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Research article

Analyzing risk factors of tuberculosis using type-2 interval-valued trapezoidal fuzzy numbers with Einstein aggregation operators extended to MCDM

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ABSTRACT

The principal motive of this work is to evolve and initiate an extension from interval-valued fuzzy sets to type-2 interval-valued fuzzy sets (T2IVFS) related to weighted aggregation functions containing the Einstein operator. The chief reason for this extension is that the constancy of the terms can also be taken into data during the aggregation operation. The main goal of this article is to compose the aggregation operators and their characteristics such as the Type-2 interval-valued fuzzy Einstein weighted arithmetic aggregating operator (T2IVFEWA), Type-2 interval-valued fuzzy Einstein weighted geometric aggregating operator (T2IVFEWG), and the characteristics are expressed. At last, to intimate the effectiveness of the suggested approach and explicate the purpose of these operators, a hybrid multi-criteria decision-making problem (MCDM) to select the best risk factor for Tuberculosis (TB) is considered and the result is compared with the outcome of the existing operators and methods. Additionally, a sensitivity analysis was conducted to verify the robustness of the proposed decision-making process.

1. Introduction

Human beings, as integral components of the ecosystem have always been susceptible to bacterial and viral infectious diseases originating from other organisms. Over the past three years, the COVID-19 virus has profoundly impacted human health, presenting unprecedented challenges to medical practitioners and scientists. The rapid spread and high mortality rate of COVID-19 have captured global attention, but it is important to recognize that numerous other infectious agents exhibit similar lethality, albeit with differing rates and durations. One such example is tuberculosis (TB), caused by Mycobacterium tuberculosis, which continues to be a significant health concern worldwide.

The World Health Organization (WHO) [42] reported that more than 1.6 million people died from TB in 2021. India has been working to reduce the incidence of TB for the last five decades, yet it continues to be one of the most severe diseases. Every year, 480,000 people die from TB in India, which translates to more than 1,400 deaths every day.

TB found worldwide, is a curable disease that impacts communities across all continents. Despite its prevalence, effective treatments are available to manage and alleviate the symptoms of TB, offering hope for those affected. Awareness and access to healthcare resources play crucial roles in combating the spread of this infectious illness on a global scale. TB affects the human lungs (Pulmonary TB) and gradually other parts of the body. In 2017, Churchyard et al. [8] explained the transmission of TB and how it spreads from

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person to person. They particularly expounded on who can get TB easily under various circumstances. The following year, Nechaeva et al. [38] conducted research on TB patients in Russia, predicting the progression of TB at various levels of infection. In 2020, Makarov et al. [33] developed the maczone injection for TB, designed to prevent bacterial infections and provide effective protection. Makarov highlighted the challenges of manufacturing injections for bacterial diseases, especially TB. Natarajan et al. [37] provided a comprehensive review of TB, classifying it into seven types: Military TB, TB lymphadenitis, pleural TB, abdominal TB, CNS TB, skeletal TB, and genito-urinary TB. They also explained the molecular methods to cure TB. In 2021, Nyarko et al. [39] presented an overview of TB, describing it as a global disease affecting people in every country. Behera [4] illustrated the impact of TB during the COVID-19 era in India, noting that the COVID-19 pandemic increased the incidence of TB.

Analysis of prior research articles reveals that TB is characterized by unpredictability, with its manifestation influenced by diverse immune systems. Understanding these complexities is essential for devising effective strategies for TB prevention and treatment. Although TB impacts the health systems of every country, it is a treatable disease. Identifying the risk of TB involves certain unidentified factors, where fuzzy systems can be effectively employed.

The concept of fuzzy sets was first established by Zadeh [59] to address the uncertainty of elements in the set, extending classical sets. Fuzzy set theory measures uncertainty using a membership function for each value in the real unit interval $[0,1]$. Zadeh extended the type-1 fuzzy set into the type-2 fuzzy set in 1975 [60], with type-2 fuzzy sets offering inconsistent membership values, allowing for a more nuanced representation of uncertainty. Type-2 fuzzy sets are crucial for handling higher levels of uncertainty and imprecision in data compared to type-1 fuzzy sets. They offer enhanced modeling flexibility by accommodating the uncertainty in the membership functions themselves. This makes them particularly valuable in complex real-world applications, such as control systems, decision-making, and medical diagnosis. Their ability to manage uncertainty leads to more robust and reliable system performance. Additionally, type-2 fuzzy sets provide a more accurate representation of ambiguous information, improving the quality of results in fuzzy logic-based systems.

In 1970, Bellman and Zadeh [5] introduced the fuzzy multi-criteria decision-making (MCDM) technique, utilizing linguistic variables for estimation. MCDM is divided into multi-attribute decision-making (MADM) and multi-objective decision-making (MODM), with significant applications in decision-making processes. MCDM is essential for evaluating and prioritizing multiple conflicting criteria in complex decision-making scenarios. It aids in systematically comparing alternatives, enhancing objectivity and transparency in the decision process. MCDM techniques are widely used in various fields, such as business, engineering, and environmental management, to optimize resource allocation and achieve balanced outcomes. They help decision-makers handle trade-offs and uncertainties effectively, leading to more informed and rational choices.

Recently, Liao et al. [30] reviewed extensions of fuzzy numbers with MCDM techniques, highlighting recent applications. Notable examples include in recent works, fuzzy relation inequalities [58] have been employed to model systems with uncertain information [56]. The authors [17] introduce a novel approach to group decision-making. The method utilizes the cubic Fermatean Einstein fuzzy weighted operator to aggregate individual opinions while effectively handling uncertainty and imprecision in the decision-making process. This technique is particularly advantageous for complex scenarios where traditional methods may fall short. The authors [34] propose a novel MCDM method tailored for cubic hesitant fuzzy sets. This method leverages Einstein's operational laws to effectively aggregate and compare multiple criteria in decision-making processes. The approach enhances the handling of hesitancy and uncertainty in fuzzy data, offering a more flexible and accurate decision-making framework. The paper provides theoretical foundations and practical applications, showcasing the method's efficiency and robustness in various decision-making scenarios. These techniques enhance decision-making based on the Einstein aggregation operator on fuzzy information.

However, these techniques have not fully addressed the complexity of medical diagnosis systems. TB is often confirmed only in later stages due to its common symptoms, such as cough, headache, and weight loss, making early detection challenging. Fuzzy techniques can predict TB stages based on symptom severity. Determining accurate risk factors for TB involves addressing the chaotic situations faced by patients and doctors in identifying the risk environment. MCDM methods are employed to find precise results for TB risk factors.

When joining type-2 fuzzy sets and MCDM methods some recent advancements offer a powerful characterization of uncertainty, enabling more accurate decision-making in complex environments. For example, Abdul et al. [2] enhanced TB detection using a GIS-based model with 76% accuracy, and De et al. [10] used a weighted linear combination to rank TB risk factors. Cui et al. [9] reviewed TB in India, China, and the US using joint point analysis, and Mousquer et al. [36] analyzed hybrid TB and COVID-19 cases in crowded areas.

According to these known reviews, the integration of type-2 fuzzy numbers with MCDM methods demonstrates significant potential. This combination effectively addresses the complexities associated with uncertainty and imprecision in decision-making scenarios. Consequently, it enhances the accuracy and reliability of the outcomes, making it a valuable approach in various multi-criteria decision-making applications. This work intended to analyze the risk factors of TB with Type-2 trapezoidal fuzzy numbers with Einstein aggregation in Analytic hierarchy process (AHP) [48], Technique for order preference by similarity to ideal solution (TOPSIS) [23], and VIsekriterijumsko KOmpromisno Rangiranje (VIKOR) [41] enhances decision-making by integrating comprehensive risk assessments, fostering more effective prevention and treatment strategies.

1.1. Review on AHP, TOPSIS and VIKOR methods

AHP provides a systematic approach for organizing and analyzing complex decisions by breaking them down into a hierarchy of subproblems. It allows for the incorporation of both qualitative and quantitative data, making it versatile for various decision contexts. The authors [6] discuss how AHP can provide insights into the factors contributing to the complexity of medical diagnoses, such as the number of symptoms, rarity of the condition, diagnostic uncertainty, and treatment options. [31] contributes to advancing both theoretical understanding and practical applications in healthcare decision-making, with a specific focus on enhancing diagnostic processes for bladder patients with hematuria. The authors [53] introduce an AHP-based model for improving the diagnostic process of typhoid fever. The study aims to systematically prioritize diagnostic criteria such as symptoms, laboratory tests, and patient history using AHP's hierarchical decision-making framework.

TOPSIS is used to identify solutions from a finite set of alternatives that are closest to the ideal solution and farthest from the negative ideal solution (NIS). It provides a simple ranking of alternatives based on their relative closeness to the ideal solution. The authors [13] integrate the Particle Swarm optimization algorithm with the TOPSIS to enhance decision-making processes and the hybrid approach aims to optimize MCDM by effectively selecting the best solutions from a set of alternatives. The study demonstrates improved accuracy and efficiency in solving complex optimization problems using this combined method.

Piegat et al. [43] using TOPSIS and AHP to prioritize liver transplants, Ali et al. [3] analyzing vector-borne diseases with geospatial techniques, and Ghorui et al. [21] identifying COVID-19 risk factors with hybrid techniques. Combined MCDM methods, such as fuzzy TOPSIS with entropy [18], and generalized interval-valued bipolar neutrosophic Einstein fuzzy aggregation operators [12], offer robust decision-making capabilities. By using the TOPSIS method the authors [63] propose a systematic approach to evaluate and select medical clinics based on multiple criteria, such as expertise, equipment, location, and patient satisfaction. By employing TOPSIS, the study aims to assist patients and healthcare providers in making informed decisions regarding clinic selection for disease diagnosis, thereby optimizing healthcare service delivery and patient outcomes. The author [15] presents the triangular cubic linguistic uncertain fuzzy TOPSIS method, an advanced decision-making approach integrating triangular cubic fuzzy numbers and linguistic variables to handle uncertainty in group decision-making contexts. The method enhances the traditional TOPSIS by incorporating fuzzy logic to better capture the ambiguity in human judgment.

VIKOR can effectively deal with incomplete information, making it robust in real-world scenarios with data limitations. It supports group decision-making by aggregating individual preferences into a collective decision. The authors [16] propose a cubic fuzzy MADM approach using an extended VIKOR method. This method incorporates cubic fuzzy numbers to better handle uncertainty and imprecision in the decision-making process. The approach is applied to a practical problem of plant location selection, demonstrating how it effectively integrates multiple criteria and group preferences to identify the most suitable site. By extending the traditional VIKOR method with cubic fuzzy logic, the authors enhance its capability to provide balanced and accurate decisions in complex, uncertain environments.

The authors [22] introduce a novel doctors ranking system utilizing the VIKOR method to evaluate and rank medical professionals. This system integrates multiple performance criteria to address conflicting aspects of doctor assessments, providing a balanced and comprehensive ranking. The VIKOR method's ability to determine compromise solutions is leveraged to rank doctors in a way that considers both the best and worst performance scenarios. The study demonstrates the effectiveness of this method through practical application, showcasing improved decision accuracy and reliability in the medical field. Through this literature review, we recognize the significance of these methods, thereby determining their potential applicability and effectiveness in the medical diagnosis field.

1.2. Motivation

- **Early detection and treatment**:
  - Understanding the risk factors for TB in India is crucial due to its endemic nature and persistent burden on public health. Unlike COVID-19, which emerged as a pandemic, TB has been a longstanding health issue in India, causing significant morbidity and mortality annually. Knowing TB risk factors helps in targeted interventions to reduce transmission, improve early diagnosis, and optimize treatment outcomes, thereby alleviating the long-term burden on healthcare resources and improving overall population health.
  - Knowing the risk factors aids in early detection and timely treatment. Health professionals can monitor high-risk populations more closely, ensuring that TB cases are identified and treated promptly, thereby reducing the spread of the disease.
- **Integration of Fuzzy Sets and Aggregation Operators**:
  - The combination of fuzzy sets with aggregation operators has expanded rapidly in recent years.
  - Type-2 fuzzy sets introduced by Zadeh [61], offers a more nuanced representation of uncertainty, leading to accurate modeling of complex systems.
- **Survey and Role of Aggregation Operators**:
  - Valdez et al. [54] emphasized the critical role of aggregation operators in systems through their survey on type-2 fuzzy logic controller design with optimization methods.
- **Research on Aggregation Operators**:
  - Various researchers have explored Einstein aggregation operators for different fuzzy sets:
    - Fermatean fuzzy sets [45]
    - Q-rung ortho-pair fuzzy sets [64]
    - Pythagorean fuzzy hypersoft sets [65]
    - Trapezoidal cubic fuzzy sets [14]
- **Extension and New Functions**:
  - Extension operators for type-2 fuzzy sets were analyzed by [25], and [57] presented a new type-2 function for decision-making problems.

- **Adaptability and Performance**:
  - The flexibility of Type-2 fuzzy logic systems with the Einstein aggregation operator allows adaptation to dynamic environments.
  - This adaptability ensures effective performance even under changing circumstances.
- **Research Gap**:
  - While various types of operators have been applied to different fuzzy sets, there is a need for higher expressiveness in capturing uncertainty in real-world systems.
  - Therefore, the Einstein aggregation operators are utilized in conjunction with type-2 interval-valued trapezoidal fuzzy numbers ($T2IVTrFN$).

1.3. Significance of the study

Analyzing and ranking the risk factors for TB, employing $T2IVTrFN$ with Einstein Aggregation Operators in MCDM Methods offers several distinct advantages:

- **Enhanced Uncertainty Modeling**:
  - **T2IVTrFN** provides a sophisticated mechanism to capture higher degrees of uncertainty and variability in data. Unlike traditional Type-1 fuzzy sets, T2IVTrFN accommodates a broader range of membership values, thus offering a more accurate representation of uncertain parameters associated with TB risk factors.
- **Superior Aggregation with Einstein Operators**:
  - **Einstein aggregation operators** enhance the integration of multiple criteria by effectively handling the non-linearity and interaction effects among criteria. This ensures that the aggregation process considers the complex relationships and dependencies between different TB risk factors, leading to more robust and reliable rankings.
- **Applicability of Advanced MCDM Methods**:
  - **AHP**: AHP helps in structuring complex decision problems into a hierarchy, making it easier to quantify the relative importance of each TB risk factor and prioritize them systematically.
  - **TOPSIS**: TOPSIS identifies solutions that are closest to the ideal solution and farthest from the worst-case scenario, thus providing a clear ranking of TB risk factors based on their relative performance.
  - **VIKOR**: VIKOR focuses on ranking and selecting from a set of alternatives, addressing conflicting criteria, and aiming for a compromise solution. It is particularly useful in scenarios where decision-makers need to balance multiple risk factors simultaneously.
- **Significance in Combatting TB**:
  - **Stopping the Spread of TB**: By accurately ranking TB risk factors, healthcare providers can prioritize interventions, allocate resources more effectively, and implement targeted strategies to mitigate the most significant risks. This proactive approach is crucial in preventing the spread of TB and reducing its incidence.
  - **Shaping a Healthy India**: Implementing these advanced decision-making tools can significantly contribute to public health efforts in India, where TB remains a major health challenge. By enhancing the precision and effectiveness of TB control measures, these methods can help achieve the goal of a TB-free India.
  - **Global Impact**: The methodologies and insights derived from this approach are not limited to India. They can be adapted and applied to other developing countries facing similar TB challenges, thus contributing to global health improvements and supporting international efforts to combat TB.

The integration of Type-2 Interval-Valued Trapezoidal Numbers with Einstein aggregation operators into MCDM methods such as AHP, TOPSIS, and VIKOR represents a significant advancement in the analysis and ranking of TB risk factors. This approach offers a precise and effective framework for decision-making, ultimately contributing to the eradication of TB and promoting healthier communities both in India and worldwide.

1.4. Contribution

A type-2 interval-valued fuzzy set, defined here requires the mirroring of trapezoidal numbers. When it comes to the real-life execution of decision-making, it proves that interdisciplinary collaboration can lead to a richer understanding of fuzzy systems and their applications. Type-2 fuzzy trapezoidal numbers introduce a higher level of complexity compared to type-1 fuzzy numbers. This complexity can be intellectually stimulating and provide opportunities for innovative problem-solving.

1. The combination of type-2 fuzzy trapezoidal number and Einstein aggregation operators can enhance the robustness of decision-making processes. By aggregating information from type-2 fuzzy sets using Einstein aggregation operators, decision-makers can obtain more reliable and comprehensive results, even in situations with conflicting and ambiguous information.
2. Aggregating operators for the $T2IVFTrN$ are derived and their properties are provided.
3. AHP, TOPSIS, and VIKOR methods utilized and modified for $T2IVFTrN$ with Einstein aggregating operators.
4. A decision-making problem is taken as a case study to find the competence and performance of the proposed method.
5. When comparing with other existing works [45] and [55], this work provides the accuracy for each alternative.

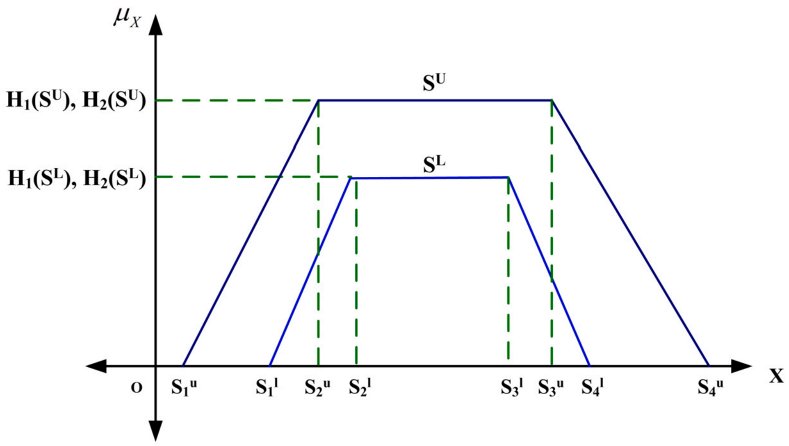


Fig. 1. Type-2 fuzzy number representation with heights.

1.5. Structure

This article is structured as: Section 2 expounds the fundamental concepts of $T2IVTrFNs$ with arithmetic operations. Section 3 explores the basic concepts of new $T2IVTrFN's$ with Einstein aggregating operators. Section 4 explains type-2 interval-valued trapezoidal fuzzy Einstein weighted arithmetic operations with examples. Section 5 describes type-2 interval-valued trapezoidal fuzzy Einstein weighted geometric operations with example. Section 6 explicates the algorithm of AHP, TOPSIS, and VIKOR to find the rankings risk factors of TB. Section 7 illustrates the case study problem which is demonstrated with type-2 fuzzy trapezoidal number with Einstein aggregating operators. At last, Section 8 clarifies the rankings of AHP, TOPSIS, and VIKOR with a fuzzy inference system.

2. Preliminaries

In this section, a few fundamental definitions and the mathematical expressions of $T2IVTrFN$ were proposed based on Einstein aggregation operators.

Definition 1. [60] proposed the origination of type-n fuzzy sets. Which is defined as follows:
A set $\tilde{\tilde{F}}$ is defined as a fuzzy set whose element has membership between [0,1].

$$\tilde{\tilde{F}} = \{(x,u), \mu_{\tilde{\tilde{F}}}(x,u) | \forall x \in X, \forall u \in U_x \subseteq [0,1],$$
$$0 \le \mu_{\tilde{\tilde{F}}}(x,u) \le 1\}.$$

Where, U_x denotes an interval in $[0,1]$. Moreover, the type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows

$$\tilde{\tilde{S}} = \int_{x\in S} \int_{u\in U_x} \mu_{\tilde{\tilde{F}}}(x,u)/(x,u)$$

Where, $U_x \subseteq [0,1]$ and $\int\int$ denotes union over all admissible x and u.

Definition 2. Let $\tilde{\tilde{S}}$ be a type-2 fuzzy set [35] in the universe of discourse S represented by the type-2 membership function $\mu_{\tilde{\tilde{S}}}$. If all $\mu_{\tilde{\tilde{S}}}(x,u) = 1$, then $\tilde{\tilde{S}}$ is called an interval type-2 fuzzy set. An interval type-2 fuzzy set $\tilde{\tilde{S}}$ could be esteemed as a particular case of a type-2 fuzzy set, illustrated as follows:

$$\tilde{\tilde{S}} = \int_{x\in S} \int_{u\in U_x} 1/(x,u).$$

Where, $U_x \subseteq [0,1]$.

The upper membership function and the lower membership function of an interval type-2 fuzzy set are type-1 membership functions, respectively. Fig. 2 shows a trapezoidal interval type-2 fuzzy number. H_1 and H_2 denotes the lower and upper heights of $T2IVTrFN$ and denoted in the Fig. 1

$$\tilde{\tilde{S}}_i = (\tilde{\tilde{S}}_i^L, \tilde{\tilde{S}}_i^U),$$

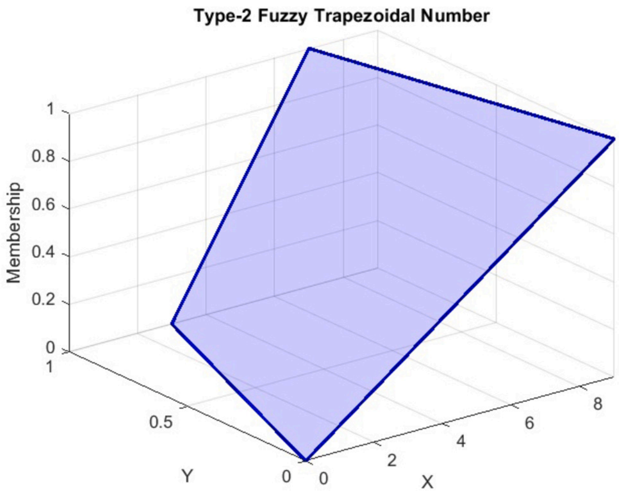


Fig. 2. 3 dimensional view of type-2 fuzzy trapezoidal number.

$$(\tilde{\tilde{S}}_i^L, \tilde{\tilde{S}}_i^U) = ((s_{i1}^l, s_{i2}^l, s_{i3}^l, s_{i4}^l; H_1(S_i^l), H_2(S_i^l)), ((s_{i1}^u, s_{i2}^u, s_{i3}^u, s_{i4}^u; H_1(S_i^u), H_2(S_i^u))).$$

2.1. Arithmetic operations on type-2 fuzzy sets

Definition 3. Let $\tilde{\tilde{S}}_1$ and $\tilde{\tilde{S}}_2$ be two $T2IVTrFN$ defined as [29] follows:

$$\begin{aligned}
\tilde{\tilde{S}}_1 &= (\tilde{\tilde{S}}_1{}^L, \tilde{\tilde{S}}_1{}^U),\\
&= (s_{11}^l, s_{12}^l, s_{13}^l, s_{14}^l; H_1(S_1^l), H_2(S_1^l)), (s_{11}^u, s_{12}^u, s_{13}^u, s_{14}^u; H_1(S_1^u), H_2(S_1^u)),\\
\tilde{\tilde{S}}_2 &= (\tilde{\tilde{S}}_2{}^L, \tilde{\tilde{S}}_2{}^U),\\
&= (s_{21}^l, s_{22}^l, s_{23}^l, s_{24}^l; H_1(S_2^l), H_2(S_2^l)), (s_{21}^u, s_{22}^u, s_{23}^u, s_{24}^u; H_1(S_2^u), H_2(S_2^u)),\\
\tilde{\tilde{S}}_1 \oplus \tilde{\tilde{S}}_2 &= ((s_{11}^l + s_{21}^l, s_{12}^l + s_{22}^l, s_{13}^l + s_{23}^l, s_{14}^l + s_{24}^l); \min(H_1(S_1^l), H_1(S_2^l)), \max(H_2(S_1^l), H_2(S_2^l))),\\
&\quad ((s_{11}^u + s_{21}^u, s_{12}^u + s_{22}^u, s_{13}^u + s_{23}^u, s_{14}^u + s_{24}^u); \min(H_2(S_1^u), H_1(S_2^u)), \max(H_2(S_1^u), H_2(S_2^u))\\
\tilde{\tilde{S}}_1 \ominus \tilde{\tilde{S}}_2 &= (\tilde{\tilde{S}}_1{}^L, \tilde{\tilde{S}}_1{}^U) \ominus (\tilde{\tilde{S}}_2{}^L, \tilde{\tilde{S}}_2{}^U),\\
&= ((s_{11}^l - s_{24}^l, s_{12}^l - s_{23}^l, s_{13}^l - s_{22}^l, s_{14}^l - s_{21}^l); \min(H_1(S_1^l), H_1(S_2^l)), \max(H_2(S_1^l), H_2(S_2^l))),\\
&\quad ((s_{11}^u - s_{24}^u, s_{12}^u - s_{23}^u, s_{13}^u - s_{22}^u, s_{14}^u - s_{21}^u); \min(H_1(S_1^u), H_1(S_2^u)), \max(H_2(S_1^u), H_2(S_2^u))),\\
\tilde{\tilde{S}}_1 \otimes \tilde{\tilde{S}}_2 &= (\tilde{\tilde{S}}_1{}^L, \tilde{\tilde{S}}_1{}^U) \otimes (\tilde{\tilde{S}}_2{}^L, \tilde{\tilde{S}}_2{}^U),\\
&= ((s_{11}^l \times s_{21}^l, s_{12}^l \times s_{22}^l, s_{13}^l \times s_{23}^l, s_{14}^l \times s_{24}^l); \min(H_1(S_1^l), H_1(S_2^l))),\\
&\quad ((s_{11}^u \times s_{21}^u, s_{12}^u \times s_{22}^u, s_{13}^u \times s_{23}^u, s_{14}^u \times s_{24}^u); \max(H_2(S_1^u), H_2(S_2^u))).\\
\tilde{\tilde{S}}_1 \otimes q &= (\tilde{\tilde{S}}_1{}^L, \tilde{\tilde{S}}_1{}^U) \otimes (q)\\
&= ((s_{11}^l \times q, s_{12}^l \times q, s_{13}^l \times q, s_{14}^l \times q); (H_1(S_1^l), H_1(S_2^l))),\\
&\quad ((s_{11}^u \times q, s_{12}^u \times q, s_{13}^u \times q, s_{14}^u \times q; (H_2(S_1^u), H_2(S_2^u))),\\
\frac{\tilde{\tilde{S}}_1}{q} &= \frac{(\tilde{\tilde{S}}_1{}^L, \tilde{\tilde{S}}_1{}^U)}{q}\\
&= (\frac{1}{q} \times s_{11}^l, \frac{1}{q} \times s_{12}^l, \frac{1}{q} \times s_{13}^l, \frac{1}{q} \times s_{14}^l; H_1(S_1^l), H_2(S_1^l)).\\
&\quad (\frac{1}{q} \times s_{11}^u, \frac{1}{q} \times s_{12}^u, \frac{1}{q} \times s_{13}^u, \frac{1}{q} \times s_{14}^u; H_1(S_1^u), H_2(S_1^u)),
\end{aligned}$$

where $q > 0$.

2.2. t-norm and t-co norm

To inspect the membership functions of the two $T2IVTrFN$, the generic Einstein t-norm and t-conorm [62] for two real numbers a_1 and a_2 are defined as,

$$T(a_1,a_2)=\frac{a_1+a_2}{1+a_1a_2}$$
$$S(a_1,a_2)=\frac{a_1a_2}{1-(1-a_1)(1-a_2)}$$

In Section 3, formulations employing the Einstein aggregation operator were developed to address the MCDM problem using $T2IVTrFN's$.

3. Arithmetic operations on $T2IVTrFN$ with Einstein aggregation operator

This section expounds on the arithmetic operations of $T2IVTrFN$ through the Einstein aggregating operator.

Definition 4. Let $\tilde{\tilde{S}}_1$ and $\tilde{\tilde{S}}_2$ be two $T2IVTrFN$ with Einstein aggregation operator is defined as, $\tilde{\tilde{S}}_1+\tilde{\tilde{S}}_2$

$$\begin{aligned}
\tilde{\tilde{S}}_1 &= (\tilde{\tilde{S}}_1{}^L,\tilde{\tilde{S}}_1{}^U),\\
&= (s^l_{11},s^l_{12},s^l_{13},s^l_{14};H_1(S^l_1),H_2(S^l_1)),(s^u_{11},s^u_{12},s^u_{13},s^u_{14};H_1(S^u_1),H_2(S^u_1)),\\
\tilde{\tilde{S}}_2 &= (\tilde{\tilde{S}}_2{}^L,\tilde{\tilde{S}}_2{}^U),\\
&= (s^l_{21},s^l_{22},s^l_{23},s^l_{24};H_1(S^l_2),H_2(S^l_2)),(s^u_{21},s^u_{22},s^u_{23},s^u_{24};H_1(S^u_2),H_2(S^u_2)),
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{S}}_1+\tilde{\tilde{S}}_2 = \{<(&\frac{s^l_{11}+s^l_{21}}{1+s^l_{11}s^l_{21}}),(\frac{s^l_{12}+s^l_{22}}{1+s^l_{12}s^l_{22}}),(\frac{s^l_{13}+s^l_{23}}{1+s^l_{13}s^l_{23}}),(\frac{s^l_{14}+s^l_{24}}{1+s^l_{14}s^l_{24}}),\\
&\min(H_1(S^l_1),H_1(S^l_2),\max(H_2(S^l_1),H_2(S^l_2))>,\\
&<(\frac{s^u_{11}+s^u_{21}}{1+s^u_{11}s^u_{21}}),(\frac{s^u_{12}+s^u_{22}}{1+s^u_{12}s^u_{22}}),(\frac{s^u_{13}+s^u_{23}}{1+s^u_{13}s^u_{23}}),(\frac{s^u_{14}+s^u_{24}}{1+s^u_{14}s^u_{24}}),\\
&\min(H_1(S^u_1),H_1(S^u_2),\max(H_2(S^u_1),H_2(S^u_2))>\}
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{S}}_1*\tilde{\tilde{S}}_2 = \{<(&\frac{s^l_{11}\times s^l_{21}}{1-(1-s^l_{11})(1-s^l_{21})}),(\frac{s^l_{12}\times s^l_{22}}{1-(1-s^l_{12})(1-s^l_{22})}),(\frac{s^l_{13}\times s^l_{23}}{1+s^l_{13}s^l_{23}}),(\frac{s^l_{14}\times s^l_{24}}{1-(1-s^l_{14})(1-s^l_{24})}),\\
&\min((H_1(S^l_1),H_1(S^l_2)),\max(H_2(S^l_1),H_2(S^l_2))>,\\
&<(\frac{s^u_{11}\times s^u_{21}}{1-(1-s^u_{11})(1-s^u_{21})}),(\frac{s^u_{12}\times s^u_{22}}{1-(1-s^u_{12})(1-s^u_{22})}),(\frac{s^u_{13}\times (s^u_{23}}{1-(1-s^u_{13})(1-s^u_{23}}),(\frac{s^u_{14}\times s^u_{24}}{1-(1-s^u_{14})(1-s^u_{24})}),\\
&\min(H_1(S^u_1),H_1(S^u_2)),\max(H_2(S^u_1),H_2(S^u_2))>\}
\end{aligned}$$

4. Type-2 interval valued fuzzy Einstein weighted arithmetic aggregating operator

This section is providing the Type-2 interval valued fuzzy weighted arithmetic aggregation operator and some properties of corresponding operator.

Definition 5. Consider a set of m $T2IVTrFNs$ $\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am}$ in a universe of discourse $\tilde{\tilde{A}}$. The vectors $\vec{\lambda}_1,\vec{\lambda}_2,\vec{\lambda}_3,\ldots,\vec{\lambda}_m$ are aggregated using the function $T2IVTrFEWA$,

$$T2IVTrFEWA=\tilde{\tilde{A}}^n\mapsto\tilde{\tilde{A}}$$

The operator is defined by

$$T2IVTrFEWA(\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am})=\sum_{j=1}^{m}(\vec{\lambda}_j\tilde{\tilde{\alpha}}_{mj})$$

Where $\vec{\lambda}=\vec{\lambda}_1,\vec{\lambda}_2,\vec{\lambda}_3,\ldots\vec{\lambda}_m$ is the weight vector in which $\vec{\lambda}_j\in[0,1]$ and $\sum_{j=1}^{m}\vec{\lambda}_j=1$. The aggregation operator can also written as

$$T2IVTrFEWA(\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am})=\sum_{j=1}^{m}\vec{\lambda}_j(\tilde{\tilde{\alpha}}^u_{1j},\tilde{\tilde{\alpha}}^l_{1j}).$$

The $T2IVTrFEWA$ operator can be made into an arithmetic average operator when we consider the weight as $\frac{1}{n}$ which is written as

$$T2IVTrFWA(\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am})=\frac{1}{n}\sum_{j=1}^{m}(\tilde{\tilde{a}}^{u}_{1j},\tilde{\tilde{a}}^{l}_{1j})$$

Einstein aggregation operators are introduced to capitalize on the distinctive characteristics of Einstein operations when managing ambiguous data. It is well-known that the Einstein operations, which are derived from the Einstein sum and product, are capable of preserving boundedness and smoothness, which are essential for addressing complex and uncertain data. These properties assist in mitigating the overestimation and underestimation issues that may arise during conventional arithmetic operations. Furthermore, Einstein's operations offer a more generalized and adaptable framework for combining imprecise data, enabling more effective consideration of the inherent uncertainties and interdependencies among attributes in decision-making scenarios.

Theorem 1. *Take m $T2IVTrFNs$ $\{\tilde{\tilde{\alpha}}_1,\tilde{\tilde{\alpha}}_2,\tilde{\tilde{\alpha}}_3,\ldots\tilde{\tilde{\alpha}}_m\}$; the valuation acquired after aggregating these m $T2IVITrFNs$ by the Einstein aggregation operator is again a $T2IVTrFN$.*

Proof. Identifying that $|\tilde{\tilde{\alpha}}_{mj}|\le 1$ for all $j=1,2,\ldots,m$ and $\sum_{j=1}^{m}\vec{\lambda}_j=1$, it is captured as follows

$$\begin{aligned}|\vec{\lambda}_1\tilde{\tilde{\alpha}}_{k1}+\vec{\lambda}_1\tilde{\tilde{\alpha}}_{k2}+\vec{\lambda}_1\tilde{\tilde{\alpha}}_{k3}+\ldots\vec{\lambda}_1\tilde{\tilde{\alpha}}_{km}| &\le \vec{\lambda}_1.|\tilde{\tilde{\alpha}}_{k1}|+\vec{\lambda}_2.|\tilde{\tilde{\alpha}}_{k2}|+\vec{\lambda}_3.|\tilde{\tilde{\alpha}}_{k3}|+\ldots+\vec{\lambda}_m.|\tilde{\tilde{\alpha}}_{km}|\\ &\le \vec{\lambda}_1+\vec{\lambda}_2+\vec{\lambda}_3+\ldots+\vec{\lambda}_m\\ &=1.\end{aligned}$$

It is simple that $T2IVTrFWA(\vec{\lambda}_1.|\tilde{\tilde{\alpha}}_{k1}|+\vec{\lambda}_2.|\tilde{\tilde{\alpha}}_{k2}|+\vec{\lambda}_3.|\tilde{\tilde{\alpha}}_{k3}|+\ldots+\vec{\lambda}_q.|\tilde{\tilde{\alpha}}_{kq}|)\le 1$ is also a $T2IVTrFN$. □

Theorem 2. *Take $\vec{\lambda}_1|\tilde{\tilde{\alpha}}_{a1}|+\vec{\lambda}_2|\tilde{\tilde{\alpha}}_{a2}|+\vec{\lambda}_3|\tilde{\tilde{\alpha}}_{a3}|\ldots+\vec{\lambda}_m|\tilde{\tilde{\alpha}}_{am}|$ to be m $T2IVTrFN$ and $\sum_{j=1}^{m}\vec{\lambda}_j=1$, then the following holds,*

1. *(Idempotency) When $\tilde{\tilde{\alpha}}_{a1}=\tilde{\tilde{\alpha}}_{a2}=\tilde{\tilde{\alpha}}_{a3}\ldots\tilde{\tilde{\alpha}}_{ak}=\tilde{\tilde{\alpha}}$, the operator becomes*

   $$T2IVTrFEWA(\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3}\ldots\tilde{\tilde{\alpha}}_{ak})=\tilde{\tilde{\alpha}}$$
2. *(Boundedness) Examine $p=\max_j|\tilde{\tilde{\alpha}}_{mj}|$, then $T2IVTrFEWA(|\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{ak}|)\le p$*

4.1. Example

Consider the example for addition with $\tilde{\tilde{\alpha}}_1$ and $\tilde{\tilde{\alpha}}_2$ with weights $\vec{\lambda}_1=0.5$ and $\vec{\lambda}_2=0.5$

$$\tilde{\tilde{\alpha}}_1=\{(0.1,0.25,0.25,0.45;(1,1),(0.24,0.3,0.3,0.45;(0.9,0.9))\}$$
$$\tilde{\tilde{\alpha}}_2=\{(0.2,0.25,0.25,0.4;(1,1),(0.15,0.35,0.35,0.4;(0.9,0.9))\}$$

$$\begin{aligned}T2IVTrFEWA(\tilde{\tilde{\alpha}}_1,\tilde{\tilde{\alpha}}_2)&=\sum_{j=1}^{2}\vec{\lambda}_j\tilde{\tilde{\alpha}}_j\\ &=\{(0.25,0.38,0.38,0.54;(1,1)),(0.31,0.46,0.46,0.55;(0.9,0.9))\}\end{aligned}$$

5. Type-2 interval valued fuzzy Einstein weighted geometric aggregating operator

Definition 6. Take a set of m $T2IVTrFNs$ $\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am}$ in a universe of discourse $\tilde{\tilde{A}}$. The vectors $\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am}$ are aggregated using the function $T2IVTrEWG$

$$T2IVTrFEWG=\tilde{\tilde{A}}\mapsto\tilde{\tilde{A}}$$

The operator is defined by,

$$T2IVTrFEWG(\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am})=\prod_{j=1}^{m}(\tilde{\tilde{\alpha}}_{mj})^{\vec{\lambda}_j}$$

Where $\vec{\lambda}=\vec{\lambda}_1,\vec{\lambda}_2,\vec{\lambda}_3,\ldots\vec{\lambda}_m$ is the weight vector in which $\vec{\lambda}_j\in[0,1]$ and $\sum_{j=1}^{m}\vec{\lambda}_j=1$. The aggregation operator can also written as

$$T2IVTrFEWG(\tilde{\tilde{\alpha}}_{a1},\tilde{\tilde{\alpha}}_{a2},\tilde{\tilde{\alpha}}_{a3},\ldots,\tilde{\tilde{\alpha}}_{am})=\prod_{j=1}^{m}(\tilde{\tilde{a}}^{u}_{1j},\tilde{\tilde{a}}^{l}_{1j})^{\vec{\lambda}_j}.$$

The $T2IVTrFEWG$ operator can be made into a geometric average operator when we consider the weight as $\frac{1}{n}$ which is written as

$$T2IVTrFEWG(\tilde{\tilde{\alpha}}_{a1}, \tilde{\tilde{\alpha}}_{a2}, \tilde{\tilde{\alpha}}_{a3}, \dots, \tilde{\tilde{\alpha}}_{am}) = \sqrt[n]{(\tilde{\tilde{\alpha}}_{a1}, \tilde{\tilde{\alpha}}_{a2}, \tilde{\tilde{\alpha}}_{a3}, \dots, \tilde{\tilde{\alpha}}_{am}}$$

Theorem 3. *Take m $T2IVTrFNs$ $\{\tilde{\tilde{a}}_1, \tilde{\tilde{a}}_2, \tilde{\tilde{a}}_3, \dots \tilde{\tilde{a}}_m\}$ the valuation acquired after aggregating these m $T2IVTrFNs$ by the Einstein multiplication operator is again a $T2IVTrFN$.*

Proof. Consider $|\tilde{\tilde{\alpha}}_{mj}| \leq 1$ for all $j = 1, 2, \dots m$ and $\sum_{j=1}^{m} \vec{\lambda}_j = 1$, it is obtained as follows

$$\begin{aligned} |(\tilde{\tilde{\alpha}}_{a1})^{\vec{\lambda}_1} \cdot (\tilde{\tilde{\alpha}}_{a2})^{\vec{\lambda}_2} \cdot (\tilde{\tilde{\alpha}}_{a3})^{\vec{\lambda}_3} \dots (\tilde{\tilde{\alpha}}_{am})^{\vec{\lambda}_m}| &= |(\tilde{\tilde{\alpha}}_{a1})^{\vec{\lambda}_1}| \cdot |(\tilde{\tilde{\alpha}}_{a2})^{\vec{\lambda}_2}| \cdot |(\tilde{\tilde{\alpha}}_{a3})^{\vec{\lambda}_3}| \dots |(\tilde{\tilde{\alpha}}_{am})^{\vec{\lambda}_m}| \\ &\leq 1^{\vec{\lambda}_1 + \vec{\lambda}_2 + \vec{\lambda}_3 + \dots + \vec{\lambda}_m} \\ &= 1 \end{aligned}$$

It is proved that the $|T2IVTrFEWG(\tilde{\tilde{a}}_1, \tilde{\tilde{a}}_2, \tilde{\tilde{a}}_3, \dots \tilde{\tilde{a}}_m)| \leq 1$ is also $T2IVTrFN$ □

Theorem 4. *Consider $T2IVTrFNs$ $\tilde{\tilde{\alpha}}_{a1}, \tilde{\tilde{\alpha}}_{a2}, \tilde{\tilde{\alpha}}_{a3}, \dots \tilde{\tilde{\alpha}}_{am}$ are aggregated using $T2IVTrFEWG$, weights to be real values which satisfied $\vec{\lambda}_j \in [0,1]$ and $\sum_{j=1}^{m} \vec{\lambda}_j = 1$ as following holds*

1. *(Idempotency) When $\tilde{\tilde{\alpha}}_{a1} = \tilde{\tilde{\alpha}}_{a2} = \tilde{\tilde{\alpha}}_{a3} = \dots = \tilde{\tilde{\alpha}}_{am}$ the operator becomes,* $T2IVTrFEWG(\tilde{\tilde{\alpha}}_{k1} = \tilde{\tilde{\alpha}}_{k2} = \tilde{\tilde{\alpha}}_{k3} = \dots = \tilde{\tilde{\alpha}}_{kq}) = \tilde{\tilde{\alpha}}$
2. *(Boundedness) Consider $p = max_j |\tilde{\tilde{\alpha}}_{mj}|$ then, $|T2IVTrFEWG(\tilde{\tilde{\alpha}}_{k1} = \tilde{\tilde{\alpha}}_{k2} = \tilde{\tilde{\alpha}}_{k3} = \dots = \tilde{\tilde{\alpha}}_{mj})| \leq p$*

Proof. Consider the values,

1. As $\tilde{\tilde{\alpha}}_{a1} = \tilde{\tilde{\alpha}}_{a2} = \tilde{\tilde{\alpha}}_{a3} = \dots = \tilde{\tilde{\alpha}}$ and $\sum_{j=1}^{m} \vec{\lambda}_j = 1$, we get

$$\begin{aligned} (\tilde{\tilde{\alpha}}_{a1})^{\vec{\lambda}_1} (\tilde{\tilde{\alpha}}_{a2})^{\vec{\lambda}_2} (\tilde{\tilde{\alpha}}_{a3})^{\vec{\lambda}_3} \dots (\tilde{\tilde{\alpha}}_{am})^{\vec{\lambda}_m} &= (\tilde{\tilde{\alpha}}_{a1})^{\vec{\lambda}_1} . (\tilde{\tilde{\alpha}}_{a2})^{\vec{\lambda}_2} . (\tilde{\tilde{\alpha}}_{a3})^{\vec{\lambda}_3} . \dots . (\tilde{\tilde{\alpha}}_{am})^{\vec{\lambda}_m} \\ &= \tilde{\tilde{\alpha}}^{(\vec{\lambda}1 + \vec{\lambda}2 + \vec{\lambda}3 + \dots \vec{\lambda}m)} \\ &= \tilde{\tilde{\alpha}} \end{aligned}$$

2. As $p = \max_j |\tilde{\tilde{\alpha}}_{kj}|$ and $\vec{\lambda}_j \in [0,1]$ for every $j = 1, 2, \dots k$ we get

$$\begin{aligned} |(\tilde{\tilde{\alpha}}_{a1})^{\vec{\lambda}_1} . (\tilde{\tilde{\alpha}}_{a2})^{\vec{\lambda}_2} . (\tilde{\tilde{\alpha}}_{a3})^{\vec{\lambda}_3} \dots (\tilde{\tilde{\alpha}}_{am})^{\vec{\lambda}_m}| &= |(\tilde{\tilde{\alpha}}_{a1})^{\vec{\lambda}_1}| . |(\tilde{\tilde{\alpha}}_{a2})^{\vec{\lambda}_2}| . |(\tilde{\tilde{\alpha}}_{a3})^{\vec{\lambda}_3}| . \dots |(\tilde{\tilde{\alpha}}_{am})^{\vec{\lambda}_m}| \\ &\leq p^{\vec{\lambda}_1} . p^{\vec{\lambda}_2} . p^{\vec{\lambda}_3} . \dots . p^{\vec{\lambda}_m} \\ &= p^{(\vec{\lambda}_1 + \vec{\lambda}_2 + \vec{\lambda}_3 + \dots + \vec{\lambda}_m)} \\ &= p \quad \square \end{aligned}$$

5.1. Example

Consider the problem with $\tilde{\tilde{\alpha}}_1$ and $\tilde{\tilde{\alpha}}_2$ with weights $\vec{\lambda}_1 = 0.4$ and $\vec{\lambda}_2 = 0.6$

$$\tilde{\tilde{\alpha}}_1 = \{(0.05, 0.23, 0.23, 0.3; (1,1), (0.2, 0.32, 0.32, 0.41; (0.9, 0.9))\}$$

$$\tilde{\tilde{\alpha}}_2 = \{(0.1, 0.25, 0.25, 0.3; (1,1), (0.15, 0.28, 0.28, 0.35; (0.9, 0.9))\}$$

$$\begin{aligned} T2IVTrFEWG(\tilde{\tilde{\alpha}}_1, \tilde{\tilde{\alpha}}_2) &= \prod_{j=1}^{2} (\tilde{\tilde{\alpha}}_j)^{\vec{\lambda}_j} \\ &= \{(0.16, 0.32, 0.32, 0.37; (1,1)), (0.25, 0.37, 0.37, 0.43; (0.9, 0.9))\} \end{aligned}$$

6. Algorithm

This section consists of the pseudocode for the weighted MCDM method AHP, distance-based MCDM method TOPSIS, and outranking method VIKOR. Algorithm 1 is providing the combined AHP and TOPSIS method and Algorithm 2 is showing the VIKOR method. Both algorithms are expounded by the flowchart in Fig. 3.

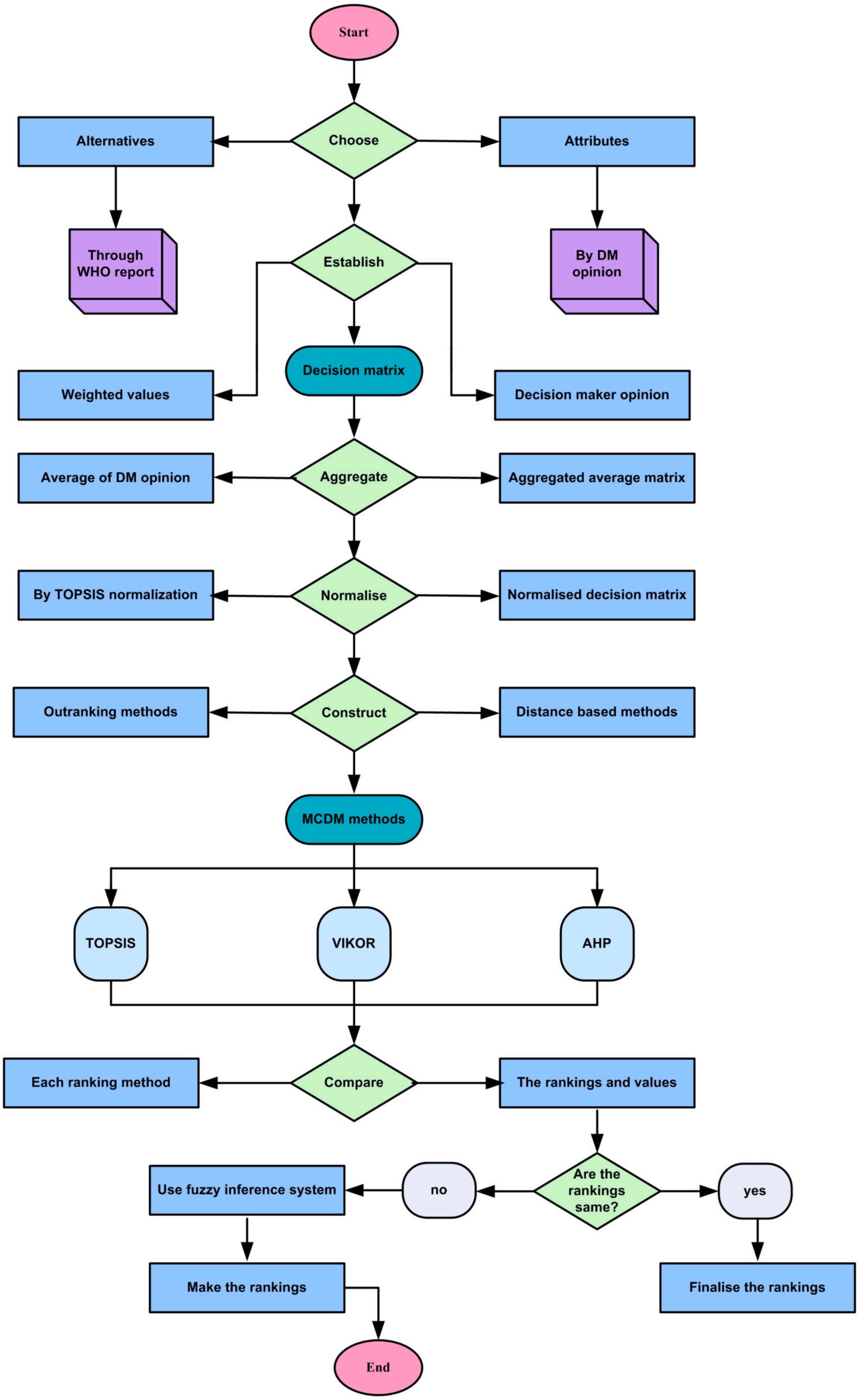


Fig. 3. Flow chart of proposed algorithm.

Algorithm 1 Choose the best risk factor from $R = R_1, R_2, \dots R_{12}$.

Require: $i, j > 0, \sum_{i=1}^{n} \tilde{\tilde{w}}_{ij} = 1$
Ensure: $R = \{R_1, R_2, \dots R_{12}\}, C = \{C_1, C_2, C_3, C_4\}$

```
Construct the weighted values
if i, j > 0 then
    Construct the relative importance with linguistic variables
    C_ij = 1/C_ji
else[C_ij = C_ji = 1]
    Calculate sum_{j=1}^{n} C_ij = w̃̃_j
    sum_{j=1}^{n} w̃̃_j = 1
end if
Frame the decision matrix S̃̃_k = (f̃̃_ij)_{m×n} 1 ≤ i ≤ m, 1 ≤ j ≤ n, 1 ≤ k ≤ p
Construct the weighted matrix W̄_p = (w̃̃_i)_{1×m}
Frame ṽ̃_ij = w̃̃_i ⊗ f̃̃_ij, 1 ≤ i ≤ m and 1 ≤ j ≤ n.
Calculate S̄*_w = (Rank(ṽ̃_ij))_{m×n}
Determine PIS and NIS
```

$$y_i^+ = \begin{cases} max\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_1 \\ min\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_2 \end{cases}$$

$$y_i^- = \begin{cases} min\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_1 \\ max\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_2 \end{cases}$$

Compute $d^+(y_j)$ and $d^-(y_j)$

$$d^+(y_j) = \sqrt{\sum_{i=1}^{m} (Rank(\tilde{\tilde{v}}_{ij}) - v_i^+)^2}, d^-(y_j) = \sqrt{\sum_{i=1}^{m} (Rank(\tilde{\tilde{v}}_{ij}) - v_i^-)^2}.$$

Calculate $C(y_j) = \frac{d^-(y_j)}{d^+(y_j)+d^-(y_j)}$

Algorithm 2 Choose the best risk factor from $R = R_1, R_2, \dots, R_{12}$ **(VIKOR)**.

Require: $i > 0 \; \forall j > 0$
Ensure: $R = \{R_1, R_2, \dots R_i\}, C = \{C_1, C_2, \dots, C_j\}$

% Determine best and worst
$v_i^* = \max(v_{ij})$ and $v_i^- = \min(v_{ij})$
% Utility measure (S_i)
$S_i = \sum_{j=1}^{n} w_j * (\frac{v_j^* - v_{ij}}{v_j^* - v_j^-})$
% Regret measure (R_i)
$R_i = \max_j (S_{ij})$
% Compute VIKOR index
$Q_i = v(\frac{S_i - S^-}{S^* - S^-}) + (1-v)(\frac{R_i - R^-}{R^* - R^-})$
% Make the rankings in ascending order.

Table 1
Risk factors of TB.

Riskfactors	Notation
Diabetes	R_1
Immune problem	R_2
Malnutrition	R_3
Alcohol	R_4
Active smoking	R_5
Crowded places	R_6
HIV infection	R_7
Air pollution	R_8
Kidney diseases and Cancer	R_9
Health care worker (In TB risk areas)	R_{10}
Medical treatments such as corticosteroids or organ transplant	R_{11}
Silicosis	R_{12}

7. Type-2 interval valued Einstein aggregating operators with extension to MCDM-case study

Fig. 4, developing a hierarchical structure by choosing the risk factor at the top level, attributes (or) criteria at the second level, and the risk factors at the third level.

The notation in Fig. 4 explained by the Table 1. Construct the relative importance between attributes. Attributes are explained through the Table 2.

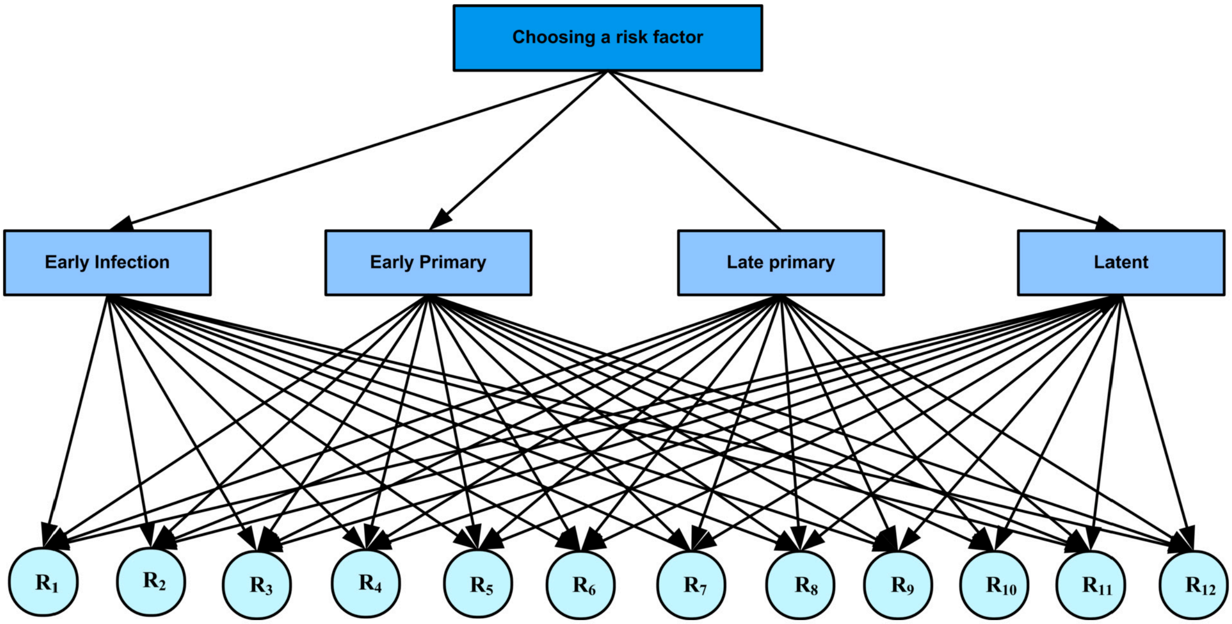


Fig. 4. Developing a hierarchical structure.

Table 2
Stages on TB.

Criteria	Notation
Early infection	C_1
Early primary progressive (Infective)	C_2
Late primary progressive (Infective)	C_3
Latent	C_4

7.1. Fuzzy AHP algorithm

In this subsection fuzzy AHP algorithm is utilized to get weights of each criterion by the way of [26].
Step-1 Developing a hierarchical structure.
Step-2 Construct the pairwise comparison matrix. In this step, construct the relative importance of different levels of attributes concerning choosing the risk factors.

$$S_k = \begin{bmatrix} 1 & \tilde{\tilde{f}}^k_{12} & \tilde{\tilde{f}}^k_{13} & \dots & \tilde{\tilde{f}}^k_{1n} \\ \tilde{\tilde{f}}^k_{21} & 1 & \tilde{\tilde{f}}^k_{23} & \dots & \tilde{\tilde{f}}^k_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}^k_{n1} & \tilde{\tilde{f}}^k_{n2} & \tilde{\tilde{f}}^k_{n3} & \dots & 1 \end{bmatrix},$$

Step-3 Find the values of the pairwise comparison matrix by using the importance values of linguistic terms.
Step-4 Normalize the pairwise comparison matrix by utilizing the geometric mean.

7.2. Fuzzy TOPSIS algorithm

In this part type-2 fuzzy TOPSIS was utilized to find the rankings through the algorithm of the article [7],
Step-5 Frame the decision matrix A_k of the k^{th} decision maker and frame the average decision matrix $\bar{A}$, subsequently, shown as follows:

$$S_k = \begin{bmatrix} \tilde{\tilde{f}}^k_{11} & \tilde{\tilde{f}}^k_{12} & \tilde{\tilde{f}}^k_{13} & \dots & \tilde{\tilde{f}}^k_{1n} \\ \tilde{\tilde{f}}^k_{21} & \tilde{\tilde{f}}^k_{22} & \tilde{\tilde{f}}^k_{23} & \dots & \tilde{\tilde{f}}^k_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}^k_{m1} & \tilde{\tilde{f}}^k_{m2} & \tilde{\tilde{f}}^k_{m3} & \dots & \tilde{\tilde{f}}^k_{mn} \end{bmatrix},$$

$$\tilde{\tilde{S}}_k = (f_{ij})_{m\times n},$$

where $\tilde{\tilde{f}}_{ij} = (\frac{\tilde{\tilde{f}}_{11} \oplus \tilde{\tilde{f}}_{12} \oplus \tilde{\tilde{f}}_{13} \oplus \ldots \oplus \tilde{\tilde{f}}_{ij}}{k})$, $\tilde{\tilde{f}}_{ij}$ is an interval type-2 fuzzy set, $1 \le i \le m$, $1 \le j \le n$, $1 \le k \le p$ and p denotes the number of decision makers.

Step-6 Construct the weighted matrix W_p of the criteria of p^{th} decision maker and frame the average weighted matrix $\bar{W}$, respectively shown as follows:

$$W_k = \begin{bmatrix} \tilde{\tilde{w}}_{11}^k & \tilde{\tilde{w}}_{12}^k & \tilde{\tilde{w}}_{13}^k & \ldots & \tilde{\tilde{w}}_{1n}^k \end{bmatrix},$$

$$\bar{W}_p = (w_i)_{1\times n},$$

where $\tilde{\tilde{w}}_i = (\frac{\tilde{\tilde{w}}_i^1 \oplus \tilde{\tilde{w}}_i^2 \oplus \tilde{\tilde{w}}_i^k \oplus \ldots \oplus \tilde{\tilde{w}}_i^k}{k})$, w_i is an interval type-2 fuzzy set, $1 \le i \le m, 1 \le k \le n$, and n denotes the number of decision makers.

Step-7 Make the weighted decision matrix $\bar{S}_w$,

$$\bar{S}_w = \begin{bmatrix} \tilde{\tilde{v}}_{11}^k & \tilde{\tilde{v}}_{12}^k & \tilde{\tilde{v}}_{13}^k & \ldots & \tilde{\tilde{v}}_{1n}^k \\ \tilde{\tilde{v}}_{21}^k & \tilde{\tilde{v}}_{22}^k & \tilde{\tilde{v}}_{23}^k & \ldots & \tilde{\tilde{v}}_{2n}^k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{v}}_{m1}^k & \tilde{\tilde{v}}_{m2}^k & \tilde{\tilde{v}}_{m3}^k & \ldots & \tilde{\tilde{v}}_{mn}^k \end{bmatrix},$$

where $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_i \otimes \tilde{\tilde{f}}_{ij}$, $1 \le i \le m$ and $1 \le j \le n$.

Step-8 Depending upon the step-3, estimate the ranking values $Rank(\tilde{\tilde{v}}_{ij})$ of type-2 interval valued fuzzy set $\tilde{\tilde{v}}_{ij}$ and frame the ranking weighted decision matrix $\bar{S}_w^*$,

$\bar{S}_w^* = (Rank(\tilde{\tilde{v}}_{ij}))_{m\times n}$.

$Rank(\tilde{\tilde{v}}_{ij})$ is obtained as,

$$\begin{aligned} Rank(\tilde{\tilde{v}}_{ij}) = {} & V_1(\tilde{v}_{ij}^U) + V_1(\tilde{v}_{ij}^L) + V_2(\tilde{v}_{ij}^L) + V_2(\tilde{v}_{ij}^L) + V_3(\tilde{v}_{ij}^U) + V_3(\tilde{v}_{ij}^L) \\ & - \frac{1}{4}(P_1(\tilde{v}_{ij}^U) + P_1(\tilde{v}_{ij}^L) + P_2(\tilde{v}_{ij}^U) + P_2(\tilde{v}_{ij}^L) \\ & + P_3(\tilde{v}_{ij}^U) + P_3(\tilde{v}_{ij}^L) + P_4(\tilde{v}_{ij}^U) + P_4(\tilde{v}_{ij}^L)) \\ & + H_1(\tilde{v}_{ij}^U) + H_1(\tilde{v}_{ij}^L) + H_2(\tilde{v}_{ij}^U) + H_2(\tilde{v}_{ij}^L) \end{aligned}$$

For instance,

$$\begin{aligned} (\tilde{\tilde{v}}_{11}) = {} & (0.63, 0.9, 0.9, 1; 1, 1), (0.76, 0.9, 0.9, 0.95; 0.9, 0.9) \\ Rank(\tilde{\tilde{v}}_{11}) = {} & 0.76 + 0.83 + 0.9 + 0.9 + 0.95 + 0.925 - \frac{1}{4}(0.135 + 0.07 + 0 + 0 + 0.05 \\ & +0.025 + 0.137 + 0.070) + 1 + 1 + 0.9 + 0.9 \\ & = 8.947 \end{aligned}$$

where $V_p(\tilde{S}_i^j)$ designates the average of the elements s_{ip}^j and $s_{i(p+1)}^j/2, 1 \le p \le 3$, $P_q(\tilde{S}_i^j)$ designates the standard deviation of the components s_{iq}^j and $s_{i(p+1)}^j$,

$P_q(\tilde{S_i^j}) = \sqrt{\frac{1}{2}\sum_{m=q}^{q+1}(s_{im}^j - \frac{1}{2}\sum_{m=q}^{q+1} s_{ik}^j)^2}$, and $1 \le q \le 3$,

$P_4(\tilde{S_i^j}) = \sqrt{\frac{1}{4}\sum_{m=1}^{4}(s_{im}^j - \frac{1}{4}\sum_{m=1}^{4} s_{ik}^j)^2}$. $H_p(s_i^j)$ designates the height of membership function of type-2 trapezoidal fuzzy number.

Step-9 Determine the positive ideal solution $y^+ = \max(y_1^+, y_2^+, y_3^+, \ldots, y_m^+)$ and negative ideal solution $y^- = \min(y_1^-, y_2^-, y_3^-, \ldots, y_m^-)$, where

$$y_i^+ = \begin{cases} \max\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_1 \\ \min\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_2 \end{cases}$$

and

$$y_i^- = \begin{cases} \min\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_1 \\ \max\{Rank(\tilde{\tilde{v}}_{(ij)})\}, & \text{if } v_i \in V_2 \end{cases}$$

where V_1 designates the set of borderline criteria and V_2 designates the set of latent line criteria and $1 \le i \le m$.

Step-10 Compute the distance $d^+(y_j)$ between every risk factor y_j and the positive ideal solution y^+, shown as follows:

$$d^+(y_j) = \sqrt{\sum_{i=1}^{m}(Rank(\tilde{\tilde{v}}_{ij}) - v_i^+)^2},$$

Table 3
Relative importance and its values.

Scale of relative importance	Values
Very low importance	$((0.0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9))$
Low importance	$((0.0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))$
Moderate importance	$((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))$
Strong importance	$((0.5, 0.7, 0.7, 0.9; 1, 1), (0.6, 0.7, 0.7, 0.8; 0.9, 0.9))$
Very strong importance	$((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))$
Extreme importance	$((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9))$
Equal importance	$((1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1))$

Table 4
Relative importance between attributes.

Alternatives	C_1	C_2	C_3	C_4
C_1	EE	S	M	M
C_2	1/S	EE	S	VS
C_3	1/M	1/S	EE	E
C_4	1/M	1/VS	1/E	EE

Table 5
Collective weights from decision makers for the Riskfactors of TB.

Alternatives	C_1	C_2	C_3	C_4
DM_1	VH	H	M	L
DM_2	VH	VH	L	L
DM_3	VH	H	M	M

$$d^-(y_j) = \sqrt{\sum_{i=1}^{m} (Rank(\tilde{\tilde{v}}_{ij}) - v_i^-)^2}.$$

Step-11 Calculate the relative degree of closeness $C(y_j)$ with respect to the positive ideal solution y^+, shown as follows:

$$C(y_j) = \frac{d^-(y_j)}{d^+(y_j) + d^-(y_j)},$$

Step-12 Arrange the values of $C(y_j)$ in descending sequence, where $1 \leq j \leq n$. The larger value of $C(y_j)$, got the higher preference of the alternative y_j.

Find the relative importance between attributes. Relative importance and its values are referred to by Table 3. Relative importance between attributes is explained in the Table 4.

where

$$\tilde{\tilde{w}}_1 = ((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9)),$$

$$\tilde{\tilde{w}}_2 = ((0.221, 0.235, 0.235, 0.248; 1, 1), (0.222, 0.235, 0.235, 0.243; 0.9, 0.9)),$$

$$\tilde{\tilde{w}}_3 = ((0.302, 0.314, 0.314, 0.278; 1, 1), (0.331, 0.314, 0.314, 0.295; 0.9, 0.9)),$$

$$\tilde{\tilde{w}}_4 = ((0.366, 0.295, 0.295, 0.271; 1, 1), (0.316, 0.295, 0.295, 0.283; 0.9, 0.9)).$$

Table 6, 7 and Table 8 provide the opinions of decision-makers about risk factors of TB. Table 9 shows the average value of three decision-makers opinion. Continuing this the average decision matrix multiplied with weighted values, explained in Table 10.

Table 5 provides opinions about the weighted values of the decision matrix. Table 10 is multiplied by the weighted values which found from Table 5.

By following this value, each alternative distance value is manifest through the coming Table 11 In the same way, we can get and substitute in Table 12. Hence we can get the values of alternatives with weightage.

7.3. Fuzzy VIKOR algorithm

In this part, type-2 fuzzy VIKOR is utilized to find the rankings through an algorithm of the work [20]

Step-13 Determine the best ϕ_j^+ and worst value ϕ_j^+ of each attribute

$$\phi_j^+ = \max(Rank(v_{ij}))\forall i = 1, 2, \ldots, 12$$

Table 6
First decision maker opinion.

Alternatives	C_1	C_2	C_3	C_4
R_1	VH	H	H	MH
R_2	H	MH	MH	M
R_3	VH	H	H	MH
R_4	VH	H	H	MH
R_5	H	MH	MH	M
R_6	VH	VH	VH	H
R_7	H	MH	MH	M
R_8	VH	MH	MH	M
R_9	H	MH	MH	M
R_{10}	H	MH	MH	M
R_{11}	H	MH	MH	M
R_{12}	H	H	H	M

Table 7
Second decision maker opinion about TB.

Alternatives	C_1	C_2	C_3	C_4
R_1	VH	VH	H	M
R_2	H	H	MH	L
R_3	MH	MH	L	VL
R_4	MH	MH	M	L
R_5	M	ML	VL	VL
R_6	MH	M	ML	L
R_7	M	ML	L	L
R_8	MH	M	L	VL
R_9	H	MH	M	L
R_{10}	M	H	L	M
R_{11}	MH	M	L	VL
R_{12}	MH	ML	L	VL

Table 8
Third decision maker opinion about TB.

Alternatives	C_1	C_2	C_3	C_4
R_1	VH	VL	MH	MH
R_2	VH	H	MH	M
R_3	VH	H	MH	L
R_4	VH	VH	MH	M
R_5	VH	H	ML	ML
R_6	VH	H	H	M
R_7	MH	H	ML	M
R_8	MH	H	ML	L
R_9	VH	MH	ML	VL
R_{10}	VH	M	ML	VL
R_{11}	VH	M	MH	VL
R_{12}	VH	H	MH	L

Table 9
Average decision matrix.

Alternatives	C_1	C_2	C_3	C_4
R_1	$\approx{f}_{11}$	$\approx{f}_{12}$	$\approx{f}_{13}$	$\approx{f}_{14}$
R_2	$\approx{f}_{21}$	$\approx{f}_{22}$	$\approx{f}_{23}$	$\approx{f}_{24}$
R_3	$\approx{f}_{31}$	$\approx{f}_{32}$	$\approx{f}_{33}$	$\approx{f}_{34}$
R_4	$\approx{f}_{41}$	$\approx{f}_{42}$	$\approx{f}_{43}$	$\approx{f}_{44}$
R_5	$\approx{f}_{51}$	$\approx{f}_{52}$	$\approx{f}_{53}$	$\approx{f}_{54}$
R_6	$\approx{f}_{61}$	$\approx{f}_{62}$	$\approx{f}_{63}$	$\approx{f}_{64}$
R_7	$\approx{f}_{71}$	$\approx{f}_{72}$	$\approx{f}_{73}$	$\approx{f}_{74}$
R_8	$\approx{f}_{81}$	$\approx{f}_{82}$	$\approx{f}_{83}$	$\approx{f}_{84}$
R_9	$\approx{f}_{91}$	$\approx{f}_{92}$	$\approx{f}_{93}$	$\approx{f}_{94}$
R_{10}	$\approx{f}_{10,1}$	$\approx{f}_{10,2}$	$\approx{f}_{10,3}$	$\approx{f}_{10,4}$
R_{11}	$\approx{f}_{11,1}$	$\approx{f}_{11,2}$	$\approx{f}_{11,3}$	$\approx{f}_{11,4}$
R_{12}	$\approx{f}_{12,1}$	$\approx{f}_{12,2}$	$\approx{f}_{12,3}$	$\approx{f}_{12,4}$

Table 10
Weighted decision matrix.

Alternatives	C_1	C_2	C_3	C_4
R_1	$\tilde{\tilde{v}}_{11}$	$\tilde{\tilde{v}}_{12}$	$\tilde{\tilde{v}}_{13}$	$\tilde{\tilde{v}}_{14}$
R_2	$\tilde{\tilde{v}}_{21}$	$\tilde{\tilde{v}}_{22}$	$\tilde{\tilde{v}}_{23}$	$\tilde{\tilde{v}}_{24}$
R_3	$\tilde{\tilde{v}}_{31}$	$\tilde{\tilde{v}}_{32}$	$\tilde{\tilde{v}}_{33}$	$\tilde{\tilde{v}}_{34}$
R_4	$\tilde{\tilde{v}}_{41}$	$\tilde{\tilde{v}}_{42}$	$\tilde{\tilde{v}}_{43}$	$\tilde{\tilde{v}}_{44}$
R_5	$\tilde{\tilde{v}}_{51}$	$\tilde{\tilde{v}}_{52}$	$\tilde{\tilde{v}}_{53}$	$\tilde{\tilde{v}}_{54}$
R_6	$\tilde{\tilde{v}}_{61}$	$\tilde{\tilde{v}}_{62}$	$\tilde{\tilde{v}}_{63}$	$\tilde{\tilde{v}}_{64}$
R_7	$\tilde{\tilde{v}}_{71}$	$\tilde{\tilde{v}}_{72}$	$\tilde{\tilde{v}}_{73}$	$\tilde{\tilde{v}}_{74}$
R_8	$\tilde{\tilde{v}}_{81}$	$\tilde{\tilde{v}}_{82}$	$\tilde{\tilde{v}}_{83}$	$\tilde{\tilde{v}}_{84}$
R_9	$\tilde{\tilde{v}}_{91}$	$\tilde{\tilde{v}}_{92}$	$\tilde{\tilde{v}}_{93}$	$\tilde{\tilde{v}}_{94}$
R_{10}	$\tilde{\tilde{v}}_{10,1}$	$\tilde{\tilde{v}}_{10,2}$	$\tilde{\tilde{v}}_{10,3}$	$\tilde{\tilde{v}}_{10,4}$
R_{11}	$\tilde{\tilde{v}}_{11,1}$	$\tilde{\tilde{v}}_{11,2}$	$\tilde{\tilde{v}}_{11,3}$	$\tilde{\tilde{v}}_{11,4}$
R_{12}	$\tilde{\tilde{v}}_{12,1}$	$\tilde{\tilde{v}}_{12,2}$	$\tilde{\tilde{v}}_{12,3}$	$\tilde{\tilde{v}}_{12,4}$

Table 11
Rankings of weighted valued decision matrix.

Alternatives	C_1	C_2	C_3	C_4
R_1	$Rank(\tilde{\tilde{v}}_{11})$	$Rank(\tilde{\tilde{v}}_{12})$	$Rank(\tilde{\tilde{v}}_{13})$	$Rank(\tilde{\tilde{v}}_{14})$
R_2	$Rank(\tilde{\tilde{v}}_{21})$	$Rank(\tilde{\tilde{v}}_{22})$	$Rank(\tilde{\tilde{v}}_{23})$	$Rank(\tilde{\tilde{v}}_{24})$
R_3	$Rank(\tilde{\tilde{v}}_{31})$	$Rank(\tilde{\tilde{v}}_{32})$	$Rank(\tilde{\tilde{v}}_{33})$	$Rank(\tilde{\tilde{v}}_{34})$
R_4	$Rank(\tilde{\tilde{v}}_{41})$	$Rank(\tilde{\tilde{v}}_{42})$	$Rank(\tilde{\tilde{v}}_{43})$	$Rank(\tilde{\tilde{v}}_{44})$
R_5	$Rank(\tilde{\tilde{v}}_{51})$	$Rank(\tilde{\tilde{v}}_{52})$	$Rank(\tilde{\tilde{v}}_{53})$	$Rank(\tilde{\tilde{v}}_{54})$
R_6	$Rank(\tilde{\tilde{v}}_{61})$	$Rank(\tilde{\tilde{v}}_{62})$	$Rank(\tilde{\tilde{v}}_{63})$	$Rank(\tilde{\tilde{v}}_{64})$
R_7	$Rank(\tilde{\tilde{v}}_{71})$	$Rank(\tilde{\tilde{v}}_{72})$	$Rank(\tilde{\tilde{v}}_{73})$	$Rank(\tilde{\tilde{v}}_{74})$
R_8	$Rank(\tilde{\tilde{v}}_{81})$	$Rank(\tilde{\tilde{v}}_{82})$	$Rank(\tilde{\tilde{v}}_{83})$	$Rank(\tilde{\tilde{v}}_{84})$
R_9	$Rank(\tilde{\tilde{v}}_{91})$	$Rank(\tilde{\tilde{v}}_{92})$	$Rank(\tilde{\tilde{v}}_{93})$	$Rank(\tilde{\tilde{v}}_{94})$
R_{10}	$Rank(\tilde{\tilde{v}}_{10,1})$	$Rank(\tilde{\tilde{v}}_{10,2})$	$Rank(\tilde{\tilde{v}}_{10,3})$	$Rank(\tilde{\tilde{v}}_{10,4})$
R_{11}	$Rank(\tilde{\tilde{v}}_{11,1})$	$Rank(\tilde{\tilde{v}}_{11,2})$	$Rank(\tilde{\tilde{v}}_{11,3})$	$Rank(\tilde{\tilde{v}}_{11,4})$
R_{12}	$Rank(\tilde{\tilde{v}}_{12,1})$	$Rank(\tilde{\tilde{v}}_{12,2})$	$Rank(\tilde{\tilde{v}}_{12,3})$	$Rank(\tilde{\tilde{v}}_{12,4})$

Table 12
Ranking values of weighted valued decision matrix.

Alternatives	C_1	C_2	C_3	C_4
R_1	8.95	9.78	8.71	8.32
R_2	8.24	9.07	6.54	8.58
R_3	7.65	9.78	8.28	7.88
R_4	7.79	9.03	8.28	6.12
R_5	7.65	9.15	8.93	7.88
R_6	7.25	9.15	8.98	6.14
R_7	7.66	9.82	9.28	7.21
R_8	7.65	9.14	7.86	6.15
R_9	7.65	9.78	8.60	7.88
R_{10}	7.25	9.54	8.84	6.59
R_{11}	7.79	9.78	8.88	8.32
R_{12}	7.25	8.91	8.31	7.65

$$\phi_j^- = \min(Rank(v_{ij}))\forall i = 1, 2, \ldots, 12$$

Step-14 Calculate the specific group utility and the individual regret. Based on the novel distance measure, calculate the specific group utility S_i and the individual regret R_i of the i^{th} alternative by calculating the distance between the best value and the i^{th} alternative as: By using the specific group utility and the individual regret, calculate the aggregating index of each alternative as:

$$Q_i = \sigma \frac{S_i - S^*}{S^+ + S^-} + (1-\sigma)\frac{R_j - R^-}{R^+ + R^-}$$

where, $S^+ = \max_i S_i$, $S^- = \min_i S_i$, $R^+ = \max_i R_i$, σ $0 \leq 1$ is the weight of the specific group utility, and $1 - \sigma$ is the weight of individual regret.

Step-15 Assign the rankings by ascending order. Based on the ranking orders, obtain the compromised solution where R_1 is ranked the best with minimum value.

7.4. Alternatives and attributes

TB is a potentially serious infectious disease that primarily affects the lungs but can also affect other parts of the body. It is caused by mycobacterium tuberculosis. TB spreads through the air when an infected person coughs, sneezes, or talks, releasing tiny infectious particles known as droplet nuclei. Several factors can increase the risk of developing TB. These risk factors can vary depending on individual circumstances and the prevalence of TB in the community. Some common risk factors include:

1. **Diabetes:** Diabetes can weaken the immune system, making individuals more susceptible to infections, including TB. High blood sugar levels can impair the function of various immune cells, such as macrophages and T-cells, which play a crucial role in defending the body against TB bacteria. Diabetes can complicate the diagnosis and management of TB. Symptoms of TB may be masked or mistaken for complications of diabetes, leading to delays in diagnosis and treatment initiation [46]. Additionally, diabetes-related complications, such as neuropathy and nephropathy, can make it more challenging for individuals to adhere to TB treatment regimens.
2. **Immune problem:** Individuals with immunodeficiency disorders, whether congenital (present from birth) or acquired (developed later in life), have weakened immune systems [27]. This impairment affects the body's ability to fight off TB bacteria effectively. TB primarily affects the lungs, and a robust immune response is crucial for containing the infection in the lungs and preventing its spread to other parts of the body.
3. **Malnutrition:** Malnutrition compromises the immune system, making individuals more susceptible to infections, including TB. Essential nutrients, such as vitamins (e.g., vitamin A, vitamin D) and minerals (e.g., zinc, iron), play crucial roles in maintaining a robust immune response against pathogens like the TB bacteria. Deficiencies in these nutrients can impair immune function, reducing the body's ability to contain TB infection and increasing the risk of progression to active TB disease [40]. Malnutrition can lead to structural and functional abnormalities in the respiratory system, including reduced lung capacity and impaired mucociliary clearance. These changes make individuals more vulnerable to respiratory infections, such as TB, as the bacteria can more easily establish an infection in the lungs and cause disease.
4. **Alcohol:** Alcohol consumption is a well-established risk factor for TB. Chronic alcohol consumption weakens the immune system, impairing the body's ability to fight off infections, including TB. Alcohol disrupts immune cell function and decreases the production of immune mediators, making individuals more susceptible to TB infection and progression to active disease. Alcohol use increases the risk of TB infection by impairing the lung's natural defenses against TB bacteria [50]. Alcohol damages respiratory epithelial cells and cilia, which are essential for clearing pathogens from the airways. This makes it easier for TB bacteria to establish an infection in the lungs.
5. **Active smoking:** Smoking tobacco weakens the immune system, making individuals more susceptible to infections, including TB. Smoking impairs the function of immune cells, such as macrophages and T-cells, which are essential for containing TB infection and preventing its progression to active disease. Smokers who develop active TB disease are more likely to experience severe forms of TB, such as cavitary TB or disseminated TB [19]. Smoking exacerbates lung inflammation and tissue damage caused by TB infection, leading to more extensive lung damage and poorer treatment outcomes.
6. **Crowded places:** Crowded places often have inadequate ventilation, which can further enhance TB transmission. Poor ventilation allows TB bacteria to remain suspended in the air for longer periods, increasing the risk of inhalation by susceptible individuals [32]. Proper ventilation, including natural ventilation and mechanical ventilation systems, is crucial for reducing TB transmission in crowded settings.
7. **HIV infection:** HIV infection increases the risk of latent TB infection (LTBI) reactivation, where TB bacteria that have been dormant in the body become active and cause TB disease. The weakened immune response in individuals with HIV allows TB bacteria to reactivate and multiply unchecked, leading to the development of active TB disease [1].
8. **Air pollution:** Air pollution has been shown to suppress the immune system, making individuals more susceptible to infections, including TB. Exposure to air pollutants can reduce the function of immune cells in the respiratory tract, impairing the body's ability to control TB infection and prevent disease progression [28]. Air pollution can increase the risk of respiratory infections, including TB, by damaging the respiratory epithelium and impairing mucociliary clearance. These changes make it easier for TB bacteria to establish an infection in the lungs and cause active TB disease.
9. **Kidney disease and Cancer:** Both kidney disease and cancer can weaken the immune system, making individuals more susceptible to infections, including TB. Chronic kidney disease (CKD) and cancer can impair the function of immune cells, such as T-cells and macrophages, which are crucial for controlling TB infection and preventing disease progression. Treatments for kidney disease and cancer, such as immunosuppressive medications, chemotherapy, and radiation therapy, can suppress the immune system and increase the risk of TB [49]. Immunosuppressive treatments weaken the body's ability to mount an effective immune response against TB bacteria, making individuals more susceptible to TB infection and disease progression.
10. **Health care worker:** Healthcare workers interact closely with patients in healthcare settings, including hospitals, clinics, and TB treatment facilities [52]. This close contact increases the risk of exposure to patients with active TB disease who may be coughing, sneezing, or talking, releasing infectious droplets into the air.
11. **Medical treatments such as corticosteroids and organ transplant:** Corticosteroids and immunosuppressive medications used in organ transplantation suppress the immune system, reducing its ability to fight off infections, including TB. Immunosuppression increases the risk of TB infection, as well as the risk of progression from latent TB infection (LTBI) to active TB disease.

12. **Silicosis:** Silicosis causes scarring and inflammation in the lungs, leading to structural changes and impaired lung function. This lung damage creates an environment conducive to TB infection and disease progression, as TB bacteria can more easily establish an infection in damaged lung tissue.

8. Discussion

TB remains a global health concern, particularly in developing nations. It is one of the top 10 global causes of mortality. Improved health and reduced mortality rates associated with this infection are significant benefits of addressing the risk factors of tuberculosis. Consequently, the objective of this investigation is to identify the most significant risk factors for tuberculosis in the general population. Initially, the risk factors of TB diseases are selected as an alternative, and the intensity of the disease is selected as the criterion for analyzing the highly vulnerable risk factors. Healthcare professionals and TB association departments are the sources of the data. Even though the data was collected in linguistic variables, it was transformed into the T2IVTrFN using a linguistic scale that offers both uncertainty and assurance perspectives. Then, we employ the Fuzzy TOPSIS method, Hybrid method, Fuzzy VIKOR method, T2IVTrFEWA, and T2IVTrFEWG to determine the ranking of the risk factors of TB. The research indicates that diabetes R_1 is the most significant risk factor. Because diabetes diminishes the body, it becomes increasingly challenging for the body to defend against tuberculosis. In nearly all of the methods, Diabetes (R_1), Immune problems (R_2), Malnutrition (R_3), and Alcohol (R_4) were regarded as high-risk factors for tuberculosis. The least risk factors were silicosis (R_{12}), medical treatments (R_{11}), and crowded locations R_6. The remaining alternatives are juggled in between rankings. It is now more common for individuals with TB-DM co-morbidity to have TB than TB-HIV co-infection, and those with DM are at a threefold increased risk of developing TB. Additionally, comparative and sensitivity analyses establish the validity of the obtained result.

8.1. Sensitivity analysis

The sensitivity analysis is explained through Fig. 8, which offers an in-depth examination of how changes in variables affect the outcome of a model decision. This analysis is important for understanding the robustness and reliability of the results, as it helps identify which risk factors have the most significant impact on the output. In arithmetic, geometric aggregation operators $\lambda = 2, 5, 10, 100$ are substituted and verify the relationship between each alternative. The results indicated that changes in the input values led to variations in the outcomes ranging from $[0, 1]$, while adjustments in criteria weights influenced the results by all risk factors and it is shown in the Tables 16, 17, 18, 19. Additionally, modifications in fuzzy parameters caused outcome shifts within the range of $[0, 1]$. These findings confirm that the model is robust and reliable, with the most significant influences identified, ensuring that the decision-making conclusions are not excessively sensitive to minor input variations. The Spearman rank coefficients analysis illustrated in Fig. 6 demonstrates that the proposed framework is highly compatible with other state-of-the-art approaches. Nevertheless, the proposed set of ranking values is more logical, as it also includes intricate ambiguous information and sub-attributes.

8.2. Comparative analysis

The result of this work provided 98% accuracy of the risk factors of TB according to the WHO report [42]. Comparison Table 15 is proving that type-2 fuzzy trapezoidal numbers make an accurate result when compared with type-1 fuzzy numbers.

The type-1 fuzzy ranking on risk factors of TB was analyzed by [11], with ten risk factors. Through Table 15, readers can understand that diabetes got first place which means it is a dangerous risk factor compared with others. The second place is the same at the type-1 and type-2 fuzzy ranking algorithm and the risk factor is an immune problem. In the result of the type-1 fuzzy MCDM method HIV infection got first place, but the WHO report and the type-2 fuzzy values given that it may be a risk factor within five ranks. In type-1 fuzzy MCDM work, they didn't consider the risk factors of organ transplant and silicosis. However, WHO reports that these two are also important risk factors. Through this work, they got eleventh and ninth positions respectively. Fig. 7 refers to understanding the values of risk factors and refers to knowing about the rankings of the TOPSIS technique and integrated technique that is the weights got from the AHP technique integrated with TOPSIS values. Fig. 9 refers to comparing the rankings of type-1 and type-2 fuzzy MCDM techniques. Hence, this type-2 fuzzy hybrid MCDM technique explains that AHP and TOPSIS provide accurate results compared with type-1 fuzzy MCDM techniques. For a clear understanding of the rankings of the TOPSIS method and the VIKOR method shown in Tables 13, 14. Figs. 5a, 5b, 5c, 5d, 5e and 5f providing the perfect rankings with values of every method. This conclusion is also corroborated by the articles [47], [24], [51], [44], [24].

8.3. Limitations

This work has few limitations. Such as,

- $T2IVTrFNs$ with Einestein aggregation operator introduces a higher level of complexity compared to traditional type-1 fuzzy sets. This complexity can make it challenging to interpret and apply the results.
- The computation involved in processing $T2IVTrFNs$ can be intensive, especially when dealing with large datasets in MCDM frameworks. This can limit the scalability of the approach to a larger population's decision scenarios.
- Implementing MCDM methods with $T2IVTrFNs$ and Einstein aggregation operators can be computationally intensive, especially when dealing with large datasets.

Table 13
Distance measures by TOPSIS.

Alternatives	$d^{+}(y_j)$	$d^{-}(y_j)$	$C(y_j)$	Rank
R_1	0	3.348	1.00	1
R_2	0.968	2.423	0.71	2
R_3	1.205	2.211	0.64	4
R_4	0.902	2.476	0.73	3
R_5	1.851	1.557	0.46	8
R_6	3.266	0.269	0.07	12
R_7	1.87	1.78	0.49	5
R_8	2.470	1.151	0.32	10
R_9	1.879	1.787	0.48	6
R_{10}	1.940	1.609	0.45	7
R_{11}	3.043	0.615	0.17	11
R_{12}	1.245	2.152	0.46	9

Table 14
Finding Q_i value for rankings of VIKOR.

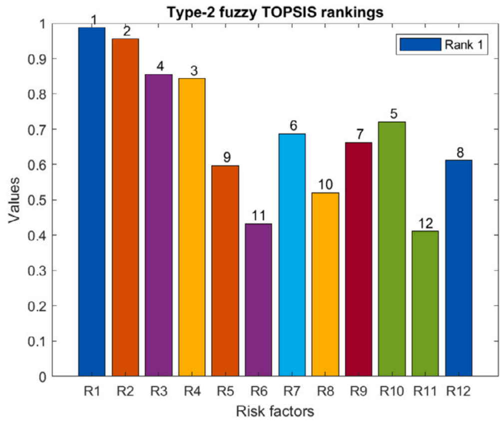
Alternatives	Q_i	Rank
R_1	0.376	1
R_2	0.412	2
R_3	0.461	4
R_4	0.454	3
R_5	0.721	8
R_6	0.821	11
R_7	0.523	5
R_8	0.743	9
R_9	0.662	6
R_{10}	0.683	7
R_{11}	0.758	10
R_{12}	0.823	12

Table 15
Final rankings of risk factors.

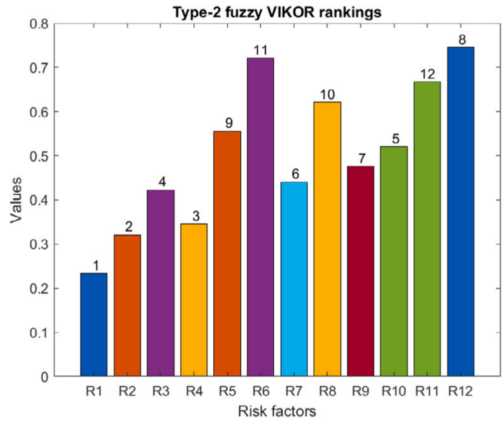
RF	T1F	T2F-TOPSIS	T2 F-Hybrid	T2 F-VIKOR	T2IVTrFEWA	T2IVTrFEWG	FIR
R_1	5	1	1	1	1	1	1
R_2	2	2	2	2	2	2	2
R_3	4	4	4	4	4	4	4
R_4	10	3	3	3	3	3	3
R_5	6	8	9	8	8	8	8
R_6	8	12	11	11	11	11	11
R_7	1	5	6	5	5	5	5
R_8	9	10	10	9	9	9	9
R_9	7	6	7	6	6	6	6
R_{10}	3	7	5	7	7	7	7
R_{11}	-	11	12	10	10	10	10
R_{12}	-	9	8	12	12	12	12

Table 16
Ranked using the aggregated values from T2IVTrFWA with $\lambda = 2$.

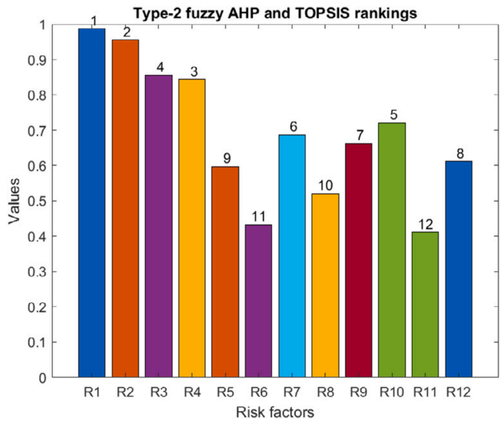
RF	TOPSIS	VIKOR	AHP & TOPSIS	T2IVTrFEWA	T2IVTrFEWG
R_1	1	1	1	1	1
R_2	2	2	2	2	2
R_3	4	4	4	4	4
R_4	3	3	3	3	3
R_5	8	8	9	9	9
R_6	12	11	11	11	11
R_7	5	5	6	6	6
R_8	10	9	10	10	10
R_9	6	6	7	7	7
R_{10}	7	7	5	5	5
R_{11}	11	10	12	12	12
R_{12}	9	12	8	8	8



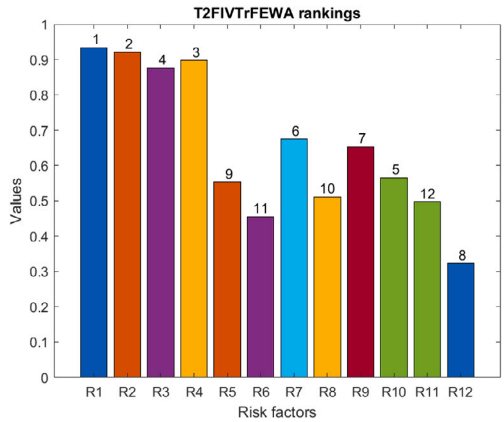
(a) Type-2 fuzzy TOPSIS rankings



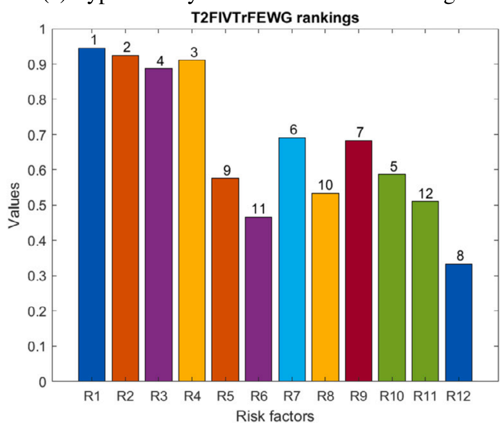
(b) Type-2 fuzzy VIKOR rankings



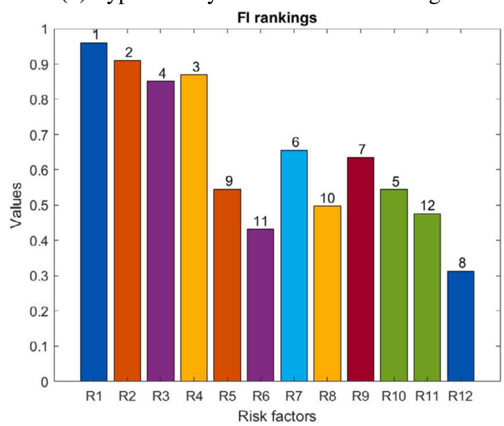
(c) Type-2 fuzzy AHP and TOPSIS rankings



(d) Type-2 fuzzy T2IVTrFEWA rankings



(e) Type-2 fuzzy T2IVTrFEWG rankings



(f) Fuzzy inference rankings

Fig. 5. Comparison of rankings obtained from different methods for evaluating the alternatives.

Table 17
Ranked using the aggregated values from T2IVTrFWA with $\lambda = 5$.

RF	TOPSIS	VIKOR	AHP & TOPSIS	T2IVTrFEWA	T2IVTrFEWG
R_1	1	1	1	1	1
R_2	2	2	2	2	2
R_3	4	4	4	4	4
R_4	3	3	3	3	3
R_5	8	8	9	9	9
R_6	12	11	11	11	11
R_7	5	5	6	6	6
R_8	10	9	10	10	10
R_9	6	6	7	7	7
R_{10}	7	7	5	5	5
R_{11}	11	10	12	12	12
R_{12}	9	12	8	8	8

Table 18
Ranked using the aggregated values from T2IVTrFWA with $\lambda = 10$.

RF	TOPSIS	VIKOR	AHP & TOPSIS	T2IVTrFEWA	T2IVTrFEWG
R_1	1	1	1	1	1
R_2	2	2	2	2	2
R_3	4	4	4	4	4
R_4	3	3	3	3	3
R_5	8	8	9	9	9
R_6	12	11	11	11	11
R_7	5	5	6	6	6
R_8	10	9	10	10	10
R_9	6	6	7	7	7
R_{10}	7	7	5	5	5
R_{11}	11	10	12	12	12
R_{12}	9	12	8	8	8

Table 19
Ranked using the aggregated values from T2IVTrFWA with $\lambda = 100$.

RF	TOPSIS	VIKOR	AHP & TOPSIS	T2IVTrFEWA	T2IVTrFEWG
R_1	1	1	1	1	1
R_2	2	2	2	2	2
R_3	4	4	4	4	4
R_4	3	3	3	3	3
R_5	8	8	9	9	9
R_6	12	11	11	11	11
R_7	5	5	6	6	6
R_8	10	9	10	10	10
R_9	6	6	7	7	7
R_{10}	7	7	5	5	5
R_{11}	11	10	12	12	12
R_{12}	9	12	8	8	8

8.4. Future work

To rectify the limitations of this work, the future works provided in the following way such as

- Developing more efficient algorithms for handling $T2IVTrFNs$ within MCDM frameworks is essential to reducing computational complexity and enhancing scalability. Future research should focus on algorithmic optimization and the integration of parallel computing techniques to streamline processing and manage large datasets effectively.
- In future works $T2IVTrFNs$ and Einstein aggregation operators involve developing efficient algorithms tailored for large datasets and exploring parallel computing and cloud-based solutions to optimize computational workflows.
- The fuzzy aggregation method can be applied to create more personalized treatment plans for TB patients. By considering individual patient data and various risk factors, the method can help healthcare providers tailor treatments to the specific needs of each patient, potentially improving treatment outcomes. Future research could explore the integration of this method into electronic health records systems to facilitate personalized healthcare.

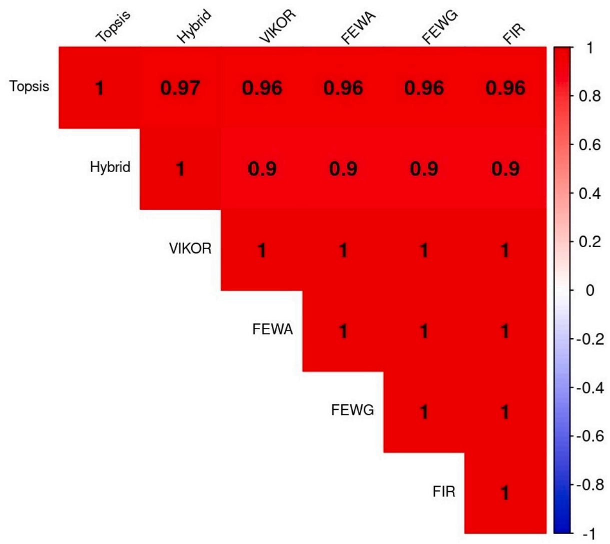


Fig. 6. Spearman correlation values.

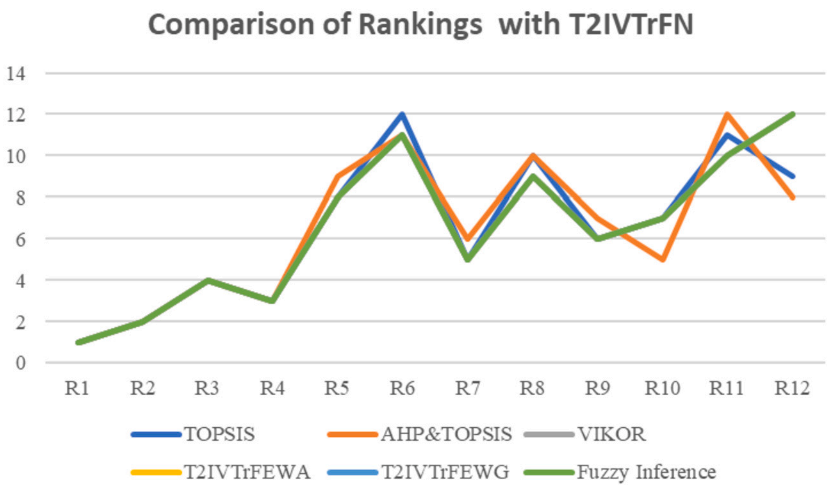


Fig. 7. Comparison of rankings.

- Effective management of healthcare resources is crucial in TB control programs. Future research could investigate the use of fuzzy aggregation methods in optimizing resource allocation, such as the distribution of medication, diagnostic tools, and healthcare personnel. This approach can help ensure that resources are used efficiently, particularly in areas with high TB prevalence.

9. Conclusion

TB is one of the deadly diseases that has not been investigated by any researchers through MCDM in the type2 interval-valued trapezoidal fuzzy context with Einstein aggregation operations. Therefore, this gap is filled by analyzing the high-risk factors of TB disease through TOPSIS, Hybrid, VIKOR, FEWA, FEWG, and FIR in a $T2IVTrFN$ with Einstein aggregation operators. These methods used a type 2 fuzzy approach to handle unclear information from the uncertainty of uncertainty. In contrast to crisp numbers and type-1 fuzzy numbers, the linguistic scale of interval type-2 fuzzy set was employed to convey the experts' assessment of alternatives in terms of criteria and the weights of each criterion. The decision process is significantly more practicable when type-2 interval-valued fuzzy sets are employed. Fuzzy simple additive weighting was implemented to determine the weights of the criteria, and the fuzzy ranking method was implemented to rank the alternatives of risk factors. The proposed method effectively resolves multi-criteria decision-making issues associated with risk factors by capturing the ambiguity of human thinking style. Additionally, a novel ranking algorithm was developed to defuzzify the $T2IVTrFN$ into a precise number. Various risk factors of tuberculosis were regarded as alternatives, and its symptoms were regarded as criteria. Diabetes (R_1) was identified as a more significant risk factor for tuberculosis based on the results of the proposed method. The least risk factor is Silicosis (R_{12}) and Crowded Places (R_6). More risk factors for tuberculosis are the combination of diabetes (R_1) immune problems (R_2), malnutrition (R_3), and alcohol (R_4). The sensitivity of the

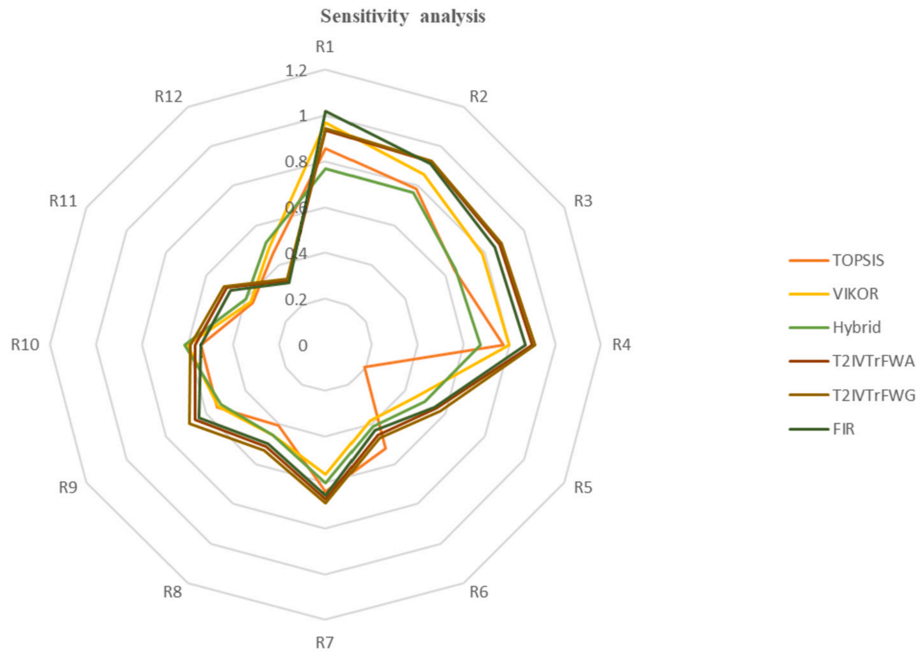


Fig. 8. Sensitivity Analysis.

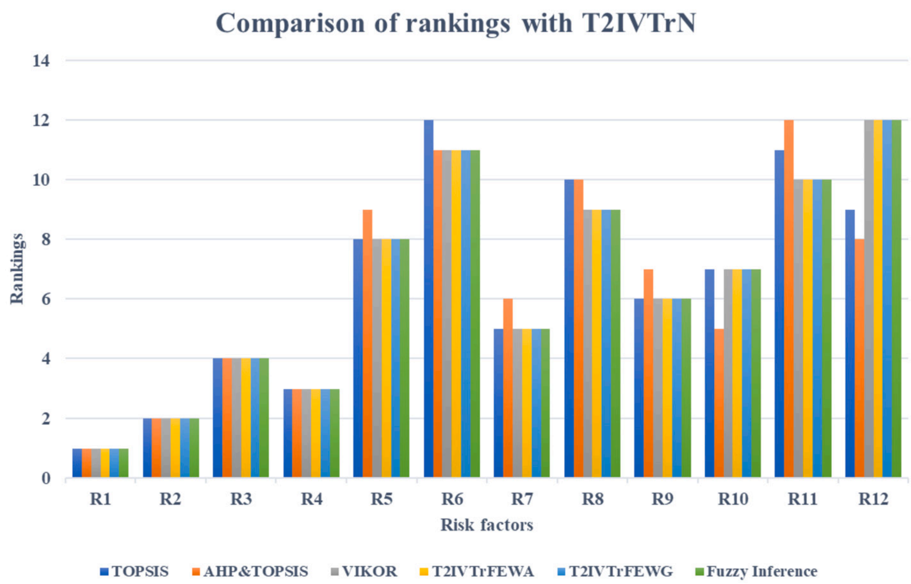


Fig. 9. Comparison of rankings with $T2IVTrFN$.

outcomes was assessed by adjusting the criteria weights in a variety of scenarios, and a comparative analysis was conducted against various MCDM methods.

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CRediT authorship contribution statement

Sheela Rani M.: Writing – original draft, Data curation, Conceptualization. **Dhanasekar S.:** Writing – review & editing, Validation, Supervision, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data obtained from experienced doctors at Sathya Sai Medical College and Research Institute, Chennai, will be provided per the recommendations.

References

[1] L. Aaron, D. Saadoun, I. Calatroni, O. Launay, N. Memain, V. Vincent, G. Marchal, B. Dupont, O. Bouchaud, D. Valeyre, et al., Tuberculosis in hiv-infected patients: a comprehensive review, Clin. Microbiol. Infect. 10 (5) (2004) 388–398.
[2] A.R. Abdul Rasam, N.M. Shariff, J.F. Dony, Identifying high-risk populations of tuberculosis using environmental factors and gis based multi-criteria decision making method, Int. Arch. Photogramm. Remote Sens. Spat. Inf. Sci. 42 (2016).
[3] Sk Ajim Ali, Ateeque Ahmad, Spatial susceptibility analysis of vector-borne diseases in kmc using geospatial technique and mcdm approach, Model. Earth Syst. Environ. 5 (2019) 1135–1159.
[4] D. Behera, Tb control in India in the covid era, Indian J. Tuberc. 68 (1) (2021) 128–133.
[5] Richard E. Bellman, Lotfi Asker Zadeh, Decision-making in a fuzzy environment, Manag. Sci. 17 (4) (1970) B-141.
[6] Ofir Ben-Assuli, Nanda Kumar, Ofer Arazy, Itamar Shabtai, The use of analytic hierarchy process for measuring the complexity of medical diagnosis, Health Inform. J. 26 (1) (2020) 218–232.
[7] Shyi-Ming Chen, Li-Wei Lee, Fuzzy multiple attributes group decision-making based on the interval type-2 topsis method, Expert Syst. Appl. 37 (4) (2010) 2790–2798.
[8] Gavin Churchyard, Peter Kim, N. Sarita Shah, Roxana Rustomjee, Neel Gandhi, Barun Mathema, David Dowdy, Anne Kasmar, Vicky Cardenas, What we know about tuberculosis transmission: an overview, J. Infect. Dis. 216 (suppl_6) (2017) S629–S635.
[9] Yiran Cui, Hui Shen, Fang Wang, Haoyu Wen, Zixin Zeng, Yafeng Wang, Chuanhua Yu, A long-term trend study of tuberculosis incidence in China, India and United States 1992–2017: a joinpoint and age–period–cohort analysis, Int. J. Environ. Res. Public Health 17 (9) (2020) 3334.
[10] Luciana Moura Mendes de Lima, Laísa Ribeiro de Sá, Ana Flávia Uzeda dos Santos Macambira, Jordana de Almeida Nogueira, Rodrigo Pinheiro de Toledo Vianna, Ronei Marcos de Moraes, A new combination rule for spatial decision support systems for epidemiology, Int. J. Health Geogr. 18 (1) (2019) 1–10.
[11] N. Ezhilarasan, A. Felix, Fuzzy electre and topsis method to analyze the risk factors of tuberculosis, in: Journal of Physics: Conference Series, vol. 2267, IOP Publishing, 2022, p. 012109.
[12] A. Fahmi, M. Aslam, R. Ahmed, Decision-making problem based on generalized interval-valued bipolar neutrosophic Einstein fuzzy aggregation operator, Soft Comput. 27 (20) (2023) 14533–14551.
[13] Aliya Fahmi, Particle swarm optimization selection based on the topsis technique, Soft Comput. 27 (14) (2023) 9225–9245.
[14] Aliya Fahmi, Saleem Abdullah, Fazli Amin, M. Sajjad Ali Khan, Trapezoidal cubic fuzzy number Einstein hybrid weighted averaging operators and its application to decision making, Soft Comput. 23 (2019) 5753–5783.
[15] Aliya Fahmi, Fazli Amin, Triangular cubic linguistic uncertain fuzzy topsis method and application to group decision making, Soft Comput. 23 (2019) 12221–12231.
[16] Aliya Fahmi, Fazli Amin, Saleem Abdullah, Muhammad Aslam, N. Ul Amin, Cubic fuzzy multi-attribute group decision-making with an application to plant location selected based on a new extended vikor method, J. Intell. Fuzzy Syst. 37 (1) (2019) 583–596.
[17] Aliya Fahmi, Aziz Khan, Thabet Abdeljawad, Group decision making based on cubic fermatean Einstein fuzzy weighted geometric operator, Ain Shams Eng. J. 15 (4) (2024) 102737.
[18] Aliya Fahmi, Aziz Khan, Thabet Abdeljawad, Manar A. Alqudah, Natural gas based on combined fuzzy topsis technique and entropy, Heliyon 10 (1) (2024).
[19] Harvir S. Gambhir, Rajeev M. Kaushik, Reshma Kaushik, Girish Sindhwani, Tobacco smoking-associated risk for tuberculosis: a case-control study, Int. Health 2 (3) (2010) 216–222.
[20] Fei Gao, Meihong Han, Siyang Wang, Jie Gao, A novel fermatean fuzzy bwm-vikor based multi-criteria decision-making approach for selecting health care waste treatment technology, Eng. Appl. Artif. Intell. 127 (2024) 107451.
[21] Neha Ghorui, Arijit Ghosh, Sankar Prasad Mondal, Mohd Yazid Bajuri, Ali Ahmadian, Soheil Salahshour, Massimiliano Ferrara, Identification of dominant risk factor involved in spread of covid-19 using hesitant fuzzy mcdm methodology, Results Phys. 21 (2021) 103811.
[22] Junhua Hu, Xiaohong Zhang, Yan Yang, Yongmei Liu, Xiaohong Chen, New doctors ranking system based on vikor method, Int. Trans. Oper. Res. 27 (2) (2020) 1236–1261.
[23] Ching-Lai Hwang, Kwangsun Yoon, Multiple Attribute Decision Making: Methods and Applications a State-of-the-Art Survey, vol. 186, Springer Science & Business Media, 2012.
[24] Christie Y. Jeon, Megan B. Murray, Diabetes mellitus increases the risk of active tuberculosis: a systematic review of 13 observational studies, PLoS Med. 5 (7) (2008) e152.
[25] Zihang Jia, Junsheng Qiao, Extension operators for type-2 fuzzy sets derived from overlap functions, Fuzzy Sets Syst. 451 (2022) 130–156.
[26] Cengiz Kahraman, Başar Öztayşi, İrem Uçal Sarı, Ebru Turanoğlu, Fuzzy analytic hierarchy process with interval type-2 fuzzy sets, Knowl.-Based Syst. 59 (2014) 48–57.
[27] G. Kaplan, V.H. Freedman, The role of cytokines in the immune response to tuberculosis, Res. Immunol. 147 (8–9) (1996) 565–572.
[28] Ting-Chun Lai, Chen-Yuan Chiang, Chang-Fu Wu, Shiang-Lin Yang, Ding-Ping Liu, Chang-Chuan Chan, Hsien-Ho Lin, Ambient air pollution and risk of tuberculosis: a cohort study, Occup. Environ. Med. 73 (1) (2016) 56–61.
[29] Qilian Liang, Jerry M. Mendel, Interval type-2 fuzzy logic systems: theory and design, IEEE Trans. Fuzzy Syst. 8 (5) (2000) 535–550.
[30] Huchang Liao, Jiayi Wang, Ming Tang, Abdullah Al-Barakati, An overview of interval analysis techniques and their fuzzy extensions in multi-criteria decision-making: what's going on and what's next?, Int. J. Fuzzy Syst. (2023) 1–28.
[31] Chia-Lun Lo, Ya-Hui Yang, Hsiao-Ting Tseng, A fact-finding procedure integrating machine learning and ahp technique to predict delayed diagnosis of bladder patients with hematuria, J. Healthc. Eng. 2021 (1) (2021) 3831453.
[32] Teresia Maina, Annie Willetts, Moses Ngari, Abdullahi Osman, Tuberculosis infection among youths in overcrowded university hostels in Kenya: a cross-sectional study, Trop. Med. Health 49 (2021) 1–11.
[33] Vadim Makarov, Katarína Mikušová, Development of macozinone for tb treatment: an update, Appl. Sci. 10 (7) (2020) 2269.
[34] Faisal Mehmood, Khizar Hayat, Tahir Mahmood, Bing-Yuan Cao, A multi criteria decision making method for cubic hesitant fuzzy sets based on Einstein operational laws, Ital. J. Pure Appl. Math. 42 (2019) 652–669.

[35] Jerry M. Mendel, Jerry M. Mendel, Type-2 fuzzy sets, in: Uncertain Rule-Based Fuzzy Systems: Introduction and New Directions, 2nd edition, 2017, pp. 259–306.
[36] Gabriel Tassi Mousquer, Alessandra Peres, Marilu Fiegenbaum, Pathology of tb/covid-19 co-infection: the phantom menace, Tuberculosis 126 (2021) 102020.
[37] Arvind Natarajan, P.M. Beena, Anushka V. Devnikar, Sagar Mali, A systemic review on tuberculosis, Indian J. Tuberc. 67 (3) (2020) 295–311.
[38] O.B. Nechaeva, Tb situation in Russia, Tuberc. Lung Dis. 96 (8) (2018) 15–24.
[39] Richard Owusu Nyarko, Amit Prakash, Nayan Kumar, Purabi Saha, Roshan Kumar, Tuberculosis a globalized disease, Asian J. Pharm. Res. Dev. 9 (1) (2021) 198–201.
[40] J. Ockenga, K. Fuhse, S. Chatterjee, R. Malykh, H. Rippin, M. Pirlich, A. Yedilbayev, K. Wickramasinghe, R. Barazzoni, Tuberculosis and malnutrition: the European perspective, Clin. Nutr. 42 (4) (2023) 486–492.
[41] Serafim Opricovic, Multicriteria optimization of civil engineering systems, Fac. Civ. Eng., Belgrade 2 (1) (1998) 5–21.
[42] World Health Organization, et al., Global Tuberculosis Report 2021: Supplementary Material, 2022.
[43] Andrzej Piegat, Wojciech Sałabun, Comparative analysis of mcdm methods for assessing the severity of chronic liver disease, in: Artificial Intelligence and Soft Computing: 14th International Conference, ICAISC 2015, Zakopane, Poland, June 14-18, 2015, Proceedings, Part I 14, Springer, 2015, pp. 228–238.
[44] Alfredo Ponce-De-Leon, Ma de Lourdes Garcia-Garcia, Ma Cecilia Garcia-Sancho, Francisco J. Gomez-Perez, Jose Luis Valdespino-Gomez, Gustavo Olaiz-Fernandez, Rosalba Rojas, Leticia Ferreyra-Reyes, Bulmaro Cano-Arellano, Miriam Bobadilla, et al., Tuberculosis and diabetes in southern Mexico, Diabetes Care 27 (7) (2004) 1584–1590.
[45] Pratibha Rani, Arunodaya Raj Mishra, Fermatean fuzzy Einstein aggregation operators-based multimoora method for electric vehicle charging station selection, Expert Syst. Appl. 182 (2021) 115267.
[46] Blanca I. Restrepo, Diabetes and tuberculosis, in: Understanding the Host Immune Response Against Mycobacterium Tuberculosis Infection, 2018, pp. 1–21.
[47] Katharina Ronacher, Simone A. Joosten, Reinout van Crevel, Hazel M. Dockrell, Gerhard Walzl, Tom H.M. Ottenhoff, Acquired immunodeficiencies and tuberculosis: focus on hiv/aids and diabetes mellitus, Immunol. Rev. 264 (1) (2015) 121–137.
[48] Thomas L. Saaty, The analytic hierarchy process (ahp), J. Oper. Res. Soc. 41 (11) (1980) 1073–1076.
[49] T-C. Shen, K-Y. Huang, C-H. Chao, Y-C. Wang, C-H. Muo, C-C. Wei, C-Y. Tu, T-C. Hsia, C-M. Shih, W-H. Hsu, et al., The risk of chronic kidney disease in tuberculosis: a population-based cohort study, QJM: Int. J. Med. 108 (5) (2015) 397–403.
[50] Evangelia Simou, John Britton, Jo Leonardi-Bee, Alcohol consumption and risk of tuberculosis: a systematic review and meta-analysis, Int. J. Tuberc. Lung Dis. 22 (11) (2018) 1277–1285.
[51] Catherine R. Stevenson, Nita G. Forouhi, Gojka Roglic, Brian G. Williams, Jeremy A. Lauer, Chirstopher Dye, Nigel Unwin, Diabetes and tuberculosis: the impact of the diabetes epidemic on tuberculosis incidence, BMC Public Health 7 (2007) 1–8.
[52] Lydia Uden, Ella Barber, Nathan Ford, Graham S. Cooke, Risk of tuberculosis infection and disease for health care workers: an updated meta-analysis, in: Open Forum Infectious Diseases, vol. 4, Oxford University Press US, 2017, p. ofx137.
[53] Faith-Michael Uzoka, Chukwudi Nwokoro, Okure Obot, Moses Ekpenyong, Aniema I.A. Udo, Boluwaji Akinnuwesi, Analytic hierarchy process model for the diagnosis of typhoid fever, in: Proceedings of the Future Technologies Conference, Springer, 2022, pp. 341–358.
[54] Fevrier Valdez, Oscar Castillo, Prometeo Cortes-Antonio, Patricia Melin, A survey of type-2 fuzzy logic controller design using nature inspired optimization, J. Intell. Fuzzy Syst. 39 (5) (2020) 6169–6179.
[55] Weize Wang, Xinwang Liu, Intuitionistic fuzzy geometric aggregation operators based on Einstein operations, Int. J. Intell. Syst. 26 (11) (2011) 1049–1075.
[56] Gang Xiao, Khizar Hayat, Xiaopeng Yang, Evaluation and its derived classification in a server-to-client architecture based on the fuzzy relation inequality, Fuzzy Optim. Decis. Mak. 22 (2) (2023) 213–245.
[57] Ting-Ting Xu, Jin-Dong Qin, A new representation method for type-2 fuzzy sets and its application to multiple criteria decision making, Int. J. Fuzzy Syst. (2023) 1–20.
[58] Xiao-bin Yang, Xiao-peng Yang, Khizar Hayat, A new characterisation of the minimal solution set to max-min fuzzy relation inequalities, Fuzzy Inf. Eng. 9 (4) (2017) 423–435.
[59] Lotfi A. Zadeh, Fuzzy sets, Inf. Control 8 (3) (1965) 338–353.
[60] Lotfi A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-iii, Inf. Sci. 9 (1) (1975) 43–80.
[61] Lotfi Asker Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets Syst. 1 (1) (1978) 3–28.
[62] Xiaofei Zhao, Guiwu Wei, Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making, Knowl.-Based Syst. 37 (2013) 472–479.
[63] Rana Muhammad Zulqarnain, Sohaib Abdal, B. Ali, L. Ali, F. Dayan, M.I. Ahamad, Z. Zafar, Selection of medical clinic for disease diagnosis by using topsis method, Int. J. Pharm. Sci. Rev. Res. 61 (1) (2020) 22–27.
[64] Rana Muhammad Zulqarnain, Rifaqat Ali, Jan Awrejcewicz, Imran Siddique, Fahd Jarad, Aiyared Iampan, Some Einstein geometric aggregation operators for q-rung orthopair fuzzy soft set with their application in mcdm, IEEE Access 10 (2022) 88469–88494.
[65] Rana Muhammad Zulqarnain, Imran Siddique, Rifaqat Ali, Jan Awrejcewicz, Hanen Karamti, Dariusz Grzelczyk, Aiyared Iampan, Muhammad Asif, Einstein ordered weighted aggregation operators for Pythagorean fuzzy hypersoft set with its application to solve mcdm problem, IEEE Access 10 (2022) 95294–95320.