

Author Response: Radial and Tangential Retinal Magnifications as Functions of Visual Field Angle Across Spherical, Oblate, and Prolate Retinal Profiles

We thank Dr Simpson for his interest in our recent publication.¹ His letter does not formulate a focal argument, but alternates between endorsements of nodal points and the chief ray. As such, we respond by first attempting to clarify the comparison of nodal points and the chief ray; thereafter, we separately discuss his various comments and the literature cited in the letter.

Nodal Points and The Chief Ray

The letter states: “The nodal point is used ... this is an excellent point to use ... this is not directly due to paraxial properties.” The letter supports the nodal point method used in the article.¹ Although comments regarding why nodal points perform well in wide-field applications are interesting and well-taken, they are supplementary to the topic of the article. The phrasing of this comment implies that an assertion was made in our article regarding why nodal points perform well, however, this is not true. Where nodal points were introduced (page 3) and discussed (page 9), the article¹ stated that their sustained popularity in modern wide-field applications, and their suitability for our methods, are despite them technically being a paraxial concept.

The letter later states: “the fundamental optical characteristics of the eye come from rays that pass through the center of the pupil, and although angular scaling at the nodal point captures the essence of this optical system, this is because of its location, and not because of its paraxial properties.” Again, no assertion was made in our article¹ that nodal points work well because of paraxial properties. All light passing through the pupil determines the fundamental optical characteristics of the eye, not only the central ray or rays; this is especially true for light entering the eye from a wide-field angle and is also indicated in Figure 1 of the letter, where rays fill the pupil, including marginal rays on both extremes. The calculation of magnification—the focus of our article—is but one

dimension of the overall optical characteristics of an eye, which, like the many objective and subjective applications cited in the article¹ (pages 3 and 9) is well-served using nodal points.

Eye model calculations of retinal magnification using nodal points and using the chief ray have similar computational burdens and both require assumptions and simplifications of the optical media and surfaces. A comparison² of retinal magnification factors using both nodal points and real ray tracing methods has shown that ocular biometry can play an important role in the degree to which these two methods agree or differ. In that study, eyes of typical biometric dimensions, such as those with emmetropia or low magnitudes of refractive error, showed very little difference (0.1%–2.0%) in retinal magnifications calculated by the two methods, while an eye with considerable myopia (–14.5 diopters) showed the greatest difference. Although the ray path modeled through the nodal points (visual axis) can differ from the path traced through the entrance pupil (line of sight),³ these two paths ultimately tend to arrive at similar retinal locations—as acknowledged in both the letter and the article.¹

Discussion of Literature and Other Statements

The letter states: “A discussion by Atchison et al [5, p80] regarding defocused images specifically emphasizes the value of using central rays for magnification calculations, with nodal rays typically being blocked for smaller pupils.” The text from Atchison and Smith⁴ (page 80) specifically emphasized by Simpson states: “In this case of a defocused retinal image we should not use the nodal ray to determine the image size as we did in Chapter 6 for focused images.” This reference to defocused retinal images is largely irrelevant; Simpson seems to have missed the passages in our article¹ (pages 2, 3, and 9) referring to the use of retinal magnification

factors in advanced retinal imaging, which inherently requires well-focused images.^{5,6} Further, the passage in Atchison and Smith⁴ (page 80) referenced by Simpson directs the reader to their Chapter 6, which in turn says (page 53), “We can use the nodal ray to find the size of the retinal image.” This is similar to the approach that was followed.

The apparent (asymmetric) decrease in pupil size with increasing field angle is interesting and may have implications for nondilated applications. Again, this factor should be largely irrelevant for retinal imaging—especially when imaging the peripheral retina—because pupil dilation is still recommended to maximize spatial frequencies passed by the optical transfer function, to reduce diffraction, and to optimize image quality.⁷

The letter states: “The nodal point scaling provides a simple concept when the retina is spherical, because visual angles are mapped linearly to increasing distances along the retinal surface [4, 6].” This statement is incorrect. Drasdo and Fowler⁸ cited as reference 4 of the letter use an eye model with a spherical retina, yet the main finding of that article is “non-linear” mapping. Suheimat et al.⁹ cited as reference 6 of the letter derive a method of determining the distance along the retina from the fovea to the image location of a peripheral object point precisely because the mapping is non-linear. In agreement with those previous publications, Figure 2 of our article¹ illustrates non-linear mapping in eyes with spherical retinas. The publication¹ also acknowledges the only theoretical case where scaling is linear, which is where the retina is spherical and the secondary nodal point is at the retinal center of curvature. As stated,¹ “In most eye models that assume a spherical retina, the nodal point is located anterior to the retinal center of curvature”—this location is true for the eye models used by Drasdo and Fowler and by Suheimat et al., which result in nonlinear mappings of visual angles to distances along the retinal surface.

The letter contains some contradictory comments. First, an assertion is made that: “it is the chief ray that passes through the center of the physical pupil that indicates the main image location.” However, that assertion is qualified later in the same paragraph: “Aberrations may also affect the exact characteristics of the image spot, but the chief ray is normally a useful reference.” The caption of Figure 1 of the letter contradicts both preceding statements: “An unrelated line drawn through NP2 at the input angle identifies the main image point.” As stated in the article¹ (page 1), the retinal image is more complicated than is represented in one-dimensional ray diagrams where the image location is simply indicated by the chief ray. Aberrations (including defocus and astigmatism)

certainly affect the characteristics of the image spot and also determine where the point of greatest intensity (centroid) forms on the retina. Retinal point spread functions are typically rotationally asymmetric and can even be multimodal (having multiple points of maximum intensity), which obviously challenges the blanket notions of either the chief ray or the nodal ray determining the most salient image location.

Two annotations on Figure 1 of the letter contradict each other. The first says: “The line (not an actual ray) joining 2nd nodal point and image centroid is approximately parallel to input rays.” A second says: “Nodal points are defined for small angles, where an input ray heading towards NP1 is refracted to become a parallel output ray that appears to come from NP2.” As such, the intention of Figure 1 of the letter is unclear. If it meant to make the point that nodal rays are not real rays, this is obvious by their definition, as was indicated by referring to them as theoretical in the first paragraph of the Discussion of our article.¹ In that case, the first annotation on Figure 1 of the letter is true and the second is false.

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