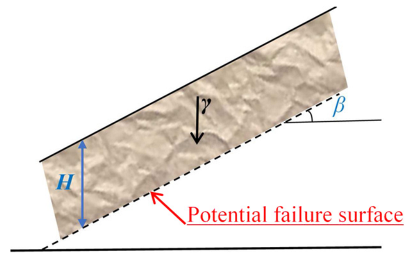


Fig. 7. The infinite slope.

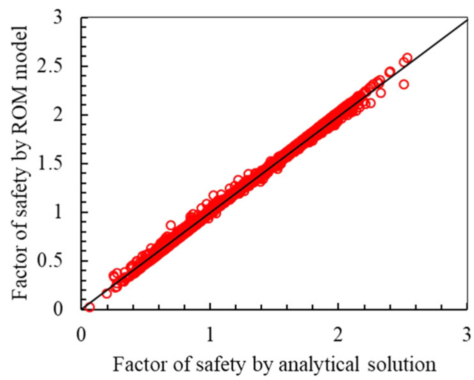


Fig. 8. Comparison of the safety factors obtained by the ROM and analytical solution.

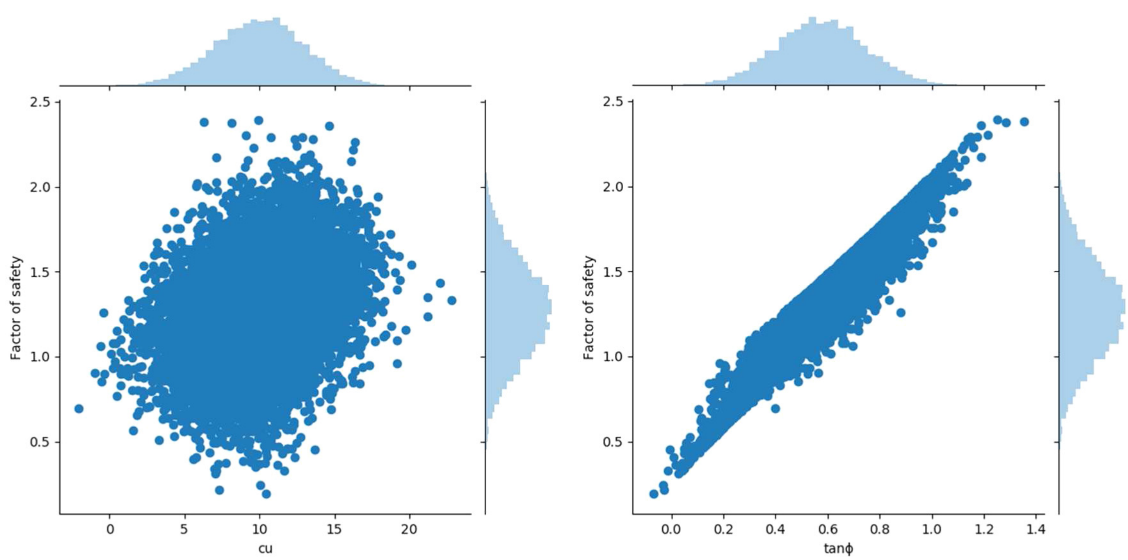
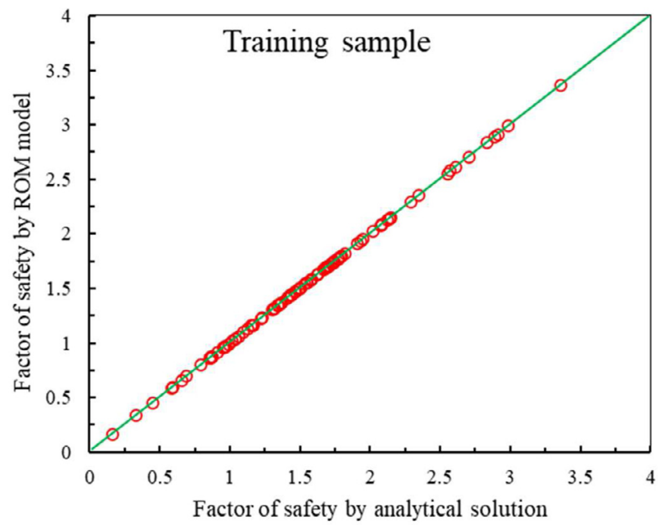


Fig. 9. The relationship between the safety factor and random variables.

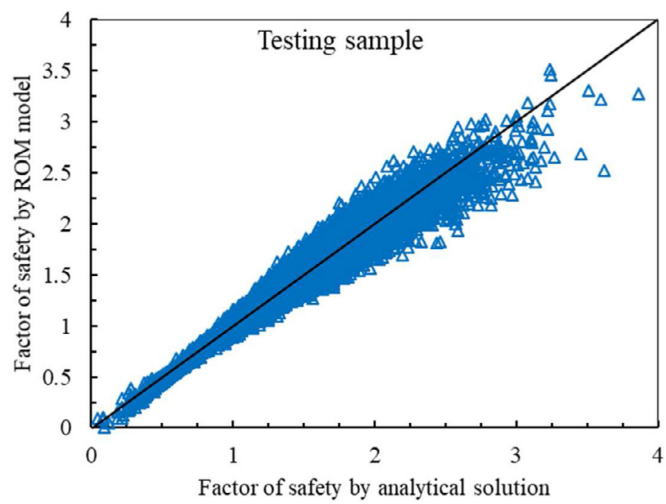
the ROM were in excellent agreement with those of the analytical solution. Fig. 9 depicts the relationship between the safety factor and random variables, revealing that the ROM presented the relationship between safety factor and random variables well. It also indicates that the friction angle had more impact on the safety factor than cohesion.

$$\mathrm{FOS} = \frac{c'}{\gamma H\, sin\beta cos\beta} + \frac{tan\varphi'}{tan\beta} \tag{28}$$

The proposed method was also adopted to determine the reliability index of the slope. Table 4 presents the results using various methods. In the FORM approach, the reliability index by Excel Solver was the same as that by SHGO (Scipy optimization) based on an analytical solution. The reliability index by SHGO was 0.8743 and was the same as the reliability index by Excel Solver. This demonstrated that SHGO has a good performance and can be utilized to determine the reliability index. The reliability index by SHGO was 0.8763 based on the ROM. It was very close to 0.8743, calculated by an analytical solution. The relative error was 0.23%. In the MCS approach, the failure probability was 19.08% based on the analytical solution and 17.75% on the ROM. The relative error was smaller than 7%. The results indicated that the ROM can replace the analytical solution in the reliability analysis and can characterize well the mechanism of the slope stability with great accuracy.



(a) Training sample



(b) Testing sample

Fig. 10. Comparison of the safety factor obtained by the ROM and analytical solution.

Table 7. The results of the reliability index and the value of limit state function using different searching ranges.

Bounds	Analytical solution		ROM	
	Reliability index	Value of LSF	Reliability index	Value of LSF
[−2, 2]	1.1037	0.0359	1.1542	2.4425E-15
[−10,10]	1.1037	0.0359	1.1542	2.4425E-15
[−100,100]	1.1037	0.0359	1.1542	2.4425E-15
[−1000,1000]	1.1037	0.0359	1.1542	2.4425E-15
[−10000,10000]	1.1037	0.0359	1.1542	2.4425E-15

Table 8. The results of the design point obtained using different searching ranges.

	$tan\beta$	$tan\phi$	γ	u	Value of LSF
Analytical solution	0.3330	0.3908	17.9941	12.0507	−0.0010
ROM	0.3255	0.3783	17.9916	12.1507	−0.0112

6.1.2. Frictional soil with pore pressure

To further verify the proposed method, we assumed $\tan\varphi'$, $\tan\beta$, γ, and u are the random variables. The other two parameters (c' and H) were deterministic variables. To simplify the computation, we assumed $c' = 0$ and $H = 5$ m. The statistical parameters of the random variables are presented in Table 5. In this case, the safety factor of the slope was obtained using the following equation (Eq. (29)):

$$\text{FOS} = \frac{tan\varphi'}{tan\beta}\left(1 - \frac{u(1 + tan^2\beta)}{\gamma H}\right) \quad (29)$$

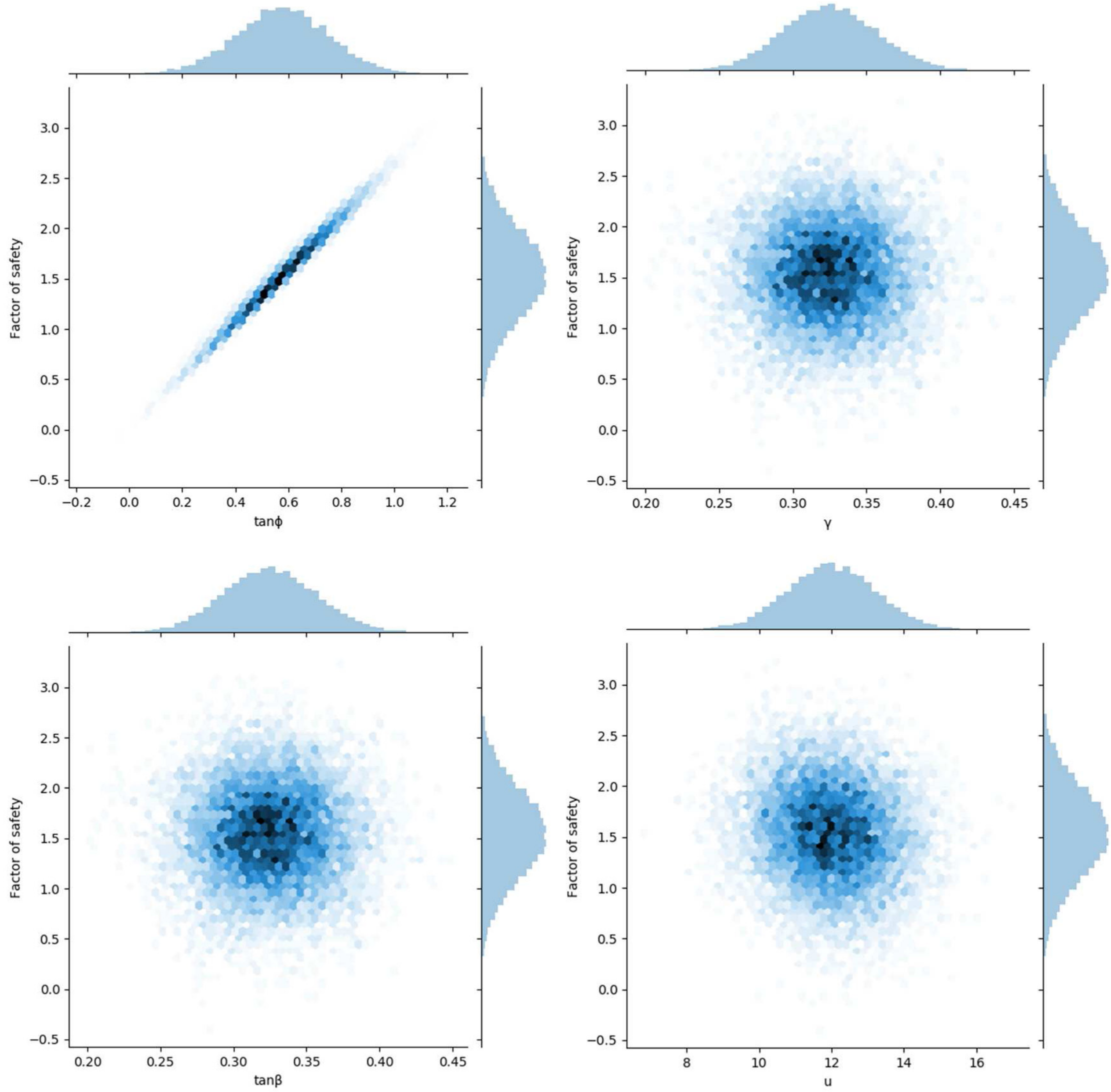


Fig. 11. Relationship between the factor of safety and random variables.

Table 9. The statistical properties of random variables.

Soil layer	c (kN/m^2)		φ (°)	
	Mean	Standard deviation	Mean	Standard deviation
I	7.2	0.2	20	3
II	5.3	0.7	23	3
III	0	0	38	5

Fig. 10 illustrates the relationship between calculated safety factors by the ROM and analytical solution. In the training samples (snapshots), the predicted safety factor was in excellent agreement with that of the analytical solution (Eq. (29)). In the testing samples, which were generated to verify the performance of the ROM, the predicted safety factor was very close to that of the analytical solution (Eq. (29)). The error of the testing samples was larger than that of the training samples. Fig. 11 displays the relationship between the factor of safety and random variables. It suggests that the strength parameters had a stronger impact on the safety factor than other parameters. The ROM presented well the relationship between safety factor and random variables, confirming that the ROM can be used to approximate the safety factor of the slope and, thus, can be an alternative for an analytical solution.

The proposed method was used to determine the reliability of the slope. Table 6 presents the results achieved using various methods. In the FORM approach, the reliability index (1.1037) by SHGO (Scipy optimization) was very close to that (1.1014) by Excel Solver based on an analytical solution. The reliability index by SHGO was 1.1542 based on the ROM. It was very close to 1.1014 calculated by an analytical solution. The relative error was smaller than 5%. In the MCS method, the failure probability was 13.53% based on the analytical solution and 11.93% on the ROM. The relative error was 11.83%. The results were similar to those of the infinite slope frictional/cohesive. This proved that the ROM could represent well the high dimensional, nonlinear, complex, and implicit relationship between the safety factor and its influencing factors. Hence, it is feasible to approximate the limit state function using the ROM.

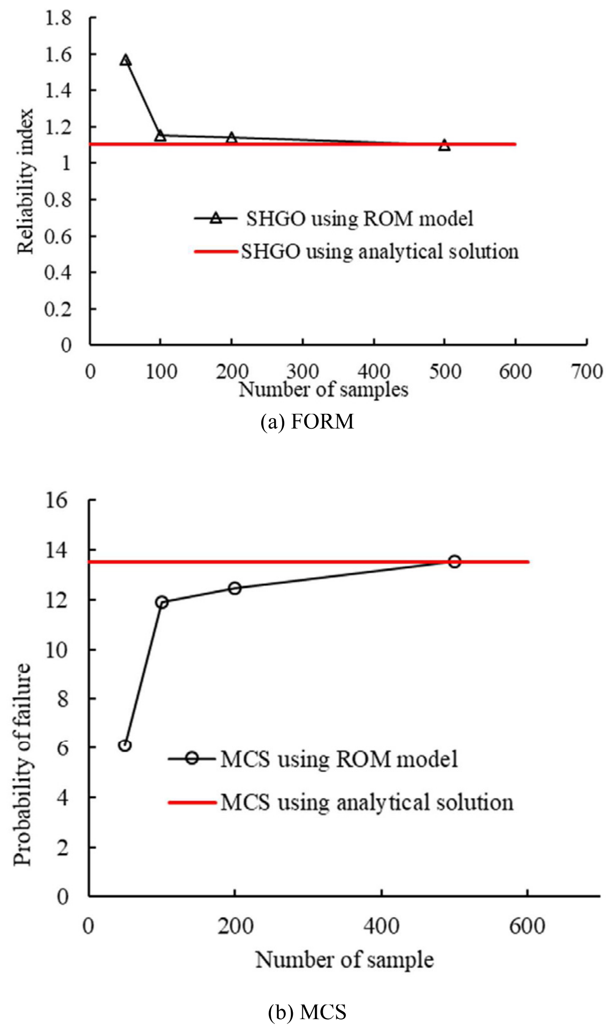


Fig. 12. The relationship between the numbers of samples, reliability index, and failure probability using different models.

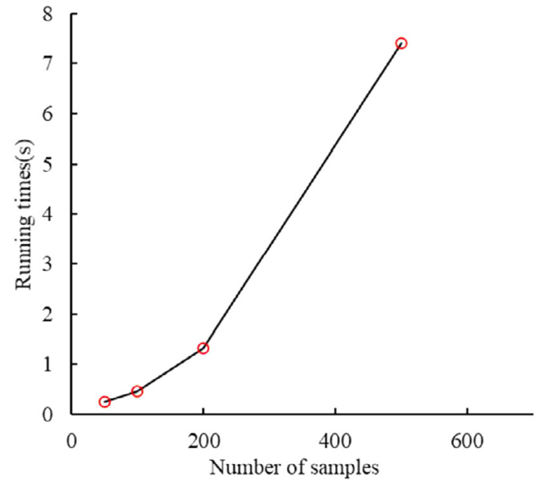
6.1.3. Discussion

The robustness and global optimization performance are essential to the optimization technology. The size of the search range of the optimization technique reflects its robustness. In order to illustrate the robustness of the ROM, the searching ranges were examined based on the SHGO optimization method in Python. The reliability index is obtained based on FORM using optimization technology. In this study, SHGO is selected as the optimization algorithm for FORM. The results are presented in Table 7 and Table 8. It can be seen from Table 7 that the developed framework doesn't depend on the searching range and is very robust for the larger searching ranges. Table 8 also shows that the design point is identical based on the ROM and analytical solution for the different searching ranges. The value of the limit state function is also close to zero. It can be observed that the SHGO had a good global searching performance, and the developed framework is feasible. This is important for geotechnical and geological engineering problems involving significant uncertainty and complex engineering system.

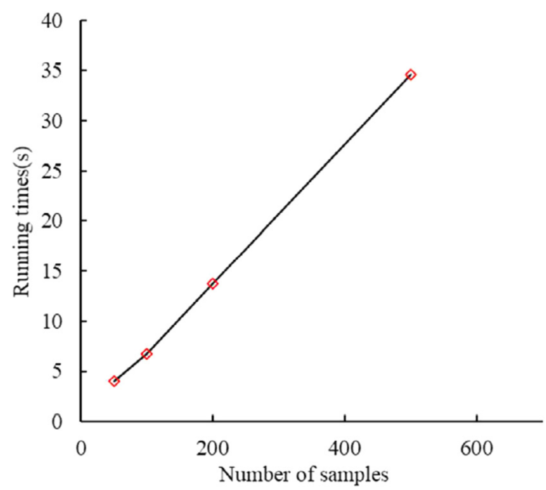
To illustrate the performance of the ROM, Fig. 12 shows the relationship between the numbers of samples, reliability index, and failure probability using different models. It demonstrates that the accuracy of the reliability analysis depends on the number of samples. With the increasing number of samples, the results were very close and almost the same as the results obtained by an analytical solution. However, the running time increased with the increasing number of samples (Fig. 13). Fig. 13 reveals that the running time increased dramatically with the number of samples in the model building stage. However, the running time increased linearly in the stage of the reliability analysis (MCS or FORM). Fig. 12 exhibits that the results achieved by the ROM were the same as the analytical solution when the number of samples was 500. The results could also meet the accuracy requirements when the number was 100, indicating that the proposed method has a reasonable convergence rate. The ROM provides a good and promising way to approximate the limit state function in the reliability analysis for practical slope engineering.

6.2. ACAD slope

In general, it is difficult to obtain an analytical solution for practical slope engineering problems. Therefore, the second slope was chosen as a complex slope with three different soil layers, and a numerical solution was adopted in the reliability analysis. The strength reduction method was adopted to compute the factor of safety (Cheng et al., 2007). The cross-section of the slope and computational mesh are shown in Fig. 14. The values of the mean and standard deviation of the random variables are presented in Table 9. The ROM was first constructed based on the ROM algorithm as in infinite slope. The reliability index was computed by the proposed method.



(a) The stage of building model



(b) The stage of the reliability analysis

Fig. 13. The relationship between running times and the number of samples in different models.

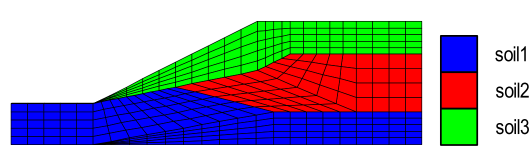


Fig. 14. The relationship between running times and the number of samples in different models.

According to the procedure elaborated in Section 4.3, the numerical model was built initially (Fig. 14). LHS was adopted to construct the 200 sets of random variables. The factor of safety of each set was calculated based on the numerical model using the strength reduced method, and 200 training samples (snapshots) were obtained. Based on the 200 snapshots, the ROM was determined using the proper orthogonal decomposition. Fig. 15 compares the safety factor acquired by the numerical model and the ROM. In the training samples, the results were in excellent agreement with those of the numerical model. To verify the performance of the ROM, the factor of safety of non-training samples, which were generated randomly using LHS, was calculated based on the numerical model and the ROM, respectively. The results were very close to those of the numerical model, proving that the ROM can present the mechanical behavior of soil slope well. Thus, it can replace the numerical model in the reliability analysis. Fig. 16 manifests the relationship between the factor of safety and random variables. It reveals that the friction angle of the third layer had more impact on the factor of safety than other random variables because the third layer had more influence on the stability of the slope. The results were similar to those of the infinite slope.

The reliability index was calculated using the ROM-based RSM, which adopted the ROM to approximate the limit state function. For the sake of comparison, the traditional RSM, which utilized the polynomial function to approximate the limit state function, was also employed to compute the reliability index and failure probability. Fig. 17 exhibits the relationship between iteration times and the reliability index. It shows that the convergence by the ROM was faster than that of the numerical model based on traditional RSM. The numerical model based on traditional RSM does not converge, which means it can not obtain the reliability index. The comparisons of the reliability index and design point are presented in Fig. 18. Direct ROM adopted ROM as the limit state function to determine the reliability index using FORM. RSM by numerical model adopted

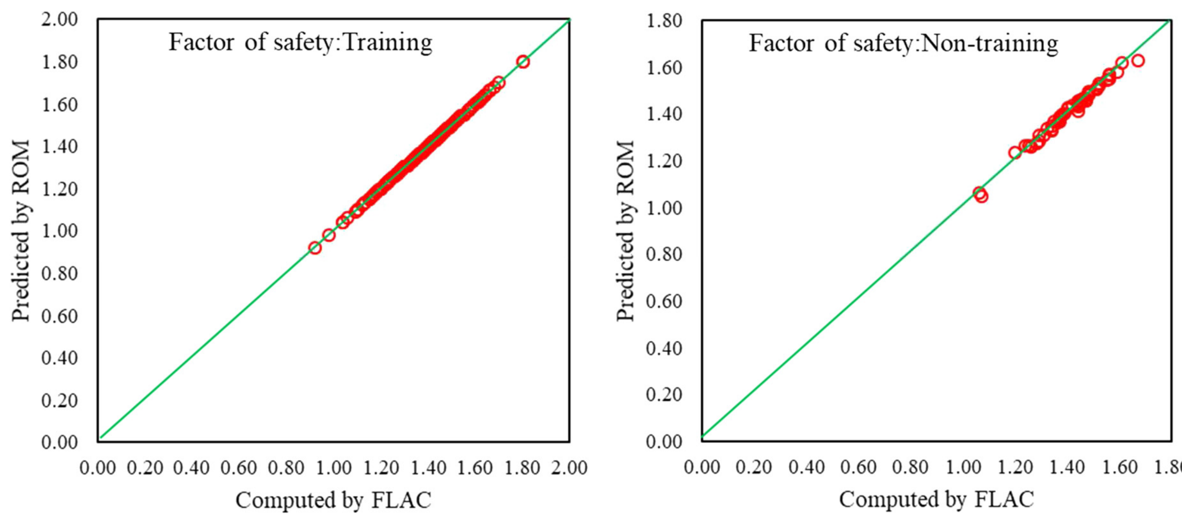


Fig. 15. The comparison of the safety factors obtained by the numerical model and the ROM.

traditional response surface as the limit state function based on numerical model. RSM by ROM model adopted ROM as the limit state function. MCS by ROM model adopted MCS to determine the failure probability based on ROM. The design point is the value of the random variable corresponding to the reliability index, which is obtained based on the optimization technology in FORM. The results were in excellent agreement with those of the various methods, confirming that the ROM is feasible for the reliability analysis. The probability distribution of the safety factor is shown in Fig. 19 (the ROM was used for the reliability analysis). The numerical model (FLAC) was utilized to evaluate the slope state at the design point. Fig. 20 displays the failure mode at the design point (the failure is indicated by shear strain rate contours). It exhibits that it is feasible and accurate to approximate the safety factor using the ROM.

7. Conclusions

This study developed a novel framework of reliability analysis to deal with the uncertainty in geotechnical and geological engineering problems by combining the ROM, FORM, and numerical models. The ROM was employed to build the response surface and approximate the limit state function. The reliability index was determined based on the ROM using SHGO optimal method. The failure probability was estimated based on the ROM using the MCS method. The developed method was verified and illustrated by three numerical examples and two slopes. The developed ROM can be utilized as an alternative for numerical modeling in the reliability analysis of geotechnical and geological engineering problems. The results of this study drew the following specific conclusions:

1) The ROM approximated the complex, nonlinear, and implicit limit state function. The ROM enhanced the efficiency of the numerical simulation and could be used as a replacement for a numerical model to solve geotechnical and geological engineering problems. Numerical modeling was costly for the reliability analysis of practical geotechnical and geological engineering problems, requiring abundant repetitive computations. The ROM provided an effective way to improve the efficiency of analyses and yielded highly accurate predictions.

2) The reliability index and failure probability were determined based on the ROM using the FORM and MCS. They were in good agreement with the results obtained based on the actual limit state function or analytical solutions. This demonstrated that the ROM-based reliability analysis could represent the uncertainty of geotechnical and geological engineering systems well. It is feasible to deal with uncertainty using the ROM. The ROM can also be applied to other geotechnical and geological engineering problems.

3) In the developed framework, the numerical model is utilized to determine the response of the practical and complex geotechnical structure, such as slope safety factor, displacement, stress, etc., because it is impossible to obtain the analytical solution. So, the numerical model is essential to the developed framework and directly affects the performance and accuracy of the developed framework. The selection of an appropriate numerical model was critical to successfully applying the developed framework.

Declarations

Author contribution statement

Hongbo Zhao: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Meng Wang: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Xu Chang: Analyzed and interpreted the data; Wrote the paper.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability statement

Data included in article/supp. material/referenced in article.

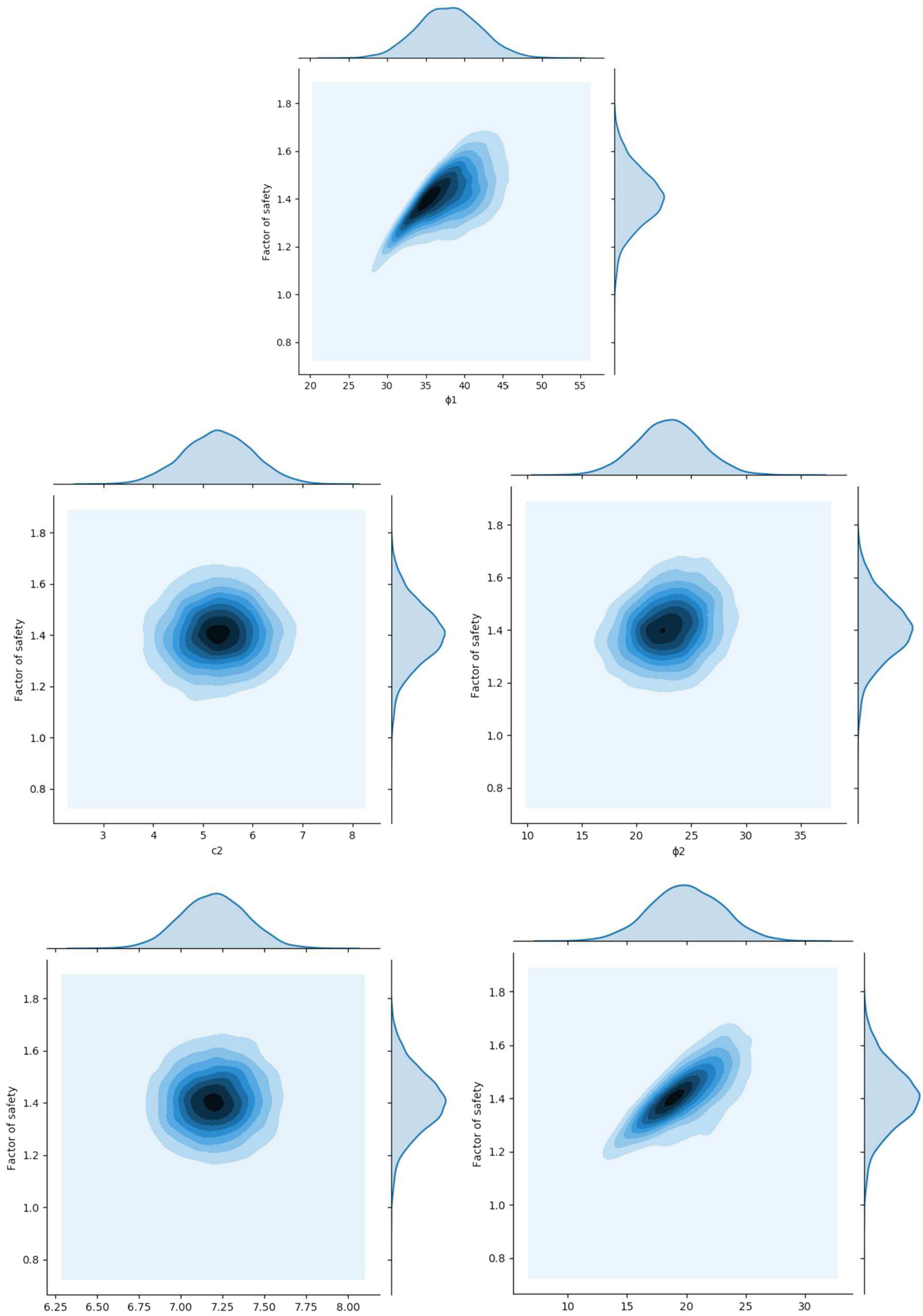


Fig. 16. The relationship between the safety factor and random variables.

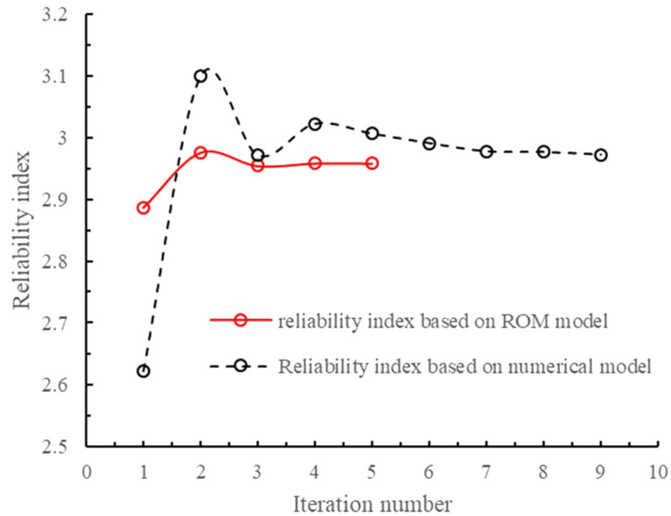
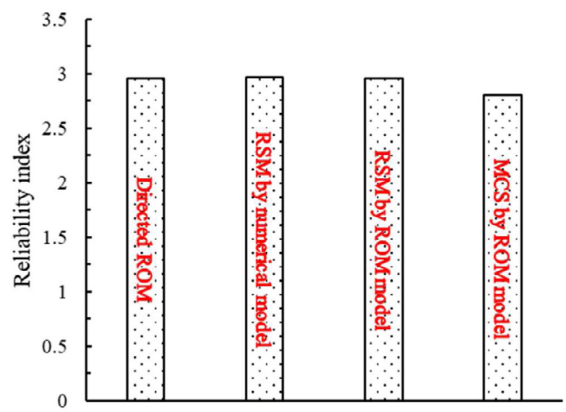
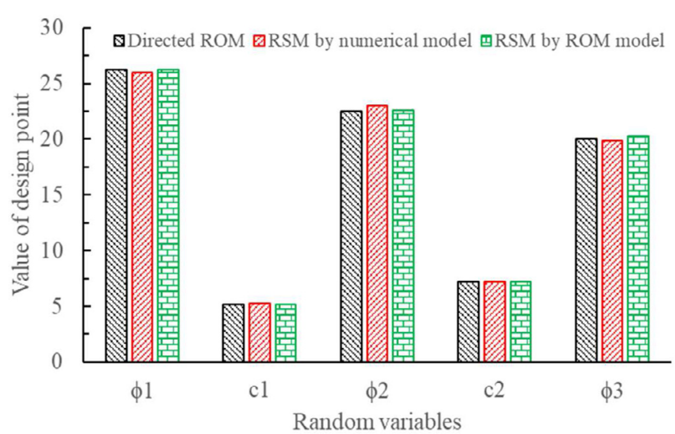


Fig. 17. The relationship between the reliability index and iteration number in different models.



(a) Reliability index



(b) Design point

Fig. 18. The comparison of reliability analysis by different methods.

Declaration of interest's statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

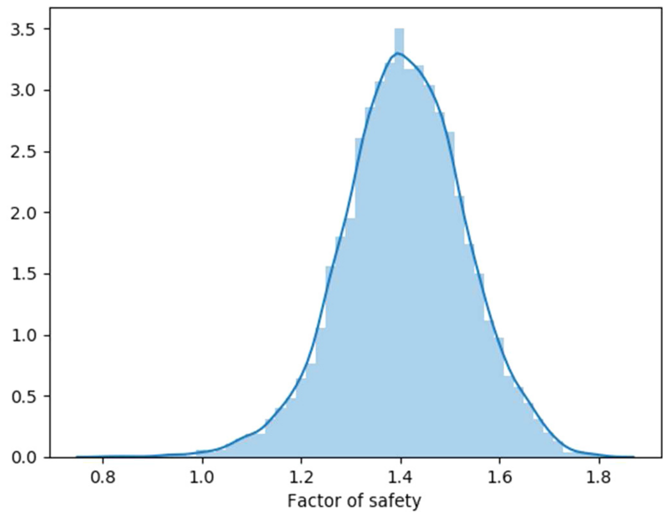


Fig. 19. The distribution of the safety factor obtained by MCS.

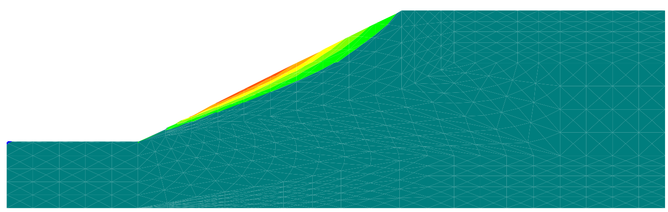


Fig. 20. The slope slip surface with soil strength parameters in the design point.

References

Alipanahi, B., Delong, A., Weirauch, M.T., Frey, B.J., 2015. Predicting the sequence specificities of DNA- and RNA-binding proteins by deep learning. Nat. Biotechnol. 33, 831–838.

Au, S.K., Beck, J.L., 1999. A new adaptive importance sampling scheme for reliability calculations. Struct. Saf. 21, 135–138.

Audouze, C., Vuyst, F.D., Nair, P.B., 2009. Reduced-order modeling of parameterized PDEs using time-space-parameter principal component analysis. Int. J. Numer. Methods Eng. 80, 1025–1057.

Bourinet, J.M., Deheeger, F., Lemaire, M., 2011. Assessing small failure probabilities by combined subset simulation and support vector machines. Struct. Saf. 33, 343–353.

Cheng, K., Lu, Z.Z., 2018. Sparse polynomial chaos expansion based on D-MORPH regression. Appl. Math. Comput. 323, 17–30.

Cheng, Y.M., Lansivaara, T., Wei, W.B., 2007. Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods. Comput. Geotech. 34 (3), 137–150.

Cho, S.E., 2009. Probabilistic stability analyses of slopes using the ANN-based response surface. Comput. Geotech. 36 (5), 787–797.

Chowdhury, R., Rao, B.N., 2010. Probabilistic stability assessment of slopes using high dimensional model representation. Comput. Geotech. 37 (7), 876–884.

Cizmas, G.A., Richardson, B.R., Brenner, T.A., O'Brien, T.J., Breault, R.W., 2008. Acceleration techniques for reduced-order models based on proper orthogonal decomposition. J. Comput. Phys. 227, 7791–7812.

Deng, J., Gu, D., Li, X., Yue, Z.Q., 2005. Structural reliability analysis for implicit performance functions using artificial neural network. Struct. Saf. 27 (1), 25–48.

Ditlevsen, O., Madsen, H.O., 1996. Structural Reliability Methods. John Wiley and Sons, Chichester.

Endres, S.C., Sandrock, C., Focke, W.W., 2018. A simplicial homology algorithm for lipschitz optimization. J. Glob. Optim. 72, 181–217.

Fic, A., Bialecki, R.A., Kassab, A.J., 2006. Solving transient nonlinear heat conduction problems by proper orthogonal decomposition and the finite element method. Numer. Heat Transf., Part B, Fundam. 48, 102–124.

Freno, B.A., Cizmas, P.G.A., 2014. A proper orthogonal decomposition method for nonlinear flows with deforming meshes. Int. J. Heat Fluid Flow 50, 145–159.

Griffiths, D.V., Huang, J., Gordon, A.F., 2011. Probabilistic infinite slope analysis. Comput. Geotech. 38, 577–584.

Hoek, E., 1998. Reliability of Hoek-Brown estimates of rock mass properties and their impact on design. Int. J. Rock Mech. Min. Sci. 35, 63–68.

Hsu, W.C., Ching, J.Y., 2010. Evaluating small failure probabilities of multiple limit states by parallel subset simulation. Probab. Eng. Mech. 25, 291–304.

Huys, Q.J.M., Maia, T.V., Frank, M.J., 2016. Computational psychiatry as a bridge from neuroscience to clinical applications. Nat. Neurosci. 19 (3), 404–413.

Ji, J., Low, B.K., 2012. Stratified response surfaces for system probabilistic evaluation of slopes. J. Geotech. Geoenviron. Eng. 138 (11), 1398–1406.

Ji, J., Zhang, C., Gao, Y., Kodikara, J., 2018. Effect of 2D spatial variability on slope reliability: a simplified FORM analysis. Geosci. Front. 9 (6), 1631–1638.

Kang, F., Li, J.J., 2016. Artificial bee colony algorithm optimized support vector regression for system reliability analysis of slopes. J. Comput. Civ. Eng. 30 (3), 04015040.

Karunanithi, N., 1992. Using neural networks in reliability prediction. IEEE Softw. 9 (4), 53–61.

LeCun, Y., Bengio, Y., Hinton, G., 2015. Deep learning. Nature 521, 436–444.

Leon, D.D., Garduno, J., 2020. Time-variant failure probability of critical slopes under strong rainfall hazard including mitigation effects. Struct. Infrastruct. Eng. Maint. Manag. Life-Cycle Des. Perform. 16 (10), 1481–1492.

Li, L., Liang, R.Y., Liu, H., 2015a. System reliability analysis for anchor-stabilised slopes considering stochastic corrosion of anchors. Struct. Infrastruct. Eng. Maint. Manag. Life-Cycle Des. Perform. 11 (10), 1294–1305.

Li, D.Q., Jiang, S.H., Cao, Z.J., Zhou, C.B., Li, X.Y., Zhang, L.M., 2015b. Efficient 3-D reliability analysis of the 530 m high abutment slope at Jinping I Hydropower Station during construction. Eng. Geol. 195, 269–281.

Li, D.Q., Zheng, D., Cao, Z.J., Tang, X.S., Phoon, K.K., 2016. Response surface methods for slope reliability analysis: review and comparison. Eng. Geol. 203, 3–14.

Li, H.Z., Low, B.K., 2010. Reliability analysis of circular tunnel under hydrostatic stress field. Comput. Geotech. 37, 50–58.

Li, S., Zhao, B., Ru, Z., 2013. Slope reliability analysis by updated support vector machine and Monte Carlo simulation. Nat. Hazards 65, 707–722.

Li, X.Q., Song, L.K., Bai, G.C., 2022. Recent advances in reliability analysis of aeroengine rotor system: a review. Int. J. Struct. Integr. 13 (1), 1–29.

Lophaven, S.N., Nielsen, H.B., Søndergaard, J., 2002. DACE-a matlab Kriging toolbox.

Low, B.K., 2004. Reliability analysis using object-oriented constrained optimization. Struct. Saf. 26 (1), 69–89.

Low, B.K., Tang, W.H., 2007. Efficient spreadsheet algorithm for first-order reliability method. J. Eng. Mech. 133 (12), 1378–1387.

Luo, C., Keshtegar, B., Zhu, S.P., Taylan, O., Niu, X.P., 2022. Hybrid enhanced Monte Carlo simulation coupled with advanced machine learning approach for accurate and efficient structural reliability analysis. Comput. Methods Appl. Mech. Eng. 388, 114218.

Lv, Q., Low, B.K., 2011. Probabilistic analysis of underground rock excavations using response surface method and SORM. Comput. Geotech. 38, 1008–1021.
Markus, R., Gustau, C., Biorn, S., Martin, J., Joachim, D., Prabhat, C.N., 2019. Deep learning and process understanding for data-driven earth system science. Nature 566, 195–203.
Mollon, G., Daniel, D., Abdul, H.S., 2009. Probabilistic analysis of circular tunnels in homogeneous soil using response surface methodology. J. Geotech. Geoenviron. Eng. 135, 1314–1325.
Panthi, K.K., Nilsen, B., 2007. Uncertainty analysis of tunnel squeezing for two tunnel cases from Nepal Himalaya. Int. J. Rock Mech. Min. Sci. 44, 67–76.
Severson, K.A., Attia, P.M., Jin, N., Perkins, N., Jiang, B., Yang, Z., Chen, M.H., Aykol, M., Herring, P.K., Fraggedakis, D., Bazant, M.Z., Harris, S.J., Chueh, W.C., Braatz, R.D., 2019. Data-driven prediction of battery cycle life before capacity degradation. Nat. Energy 4, 383–391.
Sperner, E., 1928. Neuer beweis für die invarianz der dimensionszahl und des gebietes. Abh. Math. Semin. Univ. Hamb. 6 (1), 265–272.
Tan, X.H., Bi, W.H., Hou, X.L., Wang, W., 2011. Reliability analysis using radial basis function networks and support vector machines. Comput. Geotech. 38 (2), 78–186.
Tan, X.H., Shen, M.F., Hou, X.L., Li, D., Hu, N., 2013. Response surface method of reliability analysis and its application in slope stability analysis. Geotech. Geolog. Eng. 31 (4), 1011–1025.
Tsompanakis, Y., Lagaros, N.D., Psarropoulos, P.N., Georgopoulos, E.C., 2010. Probabilistic seismic slope stability assessment of geostructures. Struct. Infrastruct. Eng. Maint. Manag. Life-Cycle Des. Perform. 6 (1–2), 179–191.
Veer, L.J., Bernards, R., 2008. Enabling personalized cancer medicine through analysis of gene-expression patterns. Nature 452, 564–570.
Wu, Z.Y., Li, Y.L., Chen, J.K., Zhang, H., Pei, L., 2013. A reliability-based approach to evaluating the stability of high rockfill dams using a nonlinear shear strength criterion. Comput. Geotech. 51, 42–49.
Xu, B., Low, B.K., 2006. Probabilistic stability analyses of embankments based on finite element method. J. Geotech. Geoenviron. Eng. 132 (11), 1444–1454.
Zhang, J., Zhang, L.M., Tang, W.H., 2011. Kriging numerical models for geotechnical reliability analysis. Soil Found. 51 (6), 1169–1177.
Zhang, W., Goh, A.T.C., 2018. Reliability analysis of geotechnical infrastructures: introduction. Geosci. Front. 9 (6), 1595–1596.
Zhao, H., 2008. Slope reliability analysis using a support vector machine. Comput. Geotech. 35 (3), 459–467.
Zhao, H., 2022. A practical and efficient reliability-based design optimization method for rock tunnel support. Tunn. Undergr. Space Technol. 127, 104587.
Zhao, H., Chen, B., 2021. Physics-based reduced order model for computational geomechanics. Springer Ser. Geomech. Geoengin. 27 (4), 361–374.
Zhao, H., Li, S., Ru, Z., 2017. Adaptive reliability analysis approach based on SVM and its application. Appl. Math. Model. 44, 508–522.
Zhao, H., Ru, Z., Chang, X., Yin, S., Li, S., 2014. Reliability analysis of tunnel using a least square support vector machine. Tunn. Undergr. Space Technol. 41, 14–23.
Zhao, Y.G., Lu, Z.H., 2007. Applicable range of the fourth-moment method for structural reliability. J. Asian Arch. Build. Eng. 6 (1), 151–158.
Zhu, S.P., Keshtegar, B., Bagheri, M., Hao, P., Trung, N.T., 2020a. Novel hybrid robust method for uncertain reliability analysis using finite conjugate map. Comput. Methods Appl. Mech. Eng. 371, 113309.
Zhu, S.P., Keshtegar, B., Chakraborty, S., Trung, N.T., 2020b. Novel probabilistic model for searching most probable point in structural reliability analysis. Comput. Methods Appl. Mech. Eng. 366, 113027.