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Original Article

The role of interaction torque and muscle torque in the control of downward squatting

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Abstract. [Purpose] The purposes of this study were first to analyze the multijoint dynamics of downward squatting, and to examine the contribution of interaction torque and muscle torque to net torque, and second, to examine mechanisms of movement control. [Subjects] The subjects were 31 healthy men with a mean age of 21.0 ± 1.2 years (range, 19–24 years). [Methods] Squatting tasks with the trunk in two positions, an erect and anterior tilt position, were performed by the subjects. Net, interaction, muscle, and gravity torque were calculated according to the Lagrange equation using 3D tracking data. [Results] The contribution ratio of interaction torque to net torque was approximately 90%, irrespective of the joint and task. In contrast, muscle torque showed complicated behavior to compensate for gravity torque. A combined muscle and gravity torque profile showed flexion or dorsiflexion immediately after the initiation of the movement, and it later changed to extension or plantar flexion. [Conclusion] The torque that contributes almost exclusively to the net torque was interaction torque. The combination of muscle and gravity torque at the knee joint and the hip joint is important for movement control, independent of the starting position.

Key words: Motor control, Interaction torque, Squatting

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INTRODUCTION

It was proposed by Bernstein[1)] that movements are organized in such a way that reactive forces at the joints not only disrupt movement but also directly support movement. Bernstein's reactive forces, in recent studies referred to as interactive or motion-dependent torque, are passive torques derived from joint reaction forces arising due to motion of segments in multijoint motion[1–3)]. The control of muscle torque (MUS) cannot be understood without considering interaction torque (INT), given that its contribution to net torque (NET) is unexpectedly large during multijoint movements.

The effects of INT have mostly been examined for arm-reaching tasks[3–8)]. Conversely, there have been few studies on the role of INT in whole-body movements, such as the squat motion. Because it is greatly dependent on segment mass and angular acceleration, larger INT arises in fast movements involving the trunk and legs than in arm movements. The contribution of INT to NET during the sit-to-stand motion was analyzed by Fujisawa et al.[9)], and it was found that the contribution of INT to NET was also very important in whole-body movements, such as the sit-to-stand motion. Moreover, hip joint movement was found to be of utmost importance to this motion, because the anterior tilt of the trunk was indispensable to the motion. It was pointed out by Bernstein[1)] that "the movements are organized so that the reactive forces not only fail to disrupt the movement but directly support it, providing it a particular kind of stability," therefore the contribution of INT to NET is expected to be large in squatting. Whole-body movements, such as squatting, may be stabilized through a crosstalk between INT and MUS. Dynamic squatting is commonly used in the clinical setting not only for lower-extremity strengthening[10–18)] but also

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for coordination training in multijoint movements, such as weight-shift exercises with knee flexion for hemiplegia[19–21]. Understanding the actual role of MUS in a closed kinetic chain exercise such as squatting is therefore essential.

Moreover, in clinical settings squatting is performed with the trunk starting the movement from an erect position (EP) or an anterior tilt position (AP)[22]. Particularly when the trunk is in AP, the gravity torque of the knee joint acts on extension, because the center of gravity (COG) above the knee joint is located considerably anterior to the rotation axis of the knee joint. Therefore, it is necessary to clarify any differences in torques caused by the starting posture.

The purposes of this study were first to analyze the multijoint dynamics of downward squatting, and to examine the contribution of interaction torque and muscle torque to net torque, and second, to examine mechanisms of movement control.

SUBJECTS AND METHODS

The subjects were 31 healthy men with a mean age of 21.0 ± 1.2 years (range, 19–24 years), height of 171.4 ± 5.6 cm (range, 158.0–182.0 cm), and weight of 65.4 ± 9.4 kg (range, 53.3–93.6 kg). Written informed consent to participate in the experiment was provided by each subject, and the study was approved by the Human Subjects Ethics Committee of Tohoku Bunka Gakuen University (No. 14-16).

Squatting tasks were performed by the subjects, starting with the trunk in two positions: EP and AP (the tasks are referred to hereafter as EP and AP tasks). In the EP task, squatting was performed by the subjects from an erect standing position with knees flexed at 0° to a squat position with knees flexed at 60°. Each subject was instructed to move by a natural movement pattern. In the AP task, squatting was performed from a 30° anterior trunk tilt with knees flexed at 0° to a squat position with knees flexed at 60° while holding a 30° anterior trunk tilt. Movement speeds were high in both tasks. In addition, the subjects crossed their arms over their chest, and the distance between the second metatarsal bone of the right and left foot was adjusted to match the distance between the right and left anterior superior iliac spine. Also, the long axis of the foot was held parallel to the sagittal plane. The subjects were asked to descend into a squat after an auditory cue, and one trial for each task was performed by the subjects. The subjects were allowed to practice the task prior to the test session.

Reflective markers were placed at both sides of the tip of the acromion process, the greater trochanter, the lateral femoral epicondyle, and the lateral malleolus of each subject. Marker positions were recorded at a sampling frequency of 250 Hz using a six-camera motion analysis system (Locus MA-5000, Anima Corp, Tokyo, Japan).

The initiation and termination of a movement were defined as the time points when the vertical velocity of the marker at the acromion process exhibited first and second zero crossing, respectively. Marker displacement data were smoothed using a moving average of 55 data points. Marker positions were used to calculate joint angles from which angular velocity and acceleration were calculated. The COG of the entire body and of each segment was calculated using marker positions. The Anthropometric data described by Winter[23] were used to calculate the COG.

NET, INT, MUS, and gravity torque (G) were calculated according to the Lagrange equation for a three-segment rigid body model, using midpoints between markers on both sides of the segments (Appendix 1). The contribution ratio of INT (CR-int), MUS (CR-mus) and G (CR-g) to NET was calculated as follows:

$$\mathrm{NET} = \mathrm{MUS} + \mathrm{G} + \mathrm{INT} \qquad (1)$$

$$\text{Contribution ratio}\,(\%) = \left[1 - \frac{\sum_{i=1}^{N}\left|NET(i) - TC(i)\right|}{\sum_{i=1}^{N}\left|NET(i)\right|}\right] \times 100 \qquad (2)$$

TC: torque component (INT, MUS, G)

Therefore, the calculated CR was 100% when the corresponding torque components varied completely in phase. Moreover, the combined value of MUS and G (MUS + G) was assumed to be an index of motor control. Data from both squatting tasks were converted into 101 data points using a spline, and data from each task were interpolated to a 100% cycle at 1% intervals. After calculating addition average values for all subjects, the peak value and timing were determined for each subject.

The Pearson's product-moment correlation coefficient (r) was used to study the dependence of NET and MUS + G peak times. The level of significance was set at $p < 0.05$.

RESULTS

The COG trajectory in both squatting tasks indicated a vertical pattern in the frontal plane and a forward moving pattern in the sagittal plane. The two tasks differed neither in the start nor at the end position in the knee joint (Fig. 1). Joint movement began almost simultaneously in all three examined joints (Fig. 1). Interjoint coordination between the hip, knee, and ankle showed a linear dependence (Fig. 1).

Figure 2 shows the profile of torque components in EP and AP tasks. In the EP task, both NET and INT were dominant, changing from flexion to extension in all three joints. Moreover, the INT profile was synchronized with the NET profile during the squatting motion. In contrast, MUS cancelled G. The NET and INT profile was the same in both the AP and EP

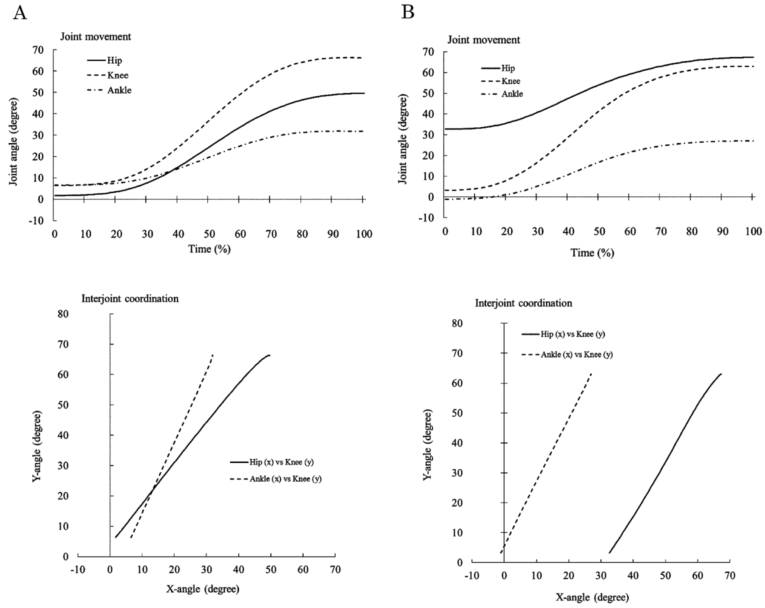

Fig. 1. Joint movement and inter-joint coordination
A: erect position (EP) task; B: anterior tilt position (AP) task

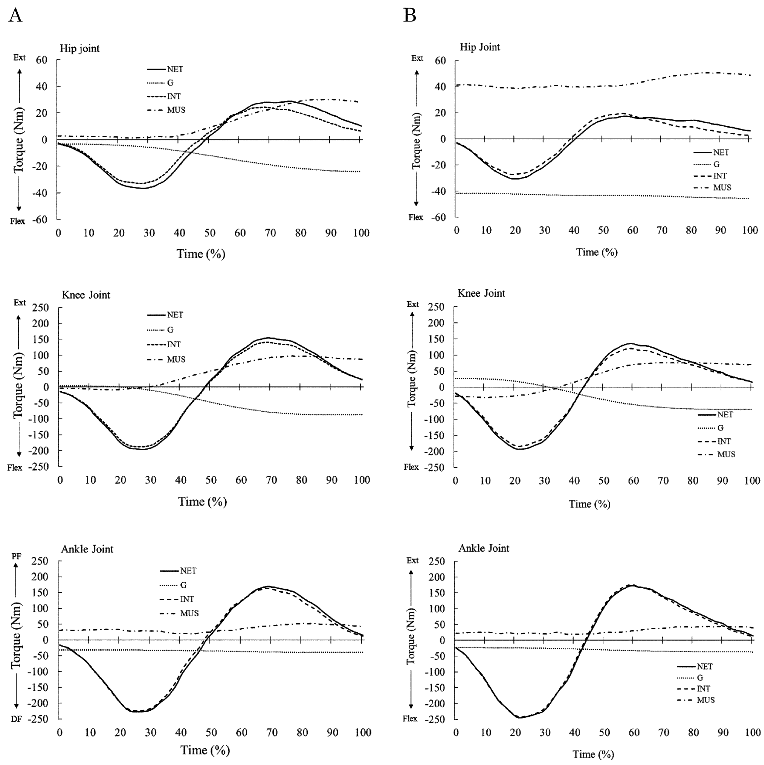

Fig. 2. Changes in each torque component
A: erect position (EP) task; B: anterior tilt position (AP) task. NET: net torque; INT: interaction torque; MUS: muscle torque; G: gravity torque

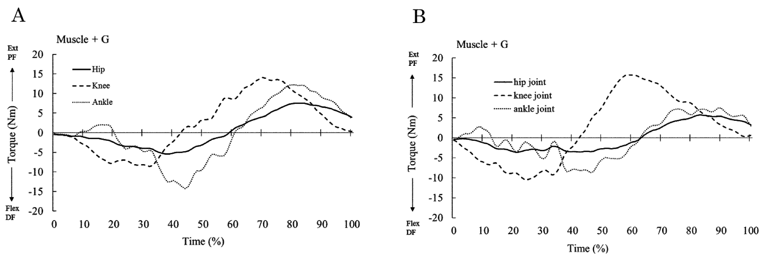

Fig. 3. Changes in muscle torque plus gravity torque
A: erect position (EP) task; B: anterior tilt position (AP) task

Table 1. The contribution ratio of interaction, muscle, and gravity torque to net torque (Mean ± SD)

	Gravity torque (%)	Interaction torque (%)	Muscle torque (%)
Erect position task (n=31)			
Hip	–25.3 ± 25.8	83.5 ± 7.8	14.8 ± 18.3
Knee	–29.0 ± 14.4	93.5 ± 1.5	24.0 ± 10.8
Ankle	–2.5 ± 7.4	92.9 ± 3.3	3.8 ± 14.5
Anterior tilt position task (n=31)			
Hip	–142.6 ± 68.1	80.1 ± 7.8	–137.4 ± 70.6
Knee	–31.3 ± 10.2	92.2 ± 1.8	26.0 ± 13.5
Ankle	–1.6 ± 5.4	94.9 ± 1.5	5.7 ± 5.1

task. However, the MUS of the hip joint was larger in AP than in the EP task. The G of the knee joint indicated an extension torque at the starting position in the AP task, even though the G of the EP task was almost neutral.

In both tasks, the MUS + G profile showed flexion or dorsiflexion immediately after the initiation of the movement, and later the profile changed to extension or plantar flexion. The MUS + G of the knee joint showed both flexion and extension peaks in the shortest time in both tasks (Fig. 3).

The CR-int was approximately 90%, irrespective of the joint and task, indicating that the torque that contributed almost exclusively to NET was INT, while both the CR-mus and CR-g showed low values: below 30% for both tasks (Table 1).

In the EP task The Pearson's product-moment correlation coefficients between MUS + G and NET at the hip, knee and ankle joint were 0.82, 0.84, and 0.69, respectively. In the AP task, these coefficients were 0.83, 0.96, and 0.54, respectively. Although all correlation coefficients were statistically significant, only the knee joint movements showed a synchronized torque profile.

DISCUSSION

The most remarkable finding of the present study was that INT was substantially larger than both MUS and G for the three joints and two tasks, with a CR-int larger than 90%, whereas both CR-mus and CR-g were below 30%. This finding is in contrast with the results of previous studies on reaching tasks, in which the contribution of MUS was larger than that of INT[24)]. As expected, a large amount of INT at adjacent joints was produced by the movement involving a large mass: the head, arm, and trunk. In squatting, INT was always in the same direction as NET, suggesting that it had a functional role in assisting, rather than in compensating for, the production of NET. Although both assistive and resistive roles of INT were suggested by Bernstein[1)], in our study INT played a completely assistive role in squatting. Of particularly importance, INT had the same effect between the two tasks. This again confirms that INT should be considered when discussing movement control in multijoint movements.

MUS was mostly used to offset G, and its contribution to actual movement was small compared with that of INT (Table 1). However, MUS appeared to be important for initiating and terminating the movement. In particular, the relationship between peak times of MUS + G and NET was considerably strong in the knee and hip joint. Moreover, MUS + G and NET in the knee joint were approximately synchronous, suggesting that the control of the knee is of utmost importance to coordination in squatting.

The effects of INT have mostly been examined for arm-reaching tasks[3–8)]. According to several reports on movement disorders, disharmony between MUS and INT represents a kinetic factor in ataxia[25–27)]. It was also reported by Beer et al.[28)] that hemiplegia shows a mismatch between these torques. In a study by Galloway et al.[24] on the shoulder, elbow, and wrist dynamics during reaching in 12 directions on the horizontal plane in normal subjects, the shoulder MUS primarily determined NET and joint acceleration, while INT was minimal for most movements. In contrast, the elbow NET and wrist NET were determined by a combination of MUS and INT. This phenomenon was termed the "shoulder-centered pattern." The same phenomenon has been observed during a fast-reaching task in the sagittal plane[27]. The shoulder-centered pattern may be the major control mechanism for upper arm movements during reaching tasks[25, 29]. We found that in sit-to-stand motion the main mechanism was hip joint movement, as it is for the reaching task, because the anterior tilt of the trunk is indispensable to the motion[9]. It was thought that the main source of joint angle acceleration was gravity forces, and that the deactivation of the semitendinosus and semimembranosus acted in conjunction with phasic activation of the tibialis anterior[30–32]. On the other hand, the deactivation of the hamstrings did not occur when the trunk tilted backward. This fact suggested that posture influences the initiation of squatting. Particularly with the trunk in AP, the gravity torque of the knee joint acts on extension because the COG above the knee joint is located considerably anterior to the rotation axis of the knee joint. In this case, it is doubtful whether the deactivation of the hamstrings is effective in movement initiation. Moreover, it is very interesting that not only the hip joint but also the knee joint was found to be of utmost importance to coordination in squatting. To elucidate the mechanism of the initiation of joint movement, it is necessary to include further muscle activity in the analysis.

In conclusion, the contribution of INT to NET was remarkably large in squatting. The source of joint torque to produce movement was primarily INT, while MUS + G at the hip and knee joints were important in controlling the movement, independent of the starting position. The control mechanisms in squatting and the functional role of INT require further examination.

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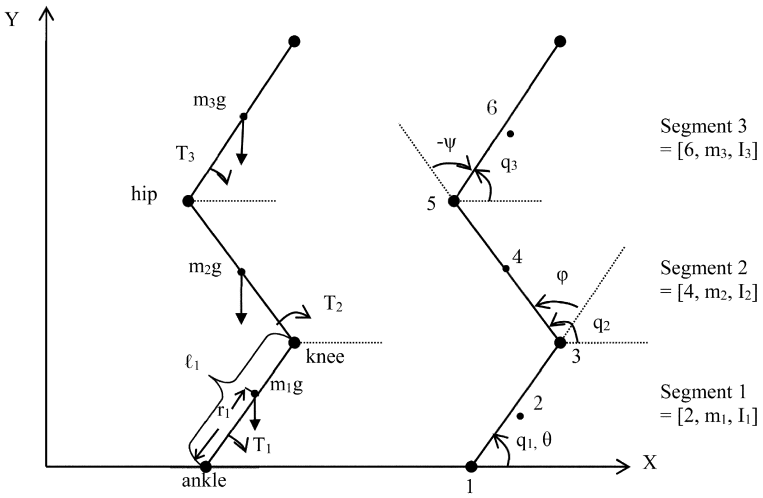
Appendix 1



Symbols:

ℓ_1: length of the lower leg

ℓ_2: length of the thigh

ℓ_3: length of the trunk

r_1: distance from the ankle to the center of gravity of the lower leg

r_2: distance from the knee to the center of gravity of the thigh

r_3: distance from the hip to the center of gravity of the head, arm, and trunk

m_1: mass of the lower legs

m_2: mass of the thighs

m_3: mass of the head, arm, and trunk

I_1: moment of inertia of the lower leg about center of gravity

I_2: moment of inertia of the thigh about center of gravity

I_3: moment of inertia of the head, arm, and trunk about the center of gravity

θ, $\dot{\theta}$, $\ddot{\theta}$: ankle angle, angular velocity, angular acceleration

ϕ, $\dot{\varphi}$, $\ddot{\varphi}$: knee angle, angular velocity, angular acceleration

ψ, $\dot{\psi}$, $\ddot{\psi}$: hip angle, angular velocity, angular acceleration

g: gravity acceleration

Basic equation:

Net torque (NET) = Muscle torque (MUS) + Gravity torque (G) + Interaction torque (INT)

MUS = NET – G – INT

Torque composition at each joint:

$$\mathrm{NET_h} = \ddot{\psi}(m_3 r_3^2 + I_3)$$

$$\mathrm{G_h} = m_3 g r_3 \cos(\theta + \varphi + \psi)$$

$$\mathrm{INT_h} = \mathrm{Centripetal_h} + \mathrm{Coriolis_h} + \mathrm{Inertial_h}$$

$$\mathrm{Centripetal_h} = m_3[\ell_1 r_3 \sin(\phi + \psi) + \ell_2 r_3 \sin\psi]\dot{\theta}^2 + m_3(\ell_2 r_3 \sin\psi)\dot{\phi}^2$$

$$\mathrm{Coriolis_h} = 2m_3(\ell_2 r_3 \sin\psi)\dot{\theta}\dot{\phi}$$

$$\mathrm{Inertial_h} = \{m_3[r_3^2 + \ell_1 r_3 \cos(\phi + \psi) + \ell_2 r_3 \cos\psi] + I_3\}\ddot{\theta} + \{m_3[r_3^2 + \ell_2 r_3 \cos\psi] + I_3\}\ddot{\phi}$$

$$\mathrm{NET_k} = \ddot{\varphi}[m_2 r_2^2 + m_3(\ell_2^2 + r_3^2 + 2\ell_2 r_3 \cos\psi) + I_2 + I_3]$$

$$\mathrm{G_k} = m_2 g r_2 \cos(\theta + \varphi) + m_3 g[\ell_2 \cos(\theta + \phi) + r_3 \cos(\theta + \phi + \psi)]$$

$$\mathrm{INT_k} = \mathrm{Centripetal_k} + \mathrm{Coriolis_k} + \mathrm{Inertial_k}$$

$$\mathrm{Centripetal_k} = \{m_2 \ell_1 r_2 \sin\varphi + m_3[\ell_1 \ell_2 \sin\phi + \ell_1 r_3 \sin(\phi + \psi)]\}\dot{\theta}^2 - m_3(\ell_2 r_3 \sin\psi)\dot{\psi}^2$$

$$\mathrm{Coriolis_k} = -2m_3(\ell_2 r_3 \sin\psi)\dot{\theta}\dot{\psi} - 2m_3(\ell_2 r_3 \sin\psi)\dot{\phi}\dot{\psi}$$

$$\mathrm{Inertial_k} = [m_2(r_2^2 + \ell_1 r_2 \cos\phi) + m_3\ell_2^2 + m_3(r_3^2 + \ell_1\ell_2 \cos\phi + \ell_1 r_3 \cos(\phi + \psi) + 2\ell_2 r_3 \cos\psi) + I_2 + I_3]\ddot{\theta} + [m_3(r_3^2 + \ell_2 r_3 \cos\psi) + I_3]\ddot{\psi}$$

$$\mathrm{NET_a} = \ddot{\theta}\{m_1 r_1^2 + m_2(\ell_2^2 + r_2^2 + 2\ell_1 r_2 \cos\phi) + m_3[\ell_1^2 + \ell_2^2 + r_3^2 + 2\ell_1\ell_2 \cos\phi + 2\ell_1 r_3 \cos(\phi + \psi) + 2\ell_2 r_3 \cos\psi] + I_1 + I_2 + I_3\}$$

$$\mathrm{G_a} = m_1 g r_1 \cos\theta + m_2 g[\ell_1 \cos\theta + r_2 \cos(\theta + \phi)] + m_3 g[\ell_1 \cos\theta + \ell_2 \cos(\theta + \phi) + r_3 \cos(\theta + \phi + \psi)]$$

$$\mathrm{INT_a} = \mathrm{Centripetal_a} + \mathrm{Coriolis_a} + \mathrm{Inertial_a}$$

$$\mathrm{Centripetal_a} = \{-m_2\ell_1 r_2 \sin\phi - m_3[\ell_1\ell_2 \sin\phi + \ell_1 r_3 \sin(\phi + \psi)]\}\dot{\phi}^2 - m_3[\ell_1 r_3 \sin(\phi + \psi) + \ell_2 r_3 \sin\psi]\dot{\psi}^2$$

$$\mathrm{Coriolis_a} = \{-2m_2\ell_1 r_2 \sin\phi - 2m_3[\ell_1\ell_2 \sin\phi + \ell_1 r_3 \sin(\phi + \psi)]\}\dot{\theta}\dot{\phi} - 2m_3[\ell_1 r_3 \sin(\phi + \psi) + \ell_2 r_3 \sin\psi]\dot{\theta}\dot{\psi} - 2m_3[\ell_1 r_3 \sin(\phi + \psi) + \ell_2 r_3 \sin\psi]\dot{\phi}\dot{\psi}$$

$$\mathrm{Inertial_a} = \{m_2(r_2^2 + \ell_1 r_2 \cos\phi) + m_3[\ell_2^2 + r_3^2 + \ell_1\ell_2 \cos\phi + \ell_1 r_3 \cos(\phi + \psi) + 2\ell_2 r_3 \cos\psi] + I_2 + I_3\}\ddot{\varphi} + \{m_3[r_3^2 + \ell_2 r_3 \cos\psi + \ell_1 r_3 \cos(\phi + \psi)] + I_3\}\ddot{\psi}$$