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Research article

A bipolar fuzzy decision-making system for assessing high-risk coexisting tuberculosis disease in pregnant women

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ABSTRACT

Tuberculosis (TB) diagnosis poses a formidable challenge in global healthcare, particularly impacting older individuals and pregnant women. Diagnosing TB disease during pregnancy and in comorbid patients is more challenging due to overlapping symptoms with normal pregnancy conditions and existing treatments for other diseases, necessitating careful assessment to differentiate TB symptoms from those of other underlying conditions. To address this issue, this study designs a novel bipolar fuzzy decision-support system by integrating the concept of complex proportional assessment (COPRAS) and a technique for order preference by similarity to the ideal solution (TOPSIS) approaches using bipolar heptagonal fuzzy numbers. The approach is utilized to assess the high-risk of TB coinfection disease in pregnant women. The bipolar fuzzy set provides positive and negative membership degrees of an element, which divulge a balanced perspective by both the presence and absence of the disease. Additionally, a defuzzification algorithm is proposed for bipolar heptagonal fuzzy numbers, converting bipolar heptagonal fuzzy into a bipolar crisp score (CBHpFBCS). The bipolar fuzzy entropy measure is utilized to weight the criteria. The findings highlight that TB+HIV (G_3) coinfection is more severe in pregnant women compared to other TB comorbidities. Finally, sensitivity and comparative analyses are executed across diverse criteria weight scenarios and with existing fuzzy multi-criteria decision-making (MCDM) methods to validate the robustness of the proposed method and its outcomes.

1. Introduction

Pregnancy brings significant transformations to a woman's body as it adapts to nourish and support the developing fetus. These changes occur both mentally and physically. Physically, the most noticeable reasons are a growing belly and tender breasts. Pregnant women undergo various issues, such as nausea and vomiting, fatigue, back pain, and mood swings, at different times during their pregnancy period. It is important to receive regular medical care, including prenatal check-ups and monitoring to ensure the health and safety of the mother and the developing fetus. Throughout this period, pregnant women can face various illnesses such as tuberculosis (TB), cardiovascular, thyroid, diabetes, blood pressure, skin allergy, hair fall, etc and they are ranging from mild to severe. One of the most threatening diseases for pregnant women is TB. TB can affect individuals of any gender or age, but pregnant

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women are particularly vulnerable due to the challenging diagnosis system. The reason for this difficulty is due to the following factors:

(i) **Similar Symptoms:** Distinguishing between TB-related symptoms and those associated with a normal pregnancy can be challenging due to the similarity of certain indicators, such as cough, fatigue, and weight loss, which are common to both conditions.
(ii) **Changes in Immune System:** Pregnancy changes a woman's immune system, which can affect the body's response to TB infection. As a result, pregnant women show typical immune responses, which leads to difficulty in detecting the infection.
(iii) **Safety Concerns:** Traditional diagnostic methods for TB, such as X-rays or certain medications, may pose potential risks to the developing fetus. Healthcare providers need to consider the safety of both the mother and the unborn baby while pursuing diagnostic approaches to TB.
(iv) **Delay in Seeking Medical Attention:** Pregnant women might delay seeking medical care due to concerns about potential harm to the fetus during diagnosis or treatment, leading to a delay in detecting and treating TB.

Due to these complexities, healthcare providers must take extra precautions and provide effective care for pregnant women suspected of TB. Suresh et al. [1] scrutinized the prevalence of TB and HIV co-infection among pregnant women and their treatment and delivery outcomes by collecting data based on pregnant women screened for TB and HIV from the National AIDS Control Programme (NACP). According to the findings, out of 3,165,729 pregnant women, only 17 were diagnosed with TB-HIV co-infection. Mathad et al. [2] studied the various consequences of TB in pregnant women based on World Health Organization (WHO) data and estimates indicate that TB affects 2,16,500 pregnant women annually. Miele et al. [3] highlighted the substantial risk of TB during pregnancy. It resulted in assessing infection risk, which is crucial for pregnant women, and suggested proper medication advice for active TB before delivery. Zenner et al. [4] analyzed the risk of TB in pregnancy. Nguyen et al. [5] analyzed the severe consequences of TB for both mother and child during pregnancy. Umoh et al. [6] aimed to develop a fuzzy inference system (FIS) that could identify risk factors for pregnant women based on their health status, lifestyle factors, and pregnancy conditions. The study applied the center of gravity defuzzification method to a small sample of 25 pregnant women. The input parameters in the study were classified into three linguistic variables: low, moderate and high, and a triangular fuzzy number (TFN) was utilized to assign values to the linguistic variables. The findings of the study demonstrated the reliability of FIS in healthcare problems for identifying risk factors and predicting outcomes across various medical fields. Therefore this work used the fuzzy system to address the most vulnerable TB comorbidities in pregnant women.
Fuzzy logic and fuzzy sets have various applications in different fields due to their ability to handle uncertain information. The idea of fuzzy logic was first introduced by Zadeh in 1965 [7] as an extension of Boolean logic. It captures the imprecision and uncertainty of natural language by assigning degrees of membership. It represents a collection of objects with membership grades assigned to each element that range between completely false (membership grade 0) and completely true (membership grade 1). Real-world judgment is always based on the positive and negative evaluations of a problem. Based on this, Hitot Zhang et al. [8] defined the notion of bipolar fuzzy set (BFS). It helps to assess real-world problems by focusing on bipolar perspectives. For instance, good-bad, benefit-drawback. The utilization of BFS provides effective outcomes for complex problems. Therefore, this study utilizes the benefits of fuzzy sets to analyze the most vulnerable TB comorbidity disease in pregnant women. Researchers have recently utilized various MCDM techniques to deal with complex real-world selection problems. The following part provides a literature review of this work.

1.1. Related works

This section is divided into two parts where the first part discusses the BFS in decision-making models, while the second part focuses on the application of fuzzy complex proportional assessment (COPRAS) and a technique for order preference by similarity to the ideal solution (TOPSIS) in various contexts.

1.1.1. Role of bipolar fuzzy set on decision-making models

In the medical field, accurate disease diagnosis is paramount, as it forms the foundation for effective treatment strategies. However, diagnosing diseases can be particularly challenging due to the presence of overlapping symptoms across various conditions. This complexity leads to difficulties in distinguishing between different diseases, resulting in misdiagnosis or delayed treatment. Here, the concept of fuzzy bipolarity emerges as a valuable tool in disease diagnosis at the presence and absence of the disease. Akram et al. [9] utilized the bipolar fuzzy (BF) MCDM methods such as BF-TOPSIS and BF-ELECTRE I to improve medical diagnosis by considering the uncertainty data. These methods aid in determining a patient's health status and evaluating influencing factors, provided useful guidance in disease diagnosis. Natarajan et al. [10] presented a novel decision support system by integrating PROMETHEE and additive ratio assessment (ARAS) techniques to evaluate TB diagnostic methods under a bipolar intuitionistic fuzzy context. As the result, sputum test found to be most accurate, and its reliability is validated using comparative and sensitivity analyses. Shumaiza et al. [11] introduced a bipolar fuzzy ELECTRE II method for business location and supplier selection scenarios. It systematically ranked alternatives using BFS and outranking relations between alternatives and criteria. The outcomes are compared with the TOPSIS and ELECTRE I. Akram et al. [12] used the bipolar fuzzy PROMETHEE method to select green suppliers, where the unclear judgments handled by asymmetrical BFS handling. The method used preference functions and weighting information to effectively evaluate and rank alternatives, ensuring the consistency in the selection process. Akram et al. [13] presented a monograph using MCDM, which covers strategies for optimizing alternatives and justifying the uncertain environments. It discusses various methods, including bipolar fuzzy TOPSIS, ELECTRE-I, ELECTRE-II, VIKOR, and PROMETHEE, making it essential for researchers. Ezhilarasan and Felix

[14] introduced the bipolar fuzzy ARAS method to identify the impact of communicable and non-communicable diseases of TB from India's top 10 affected states. Sumaiza et al. [15] utilized trapezoidal bipolar fuzzy numbers (BFN) in the VIKOR method to select the best waste treatment for a thermal power station. The TrBFNs represented the expert viewpoints about the relationship between the alternatives and criteria. It involved the compromise ranking index and the individual performance index to identify the best alternative. The plastic recycling process was analyzed by Riaz et al. [16] through the TOPSIS method and cosine similarity measures under a cubic bipolar fuzzy environment. The use of fuzzy set in graph theory has seen widely applied in various applications. Deva and Felix [17] developed a decision-making system based on the bipolar fuzzy p-competition graph to analyze the effectiveness of COVID vaccines. Akram et al. [18] introduced several methods such as α-cut, (α, β)-cut, and strong (α, β)-cut for BFNs and proposed a total ordering technique for BFNs using the double upper lower dense sequence. The method allows to assign weights to the edges of a graph in a more flexible and interpretable way by considering the uncertainty information.

The application of fuzzy set theory in bipolar complex fuzzy (BCF) sets has been applied in the MCDM. Ozer [19] introduced the complex picture fuzzy (CPF) Hamacher averaging operators for aggregation process. Additionally, it provides the operator's dependability and efficacy in comparison to the current operators and introduced a new algorithm to the CPF environment. Jaleel [20] discussed agricultural robotics system using the WASPAS method and proposed a new aggregation operators based on Schweizer Sklar tools under picture fuzzy context. Rehman and Mahmood [21] explored the BCF based operators for computer network performance assessment, showcasing their effectiveness through a decision-making mechanism and real-life example. The advantages and reliability of the proposed approach are highlighted by comparing with existing methods. Chen et al. [22] introduced a BFS based MCDM method for selecting an optimal artificial intelligence (AI) framework for a technology company's particular requirements by utilizing various bipolar fuzzy probability aggregation operators.

1.1.2. Review on COPRAS and TOPSIS method

The COPRAS focuses on handling complex decision scenarios by considering the interdependencies between criteria. It incorporates the concept of proportional assessment, i.e., the performance of alternatives is evaluated proportionally to their performance in relation to others on different criteria. On the other hand, TOPSIS calculates the distance between each alternative from the ideal and anti-ideal solutions. Many researchers explored these methods to address diverse challenges. Mishra et al. [23] analyzed different water treatment technologies, considering technical, social, environmental, and economic factors using the interval-valued hesitant Fermatean fuzzy COPRAS method. Bathrinath et al. [24] scrutinized the factors impacting the sustainability of a shipping port using COPRAS method and analyzed the 8-dimension recommended strategy of port that addresses the sustainability issues in the shipping industry. As the result, transportation and handling of petroleum products, excessive energy consumption, structural integrity issues, and the generation of dust are the critical factors affecting sustainability. Dhiman et al. [25] concentrated on penalty payments for the hybrid process of wind farms under wind turbines and battery storage, where the different penalty payments were analyzed through TOPSIS and COPRAS methods. Yilmaz et al. [26] assessed wind farm efficiency in Turkey using DEA-COPRAS method within an intuitionistic fuzzy context. The qualitative data were collected and examined using DEA and COPRAS method. Hezer et al. [27] conducted an analysis of the safe regions for COVID-19 across 100 regions. The analysis considered various criteria, including quarantine efficiency, government risk management, monitoring, health readiness, regional resilience, and emergency preparedness, and analyzed through TOPSIS, VIekriterijumsko KOmpromisno Rangiranje (VIKOR) and COPRAS methods. During the global spread of COVID-19, doctors worldwide faced immense stress during patients' treatment, leading to anxiety and fear. Fatima et al. [28] identified common stress factors through a questionnaire from doctors in both private and government hospitals. These stress factors are analyzed using TOPSIS and weighted through AHP. As the result, psychological stress is the major concern, particularly the challenges in balancing between personal and professional life and the lack of communication. Rahim et al. [29] introduced cosine similarity and distance measures for cubic Fermatean fuzzy sets. Its practical applicability was assessed by analyzing the diagnosis method based on the symptoms using the TOPSIS method. Patel et al. [30] examined medical waste treatment using stepwise weight assessment ratio analysis (SWARA) and TOPSIS methods within an intuitionistic fuzzy framework, where SWARA was utilized to determine criteria weights, while TOPSIS was employed for grading the alternative. The distance and entropy measures are also introduced based on the generalized Csiszar f-divergence in the proposed approach. Swethaa and Felix [31] utilized the AHP-TOPSIS to analyze the military robot under an intuitionistic dense fuzzy environment. Thakur et al. [32] introduced an entropy measure for fuzzy MCDM using the COPRAS method, which enables comprehensive analysis of complex decision-making problems with conflicting criteria. Ghoushchi et al. [33] presented an approach by incorporating the fuzzy COPRAS method to prioritize risks in integrated health, safety and environmental management. This approach has been addressed the limitations of failure mode and effects analysis (FMEA).

The fuzzy MCDM techniques earned applications across diverse domains within the medical field. Stephen and Felix [34] devised a fuzzy FAHP with FIS to improve the diagnosis of cardiovascular disease, outperforming other methods based on sensitivity and comparative analyses. A study by Devi et al. [35] utilized triangular IFN to identify effective COVID lockdown relaxation protocols by introducing a defuzzification algorithm for TIFNs to convert the fuzzy score into a crisp score. Kang et al. [36] employed trapezoidal IFN (TrIFN) to determine a medication service robot for COVID patients. Natarajan et al. [37] explored HpFN and interval-valued fuzzy numbers to identify stroke-prone countries using the WASPAS method and validated their findings through comparative and sensitivity analyses. Devi and Augustin [38] utilized the intuitionistic fuzzy set-double parameter for analyzing COVID vaccine effectiveness against COVID variants.

1.2. Motivation

From an in-depth review of the literature, it is evident that several researchers have investigated TB disease. The following reasons highlight the motivation of this work:

(i) TB is still a major health problem worldwide. Researchers have used different methods for TB diagnosis but haven't scrutinized TB in pregnant women. This reveals a significant research gap in the study of pregnant women with TB.
(ii) The fuzzy COPRAS and TOPSIS methods involve proportional and distance evaluations of alternatives, respectively. These approaches are considered to be an efficient way of solving decision-making problems.
(iii) Fuzzy COPRAS and TOPSIS methods have not been widely utilized in a bipolar fuzzy environment. This becomes the motivation for further exploration to understand how fuzzy COPRAS and TOPSIS methodologies perform in the decision-making context with bipolarity, where there are two contrasting perspectives.
(iv) The heptagonal fuzzy number (HpFN) effectively covers uncertain information more than other fuzzy numbers, which also not explored in bipolar fuzzy context. This inspired to analyze the TB comorbidity disease in pregnant women under bipolar heptagonal fuzzy context.

1.3. Need of bipolar fuzzy set in fuzzy MCDM

A BFS is an expansion of a fuzzy set that represents both positive and negative membership degrees. The bipolar MF assigns values in the range $[-1, 1]$ instead of the usual $[0, 1]$. The positive fuzzy membership function (MF) is indicated in the range $[0, 1]$ and the negative MF is indicated in the range $[-1, 0]$. This extension allows for analyze the data in both positive and negative influences, providing a more comprehensive representation of uncertainty in fuzzy decision-making. The COPRAS uses a proportional representation approach, which considers the relative importance of criteria and alternatives. This allows for a more balanced evaluation of alternatives. The TOPSIS uses the concept of ideal and anti-ideal solutions to evaluate alternatives. It identifies the best and worst-performing alternatives based on their distance to these ideal solutions, providing a more straightforward and intuitive ranking process. By integrating the bipolar concept into COPRAS and TOPSIS, a more nuanced analysis of the problem can be achieved. This enhancement allows for a comprehensive evaluation of alternatives, considering both positive and negative factors.

1.4. Contribution

This subsection provides the contributions of this study.

(i) This work identifies high-risk coexisting TB disease in pregnant women through the bipolar fuzzy COPRAS and TOPSIS methods.
(ii) The various comorbidity diseases of TB are chosen as alternatives, and their features are chosen as criteria.
(iii) The vagueness between these alternatives and criteria is measured using a bipolar heptagonal fuzzy context.
(iv) The notion of bipolar HpFN (BHpFN) and its arithmetic operations (AOs) are introduced.
(v) The converting bipolar heptagonal fuzzy into bipolar crisp score (CBHpFBCS) algorithm is proposed to defuzzify the BHpFN.
(vi) The robustness of the proposed methods is validated though the sensitivity and comparative analyses.

1.5. New features of the proposed method

After reviewing the literature, the MCDM has been used to prioritize various human diseases. Nonetheless, the bipolar fuzzy concept has not been widely used in fuzzy MCDM methods for medical diagnosis. Therefore, the present study designs a novel fuzzy decision-making model, and its new features are discussed below.

(i) The HpFN in bipolar view enables healthcare providers to consider the diverse range of outcomes and interpretations when diagnosing a patient with comorbidities. The arithmetic operations of BHpFN are also newly defined.
(ii) A new defuzzification CBHpFBCS algorithm is proposed for the BHpFN, which aids in converting the proposed BHpFN into bipolar crisp score.
(iii) The bipolar fuzzy decision-making system is designed using COPRAS and TOPSIS in a heptagonal fuzzy context. The COPRAS method assesses the alternatives by comparing their performance relative to each other, while TOPSIS utilizes a distance-based approach. Consequently, this study developed the COPRAS and TOPSIS techniques under a bipolar fuzzy context to analyze the adverse relationship between alternatives and criteria.
(iv) The proposed bipolar fuzzy COPRAS and TOPSIS methods further enhanced by defining a bipolar entropy measure for weighting the criteria.

Therefore, these contributions aid in making decisions in light of the uncertainty involved in real-world medical problems. Moreover, it helps to evolve with ongoing research and advancements in decision science, computational intelligence, and related discipline.
The structure of the remaining article is as follows. The basic theory that supports the proposed method is presented in section 2. The proposed bipolar fuzzy COPRAS and TOPSIS methods are demonstrated in section 3, and a numerical illustration of the proposed method is in section 4. Section 5 contains the results and discussion of the study. In section 6, the conclusion is presented.

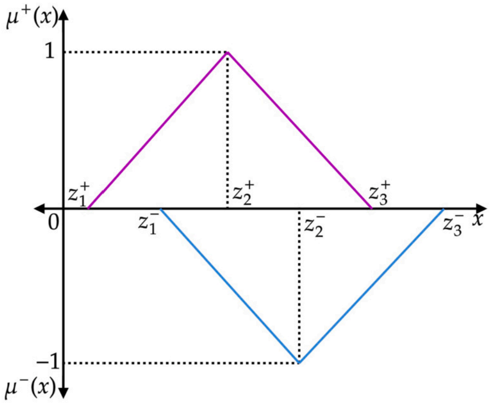

Fig. 1. Bipolar fuzzy number.

2. Preliminaries

Definition 2.1. A fuzzy subset $\tilde{Y}$ of the universal set Z is defined as $\tilde{Y} = \{(z, \mu_{\tilde{Y}}(z)) \ | z \in Z\}$, where $\mu_{\tilde{Y}}(z)$ is a MF which is a mapping $\mu_{\tilde{Y}}(z) : Z \to [0,1]$.

Definition 2.2. A *bipolar fuzzy set* (BFS) $\tilde{B}$ is defined by $\tilde{B} = \left\{(z, \mu^+_{\tilde{B}}(z), \mu^-_{\tilde{B}})(z)) \ | z \in Z\right\}$, where $\mu^+_{\tilde{B}}(z) : Z \to [0,1]$ and $\mu^-_{\tilde{B}}(z) : Z \to [-1,0]$ represent the positive and negative MFs, respectively.

Definition 2.3. A bipolar fuzzy number (BFN) is defined as $\tilde{B}$ on the real line $\mathbb{R}$, where its positive and negative MFs should satisfy the following conditions:

(i) $\tilde{B}$ is normal $\mu^+_{\tilde{B}}(z) = 1, \ \ \mu^-_{\tilde{B}}(z) = -1$.
(ii) $\mu^+_{\tilde{B}}(z), \ \mu^-_{\tilde{B}}(z)$ are piecewise continuous.
(iii) $\tilde{B}$ *is* convex $\mu^+_{\tilde{B}}(\rho z_1 + (1-\rho) z_2) \geq \min(\mu^+_{\tilde{B}}(z_1), \mu^+_{\tilde{B}}(z_2))$, for all $z_1, z_2 \in Z$, $\rho \in [0,1]$.
(iv) $\tilde{B}$ *is* concave $\mu^-_{\tilde{B}}(\rho z_1 + (1-\rho) z_2) \leq \max(\mu^-_{\tilde{B}}(z_1), \mu^-_{\tilde{B}}(z_2))$, for all $z_1, z_2 \in Z$, $\rho \in [-1,0]$.

From Fig. 1, it is clearly defined that the interval $[z_1^+, 1]$ is bounded monotonically increasing left-continuous, interval $[z_3^+, 1]$ is bounded monotonically decreasing right-continuous functions, and the interval $[-1, \ z_1^-]$ is bounded monotonically increasing left-continuous, interval $[-1, z_3^-]$ is bounded monotonically decreasing right-continuous.

Definition 2.4. *Bipolar heptagonal fuzzy number* (BHpFN)
The BHpFN is defined as $\tilde{A}_{BHpFN} = ((p_1^+, p_2^+, p_3^+, p_4^+, p_5^+, p_6^+, p_7^+), (p_1^-, p_2^-, p_3^-, p_4^-, p_5^-, p_6^-, p_7^-))$ on the real line $\mathbb{R}$. Then, the positive and negative MFs of BHpFN are defined as

$$\mu^+_{\tilde{A}}(x) = \begin{cases} \zeta_1^+ \left(\frac{x - p_1^+}{p_2^+ - p_1^+}\right) & for \quad p_1^+ < x < p_2^+ \\ \zeta_1^+ + (\zeta_2^+ - \zeta_1^+) \left(\frac{x - p_2^+}{p_3^+ - p_2^+}\right) & for \quad p_2^+ \leq x < p_3^+ \\ \zeta_2^+ + (1 - \zeta_2^+) \left(\frac{x - p_3^+}{p_4^+ - p_3^+}\right) & for \quad p_3^+ \leq x < p_4^+ \\ 1 & for \quad x = p_4^+ \\ \zeta_2^+ + (1 - \zeta_2^+) \left(\frac{p_5^+ - x}{p_5^+ - p_4^+}\right) & for \quad p_4^+ < x \leq p_5^+ \\ \zeta_1^+ + (\zeta_2^+ - \zeta_1^+) \left(\frac{p_6^+ - x}{p_6^+ - p_5^+}\right) & for \quad p_5^+ < x \leq p_6^+ \\ \zeta_1^+ \left(\frac{p_7^+ - x}{p_7^+ - p_6^+}\right) & for \quad p_6^+ < x < p_7^+ \end{cases}$$

and,

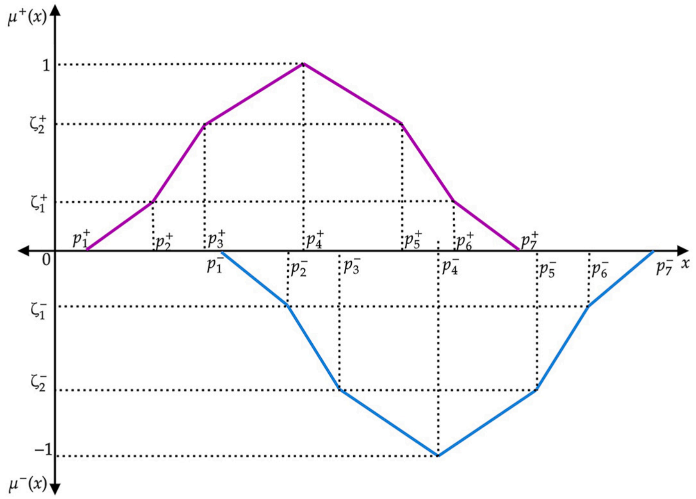

Fig. 2. Bipolar heptagonal fuzzy number.

$$\mu^-_{\tilde{A}}(x) = \begin{cases} \zeta_1^-\left(\frac{x-p_1^-}{p_2^- - p_1^-}\right) & for \quad p_1^- < x < p_2^- \\ \zeta_1^- + (\zeta_2^- - \zeta_1^-)\left(\frac{x-p_2^-}{p_3^- - p_2^-}\right) & for \quad p_2^- \le x < p_3^- \\ \zeta_2^- + (-1-\zeta_2^-)\left(\frac{x-p_3^-}{p_4^- - p_3^-}\right) & for \quad p_3^- \le x < p_4^- \\ -1 & for \quad x = p_4^- \\ \zeta_2^- + (-1-\zeta_2^-)\left(\frac{p_5^- - x}{p_5^- - p_4^-}\right) & for \quad p_4^- < x \le p_5^- \\ \zeta_1^- + (\zeta_2^- - \zeta_1^-)\left(\frac{p_6^- - x}{p_6^- - p_5^-}\right) & for \quad p_5^- < x \le p_6^- \\ \zeta_1^-\left(\frac{p_7^- - x}{p_7^- - p_6^-}\right) & for \quad p_6^- < x < p_7^- \end{cases}$$

where $p_1^+ \le p_2^+ \le p_3^+ \le p_4^+ \le p_5^+ \le p_6^+ \le p_7^+$ and $p_1^- \le p_2^- \le p_3^- \le p_4^- \le p_5^- \le p_6^- \le p_7^-$. The visual representation of the BHpFN is given in Fig. 2.

Definition 2.5. *Bipolar heptagonal fuzzy arithmetic operations:*
Let $\tilde{U} = \left((u_1^+, u_2^+, u_3^+, u_4^+, u_5^+, u_6^+ u_7^+), (u_1^-, u_2^-, u_3^-, u_4^-, u_5^-, u_6^-, u_7^-)\right)$ and
$\tilde{Z} = \left((z_1^+, z_2^+, z_3^+, z_4^+, z_5^+, z_6^+, z_7^+), (z_1^-, z_2^-, z_3^-, z_4^-, z_5^-, z_6^-, z_7^-)\right)$ be two BHpFN. Then, the AOs of $\tilde{U}$ and $\tilde{Z}$ are as follows

(i) $\left(\tilde{U} + \tilde{Z}\right) = \begin{pmatrix} \left(u_1^+ + z_1^+, u_2^+ + z_2^+, u_3^+ + z_3^+, u_4^+ + z_4^+, u_5^+ + z_5^+, u_6^+ + z_6^+, u_7^+ + z_7^+\right), \\ \left(u_1^- + z_1^-, u_2^- + z_2^-, u_3^- + z_3^-, u_4^- + z_4^-, u_5^- + z_5^-, u_6^- + z_6^-, u_7^- + z_7^-\right) \end{pmatrix}$

(ii) $\left(\tilde{U} - \tilde{Z}\right) = \begin{pmatrix} \left(u_1^+ - z_7^+, u_2^+ - z_6^+, u_3^+ - z_5^+, u_4^+ - z_4^+, u_5^+ - z_3^+, u_6^+ - z_2^+, u_7^+ - z_1^+\right), \\ \left(u_1^- - z_7^-, u_2^- - z_6^-, u_3^- - z_5^-, u_4^- - z_4^-, u_5^- - z_3^-, u_6^- - z_2^-, u_7^- - z_1^-\right) \end{pmatrix}$

(iii) $\left(\tilde{U} * \tilde{Z}\right) = \begin{pmatrix} \left(u_1^+ z_1^+, u_2^+ z_2^+, u_3^+ z_3^+, u_4^+ z_4^+, u_5^+ z_5^+, u_6^+ z_6^+, u_7^+ z_7^+\right), \\ \left(u_1^- z_1^-, u_2^- z_2^-, u_3^- z_3^-, u_4^- z_4^-, u_5^- z_5^-, u_6^- z_6^-, u_7^- z_7^-\right) \end{pmatrix}$

(iv) $\left(\tilde{U} / \tilde{Z}\right) = \begin{pmatrix} \left(u_1^+/z_7^+, u_2^+/z_6^+, u_3^+/z_5^+, u_4^+/z_4^+, u_5^+/z_3^+, u_6^+/z_2^+, u_7^+/z_1^+\right), \\ \left(u_1^-/z_7^-, u_2^-/z_6^-, u_3^-/z_5^-, u_4^-/z_4^-, u_5^-/z_3^-, u_6^-/z_2^-, u_7^-/z_1^-\right) \end{pmatrix}$

Example. The numerical example of the two BHpFNs under AOs are Fig. 3–6.
Let $\tilde{U} = ((1,2,5,7,9,12,13), (1,2,5,7,9,12,13))$ and $\tilde{Z} = ((1,3,7,10,13,17,19), (1,3,7,10,13,17,19))$.

(i) $\left(\tilde{U} + \tilde{Z}\right) = \left((2,5,12,17,22,29,32), (2,5,12,17,22,29,32)\right)$
(ii) $\left(\tilde{U} - \tilde{Z}\right) = \left((-18,-15,-8,-3,2,9,12)(-18,-15,-8,-3,2,9,12)\right)$

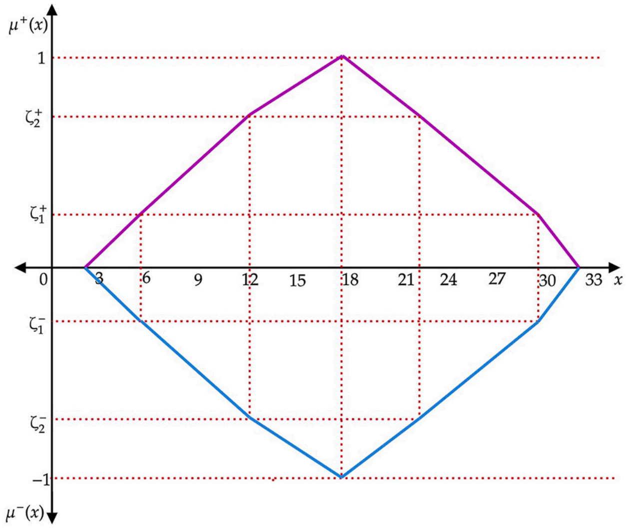

Fig. 3. Addition of two BHpFNs.

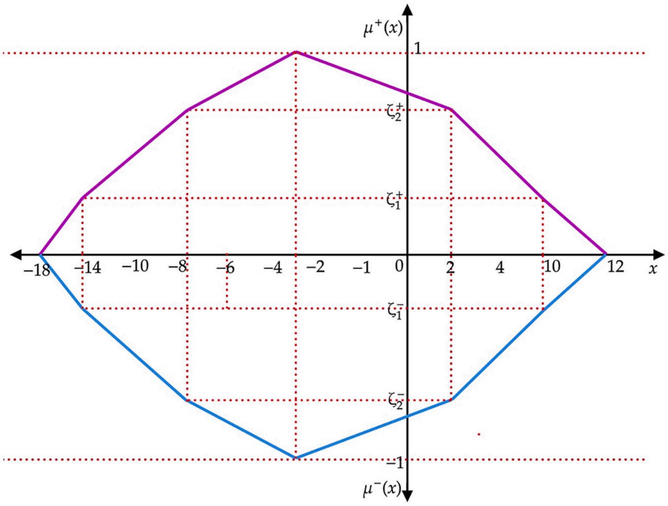

Fig. 4. Subtraction of two BHpFNs.

(iii) $\left(\tilde{U} * \tilde{Z}\right) = \left((1,6,35,70,117,204,247),(1,6,35,70,117,204,247)\right)$
(iv) $\left(\tilde{U}/\tilde{Z}\right) = \left((0.05,0.12,0.38,0.70,1.29,4.00,13.00),(0.05,0.12,0.38,0.70,1.29,4.00,13.00)\right)$

3. Proposed methodologies

3.1. The proposed CBHpFBCS algorithm

Defuzzification refers to the transformation of fuzzy or imprecise data into precise data. In fuzzy methods, it involves transforming a fuzzy number into a crisp number. Common defuzzification techniques include centre of area, centre of sum, weighted average and maxima methods. However, there is a lack of bipolar fuzzy defuzzification methods in the emerging concept. The articles [36], [39] provide defuzzification techniques for triangular intuitionistic fuzzy numbers, which serves as the foundation for proposing the CBHpFBCS algorithm.
Step 1: Normalize the elements.
Each member of the BHpFN is normalized.

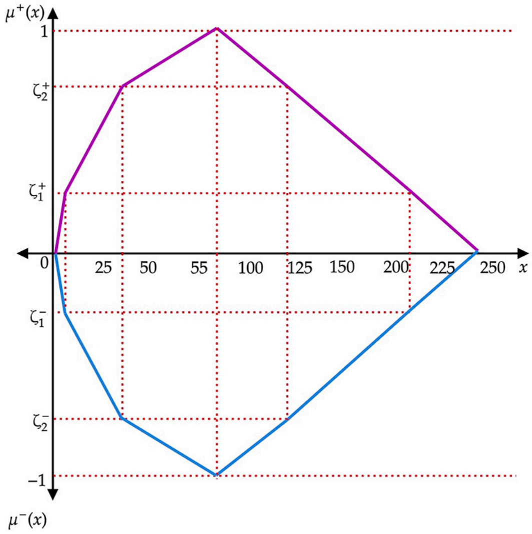

Fig. 5. Multiplication of two BHpFNs.

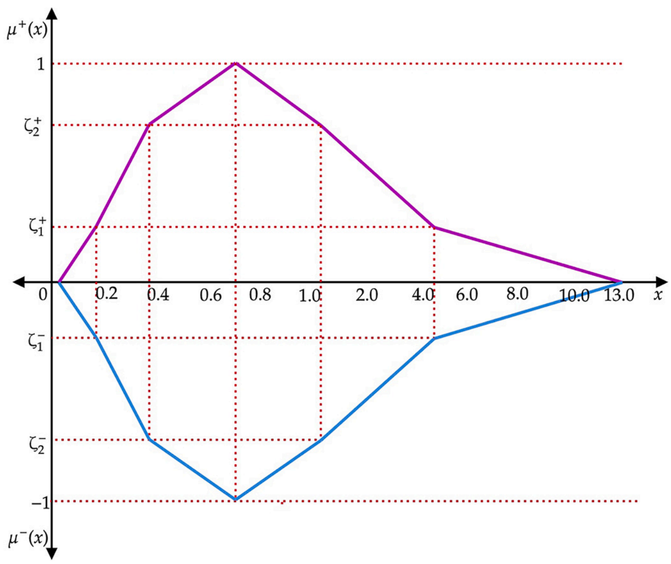

Fig. 6. Division of two BHpFNs.

$$\left(\begin{array}{c}\left(\begin{array}{c} t_{ij}^{+}=\frac{t_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}},\ u_{ij}^{+}=\frac{u_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}},\ v_{ij}^{+}=\frac{v_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}},w_{ij}^{+}=\frac{w_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}},\\ x_{ij}^{+}=\frac{x_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}},\ y_{ij}^{+}=\frac{y_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}},\ z_{ij}^{+}=\frac{z_{ij}^{+}-\min\left(t_{ij}^{+}\right)}{\Delta_{min}^{max}}\end{array}\right),\\ \left(\begin{array}{c} t_{ij}^{-}=\frac{t_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}},\ u_{ij}^{-}=\frac{u_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}},\ v_{ij}^{-}=\frac{v_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}},w_{ij}^{-}=\frac{w_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}},\\ x_{ij}^{-}=\frac{x_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}},\ y_{ij}^{-}=\frac{y_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}},\ z_{ij}^{-}=\frac{z_{ij}^{-}-\min\left(t_{ij}^{-}\right)}{\Delta_{min}^{max}}\end{array}\right)\end{array}\right)$$

Step 2: Calculate the left and right scores.

$$\left(\begin{array}{l}\left(\begin{array}{c} l_1 s_{ij}^{+} = \frac{u_{ij}^{+}}{1+u_{ij}^{+}-t_{ij}^{+}},\ l_2 s_{ij}^{+} = \frac{v_{ij}^{+}}{1+v_{ij}^{+}-u_{ij}^{+}}, l_3 s_{ij}^{+} = \frac{w_{ij}^{+}}{1+w_{ij}^{+}-v_{ij}^{+}},\ r_1 s_{ij}^{+} = \frac{x_{ij}^{+}}{1+x_{ij}^{+}-w_{ij}^{+}}, \\ r_2 s_{ij}^{+} = \frac{y_{ij}^{+}}{1+y_{ij}^{+}-x_{ij}^{+}}, r_3 s_{ij}^{+} = \frac{z_{ij}^{+}}{1+z_{ij}^{+}-y_{ij}^{+}} \end{array}\right), \\ \left(\begin{array}{c} l_1 s_{ij}^{-} = \frac{u_{ij}^{-}}{1+u_{ij}^{-}-t_{ij}^{-}},\ l_2 s_{ij}^{-} = \frac{v_{ij}^{-}}{1+v_{ij}^{-}-u_{ij}^{-}}, l_3 s_{ij}^{-} = \frac{w_{ij}^{-}}{1+w_{ij}^{-}-v_{ij}^{-}},\ r_1 s_{ij}^{-} = \frac{x_{ij}^{-}}{1+x_{ij}^{-}-w_{ij}^{-}}, \\ r_2 s_{ij}^{-} = \frac{y_{ij}^{-}}{1+y_{ij}^{-}-x_{ij}^{-}}, r_3 s_{ij}^{-} = \frac{z_{ij}^{-}}{1+z_{ij}^{-}-y_{ij}^{-}} \end{array}\right)\end{array}\right)$$

Step 3(a): Calculate the total normalized scores.

$$\left(\begin{array}{l}\left(\begin{array}{c} \alpha_{ij1}^{+} = \frac{l_1 s_{ij}^{+}\left(1-l_1 s_{ij}^{+}\right)+\left(l_2 s_{ij}^{+}\right)^2}{1-l_1 s_{ij}^{+}+l_2 s_{ij}^{+}},\ \alpha_{ij2}^{+} = \frac{l_2 s_{ij}^{+}\left(1-l_2 s_{ij}^{+}\right)+\left(l_3 s_{ij}^{+}\right)^2}{1-l_2 s_{ij}^{+}+l_3 s_{ij}^{+}},\ \alpha_{ij3}^{+} = \frac{l_3 s_{ij}^{+}\left(1-l_3 s_{ij}^{+}\right)+\left(r_1 s_{ij}^{+}\right)^2}{1-l_3 s_{ij}^{+}+r_1 s_{ij}^{+}}, \\ \alpha_{ij4}^{+} = \frac{r_1 s_{ij}^{+}\left(1-r_1 s_{ij}^{+}\right)+\left(r_2 s_{ij}^{+}\right)^2}{1-r_1 s_{ij}^{+}+r_2 s_{ij}^{+}},\ \alpha_{ij5}^{+} = \frac{r_2 s_{ij}^{+}\left(1-r_2 s_{ij}^{+}\right)+\left(r_3 s_{ij}^{+}\right)^2}{1-r_2 s_{ij}^{+}+r_3 s_{ij}^{+}} \end{array}\right), \\ \left(\begin{array}{c} \alpha_{ij1}^{-} = \frac{l_1 s_{ij}^{-}\left(1-l_1 s_{ij}^{-}\right)+\left(l_2 s_{ij}^{-}\right)^2}{1-l_1 s_{ij}^{-}+l_2 s_{ij}^{-}},\ \alpha_{ij2}^{-} = \frac{l_2 s_{ij}^{-}\left(1-l_2 s_{ij}^{-}\right)+\left(l_3 s_{ij}^{-}\right)^2}{1-l_2 s_{ij}^{-}+l_3 s_{ij}^{-}},\ \alpha_{ij3}^{-} = \frac{l_3 s_{ij}^{-}\left(1-l_3 s_{ij}^{-}\right)+\left(r_1 s_{ij}^{-}\right)^2}{1-l_3 s_{ij}^{-}+r_1 s_{ij}^{-}}, \\ \alpha_{ij4}^{-} = \frac{r_1 s_{ij}^{-}\left(1-r_1 s_{ij}^{-}\right)+\left(r_2 s_{ij}^{-}\right)^2}{1-r_1 s_{ij}^{-}+r_2 s_{ij}^{-}}, \alpha_{ij5}^{-} = \frac{r_2 s_{ij}^{-}\left(1-r_2 s_{ij}^{-}\right)+\left(r_3 s_{ij}^{-}\right)^2}{1-r_2 s_{ij}^{-}+r_3 s_{ij}^{-}} \end{array}\right)\end{array}\right)$$

Step 3(b): Calculate total normalized scores.

$$\left(\begin{array}{l}\left(\begin{array}{l} \alpha_{ij1*}^{+} = \frac{\alpha_{ij1}^{+}\left(1-\alpha_{ij1}^{+}\right)+\left(\alpha_{ij2}^{+}\right)^2}{1-\alpha_{ij1}^{+}+\alpha_{ij2}^{+}},\ \alpha_{ij2*}^{+} = \frac{\alpha_{ij2}^{+}\left(1-\alpha_{ij2}^{+}\right)+\left(\alpha_{ij3}^{+}\right)^2}{1-\alpha_{ij2}^{+}+\alpha_{ij3}^{+}}, \\ \alpha_{ij3*}^{+} = \frac{\alpha_{ij3}^{+}\left(1-\alpha_{ij3}^{+}\right)+\left(\alpha_{ij4}^{+}\right)^2}{1-\alpha_{ij3}^{+}+\alpha_{ij4}^{+}},\ \alpha_{ij4*}^{+} = \frac{\alpha_{ij4}^{+}\left(1-\alpha_{ij4}^{+}\right)+\left(\alpha_{ij5}^{+}\right)^2}{1-\alpha_{ij4}^{+}+\alpha_{ij5}^{+}} \end{array}\right), \\ \left(\begin{array}{l} \alpha_{ij1*}^{-} = \frac{\alpha_{ij1}^{-}\left(1-\alpha_{ij1}^{-}\right)+\left(\alpha_{ij2}^{-}\right)^2}{1-\alpha_{ij1}^{-}+\alpha_{ij2}^{-}},\ \alpha_{ij2*}^{-} = \frac{\alpha_{ij2}^{-}\left(1-\alpha_{ij2}^{-}\right)+\left(\alpha_{ij3}^{-}\right)^2}{1-\alpha_{ij2}^{-}+\alpha_{ij3}^{-}}, \\ \alpha_{ij3*}^{-} = \frac{\alpha_{ij3}^{-}\left(1-\alpha_{ij3}^{-}\right)+\left(\alpha_{ij4}^{-}\right)^2}{1-\alpha_{ij3}^{-}+\alpha_{ij4}^{-}},\ \alpha_{ij4*}^{-} = \frac{\alpha_{ij4}^{-}\left(1-\alpha_{ij4}^{-}\right)+\left(\alpha_{ij5}^{-}\right)^2}{1-\alpha_{ij4}^{-}+\alpha_{ij5}^{-}} \end{array}\right),\end{array}\right)$$

Step 3(c): Calculate total normalized scores.

$$\left(\begin{array}{l}\left(\alpha_{ij1\#}^{+} = \frac{\alpha_{ij1*}^{+}\left(1-\alpha_{ij1*}^{+}\right)+\left(\alpha_{ij2*}^{+}\right)^2}{1-\alpha_{ij1*}^{+}+\alpha_{ij2*}^{+}},\ \alpha_{ij2\#}^{+} = \frac{\alpha_{ij2*}^{+}\left(1-\alpha_{ij2*}^{+}\right)+\left(\alpha_{ij3*}^{+}\right)^2}{1-\alpha_{ij2*}^{+}+\alpha_{ij3*}^{+}}, \alpha_{ij3\#}^{+} = \frac{\alpha_{ij3*}^{+}\left(1-\alpha_{ij3*}^{+}\right)+\left(\alpha_{ij4*}^{+}\right)^2}{1-\alpha_{ij3*}^{+}+\alpha_{ij4*}^{+}}\right), \\ \left(\alpha_{ij1\#}^{-} = \frac{\alpha_{ij1*}^{-}\left(1-\alpha_{ij1*}^{-}\right)+\left(\alpha_{ij2*}^{-}\right)^2}{1-\alpha_{ij1*}^{-}+\alpha_{ij2*}^{-}}, \alpha_{ij2\#}^{-} = \frac{\alpha_{ij2*}^{-}\left(1-\alpha_{ij2*}^{-}\right)+\left(\alpha_{ij3*}^{-}\right)^2}{1-\alpha_{ij2*}^{-}+\alpha_{ij3*}^{-}}, \alpha_{ij3\#}^{-} = \frac{\alpha_{ij3*}^{-}\left(1-\alpha_{ij3*}^{-}\right)+\left(\alpha_{ij4*}^{-}\right)^2}{1-\alpha_{ij3*}^{-}+\alpha_{ij4*}^{-}}\right)\end{array}\right)$$

Step 3(d): Calculate total normalized scores.

$$\left(\begin{array}{l}\left(\alpha_{ij1\$}^{+} = \frac{\alpha_{ij1\#}^{+}\left(1-\alpha_{ij1\#}^{+}\right)+\left(\alpha_{ij2\#}^{+}\right)^2}{1-\alpha_{ij1\#}^{+}+\alpha_{ij2\#}^{+}},\ \alpha_{ij2\$}^{+} = \frac{\alpha_{ij2\#}^{+}\left(1-\alpha_{ij2\#}^{+}\right)+\left(\alpha_{ij3\#}^{+}\right)^2}{1-\alpha_{ij2*}^{+}+\alpha_{ij3*}^{+}}\right), \\ \left(\alpha_{ij1\$}^{-} = \frac{\alpha_{ij1*}^{-}\left(1-\alpha_{ij1*}^{-}\right)+\left(\alpha_{ij2*}^{-}\right)^2}{1-\alpha_{ij1*}^{-}+\alpha_{ij2*}^{-}}, \alpha_{ij2\$}^{-} = \frac{\alpha_{ij2\#}^{-}\left(1-\alpha_{ij2\#}^{-}\right)+\left(\alpha_{ij3\#}^{-}\right)^2}{1-\alpha_{ij2\#}^{-}+\alpha_{ij3\#}^{-}}\right)\end{array}\right)$$

Step 3(e): Calculate total normalized scores.

$$\left(\left(\alpha_{ij}^{+} = \frac{\alpha_{ij1\$}^{+}\left(1-\alpha_{ij1\$}^{+}\right)+\left(\alpha_{ij2\$}^{+}\right)^2}{1-\alpha_{ij1\$}^{+}+\alpha_{ij2\$}^{+}}\right), \left(\alpha_{ij}^{-} = \frac{\alpha_{ij1\$}^{-}\left(1-\alpha_{ij1\$}^{-}\right)+\left(\alpha_{ij2\$}^{-}\right)^2}{1-\alpha_{ij1\$}^{-}+\alpha_{ij2\$}^{-}}\right)\right)$$

Step 4: Determine the bipolar separated values.

$$\left(\left(Z_{ij}^{+} = \min(l_{ij}^{+}) + \alpha_{ij}^{+} \times \Delta_{min}^{max}\right), \left(Z_{ij}^{-} = \min(l_{ij}^{-}) + \alpha_{ij}^{-} \times \Delta_{min}^{max}\right)\right)$$

Numerical example of the proposed CBHpFBCS:

$$T = \begin{array}{c} \\ G_1 \\ G_2 \\ G_3 \end{array} \overset{P_1}{\begin{bmatrix} \begin{pmatrix} (0.360,\ 0.400,\ 0.440,\ 0.613,\ 0.720,\ 0.760,\ 0.800), \\ (0.170,\ 0.200,\ 0.240,\ 0.346,\ 0.520,\ 0.560,\ 0.600) \end{pmatrix} \\ \begin{pmatrix} (0.360,\ 0.400,\ 0.440,\ 0.613,\ 0.720,\ 0.760,\ 0.800), \\ (0.170,\ 0.200,\ 0.240,\ 0.346,\ 0.520,\ 0.560,\ 0.600) \end{pmatrix} \\ \begin{pmatrix} (0.560,\ 0.600,\ 0.640,\ 0.813,\ 0.920,\ 0.960,\ 1.000), \\ (0.000,\ 0.010,\ 0.050,\ 0.153,\ 0.320,\ 0.360,\ 0.400) \end{pmatrix} \end{bmatrix}}$$

From the matrix T, consider the relation between G_1 and P_1.
i.e., $\big((0.360,\ 0.400,\ 0.440,\ 0.613,\ 0.720,\ 0.760,\ 0.800),\ (0.170,\ 0.200,\ 0.240,\ 0.346,\ 0.520,\ 0.560,\ 0.600)\big)$. The BHpFN is defuzzified using the proposed CBHpFBCS algorithm.

Step 1: BHpFNs are normalized to calculate their corresponding values. This normalization process ensures that the values fall within the interval [0,1].

$$\left(\begin{array}{l} \begin{pmatrix} t^+_{ij} = \frac{0.360-0.170}{0.830} = 0.229,\ u^+_{ij} = \frac{0.400-0.170}{0.830} = 0.277,\ v^+_{ij} = \frac{0.440-0.170}{0.830} = 0.325, \\ w^+_{ij} = \frac{0.613-0.170}{0.830} = 0.534, x^+_{ij} = \frac{0.720-0.170}{0.830} = 0.663, \\ y^+_{ij} = \frac{0.760-0.170}{0.830} = 0.711,\ z^+_{ij} = \frac{0.800-0.170}{0.830} = 0.759 \end{pmatrix}, \\ \begin{pmatrix} t^-_{ij} = \frac{0.170-0.000}{0.800} = 0.213,\ u^-_{ij} = \frac{0.200-0.000}{0.800} = 0.250,\ v^-_{ij} = \frac{0.240-0.000}{0.800} = 0.300, \\ w^-_{ij} = \frac{0.346-0.000}{0.800} = 0.433,\ x^-_{ij} = \frac{0.520-0.000}{0.800} = 0.650, \\ y^-_{ij} = \frac{0.560-0.000}{0.800} = 0.700,\ z^-_{ij} = \frac{0.600-0.000}{0.800} = 0.750 \end{pmatrix} \end{array}\right)$$

In Step 2, the left and right scores of the normalized bipolar heptagonal fuzzy set are determined. The left score is obtained by combining the scores of the left and center, while the right score is obtained by combining the scores of the center and right.

$$\left(\begin{array}{l} \begin{pmatrix} l_1s^+_{ij} = \frac{0.277}{1+0.277-0.229} = 0.264,\ l_2s^+_{ij} = \frac{0.325}{1+0.325-0.277} = 0.310, l_3s^+_{ij} = \frac{0.534}{1+0.534-0.325} = 0.442, \\ r_1s^+_{ij} = \frac{0.663}{1+0.663-0.534} = 0.587,\ r_2s^+_{ij} = \frac{0.711}{1+0.711-0.663} = 0.678, r_3s^+_{ij} = \frac{0.759}{1+0.759-0.711} = 0.724 \end{pmatrix}, \\ \begin{pmatrix} l_1s^-_{ij} = \frac{0.250}{1+0.250-0.213} = 0.241,\ l_2s^-_{ij} = \frac{0.300}{1+0.300-0.250} = 0.286, l_3s^-_{ij} = \frac{0.433}{1+0.433-0.300} = 0.382, \\ r_1s^-_{ij} = \frac{0.650}{1+0.650-0.433} = 0.534,\ r_2s^-_{ij} = \frac{0.700}{1+0.700-0.650} = 0.667, r_3s^-_{ij} = \frac{0.750}{1+0.750-0.700} = 0.714 \end{pmatrix} \end{array}\right)$$

Step 3(a): The total normalized values are calculated.

$$\left(\begin{array}{l} \begin{pmatrix} \alpha^+_{ij1} = \frac{0.264(1-0.264)+(0.310)^2}{1-0.264+0.310} = 0.278,\ \alpha^+_{ij2} = \frac{0.310(1-0.310)+(0.442)^2}{1-0.310+0.442} = 0.362, \\ \alpha^+_{ij3} = \frac{0.442(1-0.442)+(0.587)^2}{1-0.442+0.587} = 0.516,\ \alpha^+_{ij4} = \frac{0.587(1-0.587)+(0.678)^2}{1-0.587+0.678} = 0.644, \\ \alpha^+_{ij5} = \frac{0.678(1-0.678)+(0.724)^2}{1-0.678+0.724} = 0.710 \end{pmatrix}, \\ \begin{pmatrix} \alpha^-_{ij1} = \frac{0.241(1-0.241)+(0.286)^2}{1-0.241+0.286} = 0.253,\ \alpha^-_{ij2} = \frac{0.286(1-0.286)+(0.382)^2}{1-0.286+0.382} = 0.319, \\ \alpha^-_{ij3} = \frac{0.382(1-0.382)+(0.534)^2}{1-0.382+0.534} = 0.453,\ \alpha^-_{ij4} = \frac{0.534(1-0.534)+(0.667)^2}{1-0.534+0.667} = 0.612, \\ \alpha^-_{ij5} = \frac{0.667(1-0.667)+(0.714)^2}{1-0.667+0.714} = 0.699 \end{pmatrix} \end{array}\right)$$

Step 3(b): The total normalized values are calculated.

$$\left(\begin{array}{l} \begin{pmatrix} \alpha^+_{ij1*} = \frac{0.278(1-0.278)+(0.362)^2}{1-0.278+0.362} = 0.306,\ \alpha^+_{ij2*} = \frac{0.362(1-0.362)+(0.516)^2}{1-0.362+0.516} = 0.431, \\ \alpha^+_{ij3*} = \frac{0.516(1-0.516)+(0.644)^2}{1-0.516+0.644} = 0.589,\ \alpha^+_{ij4*} = \frac{0.644(1-0.644)+(0.710)^2}{1-0.644+0.710} = 0.688 \end{pmatrix}, \\ \begin{pmatrix} \alpha^-_{ij1*} = \frac{0.253(1-0.253)+(0.319)^2}{1-0.253+0.319} = 0.273,\ \alpha^-_{ij2*} = \frac{0.319(1-0.319)+(0.453)^2}{1-0.319+0.453} = 0.373, \\ \alpha^-_{ij3*} = \frac{0.453(1-0.453)+(0.612)^2}{1-0.453+0.612} = 0.537,\ \alpha^-_{ij4*} = \frac{0.612(1-0.612)+(0.699)^2}{1-0.62+0.669} = 0.668 \end{pmatrix} \end{array}\right)$$

Step 3(c): The total normalized values are calculated.

$$\left(\begin{array}{l}\left(\begin{array}{c}\alpha^{+}_{ij1\#}=\frac{0.306(1-0.306)+(0.431)^2}{1-0.306+0.431}=0.354,\ \alpha^{+}_{ij2\#}=\frac{0.431(1-0.431)+(0.589)^2}{1-0.431+0.589}=0.511,\\ \alpha^{+}_{ij3\#}=\frac{0.589(1-0.589)+(0.688)^2}{1-0.589+0.688}=0.651\end{array}\right),\\ \left(\begin{array}{c}\alpha^{-}_{ij1\#}=\frac{0.273(1-0.273)+(0.373)^2}{1-0.273+0.373}=0.307,\ \alpha^{-}_{ij2\#}=\frac{0.373(1-0.373)+(0.537)^2}{1-0.373+0.537}=0.448,\\ \alpha^{-}_{ij3\#}=\frac{0.537(1-0.537)+(0.668)^2}{1-0.537+0.668}=0.614\end{array}\right)\end{array}\right)$$

Step 3(d): Calculate total normalized scores.

$$\left(\begin{array}{l}\left(\alpha^{+}_{ij1\$}=\frac{0.354(1-0.354)+(0.511)^2}{1-0.354+0.511}=0.423,\ \alpha^{+}_{ij2\$}=\frac{0.511(1-0.511)+(0.651)^2}{1-0.511+0.651}=0.591\right),\\ \left(\alpha^{-}_{ij1\$}=\frac{0.306(1-0.306)+(0.431)^2}{1-0.306+0.431}=362,\alpha^{-}_{ij2\$}=\frac{0.306(1-0.306)+(0.431)^2}{1-0.306+0.431}=536\right)\end{array}\right)$$

Step 3(e): Calculate total normalized scores.

$$\left(\left(\alpha^{+}_{ij}=\frac{0.423\,(1-0.423)+(0.591)^2}{1-0.423+0.591}=0.508\right),\left(\alpha^{-}_{ij}=\frac{0.362\,(1-0.362)+(0.536)^2}{1-0.362+0.536}=0.442\right)\right)$$

Step 4: Determine the bipolar separated values.

$$\left(\left(Z^{+}_{ij}=0.170+0.508\times0.830\right),\left(Z^{-}_{ij}=0.000+0.442\times0.800\right)\right)=(0.592,\ 0.353)$$

3.2. *The proposed bipolar fuzzy COPRAS and TOPSIS methods*

The proposed methods aid to analyze the finite set of alternatives and criteria. It is flexible to work on both qualitative and quantitative data. It includes the performance of each conflicting criterion. The acceptance of large-scale data measurements signifies the correlation between alternatives and criteria.

3.2.1. *Method-I: The bipolar fuzzy COPRAS method*

The specific steps of the first proposed method as follows.

Step 1: Choose the possible alternatives and criteria.

The applicable set of alternatives $G=\left\{G_1,G_2,\dots,G_m\right\}$ and criteria $P=\left\{P_1,P_2\dots,P_n\right\}$ are framed based on the decision-makers $F=\left\{F_1,F_2,\dots,F_f\right\}$ opinion.

Step 2: Compose the linguistic decision matrices (LDM).

The LDM is framed from the decision-makers view. A LDM reveals the association between the alternatives and criteria. This is shown in the Equation (1).

$$A=\left[a^r_{ij}\right]=\begin{array}{c}\\ G_1\\ G_2\\ \vdots\\ G_m\end{array}\begin{array}{c}\begin{array}{cccc}P_1 & P_2 & \cdots & P_n\end{array}\\ \begin{bmatrix}a^r_{11} & a^r_{12} & \cdots & a^r_{1n}\\ a^r_{21} & a^r_{22} & \cdots & a^r_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a^r_{m1} & a^r_{m2} & \cdots & a^r_{mn}\end{bmatrix}\end{array} \tag{1}$$

where $A=\left[a^r_{ij}\right],\ i=1,2,\dots,m,\ j=1,2,\dots,\ n,\ r=1,2,\dots,f.$

Step 3: Transform the LDM into a bipolar heptagonal fuzzy matrix (BHpFM).

Using the linguistic conversion scale, the LDM is transformed into BHpFM as illustrated in the Equation (2).

$$A=\left[\tilde{a}^r_{ij}\right]=\begin{array}{c}\\ G_1\\ G_2\\ \vdots\\ G_m\end{array}\begin{array}{c}\begin{array}{cccc}P_1 & P_2 & \cdots & P_n\end{array}\\ \begin{bmatrix}\tilde{a}^r_{11} & \tilde{a}^r_{12} & \cdots & \tilde{a}^r_{1n}\\ \tilde{a}^r_{21} & \tilde{a}^r_{22} & \cdots & \tilde{a}^r_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{a}^r_{m1} & \tilde{a}^r_{m2} & \cdots & \tilde{a}^r_{mn}\end{bmatrix}\end{array} \tag{2}$$

where $\tilde{A}=\left[\tilde{a}^r_{ij}\right]=\left(\left(t^+_{ij},u^+_{ij},v^+_{ij},w^+_{ij},x^+_{ij},y^+_{ij},z^+_{ij}\right),\left(t^-_{ij},u^-_{ij},v^-_{ij},w^-_{ij},x^-_{ij},y^-_{ij},z^-_{ij}\right)\right)^r;$
$i=1,2,\dots,m; j=1,2,\dots,\ n,\ r=1,2,\dots,f.$

Step 4: Aggregate the BHpFM.

All the BHpFM are aggregated into a single matrix, which combines the performance of alternatives and criteria from multiple decision matrices into a single matrix.

$$\begin{gathered}t^+_{ij}=t^-_{ij}=\min(t^r_{ij}),u^+_{ij}=u^-_{ij}=\min(u^r_{ij}),v^+_{ij}=v^-_{ij}=\min(v^r_{ij}),w^+_{ij}=w^-_{ij}=\frac{1}{r}(w^r_{ij}),\\ x^+_{ij}=x^-_{ij}=\max(x^r_{ij}),y^+_{ij}=y^-_{ij}=\max(y^r_{ij}),z^+_{ij}=z^-_{ij}=\max(z^r_{ij})\end{gathered} \tag{3}$$

$$A=\left[\tilde{a}_{ij}\right]=\begin{array}{c} \\ G_1\\ G_2\\ \vdots\\ G_m\end{array}\begin{array}{c}\begin{array}{cccc}P_1 & P_2 & \cdots & P_n\end{array}\\ \begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn}\end{bmatrix}\end{array} \tag{4}$$

where $A=\left[\tilde{a}_{ij}\right]=\tilde{a}_{ij}=\left(\left(t^+_{ij},u^+_{ij},v^+_{ij},w^+_{ij},x^+_{ij},y^+_{ij},z^+_{ij}\right),\left(t^-_{ij},u^-_{ij},v^-_{ij},w^-_{ij},x^-_{ij},y^-_{ij},z^-_{ij}\right)\right);i=1,2,\ldots,m;j=1,2,\ldots,n.$

Step 5: Construct the defuzzified matrix.

The aggregated matrix is defuzzified and converted into a crisp number using the proposed CBHpFBCS algorithm, which is determined in subsection 3.1.

$$U=\left[u_{ij}\right]=\begin{array}{c} \\ G_1\\ G_2\\ \vdots\\ G_m\end{array}\begin{array}{c}\begin{array}{cccc}P_1 & P_2 & \cdots & P_n\end{array}\\ \begin{bmatrix}u_{11} & u_{12} & \cdots & u_{1n}\\ u_{21} & u_{22} & \cdots & u_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ u_{m1} & u_{m2} & \cdots & u_{mn}\end{bmatrix}\end{array} \tag{5}$$

where $U=\left[u_{ij}\right]=\left(u^+_{ij},u^-_{ij}\right);i=1,2,\ldots,m,\ j=1,2,\ldots,\ n.$

Step 6: Formulate the bipolar normalized decision matrix (BNDM).

The BNDM is designed to confine the performance of alternatives and criteria within the range [0,1] using Equation (6).

$$(t^+_{ij},t^-_{ij})=\left(\frac{u^+_{ij}}{\sum_{i=1}^{m}u^+_{ij}},\frac{u^-_{ij}}{\sum_{i=1}^{m}u^-_{ij}}\right) \tag{6}$$

$$T=\left[t_{ij}\right]=\begin{array}{c} \\ G_1\\ G_2\\ \vdots\\ G_m\end{array}\begin{array}{c}\begin{array}{cccc}P_1 & P_2 & \cdots & P_n\end{array}\\ \begin{bmatrix}t_{11} & t_{12} & \cdots & t_{1n}\\ t_{21} & t_{22} & \cdots & t_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ t_{m1} & t_{m2} & \cdots & t_{mn}\end{bmatrix}\end{array}$$

where $T=[t_{ij}]=\left(t^+_{ij},t^-_{ij}\right);\ i=1,2,\ldots,m,\ j=1,2,\ldots,\ n.$

Step 7: Evaluate the bipolar weights for the criteria.

The entropy measure is employed for calculating bipolar fuzzy weights, which furnish both positive and negative weights for each criterion.

$$\left(E^+_j,E^-_j\right)=\left(-c\sum_{p=1}^{m}t^+_{ij}\log\left(t^+_{ij}\right),-c\sum_{p=1}^{m}t^-_{ij}\log\left(t^-_{ij}\right)\right) \tag{7}$$

where $c=(\log(m))^{-1}$ is a constant. The calculation for the degree of divergence (d_j) for each criterion is as follows.

$$d_j=\left(1-E^+_j,\ 1-E^-_j\right),\text{where }\ d_j=\left(d^+_j,d^-_j\right),\ j=1,2,...,n. \tag{8}$$

The bipolar objective weights for each criterion is calculated using Equation (9).

$$\left(W^+_j,W^-_j\right)=\left(\frac{d^+_j}{\sum_{j=1}^{n}d^+_j},\frac{d^-_j}{\sum_{j=1}^{n}d^-_j}\right) \tag{9}$$

Step 8: Determine the bipolar weighted normalized decision matrix (BWNDM).

The determined criteria importance level is multiplied with the BNDM to determine the BWNDM.

$$W=\left(W^+_j\times n^+_{ij},W^-_j\times n^-_{ij}\right),\quad\text{where}\quad i=1,2,\ldots,m,\quad j=1,2,\ldots,\ n. \tag{10}$$

$$W=\left[w_{ij}\right]=\begin{array}{c} \\ G_1\\ G_2\\ \vdots\\ G_m\end{array}\begin{array}{c}\begin{array}{cccc}P_1 & P_2 & \cdots & P_n\end{array}\\ \begin{bmatrix}w_{11} & w_{12} & \cdots & w_{1n}\\ w_{21} & w_{22} & \cdots & w_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ w_{m1} & w_{m2} & \cdots & w_{mn}\end{bmatrix}\end{array}$$

where $W = [w_{ij}] = \left(w_{ij}^{+}, w_{ij}^{-}\right)$; $i = 1, 2, \dots, m,\ j = 1, 2, \dots,\ n.$

Step 9: Identify the bipolar maximizing and minimizing indexes.

The bipolar maximizing and minimizing indexes are determined based on each alternative's beneficiary and non-beneficiary criteria using Equations (11) and (12).

$$M_i^{+} = \left(\sum_{j=i}^{n} w_{ij}^{+}, \sum_{j=i}^{n} w_{ij}^{+}\right), \text{ where } i = 1, 2, \dots, m,\ j = 1, 2, \dots, n, \tag{11}$$

where M_i^{+} represent the maximizing or positive attributes.

$$M_i^{-} = \left(\sum_{j=i}^{n} w_{ij}^{-}, \sum_{j=i}^{n} w_{ij}^{-}\right), \text{ where } i = 1, 2, \dots, m,\ j = 1, 2, \dots, n, \tag{12}$$

where M_i^{-} represent the minimizing or negative attributes.

Step 10: Determine the relative importance value of the alternatives.

The relative importance of each alternative are determined using Equation (13). The relative importance value express the preference level of the alternatives. The alternatives are graded in decreasing order. The flow of the proposed bipolar fuzzy COPRAS method is shown in Fig. 7.

$$\left(K_i^{+}, K_i^{-}\right) = \left(\left(M_i^{+} + \left(\frac{\sum_{j=i}^{n} M_i^{+}}{M_i^{+}\left(\sum_{j=i}^{n} \frac{1}{M_i^{+}}\right)}\right)\right)^{+}, \left(M_i^{+} + \left(\frac{\sum_{j=i}^{n} M_i^{-}}{M_i^{-}\left(\sum_{j=i}^{n} \frac{1}{M_i^{-}}\right)}\right)\right)^{-}\right); \tag{13}$$

where $i = 1, 2, \dots, m.$

Then, the final score of each alternative is obtained using Equation (14).

$$K_i = \left(\frac{1 + K_i^{+} - K_i^{-}}{2}\right) \tag{14}$$

3.2.2. Method-II: The bipolar fuzzy TOPSIS method

The second method's specific steps are constructed in the bipolar fuzzy context as follows.

Steps 1-8: The first eight steps are the same as the first proposed method.

Step 9: Construct the bipolar ideal best (B) and ideal worst (W) values.

In the bipolar view, the best $B = \left(A^{P^{+}}, A^{P^{-}}\right)$ and worst $W = \left(A^{N^{+}}, A^{N^{-}}\right)$ values are determined through Equations (15) and (16) respectively.

$$B = \begin{cases} \left(\left(\max w_{ij}^{+} \mid j \in J\right), \left(\min w_{ij\ i}^{+} \mid j \in J'\right) \mid i = 1, 2, \dots m\right) \\ \left(\left(\max w_{ij}^{-} \mid j \in J\right), \left(\min w_{ij}^{-} \mid j \in J'\right) \mid i = 1, 2, \dots m\right) \end{cases} \tag{15}$$

$$W = \begin{cases} \left(\left(\min w_{ij}^{+} \mid j \in J\right), \left(\max w_{ij}^{+} \mid j \in J'\right) \mid i = 1, 2, \dots m\right) \\ \left(\left(\min w_{ij}^{-} \mid j \in J\right), \left(\max w_{ij}^{-} \mid j \in J'\right) \mid i = 1, 2, \dots m\right) \end{cases} \tag{16}$$

where $i = 1, 2, \dots, m$; $J = 1, 2, \dots, n$; $J' = 1, 2, \dots, n$ and $j = 1, 2, \dots, n$

Step 10: Determine the bipolar separation measure.

The Euclidean distance Equations (17) and (18) is employed to evaluate the positive separation measure $\left(S_i^{P^{+}}, S_i^{P^{-}}\right)$ and negative separation measure $\left(S_i^{N^{+}}, S_i^{N^{-}}\right)$ of each alternative.

$$\left(S_i^{P^{+}}, S_i^{P^{-}}\right) = \sqrt{\left(\sum_{j=1}^{n}\left(w_{ij}^{+} - A_j^{P^{+}}\right)^2\right), \left(\sum_{j=1}^{n}\left(w_{ij}^{+} - A_j^{P^{-}}\right)^2\right)}\ i = 1, 2, \dots, m,\ j = 1, 2, \dots, n. \tag{17}$$

$$\left(S_i^{N^{+}}, S_i^{N^{-}}\right) = \sqrt{\left(\sum_{j=1}^{n}\left(w_{ij}^{-} - A_j^{N^{+}}\right)^2\right), \left(\sum_{j=1}^{n}\left(w_{ij}^{-} - A_j^{N^{-}}\right)^2\right)}\ i = 1, 2, \dots, m,\ j = 1, 2, \dots, n, \tag{18}$$

where $\left(S_i^{P^{+}}, S_i^{P^{-}}\right)$ is a bipolar positive ideal solution and $\left(S_i^{N^{+}}, S_i^{N^{-}}\right)$ is a bipolar negative ideal solution.

Step 11: Determine the rank of the alternatives.

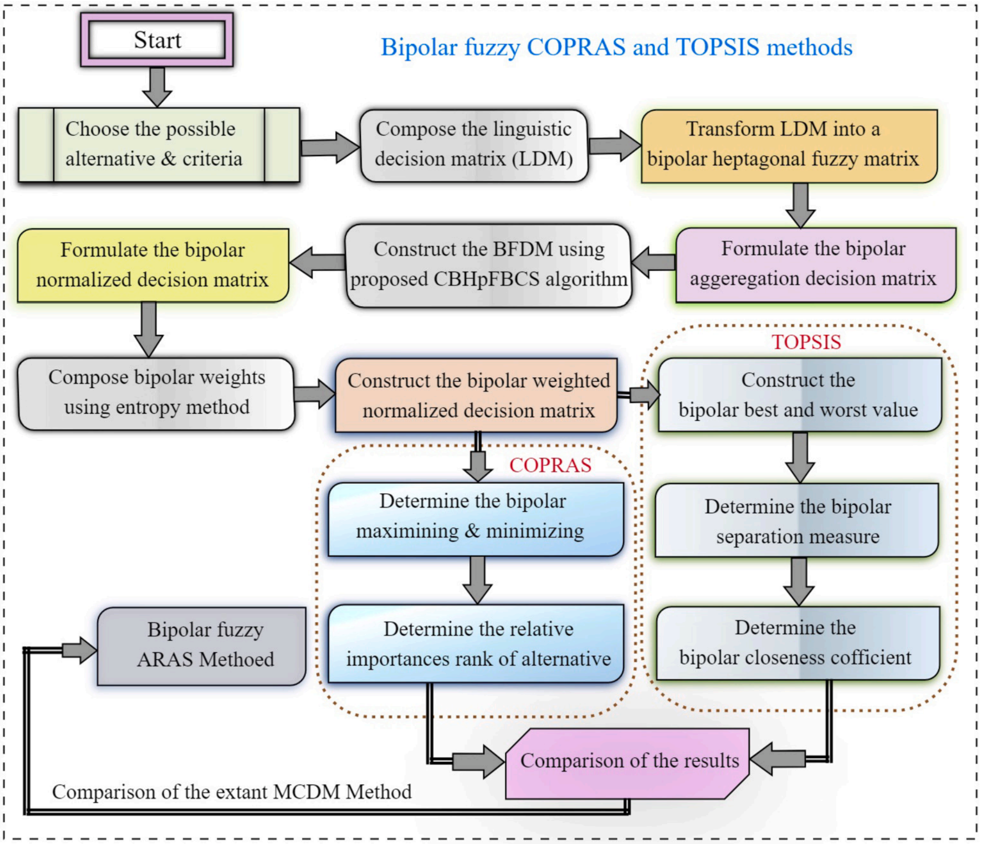

Fig. 7. The flow of the proposed method.

The closeness coefficient is utilized to ascertain the rank of the alternatives. The closeness coefficient value is determined using Equation (19) and the alternative with higher closeness coefficient value is closer to the ideal solution.

$$(C_i^+, C_i^-) = \left(\left(\frac{S_i^{N+}}{(S_i^{P+} - S_i^{N+})} \right), \left(\frac{S_i^{N-}}{(S_i^{P-} - S_i^{N-})} \right) \right), 0 < C_i^+ < 1,\ -1 < C_i^- < 0 \tag{19}$$

Then, the final score of each alternative based on method II is obtained using Equation (20).

$$C_i = \left(\frac{1 + C_i^+ - C_i^-}{2} \right) \tag{20}$$

4. Illustration of the proposed methods to detect the TB disease in pregnant women

TB is not uncommon among pregnant women, and around 2.3 per 1000 pregnant women are affected by TB in India, resulting in 44,500 cases annually. TB during pregnancy is not common, it may arise due to life circumstances. Untreated TB puts pregnant women at higher risk. Babies born to TB-infected pregnant women are more likely to have a lower birth weight than those born to non-infected mothers. To prevent transmission of TB to the baby, treatment should commence as early as possible. Pregnant women between 20-34 years old are more likely to be affected by TB. Breastfeeding is not encouraged for women undergoing TB treatment. Specialized medical care is essential for pregnant women with TB and comorbidities to receive prompt and appropriate treatment to reduce the risk of complications. This study analyzes the various TB comorbidities that impact pregnant women.

4.1. Method-I

For the first proposed bipolar fuzzy COPRAS approach,

Step 1: The possible set of alternatives $G = \{G_1, G_2, \ldots, G_6\}$ is framed, which comprises the threatening TB comorbidities diseases of pregnant women. These alternatives are analyzed through the set of criteria $P = \{P_1, P_2, \ldots, P_6\}$, are represented in Fig. 8 based on the suggestion of decision makers $F = \{F_1, F_2, F_3\}$.

Step 2: The LDM is constructed based on the decision-makers view. The two decision-makers are from the healthcare profession and

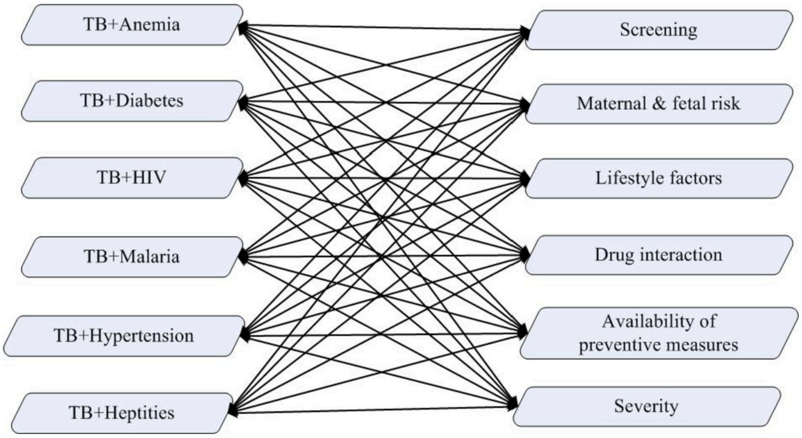

Fig. 8. Alternatives and criteria.

Table 1
Linguistic decision matrices.

Decision makers	G_m	P_1	P_2	P_3	P_4	P_5	P_6
F_1	G_1	$\daleth_3$	$\daleth_4$	$\daleth_1$	$\daleth_3$	$\daleth_1$	$\daleth_4$
	G_2	$\daleth_4$	$\daleth_3$	$\daleth_5$	$\daleth_3$	$\daleth_4$	$\daleth_5$
	G_3	$\daleth_5$	$\daleth_5$	$\daleth_4$	$\daleth_5$	$\daleth_4$	$\daleth_5$
	G_4	$\daleth_3$	$\daleth_3$	$\daleth_3$	$\daleth_1$	$\daleth_3$	$\daleth_4$
	G_5	$\daleth_3$	$\daleth_4$	$\daleth_1$	$\daleth_3$	$\daleth_3$	$\daleth_4$
	G_6	$\daleth_3$	$\daleth_4$	$\daleth_3$	$\daleth_4$	$\daleth_4$	$\daleth_4$
F_2	G_1	$\daleth_1$	$\daleth_3$	$\daleth_1$	$\daleth_3$	$\daleth_2$	$\daleth_3$
	G_2	$\daleth_3$	$\daleth_3$	$\daleth_5$	$\daleth_3$	$\daleth_4$	$\daleth_4$
	G_3	$\daleth_5$	$\daleth_5$	$\daleth_3$	$\daleth_4$	$\daleth_4$	$\daleth_5$
	G_4	$\daleth_1$	$\daleth_3$	$\daleth_1$	$\daleth_2$	$\daleth_3$	$\daleth_3$
	G_5	$\daleth_4$	$\daleth_3$	$\daleth_2$	$\daleth_1$	$\daleth_3$	$\daleth_3$
	G_6	$\daleth_1$	$\daleth_3$	$\daleth_3$	$\daleth_1$	$\daleth_3$	$\daleth_4$
F_3	G_1	$\daleth_4$	$\daleth_4$	$\daleth_3$	$\daleth_3$	$\daleth_3$	$\daleth_4$
	G_2	$\daleth_4$	$\daleth_4$	$\daleth_5$	$\daleth_4$	$\daleth_4$	$\daleth_5$
	G_3	$\daleth_4$	$\daleth_5$	$\daleth_5$	$\daleth_5$	$\daleth_3$	$\daleth_5$
	G_4	$\daleth_3$	$\daleth_3$	$\daleth_1$	$\daleth_3$	$\daleth_1$	$\daleth_4$
	G_5	$\daleth_4$	$\daleth_4$	$\daleth_3$	$\daleth_4$	$\daleth_4$	$\daleth_4$
	G_6	$\daleth_3$	$\daleth_3$	$\daleth_3$	$\daleth_3$	$\daleth_4$	$\daleth_3$

Table 2
Linguistic scale.

Linguistic variable	Bipolar heptagonal fuzzy number
$\daleth_1$ - Very Low	$\begin{pmatrix} (0.00,\ 0.01,\ 0.05,\ 0.09,\ 0.13,\ 0.17,\ 0.20), \\ (0.76,\ 0.80,\ 0.84,\ 0.88,\ 0.92,\ 0.96,\ 1.00) \end{pmatrix}$
$\daleth_2$ - Low	$\begin{pmatrix} ((0.17,\ 0.2,\ 0.24,\ 0.28,\ 0.32,\ 0.36,\ 0.40)), \\ (0.56,\ 0.60,\ 0.64,\ 0.68,\ 0.72,\ 0.76,\ 0.80) \end{pmatrix}$
$\daleth_3$ - Medium	$\begin{pmatrix} (0.36,\ 0.40,\ 0.44,\ 0.48,\ 0.52,\ 0.56,\ 0.60), \\ (0.36,\ 0.40,\ 0.44,\ 0.48,\ 0.52,\ 0.56,\ 0.60) \end{pmatrix}$
$\daleth_4$ - High	$\begin{pmatrix} (0.56,\ 0.60,\ 0.64,\ 0.68,\ 0.72,\ 0.76,\ 0.80), \\ (0.17,\ 0.20,\ 0.24,\ 0.28,\ 0.32,\ 0.36,\ 0.40) \end{pmatrix}$
$\daleth_5$ - Very High	$\begin{pmatrix} (0.76,\ 0.80,\ 0.84,\ 0.88,\ 0.92,\ 0.96,\ 1.00), \\ (0.00,\ 0.01,\ 0.05,\ 0.09,\ 0.13,\ 0.17,\ 0.20) \end{pmatrix}$

one is from the TB association department. Table 1 depict the subjective opinions of decision-makers in a linguistic manner.
Step 3: Using the linguistic conversion scale, the LDM is transformed into BHpFM as provided in Table 2.
Step 4: All the BHpFM are combined or aggregated using Equations (3) and (4). Therefore, Table 3 is viewed as a single matrix that includes all the decision-maker opinions about the performance between the alternatives and criteria.
Step 5: The BHpFN is transformed into a defuzzified matrix using CBHpFBCS algorithm, and the BHpFN is converted into a crisp number. Equation (5) is employed to generate the defuzzified matrix, which is then depicted in Table 4.
Step 6: The BNDM is calculated by dividing each element by the sum of its corresponding column elements. It aids to indicate the relation between the alternatives and criteria in the interval $[0, 1]$. Then, the obtained BNDM is represented in Table 5. **Step 7:** The

Table 3
Aggregated bipolar heptagonal fuzzy matrix.

G_i	P_1	P_2
G_1	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.360, 0.400, 0.440, 0.480, 0.520, 0.560, 0.600))
G_2	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))	((0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.413, 0.520, 0.560, 0.600))
G_3	((0.560, 0.600, 0.640, 0.813, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.153, 0.320, 0.360, 0.400))	((0.760, 0.800, 0.840, 0.880, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.153, 0.320, 0.360, 0.400))
G_4	((0.170, 0.200, 0.240, 0.413, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800))	((0.360, 0.400, 0.440, 0.480, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.680, 0.920, 0.960, 1.000))
G_5	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.480, 0.720, 0.760, 0.800))
G_6	((0.170, 0.200, 0.240, 0.413, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800))	((0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.480, 0.720, 0.760, 0.800))
	P_3	P_4
G_1	((0.000, 0.010, 0.050, 0.283, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.680, 0.920, 0.960, 1.000))	((0.360, 0.400, 0.440, 0.480, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.480, 0.520, 0.560, 0.600))
G_2	((0.760, 0.800, 0.840, 0.880, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.090, 0.130, 0.170, 0.200))	((0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.413, 0.520, 0.560, 0.600))
G_3	((0.360, 0.400, 0.440, 0.680, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.283, 0.520, 0.560, 0.600))	((0.560, 0.600, 0.640, 0.813, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.153, 0.320, 0.360, 0.400))
G_4	((0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800))	((0.000, 0.010, 0.050, 0.283, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.680, 0.920, 0.960, 1.000))
G_5	((0.000, 0.010, 0.050, 0.283, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.680, 0.920, 0.960, 1.000))	((0.170, 0.200, 0.240, 0.480, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.480, 0.720, 0.760, 0.800))
G_6	((0.360, 0.400, 0.440, 0.480, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.480, 0.520, 0.560, 0.600))	((0.170, 0.200, 0.240, 0.480, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.480, 0.720, 0.760, 0.800))
	P_5	P_6
G_1	((0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800))	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))
G_2	((0.560, 0.600, 0.640, 0.680, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.280, 0.320, 0.360, 0.400))	((0.560, 0.600, 0.640, 0.813, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.153, 0.320, 0.360, 0.400))
G_3	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))	((0.760, 0.800, 0.840, 0.880, 0.920, 0.960, 1.000), (0.000, 0.010, 0.050, 0.090, 0.130, 0.170, 0.200))
G_4	((0.170, 0.200, 0.240, 0.413, 0.520, 0.560, 0.600), (0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800))	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))
G_5	((0.360, 0.400, 0.440, 0.546, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.413, 0.520, 0.560, 0.600))	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))
G_6	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))	((0.360, 0.400, 0.440, 0.613, 0.720, 0.760, 0.800), (0.170, 0.200, 0.240, 0.346, 0.520, 0.560, 0.600))

Table 4
Defuzzified matrix.

G_i	P_1	P_2	P_3	P_4	P_5	P_6
G_1	(0.592, 0.353)	(0.517, 0.432)	(0.115, 0.825)	(0.477, 0.477)	(0.283, 0.670)	(0.517, 0.432)
G_2	(0.592, 0.353)	(0.488, 0.463)	(0.944, 0.021)	(0.604, 0.337)	(0.734, 0.224)	(0.872, 0.079)
G_3	(0.903, 0.052)	(0.934, 0.029)	(0.825, 0.115)	(0.918, 0.038)	(0.670, 0.283)	(0.934, 0.029)
G_4	(0.274, 0.676)	(0.426, 0.532)	(0.311, 0.631)	(0.115, 0.825)	(0.312, 0.640)	(0.517, 0.432)
G_5	(0.592, 0.353)	(0.517, 0.428)	(0.115, 0.825)	(0.464, 0.464)	(0.640, 0.312)	(0.517, 0.432)
G_6	(0.274, 0.676)	(0.575, 0.386)	(0.477, 0.477)	(0.464, 0.464)	(0.670, 0.283)	(0.496, 0.432)

Table 5
Normalized decision matrix.

G_i	P_1	P_2	P_3	P_4	P_5	P_6
G_1	(0.183, 0.143)	(0.153, 0.183)	(0.041, 0.285)	(0.157, 0.183)	(0.058, 0.278)	(0.134, 0.235)
G_2	(0.183, 0.143)	(0.145, 0.197)	(0.339, 0.007)	(0.199, 0.129)	(0.222, 0.093)	(0.226, 0.043)
G_3	(0.280, 0.021)	(0.277, 0.012)	(0.296, 0.040)	(0.302, 0.015)	(0.202, 0.117)	(0.242, 0.016)
G_4	(0.085, 0.274)	(0.126, 0.227)	(0.112, 0.128)	(0.038, 0.317)	(0.094, 0.266)	(0.134, 0.235)
G_5	(0.183, 0.143)	(0.153, 0.182)	(0.041, 0.285)	(0.153, 0.178)	(0.194, 0.126)	(0.134, 0.235)
G_6	(0.085, 0.274)	(0.145, 0.197)	(0.171, 0.165)	(0.153, 0.178)	(0.202, 0.117)	(0.129, 0.235)

Table 6
Objective weight of the criteria.

W	(0.169, 0.169)	(0.163, 0.168)	(0.167, 0.156)	(0.172, 0.166)	(0.173, 0.171)	(0.155, 0.167)

Table 7
Weighted normalized decision matrix.

G_i	P_1	P_2	P_3	P_4	P_5	P_6
G_1	(0.031, 0.024)	(0.025, 0.031)	(0.007, 0.045)	(0.027, 0.031)	(0.015, 0.048)	(0.021, 0.039)
G_2	(0.031, 0.024)	(0.024, 0.033)	(0.057, 0.001)	(0.034, 0.022)	(0.038, 0.016)	(0.035, 0.007)
G_3	(0.047, 0.004)	(0.045, 0.002)	(0.049, 0.006)	(0.052, 0.002)	(0.035, 0.020)	(0.038, 0.003)
G_4	(0.014, 0.046)	(0.021, 0.038)	(0.019, 0.034)	(0.007, 0.053)	(0.016, 0.045)	(0.021, 0.039)
G_5	(0.031, 0.024)	(0.025, 0.031)	(0.007, 0.045)	(0.026, 0.030)	(0.033, 0.022)	(0.021, 0.039)
G_6	(0.014, 0.046)	(0.024, 0.033)	(0.029, 0.026)	(0.026, 0.030)	(0.035, 0.020)	(0.020, 0.039)

Table 8
Maximizing and minimizing indices.

Alternatives	Bipolar values	Final scores value	Rank
G_1	(0.1256, 0.2174)	0.4541	5
G_2	(0.2190, 0.1034)	0.5578	2
G_3	(0.2668, 0.0371)	0.6148	1
G_4	(0.0973, 0.2565)	0.4204	6
G_5	(0.1436, 0.1946)	0.4764	4
G_6	(0.1478, 0.1909)	0.4766	3

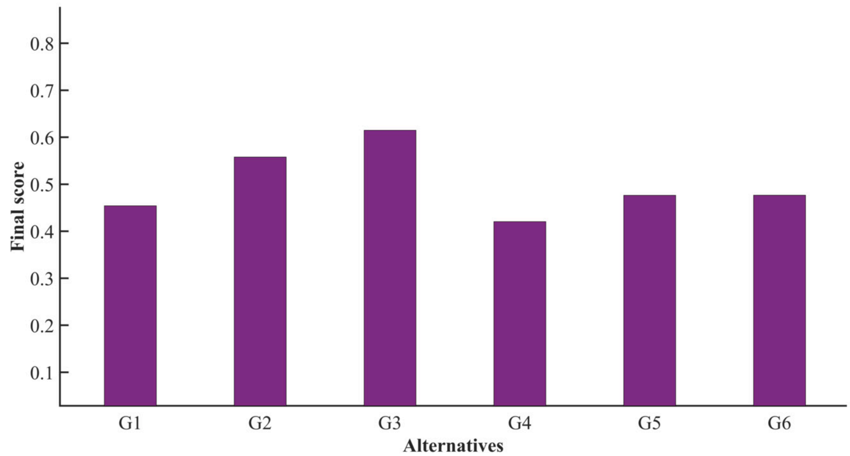

Fig. 9. Final score of the COPRAS method.

entropy technique is demonstrated from Equations (7)-(9) to determine the criteria weights in Table 6.
Step 8: The BWNDM is evaluated by multiplying the weight of the criteria with each element in the corresponding column. This matrix reveals the alternatives performance by calculating the involvement of the criteria using Equation (10). The BWNDM is expressed in Table 7.
Step 9: All the considered criteria are beneficiary criteria so evaluated the maximizing and minimizing indexes using Equations (11) and (12). After finding the maximizing and minimizing indices, the final scores are indicated in Table 8 and Fig. 9.
Step 10: Using the obtained bipolar score, the crisp score is determined through Equation (14) for alternatives. The rank are assigned in descending order. As a result, the alternative G_3 obtained rank one and G_2 achieved the second position, followed by this $G_6 > G_5 > G_1 > G_4$.

4.2. Method-II

For the second proposed bipolar fuzzy TOPSIS approach first eight steps are the same as the first proposed method.
Step 9: The bipolar ideal best $B = \left(A^{P^+}, A^{P^-}\right)$ and worst $W = \left(A^{N^+}, A^{N^-}\right)$ values is obtained from BWNDM. The maximum value of each column is chosen as the best and the minimum value is chosen as the worst value depicted in Table 9.

Table 9
Best and worst values.

	P_1	P_2	P_3	P_4	P_5	P_6
B	(0.047, 0.046)	(0.045, 0.038)	(0.057, 0.045)	(0.052, 0.053)	(0.038, 0.048)	(0.038, 0.039)
W	(0.014, 0.004)	(0.021, 0.002)	(0.007, 0.001)	(0.007, 0.002)	(0.015, 0.016)	(0.020, 0.003)

Table 10
Separation measure.

Alternatives	(S^{P^+}, S^{P^-})	(S^{N^+}, S^{N^-})
G_1	(0.0679, 0.0323)	(0.0268, 0.0794)
G_2	(0.0325, 0.0737)	(0.0656, 0.0423)
G_3	(0.0079, 0.0963)	(0.0793, 0.0066)
G_4	(0.0773, 0.0107)	(0.0118, 0.0949)
G_5	(0.0641, 0.0416)	(0.0322, 0.0726)
G_6	(0.0577, 0.0409)	(0.0357, 0.0743)

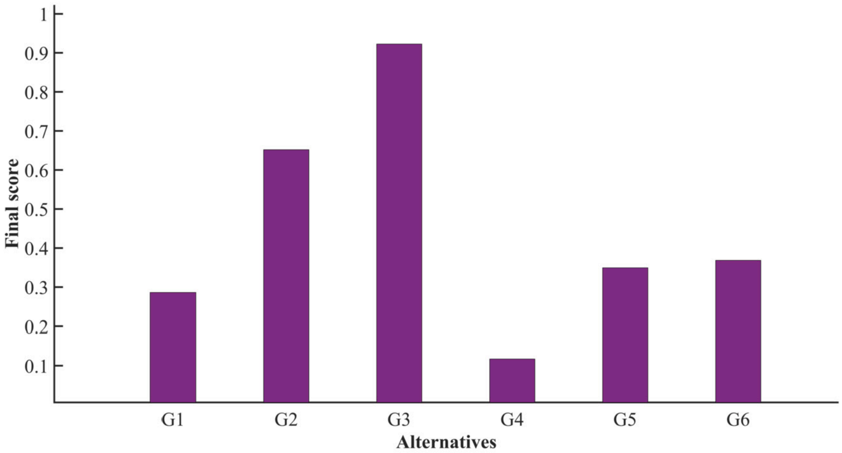

Fig. 10. Final score of the TOPSIS method.

Table 11
Outcome of the TOPSIS method.

Alternatives	Bipolar values	Final scores value	Rank
G_1	(0.2831, 0.7108)	0.2862	5
G_2	(0.6685, 0.3648)	0.6518	2
G_3	(0.9097, 0.0645)	0.9226	1
G_4	(0.1328, 0.8984)	0.1172	6
G_5	(0.3345, 0.6359)	0.3493	4
G_6	(0.3822, 0.6451)	0.3685	3

Step 10: The bipolar separation measures positive $\left(S^{P^+}, S^{P^-}\right)$ and negative $\left(S^{N^+}, S^{N^-}\right)$ ideal solutions are determined for each alternative in Table 10.
Step 11: The bipolar closeness coefficient is determined using Equation (19). The optimum alternative is obtained using Equation (20), which provides the final score of the alternatives. The alternatives with a higher value is considered to be more preferred than those with a lower coefficient value (Table 11 and Fig. 10).

5. Result and discussion

TB is still a global health issue, especially in developing countries. It is one of the top 10 causes of death worldwide, and the impact of TB comorbidity on pregnant women is profound, leading to increased risks of obstetric complications, maternal mortality, adverse maternal and neonatal outcomes, synergistic effects with HIV. Addressing TB in pregnant women is significant to improve the maternal and neonatal health consequences and reducing mortality rates associated with this dual infection.
Therefore, this study aims to identify the most high-risk coexisting TB disease in pregnant women. Initially, the TB co-infection diseases are chosen as alternative and its features are chosen as criteria to analyze the highly vulnerable TB comorbidities disease in

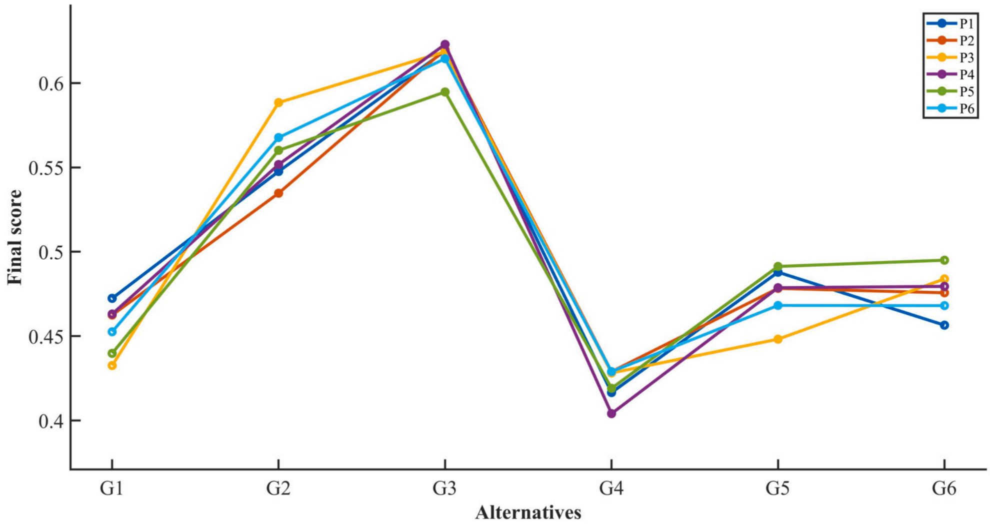

Fig. 11. Sensitive analysis (Method-I).

pregnant women. The data are collected from healthcare professionals and TB association departments. Though the collected data are in linguistic variables it was transformed into the proposed BHpFN using linguistic scale, which provides both positive and negative perspectives. Then, the BHpFM were aggregated into a single BHpFM using the aggregation operator. The aggregated matrix was defuzzified using the proposed CBHpFBCS algorithm. The normalization process was performed to assess the scattering scale, and the resulting matrix was in the interval [0,1]. The entropy method was used to evaluate the performance of the criteria, and the criteria weight values were multiplied with the normalized matrix to calculate a weighted normalized matrix. Though the considered criteria were beneficiary type, maximizing and minimizing indices are determined. The final score value for each alternative is demonstrated using Equation (14). In method-II, the first eight processes were similar to method-I, and the procedures of the bipolar fuzzy TOPSIS method were followed from the weighted normalized matrix. Then, the optimal score of each alternative was determined to rank the alternatives.

The study demonstrates that the most significant impact on the health of pregnant women is TB+HIV (G_3). Because HIV weakens the immune system, it becomes more difficult for the body to defend against TB. Pregnant women with HIV are particularly susceptible to develop TB and face an increased risk of disease complications. This also increases the risk of mother-to-child transmission of both diseases. The remaining alternatives are ranked as follows: $G_2 > G_6 > G_5 > G_1 > G_4$.

The ranking order of the alternatives are discussed as follows: Pregnant women with TB+Diabetes (G_2) are at second higher risk because the management of diabetes during pregnancy is challenging, and the combination of the two conditions may increase the risk in immune system. TB+Hepatitis (G_6) affect the liver and cause complications during pregnancy. Hypertension is also known as high blood pressure. The TB+Hypertension (G_5) lead to complications such as preeclampsia and preterm birth. TB+anemia (G_1) may cause shortage of red blood cells in the body. This leads to fatigue, weekend immune system and shortness of breath. Anemia during pregnancy increases the risk of complications for both the mother and the baby, such as cause preterm birth and less baby birth weight. TB+Malaria (G_4) also lead to complications such as preterm birth, less baby birth weight and anemia. Therefore, TB comorbidities diseases are threatening diseases for pregnant women, and TB+HIV being the most complicated. The result is also supported by the article [40], which indicates that the co-infection of TB and HIV poses a high risk in pregnant women. Moreover, the obtained result is also validated through the sensitivity and comparative analyses.

5.1. Sensitivity analysis

Sensitivity analysis is essential because it helps to assess the robustness and stability of the proposed method. It also aids in exploring the range of potential outcomes based on different scenarios, which helps to gain insights into the proposed method's behavior. Here, the impact on criteria is analysed in the six different situations. Through the simulation of criteria weights, it is possible to identify the criteria that have the most influence on the decision outcomes.

Due to the changes in preference rating criteria, the result differs in a few scenarios in sensitivity analysis. In method-I, alternative G_3 secured the first position, and G_2 obtained the second position consistently in all cases. G_6, G_5, and G_1 achieved the third, fourth, and fifth positions, respectively, in two out of six cases. However, G_4 consistently maintained the last position in all cases. The criteria P_1, P_2, P_5 and P_6 have a marginal influence on the outcome when given higher priority or preference. In method-II, alternative G_3 secured the first position, and G_2 obtained the second position consistently in all cases. G_6, G_5, and G_1 achieved the third, fourth, and fifth positions, respectively, in three out of six cases. However, G_4 consistently maintained the last position in all cases. The criteria P_1, P_2 and P_3 have a slight influence on the outcome when given higher priority or preference. Figs. 11, 12 and Table 12 present the sensitivity results of methods I and II, respectively.

Table 12
Sensitive analysis.

Various scenario	COPRAS-Criteria weight changes	Final rank
Scenario 1	$P_1 = 40\%$ P_2-$P_6 = 60\%$	$G_3 < G_2 < G_5 < G_1 < G_6 < G_4$
Scenario 2	$P_2 = 40\%$ P_1, P_3-$P_6 = 60\%$	$G_3 < G_2 < G_5 < G_6 < G_1 < G_4$
Scenario 3	$P_3 = 40\%$ $P_1, P_2, P_4 - P_6 = 60\%$	$G_3 < G_2 < G_6 < G_5 < G_1 < G_4$
Scenario 4	$P_4 = 40\%$ P_1-$P_3, P_5, P_6 = 60\%$	$G_3 < G_2 < G_6 < G_5 < G_1 < G_4$
Scenario 5	$P_5 = 40\%$ P_1-$P_4, P_6 = 60\%$	$G_3 < G_2 < G_5 < G_1 < G_6 < G_4$
Scenario 6	$P_6 = 40\%$ P_1-$P_5, P_6 = 60\%$	$G_3 < G_2 < G_5 < G_6 < G_1 < G_4$
Various scenario	**TOPSIS-Criteria weight changes**	**Final rank**
Scenario 1	$P_1 = 40\%$ P_2-$P_6 = 60\%$	$G_3 < G_2 < G_5 < G_1 < G_6 < G_4$
Scenario 2	$P_2 = 40\%$ P_1, P_3-$P_6 = 60\%$	$G_3 < G_2 < G_5 < G_6 < G_1 < G_4$
Scenario 3	$P_3 = 40\%$ $P_1, P_2, P_4 - P_6 = 60\%$	$G_3 < G_2 < G_6 < G_4 < G_5 < G_1$
Scenario 4	$P_4 = 40\%$ P_1-$P_3, P_5, P_6 = 60\%$	$G_3 < G_2 < G_6 < G_5 < G_1 < G_4$
Scenario 5	$P_5 = 40\%$ P_1-$P_4, P_6 = 60\%$	$G_3 < G_2 < G_6 < G_5 < G_1 < G_4$
Scenario 6	$P_6 = 40\%$ P_1-$P_5, P_6 = 60\%$	$G_3 < G_2 < G_6 < G_5 < G_1 < G_4$

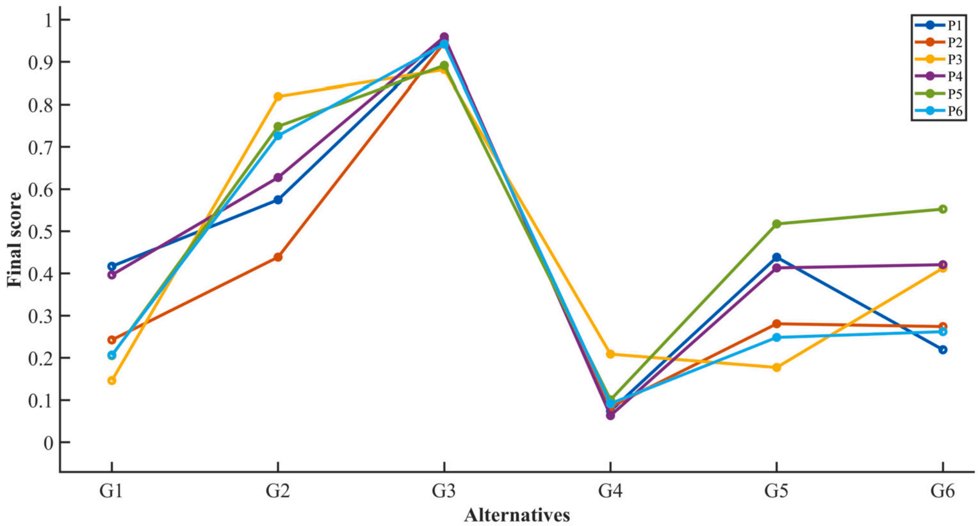

Fig. 12. Sensitive analysis (Method-II).

5.2. Comparative analysis

The ARAS method gauges the degree of utility of each alternative to identify the most advantageous alternative among others. In the ARAS method, the rank of the alternatives remains the same. The result follows the order $G_3 > G_2 > G_6 > G_5 > G_1 > G_4$ in the comparative analysis. Therefore, the result of the proposed method exhibits its robustness and collides with the real-world scenario. Table 13 and Fig. 13 divulge the final value of the alternatives through the ARAS method.

5.3. Advantages of the work

(i) The use of COPRAS and TOPSIS in the bipolar fuzzy environment enables a comprehensive evaluation of high-risk coexisting TB in pregnant women by considering multiple criteria in the decision-making process.

Table 13
Comparative analysis.

G_i	COPRAS scores	Rank	TOPSIS scores	Rank	ARAS scores	Rank
G_1	0.4541	5	0.2862	5	0.3236	5
G_2	0.5578	2	0.6418	2	0.7041	2
G_3	0.6148	1	0.9226	1	0.9121	1
G_4	0.4204	6	0.1172	6	0.2020	6
G_5	0.4764	4	0.3493	4	0.4071	4
G_6	0.4766	3	0.3685	3	0.4080	3

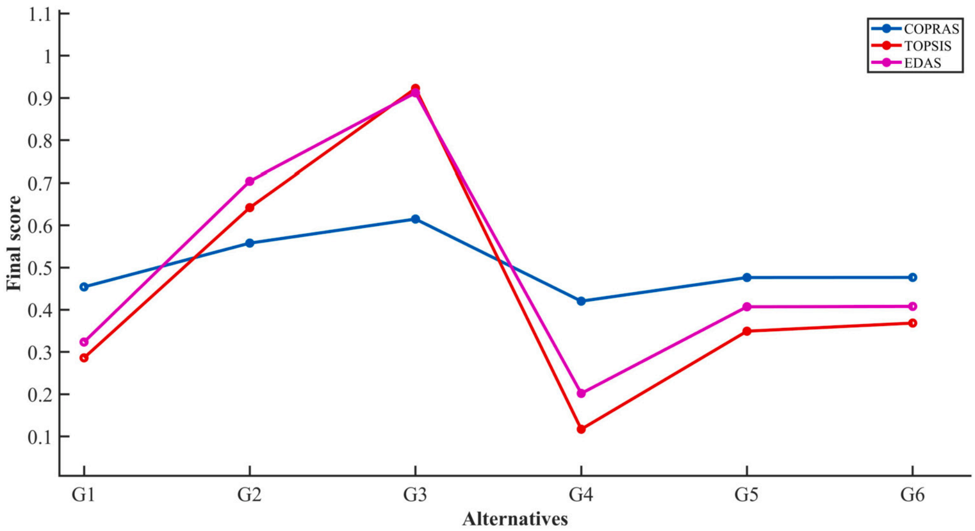

Fig. 13. Comparative analysis rank on final scores.

(ii) The COPRAS method ensures decision consistency by balancing the importance weights of criteria, eliminating subjective biases, and TOPSIS ranks alternatives based on their similarity to the ideal solution. Therefore, both methods lead to more reliable and consistent results.
(iii) The incorporation of entropy in the bipolar fuzzy environment allows for precise decision-making by measuring the uncertainty associated with the data.
(iv) The proposed method is adaptable to problems of $m \times n$ dimensions and incorporates both qualitative and quantitative data.

5.4. Limitations of the work

(i) This study utilized the data collected from India, so the results might differ for other countries.
(ii) Moreover, the results could differ when only positive views are considered, due to the lack of information about negative views.
(iii) Modifications in input parameters may lead to diverse outcomes.

6. Conclusion

TB is one of the deadly diseases that has not been investigated by any researchers through MCDM in the bipolar fuzzy context. Therefore, this gap is filled by analyzing the high-risk coexisting TB disease during pregnancy through COPRAS and TOPSIS in a BHpFE. These methods were used a bipolar fuzzy approach to handle unclear information from both positive and negative views. Also, a new CBHpFBCS algorithm was designed to defuzzify the BHpFN into a crisp number. Various TB comorbidities diseases were considered as alternatives and its features were considered as criteria. From the outcome of the proposed method, the coinfection TB+HIV (G_3) had identified as a more significant impact on pregnant women and caused high complications during pregnancy. The combination of TB and HIV significantly impacts pregnant women, elevating the risks of obstetric complications and mortality. The TB and HIV exacerbates immune compromise in pregnant women. Overall, TB+HIV coinfection during pregnancy pose substantial health risks. The outcomes were validated through sensitivity by varying the criteria weights under various scenarios and comparative analysis was done against different MCDM methods.
This piece of work can be further extended into:

(i) The BFS can be extended into bipolar intuitionistic complex fuzzy sets to enhance the TB diagnosis.
(ii) The study will also concentrate on the aggregation and defuzzification techniques of diverse bipolar fuzzy contexts.

(iii) The study will concentrate on the methodology of different types of fuzzy numbers, such as intuitionistic, and Pythagorean within a bipolar fuzzy context.
(iv) The proposed bipolar fuzzy MCDM methods can also be utilized for diagnosing diseases like COVID-19, cardiovascular, etc.
(v) The proposed MCDM methods can be emphasized by integrating artificial intelligence, enabling real-time decision-making, and improving risk assessment.
(vi) The similarity and entropy measures will be focused for bipolar intuitionistic fuzzy sets.
(vii) Moreover, the limitations of the proposed study will be addressed in the future works.

CRediT authorship contribution statement

Ezhilarasan Natarajan: Writing – original draft, Visualization, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Felix Augustin:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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