Journal of Advanced Research 25 (2020) 217–225

Contents lists available at ScienceDirect

Journal of Advanced Research

journal homepage: www.elsevier.com/locate/jare

Fractional-order autonomous circuits with order larger than one

Yanwei Jiang [a], Bo Zhang [b,*], Xujian Shu [b], Zhihao Wei [b]

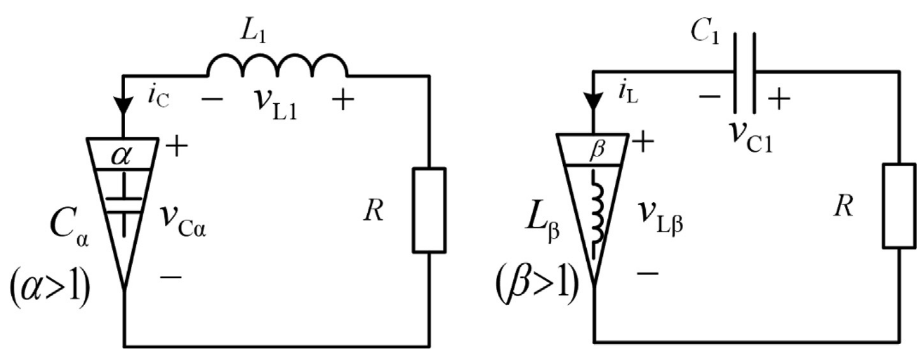
[a] College of Electric Engineering and Automation, Fuzhou University, 350108, China
[b] School of Electric Power Engineering, South China University of Technology, Guangzhou 510641, China

HIGHLIGHTS

- Two kinds of new fractional-order autonomous circuits are proposed.
- The fractional-order autonomous circuits are based on fractional-order elements with order larger than one.
- The operating frequency or resonant frequency of the circuits can be changed by adjusting the resistance.
- The current and voltage of the circuits can be controlled by adjusting the orders of fractional-order elements.
- The available simulations verify the effectiveness of the theoretical analysis.

GRAPHICAL ABSTRACT

Two kinds of fractional-order autonomous circuits are constructed by using fractional-order capacitor and fractional-order inductor respectively. The orders of the adopted fractional-order elements must be greater than one. The corresponding circuit simulations were developed and verified the proposed fractional-order autonomous circuits.



ARTICLE INFO

Article history:
Received 15 February 2020
Revised 29 April 2020
Accepted 3 May 2020
Available online 22 May 2020

Keywords:
Fractional calculus
Fractional-order element
Fractional-order autonomous circuit

ABSTRACT

Fractional-order circuit is a kind of circuit which contains fractional-order elements. It has been proved that the fractional-order circuit has some characteristics which are hard to be achieved by integer-order circuits, such as higher degree of freedom in circuit design. For integer-order circuits, there are not only non-autonomous circuits, but also autonomous circuits. Since there are many applications of integral-order autonomous circuits in real world, it is also necessary to explore fractional-order autonomous circuits. However, few research focuses on fractional-order autonomous circuits. Therefore, this paper proposes two kinds of fractional-order autonomous circuits based on fractional-order elements with order larger than one. The corresponding mathematical models are also established based on fractional calculus and their characteristics are analyzed based on circuit theory. Finally, circuit simulation are performed to verify the correctness of theoretical analysis.

Introduction

Recent years have witnessed a continuous progress of fractional-order calculus, which can be applied rheology, electrochemistry, mechanics, bioengineering, circuit systems and other fields [1–5]. Fractional-order calculus is defined as the extension of traditional integer-order calculus to arbitrary

Peer review under responsibility of Cairo University.
* Corresponding author.
E-mail address: epbzhang@scut.edu.cn (B. Zhang).

https://doi.org/10.1016/j.jare.2020.05.005
2090-1232/

non-integer-order calculus. The initial application of fractional-order calculus in circuit systems is to accurately describe the models of capacitors, because there is no ideal integer-order capacitor [6]. Since the orders of most capacitors are normally close to 1, they are often treated as 1 with neglecting their fractional-order characteristics. However, some capacitors and inductors are found to have strong fractional-order characteristics, for example, the order of supercapacitors and relay coils are far less than one [7,8]. These elements with fractional-order characteristics are generally called fractional-order elements, mainly including fractional-order capacitor (FOC) and fractional-order inductor (FOI). Assuming that $i_C(t)$ and $v_C(t)$ are the current and voltage of FOC respectively, then the model that involves both characteristics can be described by the following relationship, given in [9]

$$i_C(t) = C_\alpha \frac{d^\alpha}{dt^\alpha} v_C(t) \tag{1}$$

where α and C_α are the order and capacitance of FOC respectively and d^α/dt^α is the fractional-order derivative operator. For FOC, its current leads the voltage $0.5\pi\alpha$ degree, so the order α is generally $0 < \alpha < 2$. This is because if the order is greater than 2, the degree of current lead voltage will be more than 180°, at this time, the element would become inductive and no longer capacitive. Similarly, assuming that $i_L(t)$ and $v_L(t)$ are the current and voltage of FOI respectively, then the model that involves the characteristics can be described by the following relationship, given in [9]

$$v_L(t) = L_\beta \frac{d^\beta}{dt^\beta} i_L(t) \tag{2}$$

where β and L_β are the order and inductance of FOI respectively, and $0 < \beta < 2$. From (1) and (2), the fractional order element has one more parameter than the integer-order element. This extra parameter makes fractional-order elements have different properties from integer-order elements. For example, fractional-order elements possess both real and imaginary impedance part, while an ideal capacitor or inductor has only an imaginary part [10]. Although fractional-order elements have not been commercialized, various FOCs and FOIs have been fabricated in the laboratory [11–19], paving the way for the application of fractional-order elements.

Fractional-order elements can be used to construct a variety of fractional-order circuits such as fractional-order DC-DC converter, fractional-order impedance network, fractional-order RLC resonant circuit, wireless power transfer system, fractional-order PID controller and so on [20–31]. In [20], the FOIs are used in DC-DC converters, and the result shows that the output voltage gain can be adjusted by changing the order of the FOI. In fractional-order impedance matching networks, only a single FOI or FOC match any inductive or capacitive impedance, so extra resistance does not needed [24]. Reference [26] presents a fractional-order RLC resonant circuit, which demonstrates that the resonant frequency can be controlled not only by inductance and capacitance but also by the fractional-order α and β. FOC can also be used to realize a fractional-order wireless power transfer circuit, whose output characteristic is determined by the order of FOC, and when the order is constant, constant current output of the circuit can be realized, while integer-order circuits is difficult to achieve [29,30]. In [31], using simple analog circuits can realize a fractional-order PID controller, which has more control freedom than the integer-order PID controller. Therefore, the fractional-order circuits and have shown more design flexibility and beneficial performance than the integer-order circuit.

The above fractional-order circuits are all non-autonomous. Just like the integer-order circuits have non-autonomous circuits and autonomous circuits, the fractional-order circuits also have non-autonomous circuits and autonomous circuits. The integer-order autonomous circuits have been proved to play an important role in signal processing, aerospace, chaotic secure communications and other fields [32,33]. At the same time, although fractional-order non-autonomous circuits have demonstrated more characteristics than integer-order non-autonomous circuits. However, there are few researches on fractional-order autonomous circuits, as the work done by Ana Dalia Pano-Azucena in [34]. Therefore, it is of great significance to study fractional-order autonomous circuits.

In [35], the characteristics of fractional-order autonomous system is analyzed, but only stays at the mathematical level. In [36], a fractional-order autonomous wireless power transfer system constructed by FOC is proposed for the first time, and its experiment demonstrates that fractional-order autonomous system has better anti-interference performance than integer-order system. However, reference [36] mainly analyzes the energy transfer characteristics based on coupled-mode theory, and lacks the analysis of the basic circuit characteristics of a single fractional-order autonomous circuit.

This paper focuses on the topologies and properties of fractional-order autonomous circuits and is organized as follows. The characteristics of fractional-order elements are introduced in Section2. The structures and mathematical models of the proposed fractional-order autonomous circuits are described in Section 3. The circuit characteristics are analyzed in Section 4. Circuit simulations are demonstrated in Section 5. Final conclusions are offered in Section 6.

Fractional-order elements with order bigger than one

By processing Laplace transformation of (1) and (2) respectively, the impedances of FOI and FOC can be derived as $Z_C(s) = 1/(s^\alpha C_\alpha)$ and $Z_L(s) = s^\beta L_\beta$, where s is the Laplace operator. Let $s = j\omega$, the impedance expressions can be respectively expressed as

$$Z_C = \frac{1}{(j\omega)^\alpha C_\alpha} \tag{3}$$

$$Z_L = (j\omega)^\beta L_\beta \tag{4}$$

where ω is the operating angle frequency of the fractional-order elements. According to Euler formula, we have $e^{j0.5\pi} = \cos(0.5\pi) + j\sin(0.5\pi) = j$. Then the following equation can be obtained.

$$j^\alpha = e^{j0.5\pi\alpha} = \cos(0.5\pi\alpha) + j\sin(0.5\pi\alpha) \tag{5}$$

By substituting (5) into (3), the impedance of FOC can be described as

$$Z_C = \frac{1}{\omega^\alpha C_\alpha}\cos(0.5\pi\alpha) - j\frac{1}{\omega^\alpha C_\alpha}\sin(0.5\pi\alpha) \tag{6}$$

Similarly, the impedance of FOI can be derived as

$$Z_L = \omega^\beta L_\beta \cos(0.5\pi\beta) + j\omega^\beta L_\beta \sin(0.5\pi\beta) \tag{7}$$

From (6) and (7), the real part of the FOC and FOI impedance can be described as

$$R_{Ceq} = \frac{1}{\omega^\alpha C_\alpha}\cos(0.5\pi\alpha) \tag{8}$$

$$R_{Leq} = \omega^\beta L_\beta \cos(0.5\pi\beta) \tag{9}$$

From (8) and (9), when $\alpha > 1$ and $\beta > 1$, $R_{Ceq} < 0$ and $R_{Leq} < 0$, so such a FOC or FOI has the characteristic of negative resistor.

Assuming that the voltage of FOC is $v_C(t) = V_{Cm}\sin(\omega t)$, so the stead state current of FOC can be derived as

$$i_C(t) = \omega^\alpha C_\alpha V_{Cm} \sin(\omega t + 0.5\pi\alpha) \tag{10}$$

Therefore, the instantaneous power of FOC can be acquired as

$$p_{C\alpha}(t) = v_C(t)i_C(t) = \frac{\omega^\alpha C_\alpha V_{Cm}^2}{2}\cos(0.5\pi\alpha)[1-\cos(2\omega t)] + \frac{\omega^\alpha C_\alpha V_{Cm}^2}{2}\sin(0.5\pi\alpha)\sin(2\omega t) \quad (11)$$

Then, the average power of fractional capacitor in a sinusoidal period Ts is

$$P_{C\alpha} = \frac{1}{T_S}\int_0^{T_S} p_{C\alpha}(t)dt = \frac{\omega^\alpha C_\alpha V_{Cm}^2}{2}\cos(0.5\pi\alpha) \quad (12)$$

where $T_S = 2\pi/\omega$. From (12), when $\alpha < 1$, $P_{C\alpha} > 0$ which means that the FOC consumes power. However, when $\alpha > 1$, $P_{C\alpha} < 0$ which means that the FOC supply power. Similarly, FOI with $\beta > 1$ also can supply power. Therefore, fractional-order element with order larger than one is an active element. In fact, this conclusion is also consistent with the existing experiments, because the existing fractional-order elements with order greater than one in the laboratory are all also constructed by active methods [17,18].

Proposed factional-order autonomous circuit and model

Circuit topology

An negative resistor can be used to construct an integer-order autonomous oscillation circuit together with integral-order inductor and capacitor [37]. Since fractional-order element of order greater than one has a part of negative resistance, it can also be applied to realize autonomous circuits. In this paper, two types of fractional-order autonomous circuits are proposed as shown in Fig. 1.

The first type is FOC-based fractional-order autonomous circuit, which is composed of a FOC, an integer-order inductor and a resistor in series, as shown in Fig. 1a. The second is FOI-based fractional-order autonomous circuit, which is composed of a FOI, an integer-order capacitor and a resistor as shown in Fig. 1b. By using the negative resistance characteristic of order greater than 1, the fractional-order autonomous circuit can be realized without additional single negative resistance, while the traditional integer-order autonomous circuit needs a single negative resistance to excite the circuit and continuously provide the required energy. For the fractional-order elements with order less than 1, they have only the characteristic of positive resistance but no negative resistance. Therefore, the autonomous circuit proposed in this paper can only be constructed by using fractional-order elements with order larger than 1.

According to Fig. 1, the total impedances of the FOC-based circuit and the FOI-based circuit can be respectively described as

$$Z_{T_FOC} = R + \frac{1}{\omega^\alpha C_\alpha}\cos(0.5\pi\alpha) - j\frac{1}{\omega^\alpha C_\alpha}\sin(0.5\pi\alpha) + j\omega L_1 \quad (13)$$

$$Z_{T\_FOI} = R + \omega^\beta L_\beta \cos(0.5\pi\beta) + j\omega^\beta L_\beta \sin(0.5\pi\beta) - j\frac{1}{\omega C_1} \quad (14)$$

When the imaginary part of the circuit impedance is zero, the circuits resonate. Therefore, using (13) and (14), the resonant frequency ω_{R_FOC} of FOC-based circuit can be derived as

$$\omega_{R\_FOC} = \left[\frac{\sin(0.5\pi\alpha)}{L_1 C_\alpha}\right]^{\frac{1}{\alpha+1}} \quad (15)$$

and the resonant frequency ω_{R_FOI} of FOI-based circuit is

$$\omega_{R\_FOI} = \left[\frac{1}{L_\beta C_1 \sin(0.5\pi\beta)}\right]^{\frac{1}{\beta+1}} \quad (16)$$

As can be observed from (15) and (16), the resonant frequencies of fractional-order circuits depend not only on the inductance and capacitance but also on the order.

Mathematical model

For the fractional-order autonomous circuit based on FOC as shown in Fig. 1(a), the following voltage equations can be acquired based on KVL.

$$0 = v_{C\alpha} + Ri_C + v_{L1} \quad (17)$$

By substituting $v_{L1} = L_1 di_C/dt$ and (1) into (17), the model of Fig. 1a can be deduced as

$$0 = L_1 C_\alpha \frac{d^{\alpha+1}}{dt^{\alpha+1}} v_{C\alpha} + RC_\alpha \frac{d^\alpha}{dt^\alpha} v_{C\alpha} + v_{C\alpha} \quad (18)$$

From (18), it can be observed that the circuit of Fig. 1b is an autonomous circuit

For the fractional-order autonomous circuit based on FOI as shown in Fig. 1(b), its voltage equations can be acquired as

$$0 = v_{C1} + Ri_L + v_{L\beta} \quad (19)$$

By substituting $i_L = C_1 dv_{C1}/dt$ and (2) into (19), the model of Fig. 1a can be derived as

$$0 = L_\beta C_1 \frac{d^{\beta+1}}{dt^{\beta+1}} v_{C1} + RC_1 \frac{d}{dt} v_{C1} + v_{C1} \quad (20)$$

From (20), it also can be observed that the circuit of Fig. 1b is an autonomous circuit.

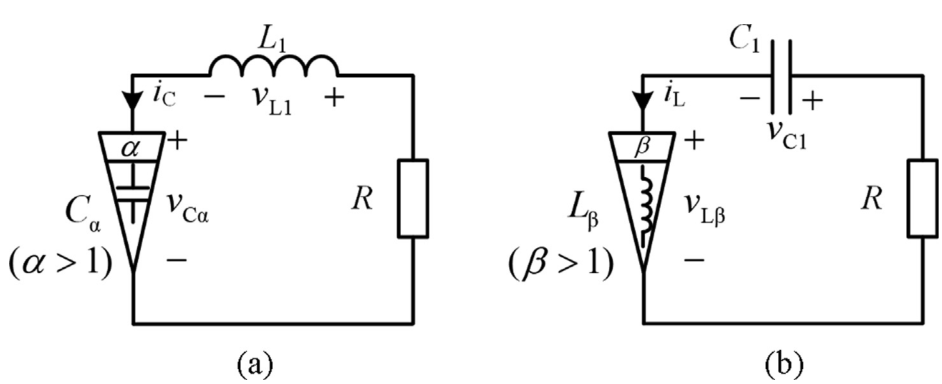


Fig. 1. The proposed fractional-order autonomous circuits based on (a) FOC and (b) FOI.

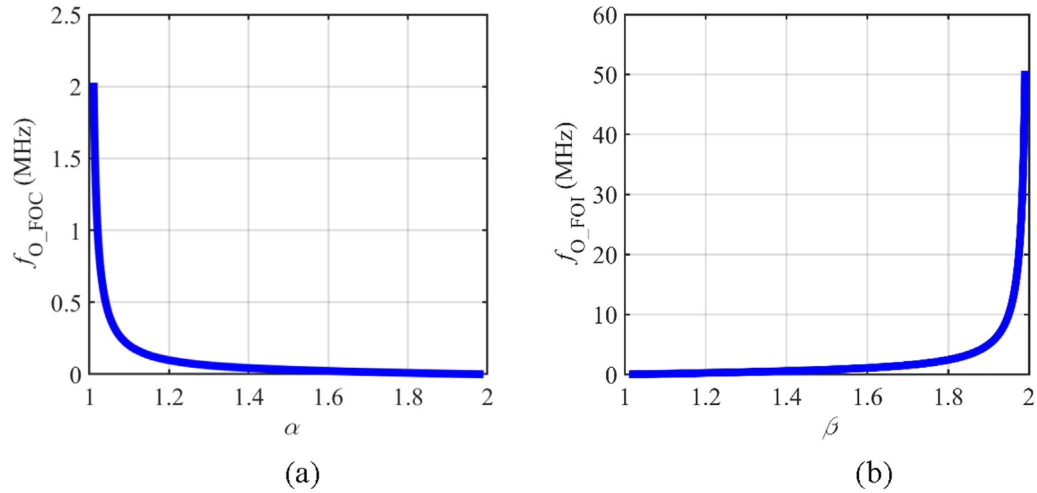


Fig. 2. The operating frequency as a function of order (a) FOC- based fractional-order autonomous circuits (b) FOI- based fractional-order autonomous circuits.

Characteristics analysis

Unlike the non-autonomous circuit that is influenced by the external power supply, the autonomous circuit is a free oscillation circuit. For example, the operating frequency of an non-autonomous circuit is determined by the external power supply, while the operating frequency of an autonomous circuit is determined by the circuit parameters. Moreover, for an integer-order autonomous circuit, the value of the negative resistance needs to be able to change online, which depends on the circuit parameters [32]. In an fractional-order autonomous circuit, in analogy with integer-order autonomous circuit, the fractional-order elements have fixed order, but should allow capacitance or inductance to change online. The capacitance or inductance is also determined by other circuit parameters. In addition, since the energy of the fractional-order autonomous circuits come from FOC or FOI, the current and voltage of the circuits are decided by the power capacity of fractional-order element and circuit parameters. Therefore, this section mainly analyzes the effect of circuit parameters on circuit characteristics, including operating frequency, values of fractional-order element, voltage and current. In addition, the stability of the proposed autonomous circuits is also discussed in this section.

Operating frequency

Assuming that $v_{C\alpha}(0) = v_{C1}(0) = 0$, the Laplace transformation of (18) and (20) can be respectively obtained as

$$0 = \left(s^{\alpha+1}L_1C_\alpha + s^\alpha RC_\alpha + 1\right)V_{C\alpha}(s) \tag{21}$$

$$0 = \left(s^{\beta+1}L_\beta C_1 + sRC_1 + 1\right)V_{C1}(s) \tag{22}$$

Let $s = j\omega$, we have

$$0 = (j\omega)^{\alpha+1}L_1C_\alpha + (j\omega)^\alpha RC_\alpha + 1 \tag{23}$$

$$0 = (j\omega)^{\beta+1}L_\beta C_1 + j\omega RC_1 + 1 \tag{24}$$

Using (5) and separating the real and imaginary part of (23) and (24), we can obtain

$$\begin{cases} \omega^{\alpha+1}L_1C_\alpha\cos(0.5\pi\alpha) + \omega^\alpha RC_\alpha\sin(0.5\pi\alpha) = 0 \\ 1 - \omega^{\alpha+1}L_1C_\alpha\sin(0.5\pi\alpha) + \omega^\alpha RC_\alpha\cos(0.5\pi\alpha) = 0 \end{cases} \tag{25}$$

and

$$\begin{cases} \omega^{\beta+1}L_\beta C_1\cos(0.5\pi\beta) + \omega RC_1 = 0 \\ 1 - \omega^{\beta+1}L_\beta C_1\sin(0.5\pi\beta) = 0 \end{cases} \tag{26}$$

Hence, from (25), the operating frequency f_{O_FOC} of the FOC-based circuit can be derived as

$$f_{O\_FOC} = -\frac{R}{2\pi L_1}\mathrm{tg}(0.5\pi\alpha) \tag{27}$$

From (26), the operating frequency f_{O_FOI} of the FOI-based circuit is deduced as

$$f_{O\_FOI} = \frac{-1}{2\pi RC_1}\mathrm{ctg}(0.5\pi\beta) \tag{28}$$

According to (27) and (28), the operating frequencies of the fractional-order autonomous circuits are determined not only by integer-order elements but also by orders of FOC or FOI. Fig. 2 shows the curves of the operating frequencies with different orders.

As can be seen from Fig. 2(a), the operating frequency of FOC-based autonomous circuit decreases with increase order α, while from Fig. 2(b), the operating frequency of FOI-based autonomous circuit increases with increase order β.

Solutions of fractional-order elements

When the fractional-order autonomous circuits operate at steady-state, the capacitance of FOC and the inductance of FOI can be respectively derived as follow equations by solving (25) and (26).

$$C_\alpha = -\frac{1}{R}\left[-\frac{R}{L_1}\mathrm{tg}(0.5\pi\alpha)\right]^{-\alpha}\cos(0.5\pi\alpha) \tag{29}$$

$$L_\beta = \frac{1}{C_1\sin(0.5\pi\beta)}\left[\frac{-1}{RC_1}\mathrm{ctg}(0.5\pi\beta)\right]^{-(\beta+1)} \tag{30}$$

Fig. 3 shows the curves of the C_α and L_β with different orders. As can be observed from Fig. 3(a), the capacitance of FOC is not monotonic with increase order α, while from Fig. 3(b), the inductance of FOI-based autonomous circuit decreases with increase order β.

In addition, substituting (29) into (15) and substituting (30) into (16), the resonant angle frequencies of the FOC-based circuit and FOI-based circuit can be rewritten as follows.

$$\omega_{R\_FOC} = -\frac{R}{L_1}\mathrm{tg}(0.5\pi\alpha) \tag{31}$$

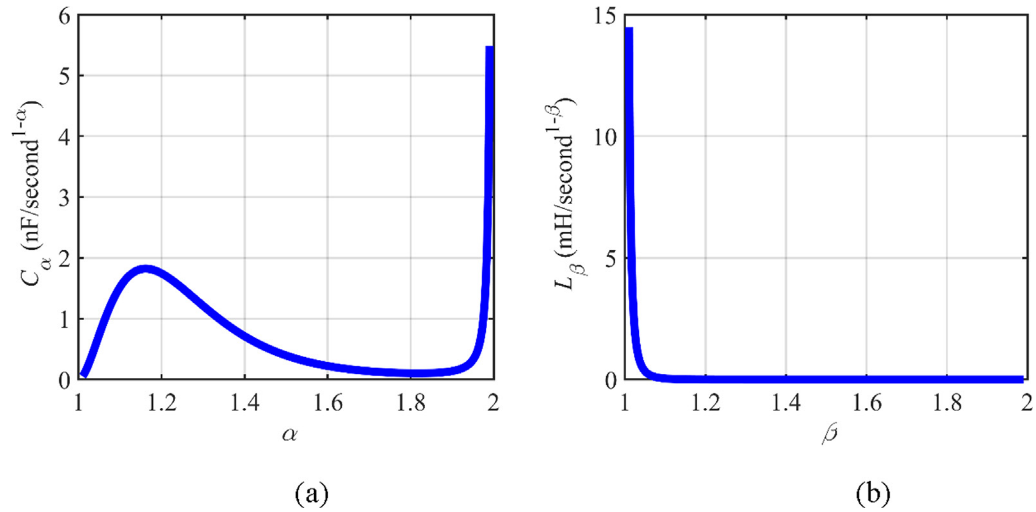


Fig. 3. (a) The capacitance of FOC as a function of order (b) The inductance of FOI as a function of order.

$$\omega_{\mathrm{R\_FOI}} = \frac{-1}{RC_1}\mathrm{ctg}(0.5\pi\beta) \tag{32}$$

By comparing (27) and (31), and by comparing (28) and (32), it can be seen that the resonant frequency of fractional order autonomous circuit is consistent with the operating frequency. Moreover, from (31) and (32), the resonant frequency of fractional-order autonomous circuit can be adjusted by changing the resistance R.

Current and voltage

The current of fractional-order autonomous circuit depends on the active power released by fractional-order elements. Assume that the apparent power of fractional-order elements is S and $\alpha = \beta = \Upsilon$, so the released active power of FOC or FOI is $P_{\mathrm{FOC}} = P_{\mathrm{FOI}} = -S\cos(0.5\pi\Upsilon)$. Therefore, the current of the fractional-order autonomous circuits can be derived as

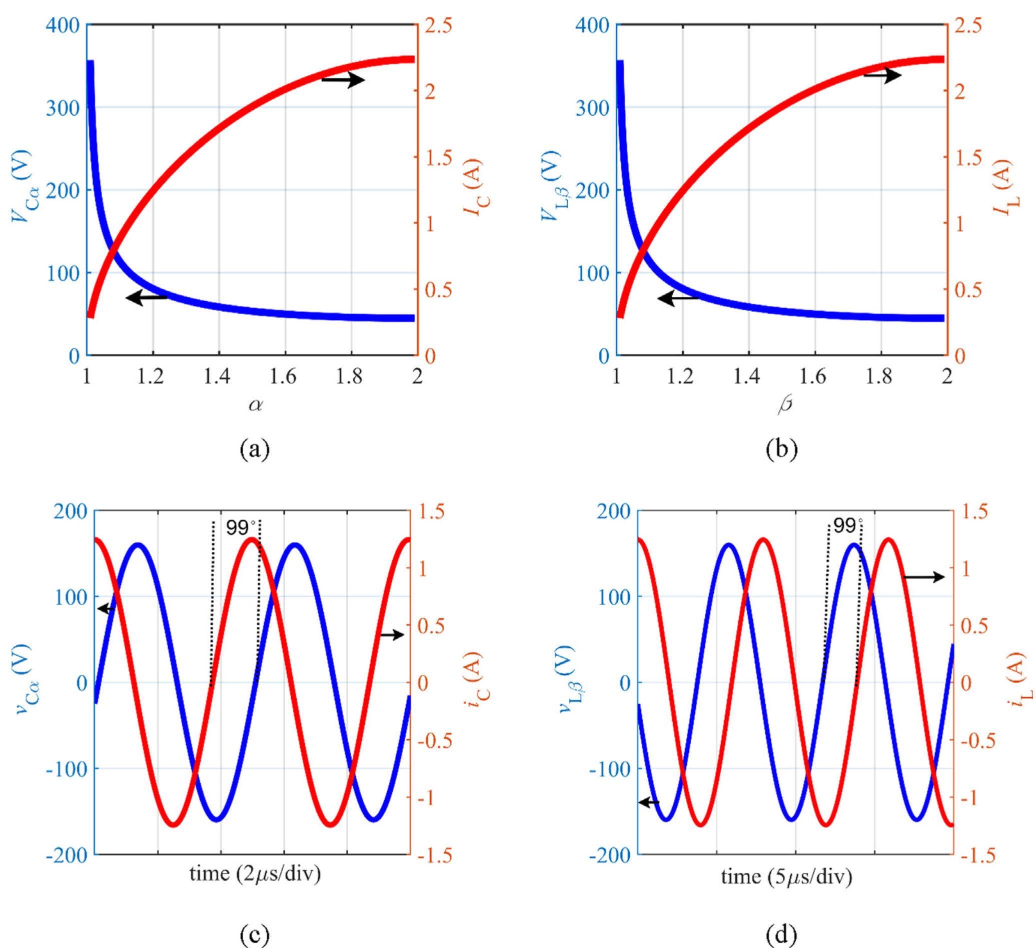


Fig. 4. The RMS values of current and voltage of (a) FOC and (b) FOI as a function of order. The time-domain waveform of current and voltage of (c) FOC and (d) FOI.

$$I_C = I_L = \sqrt{-\frac{S\cos(0.5\pi\gamma)}{R}} \tag{33}$$

where I_C is the RMS value of FOC-based fractional-order autonomous circuit and I_L is the RMS value of FOI-based fractional-order autonomous circuit. From (33), the current of fractional-order autonomous circuit is only related to the apparent power, the order and the resistance, but not to the inductance or capacitance and frequency.

As can be seen from Fig. 1(a), the voltage of FOC is equal to the voltage of the series branch of L_1 and R, so the RMS value of FOC voltage $V_{C\alpha}$ can be deduced as

$$V_{C\alpha} = \sqrt{-\frac{S\cos(0.5\pi\alpha)}{R}}\sqrt{(2\pi f_{O\_FOC}L_1)^2 + R^2} \tag{34}$$

Similarly, the RMS value of FOI voltage $V_{L\beta}$ can be derived as

$$V_{L\beta} = \sqrt{-\frac{S\cos(0.5\pi\beta)}{R}}\sqrt{\left(\frac{1}{2\pi f_{O\_FOC}C_1}\right)^2 + R^2} \tag{35}$$

It is worth noting that not only the RMS but also the phase of voltage and current are related to the orders. The phase between voltage and current of fractional-order elements is equal to $0.5\pi\Upsilon$. Fig. 4 shows the currents and voltages of FOC and FOI. As can be seen from Fig. 4(a), when the order of FOC increases, the RMS of current also increases monotonously, while the voltage decreases. From Fig. 4(b), the RMS values of voltage and current of FOI-based autonomous circuit have the same characteristics with FOC-based autonomous circuit. Fig. 4(c) and (d) show the steady-state time-domain waveform of FOC and FOI with Υ = 1.1. It can be observed that the current of FOC has a leading degree of 99° from FOC voltage, while the current of FOI lags the voltage 99°.

Stability analysis

The stability of fractional-order autonomous circuits can be analyzed by the method of reference [38]. From the math model of (18), the characteristic equation in the s-domain of the autonomous circuit based on FOC can be acquired as

$$0 = s^{\alpha+1}L_1C_\alpha + s^\alpha RC_\alpha + 1 \tag{36}$$

Assuming α can be represented as a rational number $\alpha = k/m$, where k and m are positive integers. Let us define $W = s^{1/m}$, so equation (36) can be transferred to W-plane and is rewritten as

$$0 = W^{k+m}L_1C_\alpha + W^kRC_\alpha + 1 \tag{37}$$

Therefore, the $\pm j\omega$ axes of the s-plane can be mapped onto the lines $|\theta_W| = \pi/2m$ in W-plane. By using numerical calculation, the roots of equation (37) with different orders can be obtained, as shown by the triangle mark in Fig. 5.

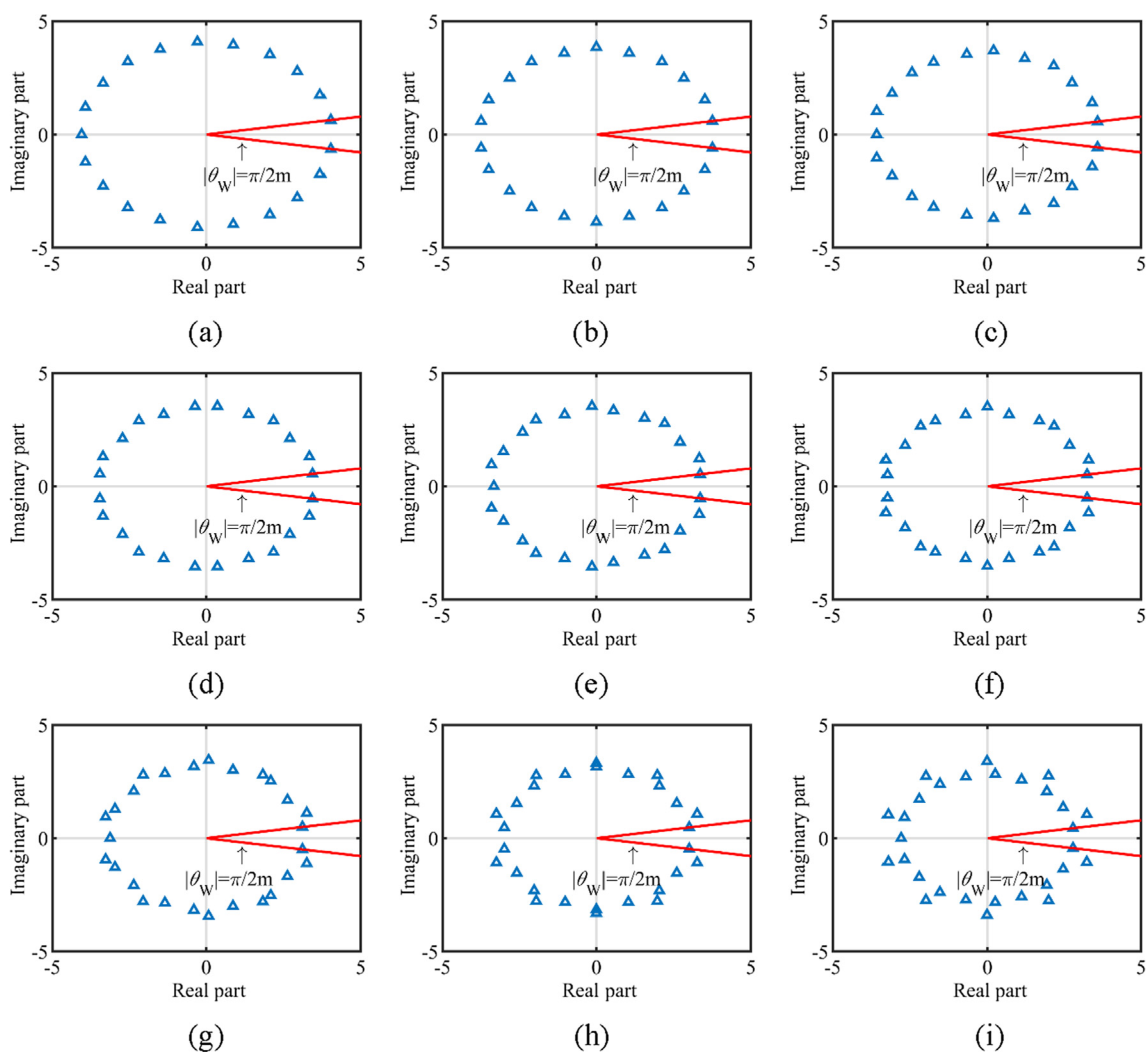


Fig. 5. The roots location in W-plane of the characteristic equation when (a) α = 1.1, (b) α = 1.2, (c) α = 1.3, (d) α = 1.4, (e) α = 1.5, (f) α = 1.6, (g) α = 1.7, (h) α = 1.8, (i) α = 1.9.

As can be seen from Fig. 5, all cases of order have roots on the lines $|\theta_W| = \pi/2m$. According to [38], the system will be stable only if all roots in the W-plane lie in the region $|\theta_W| > \pi/2m$, and will oscillate if at least one root is on the lines $|\theta_W| = \pi/2m$. Hence, the FOC-based fractional-order autonomous circuit is a sinusoidal oscillation circuit. The same characteristic can be obtained for FOI-based fractional-order autonomous circuit.

The poles distribution of FOC-based circuit with order less than 1 in the W-plane are also given by the circle mark in Fig. 6. As can be seen from Fig. 6, all roots lie in the region $|\theta_W| > \pi/2m$, while the case of order larger than 1 have roots on the lines $|\theta_W| = \pi/2m$ from Fig. 5. Therefore, the circuit with order less than 1 is also stable. However, since the fractional-order element with order larger than 1 has the characteristic of negative resistance which is necessary to provide required energy for the circuit continuously, the proposed autonomous circuit must adopt the fractional-order elements with order larger than 1.

Circuit simulations

To verify the characteristics of the proposed fractional-order autonomous circuits, circuit simulations based on Power Simulation Software are performed. Power Simulation Software can provide a powerful simulation environment for the analysis and research of power electronic circuits. It has the advantages of high-speed simulation, user-friendly interface, waveform analysis. Moreover, it also has a huge component library, which can satisfy the simulation requirements of fractional-order autonomous circuits.

FOC-based Fractional-order autonomous circuit

A FOC with order larger than one for autonomous circuit constructed in [36] is adopted. The FOC in [36] have a constant order α and apparent power S, but enable the its capacitance to vary. The corresponding realization schematic diagram is also shown in Fig. 7. The FOC circuit consists of a half-bridge converter and a capacitor C_0, S_1 and S_2 are a pair of power switches that turn ON and OFF complementarily, V_{GS1} and V_{GS2} are the drive signal of switch S_1 and S_2, respectively. By controlling the phase difference of the input voltage $v_{C\alpha}$ and current i_C with phase-lock loop technology, the designed order α can be acquired. Meanwhile, By controlling the duty of switch, the required apparent power S can be realized. The specific design process can be seen in the reference [36].

Fig. 8 shows the simulation waveforms of current and voltage of the FOC with designed order α = 1.1 in fractional-order autonomous circuit. The parameters of fractional-order autonomous circuit in Fig. 1(a) can be selected by (26), (28), (32) and (33). The adopted circuit parameters are L_1 = 100 μH, R = 20 Ω, S = 100 VA. From Fig. 8, the current i_C leads the voltage $v_{C\alpha}$ 1.36 μs, and

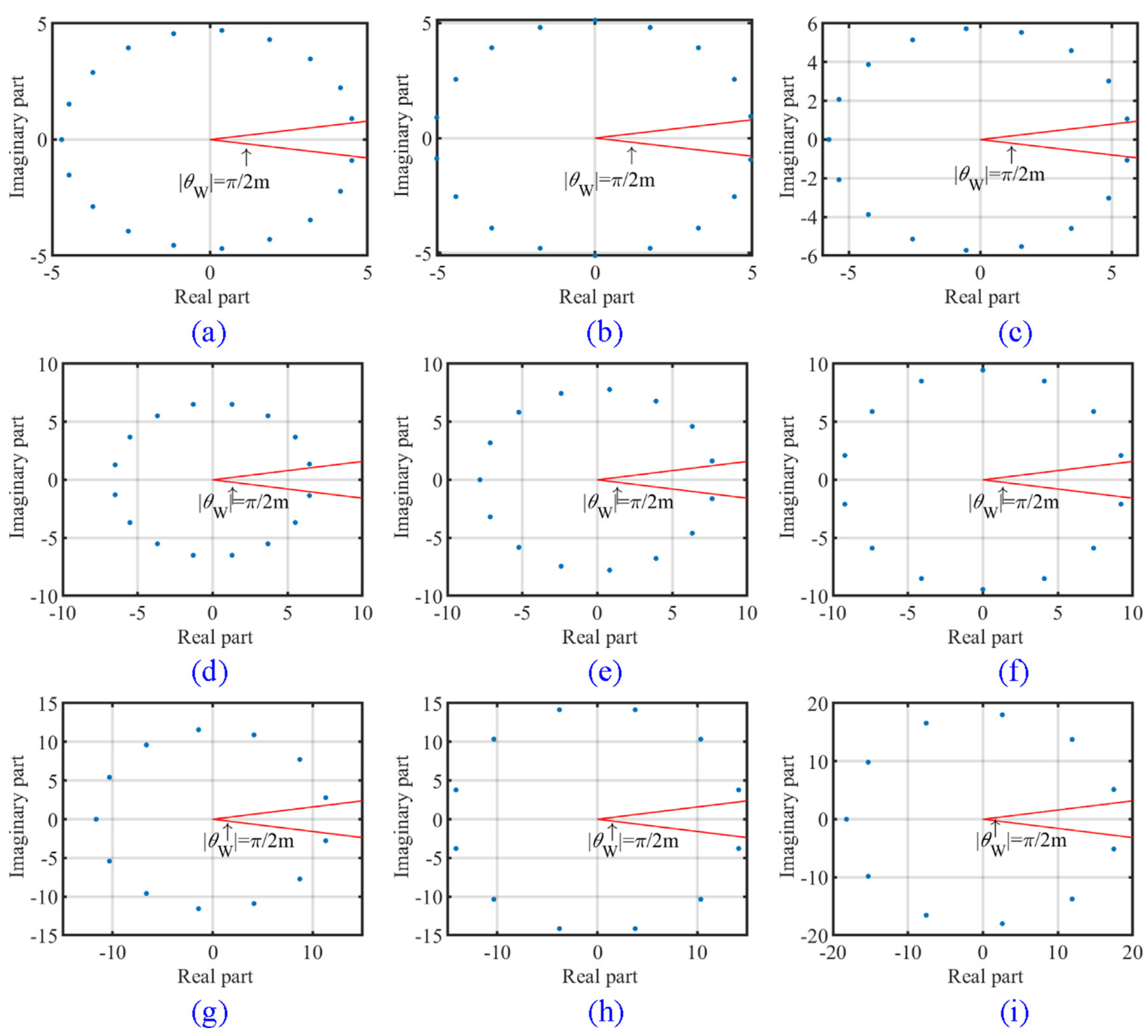


Fig. 6. The roots location in W-plane of the characteristic equation when (a) α = 0.9, (b) α = 0.8, (c) α = 0.7, (d) α = 0.6, (e) α = 0.5, (f) α = 0.4, (g) α = 0.3, (h) α = 0.2, (i) α = 0.1.

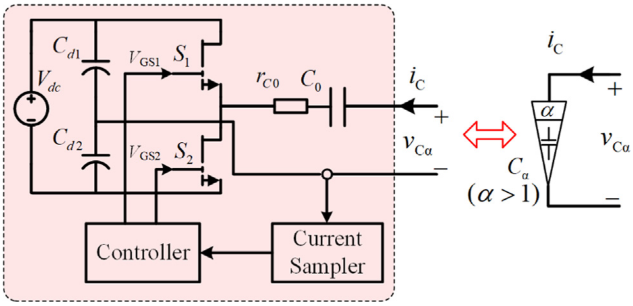


Fig. 7. Realization schematic diagram of the constructed FOC in [36].

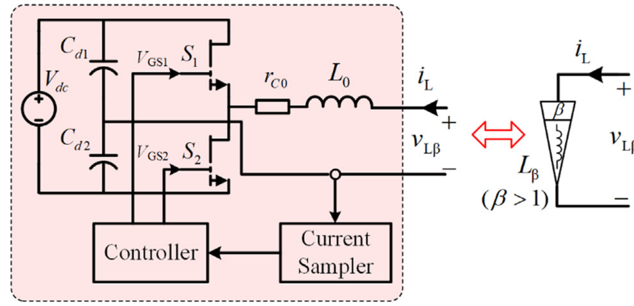


Fig. 9. Realization schematic diagram of the constructed FOI.

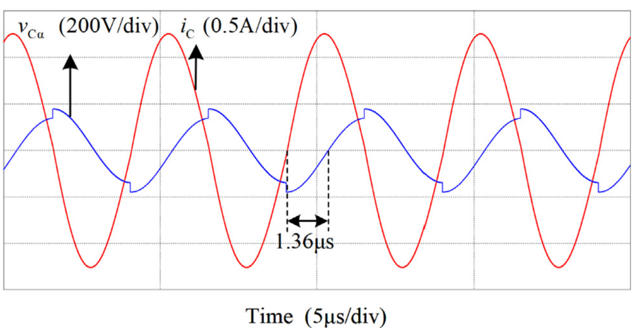


Fig. 8. The current and voltage of the FOC in fractional-order autonomous circuit.

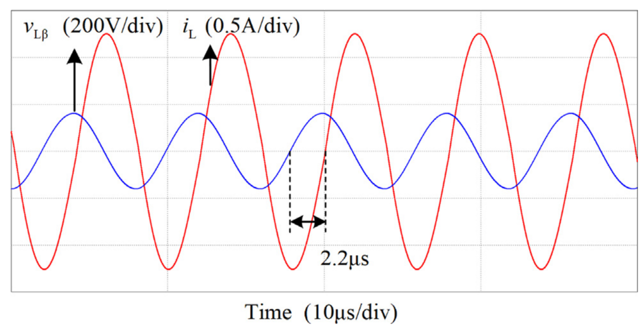


Fig. 10. The current and voltage of the FOI in fractional-order autonomous circuit.

the operating frequency is 200.97 kHz, so the phase of current leading voltage can be obtained as 98.39°. Therefore, the actual order of the FOC can be calculated as α = 98.39°/90 = 1.0932. The corresponding simulation current and voltage of FOC are I_C = 0.88A and $V_{C\alpha}$ = 113.2 V, so the capacitance of the FOC can be acquired as $C_\alpha = I_C/(\omega^\alpha V_{C\alpha})$ = 1.66nF/(second)$^{1-\alpha}$. As can be observed from Fig. 8, the simulation results are all consistent with the theoretical analysis.

In addition, it can be seen from Fig. 8 that there is a little harmonic in the voltage. The construction circuit of the FOC used in this paper is composed of a switching converter from Fig. 7, and the switches in the converter would produce harmonics when they operate. Therefore, the harmonics in the voltage waveforms of the fractional-order capacitor is generated by the switches of the converter. Nevertheless, the phase and amplitude of voltage and current in Fig. 8 can approximately describe the relationship between the voltage and current of FOC, because the actual order and capacitance of FOC calculated from Fig. 8 are consistent with the theoretical values.

FOI-based Fractional-order autonomous circuit

Referring to the construction method of FOC in [36], FOI suitable for autonomous circuit is also constructed as shown in Fig. 9. Different with the constructed circuit of FOC, the FOI is a converter in series with an integer-order inductor. This inductor can not only provide inductive reactive power for FOI, but also block high frequency harmonics.

The simulation waves of current and voltage of FOI with different orders in fractional-order autonomous circuit are shown in Fig. 10. The parameters of fractional-order autonomous circuit in Fig. 1(b) can be selected by (27), (29), (32) and (34). The adopted circuit parameters are C_1 = 10nF, R = 20 Ω, S = 100VA. From Fig. 10, the current i_L lags the voltage $v_{L\beta}$ 2.2 μs, and the operating frequency is 126 kHz, so the phase of current lagging voltage can be obtained as 99.79°, which means that the order is β = 99.79°/90 = 1.108. The corresponding simulation current and voltage of FOI are I_L = 0.88 and $V_{L\beta}$ = 113, so the inductance of the FOI is $L_\beta = V_{L\beta}/(\omega^\beta I_L)$ = 37.41 μH/(second)$^{1-\beta}$. From Fig. 10, the simulation results are consistent with the theoretical analysis.

Conclusion

In this paper, two kinds of fractional-order autonomous circuits based on FOC and FOI are proposed. Firstly, the characteristics of negative resistance in fractional order elements with order greater than 1 are analyzed. Then, by utilizing the characteristic of negative resistance, FOC and FOI are adopt to construct fractional-order autonomous circuits, and the models of the circuits are established using fractional calculus. On this basis, the properties of the fractional-order autonomous circuit are explored. Theoretical analysis demonstrates that that the operating frequency or resonance frequency, the voltage and current of the autonomous circuits can be changed by adjusting the resistance or the orders. Moreover, the stability analysis of the fractional-order autonomous circuits proves that the circuits are sinusoidal oscillation system. Finally, two circuit simulations were developed to validate the proposed fractional-order autonomous circuits. The simulation results verified the theoretical analysis.

The proposal of fractional-order autonomous circuits may promote the development of fractional-order circuit theory. Meanwhile, their potential applications include wireless power transfer system, communication system, automatic control system and so on. The future work on this topic might include transient characteristic analysis, experimental study and application of fractional-order autonomous circuits.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

Declaration of Competing Interest

The authors declare no conflict of interest.

Acknowledgements

The authors would like to thank the Key Program of the National Natural Science Foundation of China (51437005) for funding the project.

References

[1] Martynyuk V, Ortigueira M, Fedula M, Savenko O. Methodology of electrochemical capacitor quality control with fractional order model. Adv Eng Softw 2010. doi: https://doi.org/10.1016/j.aeue.2018.05.005.

[2] Hidalgo-Reyes JI, Gómez-Aguilar JF, Escobar-Jimenez RF, Alvarado-Martinez VM, Lopez-Lopez MG. Determination of supercapacitor parameters based on fractional differential equations. Int J Circuit Theory Appl 2019. doi: https://doi.org/10.1002/cta.2640.

[3] Magin RL. Fractional calculus models of complex dynamics in biological tissues. Comput Math Appl 2010. doi: https://doi.org/10.1016/j.camwa.2009.08.039.

[4] Atangana A, Gómez-Aguilar JF. Fractional derivatives with no-index law property: application to chaos and statistics. Chaos, Solitons Fractals 2018. doi: https://doi.org/10.1016/j.chaos.2018.07.033.

[5] Chen D, Zhao W, Sprott JC, Ma X. Application of Takagi-Sugeno fuzzy model to a class of chaotic synchronization and anti-synchronization. Nonlinear Dyn 2013. doi: https://doi.org/10.1007/s11071-013-0880-1.

[6] Westerlund S, Ekstam L. Capacitor theory. IEEE Trns Dielectr Electr Insul 1994. doi: https://doi.org/10.1109/94.326654.

[7] Freeborn TJ, Maundy B, Elwakil AS. Measurement of supercapacitor fractional-order model parameters from voltage-excited step response. IEEE J Emerging Sel Top Circuits Syst 2013. doi: https://doi.org/10.1109/JETCAS.2013.2271433.

[8] Schäfer Ingo. Krüger, Klaus, Modelling of lossy coils using fractional derivatives. J Phys D Appl Phys 2008. doi: https://doi.org/10.1088/0022-3727/41/4/045001.

[9] Radwan AG. Resonance and quality factor of the RL alpha C alpha fractional circuit. IEEE J Emerging Sel Top Circuits Syst 2013. doi: https://doi.org/10.1109/JETCAS.2013.2272838.

[10] Kartch A, Agambayev A, Herencsar N, Salama KN. Series-parallel-and inter-connection of solid-state arbitrary fractional-order capacitors: theoretical study and experimental verification. IEEE Access 2018. doi: https://doi.org/10.1109/ACCESS.2018.2809918.

[11] Peng Chen, Kai Yang, Tianliang Zhang. A dualband impedance transformer realized by fractional-order inductor and capacitor. In: IEEE Asia Pacific conference on circuits and systems. doi: https://doi.org/10.1109/APCCAS.2016.7804045.

[12] Carlson G, Halijak C. Approximation of fractional capacitors (1/s)^(1/n) by a regular newton process. IEEE Trans Circuit Theory 1964. doi: https://doi.org/10.1109/TCT.1964.1082270.

[13] Valsa J, Vlach J. RC models of a constant phase element. Int J Circ Theory Appl 2013. doi: https://doi.org/10.1002/cta.785.

[14] Semary MS, Fouda ME, Hassan HN, Radwan AG. Realization of fracitonal-order capacitor based on passive symmetric network. J Adv Res 2019;18:147–59. doi: https://doi.org/10.1016/j.jare.2019.02.004.

[15] Adhikary A, Sen P, Sen S, Biswas K. Design and performance study of dynamic factors in any of the four quadrants. Circuits Syst Signal Process 2016. doi: https://doi.org/10.1007/s00034-015-0213-3.

[16] Sivarama Krishna M, Das S, Biswas K, Goswami B. Fabrication of a fractional order capacitor with desired specifications: a study on process. Identificat Charact, IEEE Trans Electron Devices 2011. doi: https://doi.org/10.1109/ted.2011.2166763.

[17] Tsirimokou G, Psychalinos C, Elwakil AS. Emulation of a constant phase element using operational transconductance amplifiers. Analog Integr Circ Sig Process 2015. doi: https://doi.org/10.1007/s10470-015-0626-8.

[18] Jiang Y, Zhang B. High-Power fracitonal-order capacitor with 1<α<2 based on power electronics. IEEE Trans Ind Electron 2018. doi: https://doi.org/10.1109/TIE.2017.2756581.

[19] Kapoulea S, Tsirimokou G, Psychalinos C, Elwakil AS. Generalized fully adjustable structure for emulating fractional-order capacitors and inductors of orders less than two. Circ Syst Signal Process 2020. doi: https://doi.org/10.1007/s00034-019-01252-5.

[20] Radwan AG, Emira AA, AbdelAty AM, et al. Modeling and analysis of fractional order DC-DC converter. ISA Trans 2018. doi: https://doi.org/10.1016/j.isatra.2017.06.024.

[21] Wang F, Ma X. Fractional order Buck-Boost converter in CCM: modelling, analysis and simulations. Int J Electron 2014. doi: https://doi.org/10.1080/00207217.2014.888779.

[22] Chen X, Chen Y, Zhang B, Qiu D. A method of modeling and analysis for fractional-order DC-DC Converters. IEEE Trans Pow Electron 2016. doi: https://doi.org/10.1109/TPEL.2016.2628783.

[23] Freeborn Todd J, Elwakil Ahmed, Psychalinos Costas. Analysis of a rectifier circuit realized with a fractional-order capacitor. In: 28th IEEE international conference on microelectronics; 2016.

[24] Radwan AG, Shamim A, Salama KN. Theory of fractional order elements based impedance matching networks. IEEE Microw Wirel Compon Lett 2011. doi: https://doi.org/10.1109/LMWC.2010.2103051.

[25] Radwan AG, Shamim A, Salama KN. Impedance matching through a single passive fractional element. Antennas and propagation society international symposium, 2012.

[26] Diao LJ, Zhang XF, Chen DY. Fractional-order multiple RL alpha C beta circuit. Acta Physica Sinica 2014. doi: https://doi.org/10.7498/aps.63.038401.

[27] Ahmadi P, Maundy B, Elwakil AS, Belostotski L. High-quality factor asymmetric-slope band-pass filters: a fractional-order capacitor approach. Circ, Dev Syst IET 2012. doi: https://doi.org/10.1049/iet-cds.2011.0239.

[28] Ahmad W. Power factor correction using fractional capacitors. Circuits and systems proceedings of the international symposium, 2003.

[29] Jiang Y, Zhang B, Zhou J. A fractional-order resonant wireless power transfer system with inherently constant current output. IEEE Access 2020. doi: https://doi.org/10.1109/ACCESS.2020.2970199.

[30] Rong C, Zhang B, Jiang Y. Analysis of a fractional-order wireless power transfer system. IEEE Trans Circ Syst II: Exp Briefs 2019. doi: https://doi.org/10.1109/TCSII.2019.2949371.

[31] Muniz-Montero C, Garcia-Jimenez LV, Sanchez-Gaspariano LA, et al. New alternatives for analog implementation of fractional-order integrators, differentiators and PID controllers based on integer-order integrators. Nonlinear Dynam 2017. doi: https://doi.org/10.1007/s11071-017-3658-z.

[32] Niederberger D, Morari M. An autonomous shunt circuit for vibration damping. Smart Mater Struct 2006. doi: https://doi.org/10.1088/0964-1726/15/2/016.

[33] Tlelo-Cuautle E, Carbajal-Gomez VH, Obeso-Rodelo PJ. FPGA realization of a chaotic communication system applied to image processing. Nonlinear Dynam 2015. doi: https://doi.org/10.1007/s11071-015-2284-x.

[34] Pano-Azucena AD, Martinez BO, Tlelo-Cuautle E, et al. FPGA-based implementation of different families of fractional-order chaotic oscillators applying Grünwald-Letnikov method. Commun Nonlinear Sci Numeric Simul 2019. doi: https://doi.org/10.1016/j.cnsns.2019.01.014.

[35] Xin G. Chaos in fractional-order autonomous system and its control. In: International conference on communications, circuits and systems proceedings; 2006.

[36] Jiang Yangwei, Zhang Bo. A fractional-order wireless power transfer system insensitive to resonant frequency. IEEE Trans Pow Electron 2019. doi: https://doi.org/10.1109/TPEL.2019.2946964.

[37] Assawaworrarit S, Yu X, Fan S. Robust wireless power transfer using a nonlinear parity–time-symmetric circuit. Nature 2017. doi: https://doi.org/10.1038/nature22404.

[38] Radwan AG, Soliman AM, Elwakil AS, et al. On the stability of linear systems with fractional-order elements. Chaos, Solitons Fractals 2007. doi: https://doi.org/10.1016/j.chaos.2007.10.033.