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Method Article

Measuring dynamic dependency using time-varying copulas with extended parameters: Evidence from exchange rates data

Atina Ahdika[a,b,*], Dedi Rosadi[a], Adhitya Ronnie Effendie[a], Gunardi[a]

[a] Department of Mathematics, Universitas Gadjah Mada, Yogyakarta, Indonesia
[b] Department of Statistics, Universitas Islam Indonesia, Yogyakarta, Indonesia

A B S T R A C T

This study proposes a novel approach that investigates the dynamic dependency among exchange rates by extending time-varying copulas' parameters following an autoregressive moving average (ARMA) process. The process consists of an autoregressive part that explains the effect of the previous parameters and a forcing variable that measures the dependence structure between marginal variables. We apply this model to the daily data of the exchange rates of five Asian countries with the strongest economies before and during the 2020 pandemic, namely CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD. The ARIMA-GARCH model was used to model the exchange rates data and estimate the dynamic dependence using time-varying copulas with the extended parameters. The dynamic dependencies between Chinas and the four countries' exchange rates before and during the 2020 pandemic was evidenced. Moreover, India is the country whose exchange rate has been most strongly affected by the pandemic. Some of the highlights of the proposed approach are:

- This paper provides two algorithms to investigate the dynamic dependencies among exchange rates data during a crisis and forecast the data using time-varying copulas with the extended parameters.
- There are four extended time-varying copulas' parameters which can measure the dynamic dependencies between variables.
- The computation procedure is easy to implement.

A R T I C L E   I N F O
Method name: Time-varying copulas with extended parameters
Keywords: ARIMA-GARCH, Dynamic parameter, Forcing variable, Time-varying copulas, Exchange rates
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* Corresponding author.
E-mail address: atina.a@uii.ac.id (A. Ahdika).

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Specifications table

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More specific subject area:	Dynamic dependencies modeling, time-varying copula modeling
Method name:	Time-varying copulas with extended parameters
Name and reference of original method:	Patton, A. J. (2006). Modelling Asymmetric Exchange Rate Dependence. International Economic Review, 47(2), 527–556.
Resource availability:	Exchange rate data from Yahoo Finance

Introduction

The possibility of changes in the interdependence between variables in the financial markets, especially during a crisis, is one of the fundamental issues in economic and financial fields. There has been plenty of evidence regarding this issue. Caporale, Cipollini, & Demetriades (2005) examined the relationship between the interest rate and the exchange rate and found that the increase in domestic interest rates was in contrast with the depreciated exchange rate during the Asian financial crisis [1]. Rodriguez (2007) also showed that there had been changes of dependence within the daily returns of the stock indices of East Asian and Latin American countries during the disruption period of the Asian and Mexican crisis [2]. Similar behavior was also found in the analysis of dependence between the exchange rate of JPY/USD and EUR/USD before, during, and after the crisis [3]. Recent studies show that a crisis, especially the Covid-19 pandemic, provides valuable information on asset price or return discovery and financial market behavior both before and during a pandemic. Iyke (2020) showed that disease outbreaks can be an alternative channel of the exchange rate behavior because public panic can affect the dynamics of financial markets [4]. Narayan (2020) evidenced the non-stationarity of the Yen/USD exchange rate prior to the Covid-19 pandemic and the high stationarity during the pandemic which proved that the global shock has an effect on the exchange rate behavior [5]. Other studies showed that Covid-19 pandemic has influenced the relationship between exchange rate and stock return. Generally, the dynamic relationship between exchange rate and stock return strengthened during the Covid-19 pandemic [6]. Narayan et al. (2020) also showed that the depreciation on the standard deviation of Yen/USD improved stock market returns during Covid-19 pandemic [7].

One of the popular methods used to identify the structure of dependency between variables is the copula function, introduced by Sklar (1959) [8], i.e., a function that combines one-dimensional marginal distribution functions into a multivariate distribution function. Copula overcomes some limitations of the Pearson coefficient correlation as the most straightforward dependence measure [9].

Copula modeling that can identify temporal dependence is known as a time-varying copula, introduced by Patton (2006), which captures the possible time variation in the conditional copula [10]. Temporal dependence defines the intervention of previous parameters. Two approaches in identifying temporal dependence using the time-varying copula model are observation- and parameter-based methods. The first approach allows time variations on the copula's functional form using a regime-switching model [2,11–15]. The second approach is more straightforward; that is, the copula's functional form is assumed to remain the same throughout the observations, whereas the copula parameters vary over time. The time variation consists of an autoregressive coefficient explaining the effect of the parameters in previous times and a forcing variable representing the dependence structure between the marginal variables through the latest observations [3,16–20]. Patton (2006) [10] reported that identifying the forcing variable is somewhat problematic. Thus, the simplest model of the mean absolute difference between the last ten observations of the marginal cumulative distribution function is selected. However, information regarding the selection of the previous ten observations is lacking. Therefore, several possibilities can still be explored from the aforementioned dynamic parameters, such as selecting the forcing variable and estimating the optimal window length of past observations.

By considering this problem, the present study aims to propose an approach to investigate the best parameter of time-varying copulas by extending the dynamic parameters into four functional forms, which is explained in the following sections. The past observations in the forcing variable are considered as a new parameter to be estimated in these extended functions. The proposed approach is applied to the daily data of the exchange rates of the five most influential countries in Asia before and during the 2020 pandemic, namely CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD. Based on previous studies, there has been substantial evidence of the dynamic dependence among several countries' exchange rates. Patton (2006) found evidence of the dynamic dependence between the Deutsche mark – US dollar and yen – US dollar from January 1991 to December 2001 using time-varying Normal and SJC copula [10]. Pelletier (2006) found the dynamic correlation within Pound, Deutsche mark, yen, and Swiss – Franc against the US dollar between October 1981 and June 1985 using the RSDC model [12]. Dias & Embrechts (2010) found significant decreases in the dependence between EUR/USD and JPY/USD from the beginning of 2007 until mid-2007, which returned to its pre-crisis level after mid-2007 [3]. Acar, Czado, & Lysy (2019) found that there are significant time variations in the dependence between seven major currencies, i.e., EUR, GBP, CAD, AUD, CHF, JPY/100, and CNY/10, which often coincide with the occurrence of the financial crisis or other economic events [21].

Based on the previous studies, it may be presumed that there have been changes in the dependence among the five most influential currencies in Asia, particularly during the 2020 pandemic. Therefore, the dynamic dependence between the four exchange rates and those of China as the country with the strongest economy in Asia and the epicenter of the 2020 pandemic is investigated using the proposed approach. The intention is to study the impact of the 2020 pandemic that initially occurred in China on the dependence of China's exchange rate against the other four major exchange rates in Asia. A hybrid ARIMA-GARCH is used to model the exchange rates data and the extended functions of the dynamic parameter of time-varying copulas is employed in order to obtain the dynamic dependence. The accuracy of the proposed approach is measured by calculating the mean absolute percentage errors (MAPEs) of the actual and forecasted data of the in-sample and out-sample exchange rates.

Model Specifications and Estimation Procedures

ARIMA-GARCH Model

The exchange rates data are modeled using ARIMA and the volatility clustering of the residuals is evaluated using GARCH. Suppose that the exchange rate data is denoted by $\mathrm{X}_t = \{X_{1,t}, \ldots, X_{5,t}\}$, $t = 1, \ldots, T$, and the index listed below, namely {1,2,3,4,5}, represents the exchange rates of CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD, respectively. Because the exchange rate data are non-stationary, we compute the exchange rate's log return to smooth out and stabilize the variance of the series. The $\mathrm{ARIMA(p, d, q)}$ – $\mathrm{GARCH(r, s)}$ model for the exchange rates is defined by

$$R_{i,t} = \mu_i + \sum_{j=1}^{p} \varphi_{ij} R_{i,t-j} + \sum_{k=1}^{q} \theta_{ik} \varepsilon_{i,t-k} + \varepsilon_{i,t}, \;\; i = 1, \ldots, 5, \;\; t = 1, \ldots, T \tag{1}$$

$$\varepsilon_{i,t} = \sigma_{i,t} Z_{i,t} \tag{2}$$

$$\sigma^2_{i,t} = \alpha_{i0} + \sum_{l=1}^{r} \alpha_{il}\, \varepsilon^2_{i,t-l} + \sum_{m=1}^{s} \beta_{im}\, \sigma^2_{i,t-m} \tag{3}$$

where $R_{i,t} = \ln\left(\frac{X_{i,t}}{X_{i,t-1}}\right)$ is the log return of the exchange rates, μ_i is a constant, ϕ_{ij} and θ_{ik} are the autoregressive and moving average coefficient, respectively, $\varepsilon_{i,t}$ and $\sigma_{i,t}$ are the residual and variance of the log return of the exchange rates, respectively. α_{i0},α_{i1}, and β_{im} are the non-negative parameters of the GARCH model where $\sum_{i=1}^{r} \alpha_{il} + \sum_{m=1}^{s} \beta_{im} < 1$, and the innovation variable $Z_{i,t} \sim N(0, 1)$.

Time-Varying Copulas Model

This research investigates the dynamic dependence among the residuals of the log return of the exchange rates, which show volatility clustering. Suppose that each residual has a continuous distribution function $F_i(\varepsilon_{i,t})$, $i = 1, \dots, 5$ denoting CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD, respectively. Suppose that the bivariate distribution function between two different marginal variables is denoted by $F(\varepsilon_{i,t}, \varepsilon_{j,t})$, $i \neq j$, then by Sklar's Theorem, there exists a copula $C(\cdot)$ such that

$$F\left(\varepsilon_{i,t}, \varepsilon_{j,t}\right) = C\left(F_i\left(\varepsilon_{i,t}\right), F_j\left(\varepsilon_{j,t}\right)\right), \quad i \neq j \tag{4}$$

The copula types considered in this study include Normal, Clayton, Gumbel, and SJC, which correspond to copulas having zero tail dependence, lower tail dependence, upper tail dependence, and both lower and upper tail dependence, respectively. The marginal variables used are time-series data; thus, the time-varying copula with a dynamic parameter is assumed to be the optimal model.

Let C_1, C_2, C_3, and C_4 be the time-varying Normal, Clayton, Gumbel, and SJC copulas, and $u_{i,t} = F_i(\varepsilon_{i,t})$ is the cumulative distribution function of the residuals. The formula of each copula is given by

$$C_1\left(u_{i,t}, u_{j,t}\right) = \int_{-\infty}^{\Phi^{-1}(u_{i,t})} \int_{-\infty}^{\Phi^{-1}(u_{2,t})} \frac{1}{2\pi\sqrt{1-\rho_t}} \exp\left\{\frac{-\left(r^2 - 2\rho_t rs\right)}{2\left(1-\rho_t^2\right)}\right\} dr\, ds, \quad -1 < \rho_t < 1 \tag{5}$$

$$C_2\left(u_{i,t}, u_{j,t}\right) = \left(u_{i,t}^{-\theta_t^{Cl}} + u_{j,t}^{-\theta_t^{Cl}} - 1\right)^{-\frac{1}{\theta_t^{Cl}}}, \quad \theta_t^{Cl} \in (0, +\infty) \tag{6}$$

$$C_3\left(u_{i,t}, u_{j,t}\right) = \exp\left\{-\left[\left(-\ln u_{i,t}\right)^{\theta_t^{Gb}} + \left(-\ln u_{j,t}\right)^{\theta_t^{Gb}}\right]^{\frac{1}{\theta_t^{Gb}}}\right\}, \quad \theta_t^{Gb} \in [1, +\infty) \tag{7}$$

$$C_4\left(u_{i,t}, u_{j,t}\right) = 0.5\left(C_5\left(u_{i,t}, u_{j,t}\right) + C_5\left(1 - u_{i,t}, 1 - u_{j,t}\right) + u_{i,t} + u_{j,t} - 1\right) \tag{8}$$

where $C_5(\cdot)$ is the Joe-Clayton (JC) copula defined as

$$C_5\left(u_{i,t}, u_{j,t}\right) = 1 - \left(1 - \left\{\left[1 - \left(1 - u_{i,t}\right)^{\kappa_t}\right]^{-\gamma_t} + \left[1 - \left(1 - u_{j,t}\right)^{\kappa_t}\right]^{-\gamma_t} - 1\right\}^{-\frac{1}{\gamma_t}}\right)^{\frac{1}{\kappa_t}} \tag{9}$$

with $\kappa_t = \frac{1}{\log_2(2-\lambda_{U,t})}$, $\gamma_t = \frac{1}{\log_2(\lambda_{L,t})}$, *and* $\lambda_{U,t}, \lambda_{L,t} \varepsilon (0, 1)$ [10].

In a general framework, the parameter of the copula functions is static. In Normal, Clayton, Gumbel, and SJC copulas, the parameter of the static copulas is $\Theta = \{\rho, \theta^{Cl}, \theta^{Gb}, (\lambda_U, \lambda_L)\}$. However, in the context of the time-varying copula, the parameter is always time-dependent and defined by $\Theta_t = \{\rho_t, {\theta_t}^{Cl}, {\theta_t}^{Gb}, (\lambda_{U,t}, \lambda_{L,t})\}$, for time-varying Normal, Clayton, Gumbel, and SJC copulas, respectively.

Extended Dynamic Parameters

We extend the functional forms of the forcing variable in the ARMA process defined by Patton (2006) [10] and Manner & Reznikova (2012) [19] and generalize the window length order. The autoregressive function explains the interdependence of time through a parameter Θ_{t-1}. Furthermore, the difference in the cumulative probabilities defines the dependence between the marginal variables. The premise that the smaller the difference between the two marginal variables' cumulative probabilities, the higher the dependence between the two variables [22] becomes the basis for selecting the proposed functions. The extension of the dynamic parameters is offered as follows:

Let Θ_t be a function of the dynamic parameter of time-varying copulas, which follows an ARMA process. The process contains a constant ω, an autoregressive coefficient β, and a forcing variable. The coefficient β is multiplied by the last parameter and shows the interdependence between parameters at different lag times. Meanwhile, a forcing variable represents the dependence between the two marginal variables. The first three forcing variables are defined as the positive-definite functions of the difference between the two marginal variables' cumulative probabilities. In contrast, the fourth forcing variable is determined by the concordance-discordance-like property between the two marginal variables, which describes the nature of the dependence between two variables. Mathematically, the extended functions of the forcing variable are defined by

$$\Theta_{k,t} = \begin{cases} \Lambda\left(\omega + \beta\,\Theta_{k,t-1} + \alpha\,\frac{1}{m}\sum_{j=1}^{m}\left|u_{1,t-j} - u_{2,t-j}\right|\right), k = 1 \\ \Lambda\left(\omega + \beta\,\Theta_{k,t-1} + \alpha\,\frac{1}{m}\sum_{j=1}^{m}\left(u_{1,t-j} - u_{2,t-j}\right)^2\right), k = 2 \\ \Lambda\left(\omega + \beta\,\Theta_{k,t-1} + \alpha\sqrt{\frac{1}{m}\sum_{j=1}^{m}\left(u_{1,t-j} - u_{2,t-j}\right)^2}\right), k = 3 \\ \Lambda\left(\omega + \beta\,\Theta_{k,t-1} + \alpha\,\frac{1}{m}\sum_{j=1}^{m}\left(\varepsilon_{1,t-j+1} - \varepsilon_{1,t-j}\right)\left(\varepsilon_{2,t-j+1} - \varepsilon_{2,t-j}\right)\right), k = 4 \end{cases} \quad (10)$$

where $\Theta_{k,t} = \{\rho_{k,t}, {\theta_{k,t}}^{Cl}, {\theta_{k,t}}^{Gb}, (\lambda_{U,k,t}, \lambda_{L,k,t})\}$ are the parameters of time-varying Normal, Clayton, Gumbel, and SJC copulas, respectively, $u_{i,t} = F_i(\varepsilon_{i,t})$ with $i = 1, \ldots, 5$ is the cumulative distribution function of the marginal variables, and

$$\Lambda(x) = \begin{cases} (1 - e^{-x})(1 + e^{-x})^{-1}, & \text{Normal copula} \\ e^x, & \text{Clayton copula} \\ e^x + 1, & \text{Gumbel copula} \\ (1 + e^{-x})^{-1}, & \text{SJC copula} \end{cases} \quad (11)$$

The first three functions of the dynamic parameter in Eq. 10 define the mean of the absolute deviation, the mean of the squared deviation, and the root of the mean squared difference between the two marginal data series of m past observations, respectively. These three functions are positive-definite, where the dependence between marginal variables is only indicated by the distance (which is non-negative) between the cumulative probabilities of the marginal variables. In contrast, the fourth function defines the concordance-discordance-like property in order to identify the two marginal variables' dependence. This function shows the dependent nature of the two marginal variables; it is positive if the multiplication between $(\varepsilon_{i,t-l+1} - \varepsilon_{i,t-l})$ and $(\varepsilon_{j,t-l+1} - \varepsilon_{j,t-l})$ is more than zero. Contrarily, it is negative if the multiplication of both is less than zero.

Estimation Procedures

Patton (2006) [10] and Manner & Reznikova (2012) [19] estimated three parameters consisting of $\{\omega, \beta, \alpha\}$ and set the window length as $m = 10$. In this paper, it is assumed that m is also a parameter. Therefore, the parameters $\{\omega, \beta, \alpha, m\}$ are estimated in order to obtain the optimal parameters.

Suppose that a joint distribution function of the two different residual variables with parameter Ω_i and Ω_j, $i \neq j$, is written as

$$F\left(\varepsilon_{i,t}, \varepsilon_{j,t}; \Theta_{k,t}\right) = C\left(F_i\left(\varepsilon_{i,t}; \Omega_i\right), F_j\left(\varepsilon_{j,t}; \Omega_j\right); \Theta_{k,t}\right) \quad (12)$$

where $C(\cdot)$ is the time-varying copula and $\Theta_{k,t}$ is the dynamic parameter of the time-varying copula defined by Eq. 10. The log-likelihood function of the model is given by

$$\begin{aligned} \ell\left(\Omega_i, \Omega_j, \Theta_{k,t}\right) &= \log\left(\prod_{t=1}^{T} f\left(\varepsilon_{i,t}, \varepsilon_{j,t}; \Theta_{k,t}\right)\right) \\ &= \log\left(\prod_{t=1}^{T} \frac{\partial^2 F\left(\varepsilon_{i,t}, \varepsilon_{j,t}; \Theta_{k,t}\right)}{\partial\varepsilon_{i,t}\,\partial\varepsilon_{j,t}}\right) \\ &= \log\left(\prod_{t=1}^{T} \frac{\partial^2 C\left(F_i\left(\varepsilon_{i,t}; \Omega_i\right), F_j\left(\varepsilon_{j,t}; \Omega_j\right); \Theta_{k,t}\right)}{\partial\varepsilon_{i,t}\,\partial\varepsilon_{j,t}}\right) \end{aligned}$$

$$= \log\left(\prod_{t=1}^{T} f_i\left(\varepsilon_{i,t};\Omega_i\right) f_j\left(\varepsilon_{j,t};\Omega_j\right) c\left(F_i\left(\varepsilon_{i,t};\Omega_i\right), F_j\left(\varepsilon_{j,t};\Omega_j\right);\Theta_{k,t}\right)\right)$$

$$= \sum_{t=1}^{T}\log f_i\left(\varepsilon_{i,t};\Omega_i\right) + \sum_{t=1}^{T}\log f_j\left(\varepsilon_{j,t};\Omega_j\right) + \sum_{t=1}^{T}\log c\left(F_i\left(\varepsilon_{i,t};\Omega_i\right), F_j\left(\varepsilon_{j,t};\Omega_j\right);\Theta_{k,t}\right) \quad (13)$$

By considering the maximum likelihood estimate (MLE) complexity due to the many unknown parameters, the estimation procedure can be carried out in two steps [3,10,13]. First, the marginal variables' parameters (Ω_i, Ω_j), and then the parameter of the time-varying copulas $\Theta_{k,t}$ are estimated. Thus, Eq. 14 can be written as

$$\ell\left(\Omega_i, \Omega_j, \Theta_{k,t}\right) = \ell_{f_i}(\Omega_i) + \ell_{f_j}\left(\Omega_j\right) + \ell_{c_k}\left(\Theta_{k,t}\right) \quad (14)$$

where $\ell_{f_i}(\Omega_i)$ and $\ell_{f_j}(\Omega_j)$ are the log-likelihood function of the marginal variables and $\ell_{c_k}(\Theta_{k,t})$ is the log-likelihood function of the time-varying copulas with $k = 1, 2, 3, 4$.

Finally, $\Theta_{k,t}$ is estimated by maximizing $\ell_{c_k}(\Theta_{k,t})$

$$\hat{\Theta}_{k,t} = \arg\max_{\Theta_{k,t}} \ell_{c_k}\left(\Theta_{k,t}\right) \quad (15)$$

AIC is used as the performance measure to select the best copula model.

Algorithm 1 and 2 represent the procedures to estimate the time-varying copula's best parameter and forecast the exchange rates using ARIMA-GARCH-time-varying copulas with extended dynamic parameters.

Algorithm 1 – Modeling the dynamic dependency of the exchange rates data.

Step 1. Estimate the parameters of ARIMA-GARCH for the marginal variables $X_{i,t}$, $i = 1, \ldots, 5$, $t = 1, \ldots, T$ and obtain the residuals $\varepsilon_{i,t}$
Step 2. Calculate the cumulative distribution function of the residuals, i.e., $u_{i,t} = F_i(\varepsilon_{i,t})$.
Step 3. Estimate the static copula's parameters $\hat{\Theta} = \{\hat{\rho}, \hat{\theta}^{Cl}, \hat{\theta}^{Gb}, (\hat{\lambda}_U, \hat{\lambda}_L)\}$, which generate the joint distribution of $\varepsilon_{i,t}$ by using the MLE method, and use it as the initial value $\hat{\Theta}_{k,0}$ of the dynamic parameter to be estimated.
Step 4. Define the function $\Lambda(\cdot)$ based on Eq. 11. Set the initial value of the first three parameters of the ARMA process

$$\omega_0 = \Lambda^{-1}(\hat{\Theta}_{k,0})$$
$$\beta_0 = 0$$
$$\alpha_0 = 0$$

Step 5. Set the value of the window length $m = 1, 2, \ldots$
Step 6. Define the log-likelihood function of each copula and estimates $\{\hat{\omega}, \hat{\beta}, \hat{\alpha}, \hat{m}\}$ to obtain $\hat{\Theta}_{k,t}$ (Eq. 15).
Step 7. Select the optimal dynamic parameter estimates, which have the smallest AIC value.
Step 8. Obtain the dynamic dependence of the best time-varying copula.

Algorithm 2 Forecasting the exchange rates data.

Step 9. Generate a joint vector of the residuals $\{\hat{\varepsilon}_{i,t}, \hat{\varepsilon}_{j,t}\}$, $i \neq j$ using the best time-varying copula.
Step 10. Obtain the new series of $\hat{X}_{i,t}$ by adding the copula-based residuals $\hat{\varepsilon}_{i,t}$

$$\hat{R}_{i,t} = \hat{\mu}_i + \sum_{j=1}^{p} \hat{\varphi}_{ij} R_{i,t-j} + \sum_{k=1}^{q} \hat{\theta}_{ik}\, \varepsilon_{i,t-k} + \hat{\varepsilon}_{i,t}, \quad i = 1, \ldots, 5, \ t = 1, \ldots, T$$

Step 11. Calculate the MAPEs between the actual and the forecasted data.

Data

The data used in this research consist of the daily exchange rates of five Asian countries that have the strongest economies, namely CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD, before and during the 2020 pandemic, obtained from Yahoo Finance. The sample data cover the period between January 1, 2018, and July 15, 2020. The exchange rate data for all five countries tend to have a trend

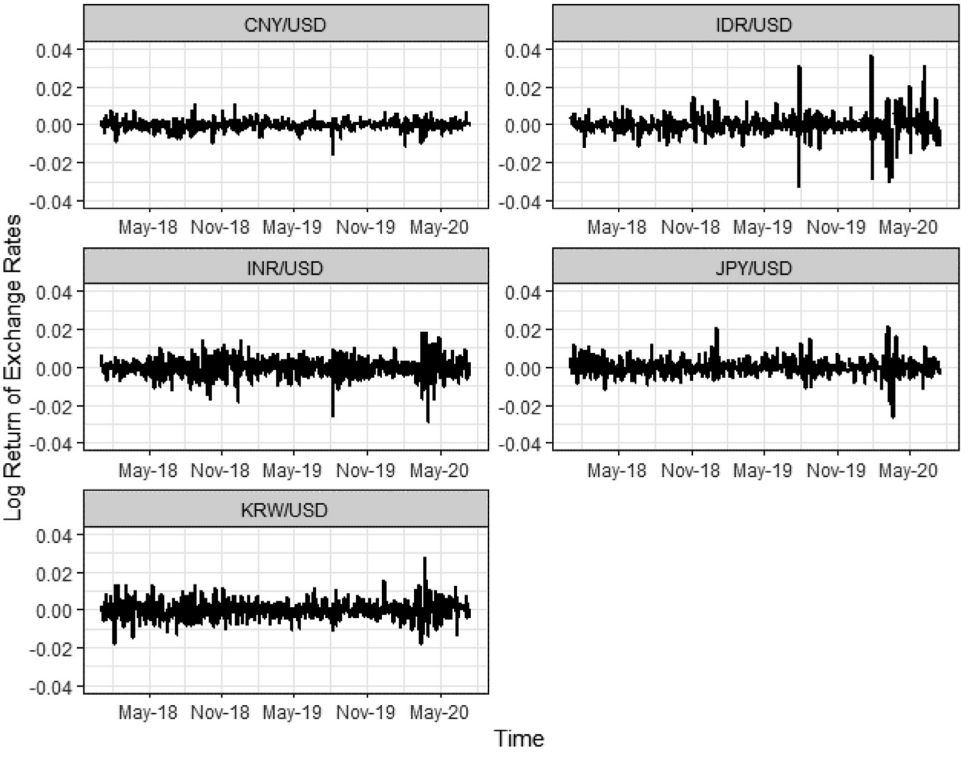

Fig. 1. Log Return of Exchange Rates Data. This figure depicts the log return series of the exchange rates data which is calculated by $R_t = \ln(\frac{X_{i,t}}{X_{i,t-1}})$. There are some volatility clusters of the log return of exchange rates, especially in pandemic period between December 2019 and July 2020, which indicates the possibility of interdependence between these exchange rates.

and are non-linear. This presumption is supported by the ACF and PACF of the exchange rates and the stationarity testing by Augmented-Dickey Fuller (ADF) presented in Appendix (see Table A.1). The ADF test results in no rejection of the unit root for all the exchange rates, which means that the exchange rates data are non-stationary. Therefore, the log transformation is computed so as to smooth out the data and the differencing process, thus stabilizing the series' variance. The exchange rates data are then modelled using a hybrid ARIMA-GARCH model. Fig. 1 shows the log return of the exchange rates.

Fig. 1 shows that despite being stationary, some volatility clusters of the log return of exchange rates exist, particularly in the pandemic period between December 2019 and July 2020. It shows a time effect on the variance of the log return of the exchange rates. The tendency of significant fluctuations in pandemic periods in almost all countries' exchange rates also indicates the possibility of interdependence between these exchange rates, particularly against the CNY/USD as China is the country with the strongest economy in Asia and it was the epicenter of pandemic. Table 1 shows the summary statistics of the log return of the exchange rates. The skewness and kurtosis values for all series indicate that the data do not exhibit normality assumption. The non-zero skewness shows that the data is asymmetrical, even though it is not prominent.

Furthermore, the kurtosis values greater than zero indicate that the log return data is heavy-tailed, with the order of tail thickness being IDR/USD, JPY/USD, CNY/USD, INR/USD, and KRW/USD. Therefore, a normality test using a Jarque-Bera (J-B) test was performed. Table 1 shows that the J-B test rejects the normality assumption because the J-B statistics are higher than the chi-square value with the degree of freedom of 2 at the 0.05 significance level. The last two rows of Table 1 provide the empirical linear correlation and Kendall's tau of the log return between CNY/USD and other currencies.

Table 1
Summary Statistics of the Log Return of the Exchange Rates.

Statistics	CNY/USD	IDR/USD	INR/USD	JPY/USD	KRW/USD
Mean	-1.097 $\times$ 10^{-4}	-1.116 $\times$ 10^{-4}	-2.548 $\times$ 10^{-4}	7.438 $\times$ 10^{-5}	-1.829 $\times$ 10^{-4}
Standard Deviation	2.784 $\times$ 10^{-3}	5.929 $\times$ 10^{-3}	5.368 $\times$ 10^{-3}	4.561 $\times$ 10^{-3}	5.033 $\times$ 10^{-3}
Skewness	-0.440	-0.480	-0.360	0.140	0.230
Kurtosis	3.330	12.290	2.480	4.240	1.920
J-B Test ($p-value$)	327.010 (0.000)	4198.400 (0.000)	184.150 (0.000)	498.600 (0.000)	107.690 (0.000)
Linear Correlation	-	0.083	0.267	0.063	0.472
Kendall's Tau	-	0.058	0.179	0.019	0.320

This table provides the descriptive statistics of the log return of the exchange rates of CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD. The descriptive measures include Mean, Standard Deviation, Skewness, Kurtosis, Jarque-Bera (J-B) Test for Normality, Empirical Linear Correlation, and Kendall's Tau between CNY/USD and the other four exchange rates.

The results indicate that the countries with the most dependence on China's exchange rates are South Korea and India.

Results and Analysis

According to the summary statistics, the appropriate time series model for modeling the exchange rates' log return is $\mathrm{ARIMA(p, d, q)}$ - $\mathrm{GARCH(r, s)}$ model. Table 2 provides the order of the model and the parameter estimates of the $\mathrm{ARIMA(p, d, q)}$ - $\mathrm{GARCH(r, s)}$ and the residuals modeling.

Due to the presence of the autocorrelation between residuals, a dynamic dependence appears to exist. Some literature discussed the dynamic dependence modeling, explaining that it could be identified by engaging time variation with its parameters (Hafner & Reznikova, 2010; Manner & Reznikova, 2012; Patton, 2006). In the earlier studies, there have been limitations relating to the dynamic parameters of the time-varying copula. Therefore, it is estimated the possibility of the dynamic dependence among the exchange rates using our extended parameter functions is provided in Eq. 10. First, the static copula's parameter $\hat{\Theta} = \{\hat{\rho}, \hat{\theta}^{Cl}, \hat{\theta}^{Gb}, (\hat{\lambda}_U, \hat{\lambda}_L)\}$ is evaluated, then, following procedures in Algorithm 1, the dynamic parameter of time-varying copulas is estimated. Table 3 shows the parameter estimates of the time-varying copulas with four extended dynamic parameters that fit the bivariate residuals of the log return of the exchange rates along with the static parameter estimates used as a comparison for each pair of exchange rates data.

The results in bold show the estimation results with the smallest AIC value for each copula in each residual pair. Meanwhile, the addition of asterisks shows the estimation results with the smallest AIC value among all copula in each residual pair. The results show that, of all the copulas used, the fourth dynamic parameter function gives the best estimation result because it has the smallest AIC value. It indicates that, of the four extended dynamic parameter functions, a function that uses the concordance-discordance property can best describe the dynamic dependence between exchange rates. Furthermore, among all the copulas, the best-fitting time-varying copula for all residual pairs of exchange rates is the time-varying Normal copula with the fourth extended dynamic parameter. The estimation results also show that, the shorter the length of the past observations involved is, the higher the influence on the dynamic parameters' calculation is. Most of the time-varying copula with the smallest AIC value on almost all residual pairs gives a window length of 1, except for time-varying SJC for the pairs of CNY/USD - INR/USD and CNY/USD - KRW/USD. The window length that gives the best effect is from the two previous observations for the two pairs. However, it shows that the best window length involved in calculating the dynamic parameter is the short period of past observations. The comparison between the static and time-varying Normal copula for all residual pairs is presented in Fig. 2.

The black line represents the time-varying copula's dynamic parameter, while the red line represents the static copula parameter. For the CNY/USD – IDR/USD pair, the static parameter has a value of 0.1180, which is a small value of the relationship. However, the dynamic dependence shows an intense relationship at some point in time. We can see no significant changes in the dynamic dependence fluctuation before and during the 2020 pandemic. The dependence differs

Table 2
ARIMA-GARCH Model of Asian Countries' Exchange Rates.

Parameter Estimates	CNY/USD	IDR/USD	INR/USD	JPY/USD	KRW/USD
AR(1), $\hat{\phi}_1$	0.844 (0.064)	0.673 (0.085)	-	0.295 (0.061)	-0.374 (0.041)
AR(2), $\hat{\phi}_2$	0.112 (0.039)	0.171 (0.039)	-	-0.905 (0.072)	-0.923 (0.046)
MA(1), $\hat{\theta}_1$	-0.933 (0.052)	-0.776 (0.079)	-0.255 (0.038)	-0.317 (0.045)	0.330 (0.030)
MA(2), $\hat{\theta}_1$	-	-	-	0.945 (0.059)	0.950 (0.045)
GARCH constant, $\hat{\alpha}_0$	6.374×10^{-6} (2.926×10^{-7})	9.711×10^{-6} (1.272×10^{-6})	1.500×10^{-6} (3.249×10^{-7})	2.398×10^{-6} (6.247×10^{-7})	8.693×10^{-7} (4.962×10^{-7})
Lag 1 ε_{t-1}, $\hat{\alpha}_1$	0.122 (0.025)	0.397 (0.035)	0.145 (0.037)	0.215 (0.053)	0.050 (0.015)
Lag 1 σ^2_{t-1}, $\hat{\beta}_1$	-	0.315 (0.068)	0.806 (0.038)	0.698 (0.059)	0.914 (0.032)
Goodness of fit for residuals, *AIC*					
Normal	-5927.806	-4935.298	-5088.898	-5927.806	-5145.002
Student-*t*	1221.327	1221.345	1221.341	1221.327	1221.339
$\hat{\mu}$	-6.990×10^{-5}	-1.010×10^{-4}	-3.460×10^{-4}	6.590×10^{-5}	-1.950×10^{-4}
$\hat{\sigma}$	2.760×10^{-3}	5.830×10^{-3}	5.190×10^{-3}	4.530×10^{-3}	4.980×10^{-3}

This table shows the parameter estimates of the ARIMA-GARCH model for the log return of each exchange rate data. The models are ARIMA(2,1,1) - GARCH(1,0), ARIMA(2,1,1) - GARCH(1,1), ARIMA(0,1,1) - GARCH(1,1), ARIMA(2,1,2) - GARCH(1,1), and ARIMA(2,1,2) - GARCH(1,1) for CNY/USD, IDR/USD, INR/USD, JPY/USD, and KRW/USD, respectively. In addition, the goodness of fit for residuals of the log return of the exchange rates show that the residuals follow Normal distribution.

Table 3
Parameter Estimates of Time-Varying Copula Models with Extended Parameter Functions.

Bivariate Variables	Copula (Static Parameter Estimates)	Dynamic Parameters					
		k	$\hat{\omega}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{m}$	AIC
CNY/USD	Normal	1	1.711	−1.895	−4.029	8	−16.566
-	0.118	2	0.507	−1.445	−0.584	1	−13.281
IDR/USD		3	0.617	−1.535	−0.651	1	−16.309
		4	**0.348**	**−0.348**	**0.225**	**1**	**−86.843***
	Clayton	1	−0.708	−4.392	−5.000	1	−12.213
	0.114	2	−1.251	−3.179	−5.000	1	−10.603
		3	−0.708	−4.392	−5.000	1	−12.213
		4	**−1.386**	**−1.972**	**0.191**	**1**	**−23.794**
	Gumbel	1	4.183	−5.000	−2.897	1	−18.867
	1.100	2	−4.974	1.959	3.709	2	−21.161
		3	4.183	−5.000	−2.897	1	−18.867
		4	**0.369**	**−2.137**	**0.283**	**1**	**−48.124**
	SJC	1	(-0.419,-1.707)	(-4.716,-15.251)	(-24.999,-25.000)	1	−21.875
	(0.032,0.003)	2	(-0.703,-2.803)	(-10.959,-24.999)	(-24.998,-24.987)	1	−22.165
		3	(-0.419,-1.707)	(-4.716,-15.252)	(-24.999,-25.000)	1	−21.875
		4	**(-1.931,-4.972)**	**(0.409,0.341)**	**(-3.383,-19.364)**	**1**	**−35.766**
CNY/USD	Normal	1	2.063	−2.001	−3.336	7	−59.494
-	0.262	2	1.330	−1.980	−1.735	3	−56.633
INR/USD		3	2.016	−1.967	−2.626	7	−56.879
		4	**0.497**	**−0.215**	**0.141**	**1**	**−103.096***
	Clayton	1	0.461	−0.122	−5.000	15	−55.371
	0.361	2	0.197	−2.847	−2.050	7	−58.465
		3	−0.659	0.990	−2.037	15	−54.775
		4	**−1.049**	**−0.285**	**0.214**	**1**	**−84.024**
	Gumbel	1	0.145	−0.431	−5.000	11	−38.574
	0.168	2	−3.889	1.995	−5.000	3	−40.289
		3	−2.361	0.975	−3.842	3	−40.799
		4	**−1.567**	**−0.193**	**0.108**	**1**	**−48.889**
	SJC	1	(3.411,-1.885)	(-24.999,-1.773)	(-2.839,4.526)	13	−63.649
	(0.018,0.183)	2	(-1.097,-0.035)	(-24.394,-2.560)	(3.303,-8.173)	5	−67.231
		3	(4.215,0.662)	(-24.653,-2.854)	(-3.474,-7.261)	9	−68.139
		4	**(-6.996,-0.457)**	**(0.106,0.509)**	**(-0.077,-5.526)**	**2**	**−79.514**

(*continued on next page*)

Table 3 (*continued*)

Bivariate Variables	Copula (Static Parameter Estimates)	Dynamic Parameters					
		k	$\hat{\omega}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{m}$	AIC
CNY/USD	Normal	1	−0.057	2.049	0.176	15	−20.166
-	0.046	2	−0.024	2.026	0.152	15	−8.596
JPY/USD		3	−0.069	2.059	0.172	13	−17.954
		4	**0.029**	**−0.325**	**0.203**	**1**	**−85.191***
	Clayton	1	0.044	−5.000	−5.000	4	−7.213
	0.075	2	−2.155	3.074	−5.000	1	−6.317
		3	−1.594	2.221	−4.806	1	−6.892
		4	**−1.872**	**−0.457**	**0.311**	**1**	**−38.805**
	Gumbel	1	−5.000	3.015	−5.000	3	−4.908
	1.100	2	2.371	−5.000	−5.000	4	−4.998
		3	2.855	−4.999	−4.999	6	−5.782
		4	**−1.983**	**−0.665**	**0.239**	**1**	**−26.190**
	SJC	1	(-19.225,2.073)	(-2.557,-16.643)	(0.003,-14.575)	4	−10.870
	($5.045 \cdot 10^{-6}$,0.011)	2	(-21.555,-13.721)	(-1.978,24.973)	(-0.006,7.865)	10	−10.652
		3	(-18.950,4.372)	(-2.055,-25.000)	(-0.018,-4.727)	6	−10.707
		4	**(-5.866,-2.491)**	**(0.441,0.412)**	**(7.227,-2.812)**	**1**	**−29.275**
CNY/USD	Normal	1	3.143	−2.576	−3.782	12	−177.208
-	0.477	2	0.584	1.254	−1.393	4	−176.936
KRW/USD		3	0.505	1.532	−0.663	3	−176.578
		4	**1.677**	**−1.587**	**0.167**	**1**	**−235.017***
	Clayton	1	−0.456	−0.261	1.195	1	−143.159
	0.686	2	−0.449	−0.098	1.682	1	−143.718
		3	−0.067	−1.133	1.956	2	−143.289
		4	**0.068**	**−0.843**	**0.159**	**1**	**−173.521**
	Gumbel	1	2.092	−1.225	−4.952	5	−182.143
	1.450	2	−2.361	1.151	−4.576	3	−178.483
		3	−1.912	0.920	−2.158	4	−178.705
		4	**0.711**	**−1.111**	**0.115**	**1**	**−204.825**
	SJC	1	(-0.614,-1.329)	(-3.549,6.102)	(1.820,-3.984)	2	−199.046
	(0.310,0.294)	2	(-1.016,-0.664)	(-5.7696,6.796)	(2.153,-3.541)	2	−200.185
		3	(-0.617,-1.471)	(-3.083,4.823)	(1.885,-3.265)	2	−198.387
		4	**(-0.138,0.501)**	**(-0.324,0.321)**	**(-5.686,-5.044)**	**2**	**−212.747**

This table provides parameter estimates of time-varying copula models with extended parameter functions along with the estimates for static copula as a comparison. For each pair of residual of the log return of exchange rates, the optimal parameters of time-varying copula obtained are from the extended parameter function 4 which elaborate the concordance-discordance like property. Furthermore, among all type of time-varying copulas, the best copula selected are the time-varying Normal copula which has the smallest AIC values.

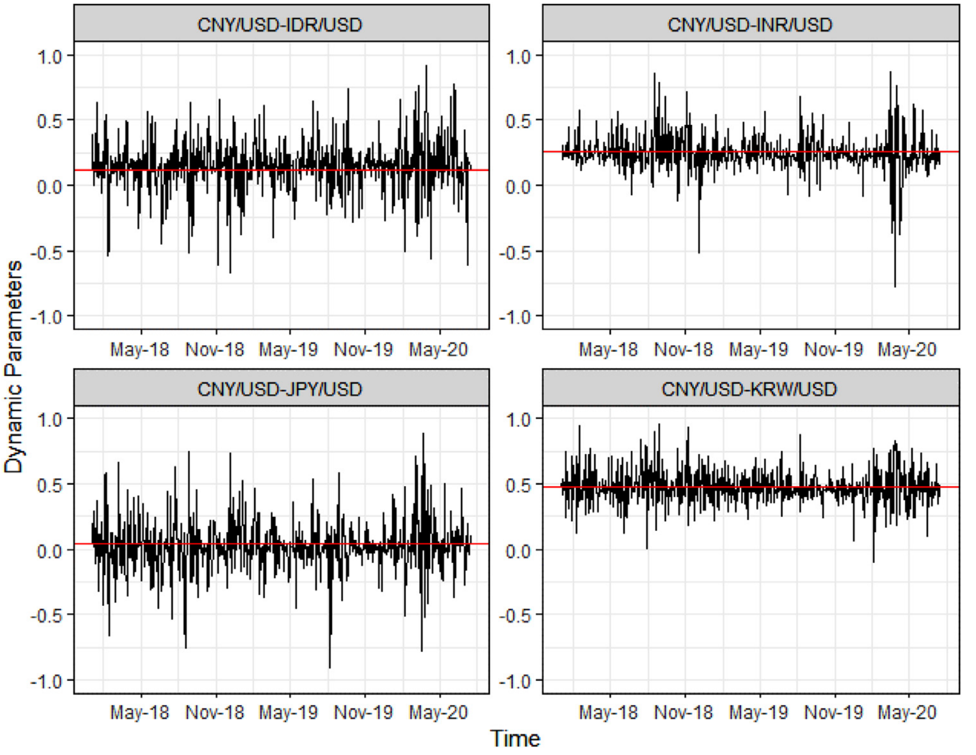

Fig. 2. Parameter Estimates of the Selected Static and Time-Varying Copula. This figure depicts the comparison between the parameter estimates of the static and the best time-varying copula for each pair of residuals of the log return of exchange rates. Each figure show that the parameter estimates fluctuate throughout the observations which means that the dependence between each pair of residuals is time-dependent. Furthermore, there are some significant changes in the dynamic dependence fluctuation during the 2020 pandemic (around after December 2019), especially for the pairs of CNY/USD – INR/USD.

almost constantly around the interval of [-0.7,0.7]. The positive (negative) value of dependence means that the IDR/USD moves together (against) with the CNY/USD. However, the highest value of dependence happened on March 30, 2020, with a value of 0.915, shortly after the first case in Indonesia on March 2, 2020, was announced. On that date, both exchange rates depreciated against the US dollar. The dependence fluctuation between CNY/USD and IDR/USD, which did not experience a significant difference between the pre-pandemic and the pandemic period, implies that the 2020 pandemic does not substantially affect the Indonesian exchange rate concerning China's exchange rate. The dynamic dependence between the two exchange rates is in a relatively substantial range of relationship, indicating that the economic dynamics in Indonesia are entirely related to those in China.

For the CNY/USD – INR/USD pair, the dynamic dependence between the two exchange rates had insufficient interdependence before the pandemic. However, at one point, it has shown a reasonably substantial change of dependence. An exceptionally significant drop occurred in the pre-pandemic period on December 4, 2018, with the dependence value of -0.509. It happened when CNY/USD appreciated against the US dollar, while INR/USD depreciated against the US dollar. There are significant changes in the dependence during the pandemic period, which predominantly happened in March 2020. The most significant positive and negative dependence occurred on March 16 and 26, 2020, with a value of 0.868 and -0.773, showing evidence of these changes. On March 16, 2020, both the CNY/USD and INR/USD exchange rates experienced increased purchasing power against the US

Table 4
Accuracy Measure of the Model.

Exchange Rates	MAPE (%)	
	In Sample	Out Sample
CNY/USD	0.194	2.972
IDR/USD	0.361	1.758
INR/USD	0.392	1.895
JPY/USD	0.324	1.938
KRW/USD	0.381	4.459

This table provides the value of the Mean Absolute Percentage Error (MAPE) for the in-sample and out-sample data as the accuracy measure of the model. The results show that the prediction of the exchange rate using ARIMA-GARCH and time-varying Normal copula with the concordance-discordance-like extended parameter function gives a good estimation result because the value of the MAPEs are quite small, i.e. less than 5%.

dollar. Meanwhile, on March 26, 2020, CNY/USD depreciated significantly, while INR/USD appreciated against the US dollar.

For the CNY/USD – JPY/USD pair, almost similar to China – Indonesia dependency, there are no significant dynamic dependence changes before and during the pandemic. However, the interval of dependency between China and Japan exchange rates is in [-0.5,0.5], which is weaker than the dependency between China and Indonesia, although there is quite a strong dependency at some point in time. A significant drop of dependency occurred in the pre-pandemic period on August 7, 2019, where the CNY/USD experienced a considerable appreciation, while the JPY/USD experienced a significant depreciation. The cluster of dependence changes also appears to emerge around March 2020. There was a substantial decrease on March 16, 2020, followed by the highest increase in dependence on March 23, 2020. On March 16, 2020, the CNY/USD appreciated against the US dollar, while JPY/USD depreciated against the US dollar. On March 23, 2020, the highest increase in dependence with the value of 0.886 occurred when both CNY/USD and JPY/USD were appreciated against the US dollar.

For the CNY/USD – KRW/USD pair, there have been no significant changes in the dynamic dependence, except that it had converged around the value of 0.5 just before the pandemic. Yet, it again experienced dynamics after entering the pandemic with the same movement as before the pandemic. The dependence in the exchange rates between China and South Korea is quite strong. It is almost always in a positive relationship, as it ranges at [0.25,0.9] both before and during the pandemic. However, a significant drop in dependence occurred on February 4, 2020, with the value of -0.102, when the CNY/USD depreciated against the US dollar, while KRW/USD appreciated against the US dollar. This significant change came shortly after the first case in January 2020 was announced in South Korea.

Using the dynamic parameter estimates obtained from the last step in Algorithm 1 and following the steps in Algorithm 2, the new series of the exchange rates for all countries is built using the optimal time-varying copula obtained from the previous modeling. Fig. 3 shows the lower and upper prediction of the in-sample data at a 95% confidence interval based on the best time-varying copula with the best-extended parameter function.

The blue and the red lines present the upper and lower prediction of the in-sample data. From the figure, it can be seen that the time-varying copula with the extended parameter gives a relatively good forecast for the exchange rate data. Furthermore, Table 4 provides the model accuracy of the in-sample and out-sample data (from July 16, 2020, to December 25, 2020). The values show that the ARIMA-GARCH-time-varying Normal copula with the concordance-discordance-like extended function provides a good estimation result to the dynamic dependency of the Asian countries' exchange rates.

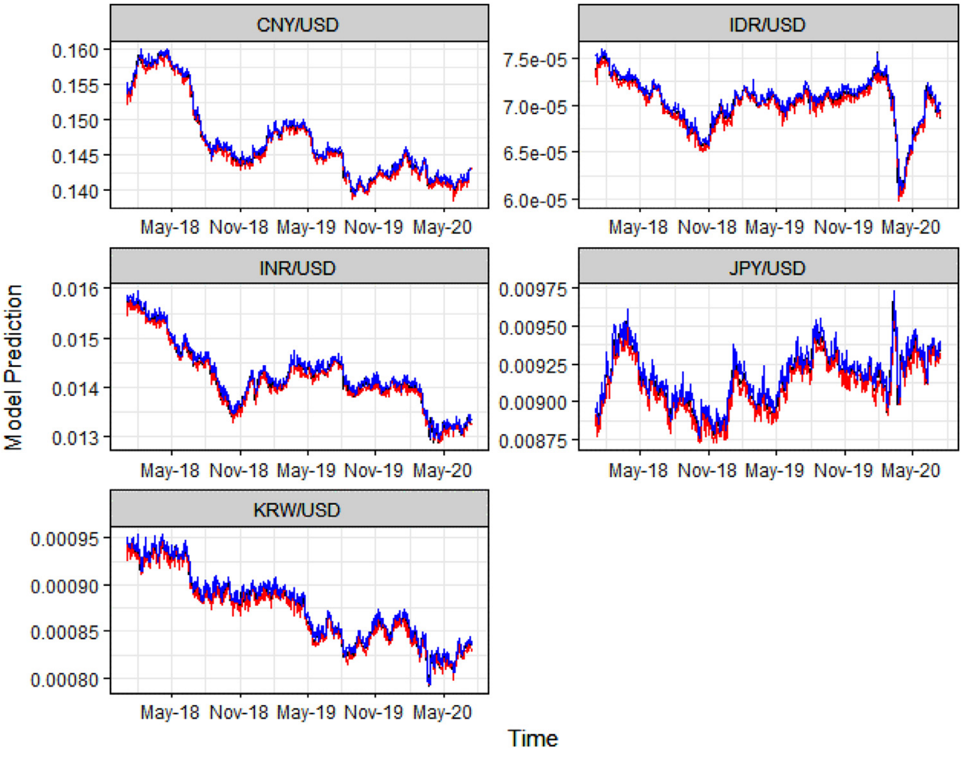

Fig. 3. Model Prediction. This figure shows the upper and lower prediction of the in-sample data between January 1, 2018, and July 15, 2020. The figure shows that, in general, the time-varying Normal copula with the extended parameter function 4 gives a relatively good forecast for all the exchange rates data.

Conclusion

There are three main contributions in this paper. First, four extended functions of the time-varying copula's dynamic parameter are proposed, which elaborate the positive-definite function of the distance between the cumulative probabilities of the marginal variables and the concordance-discordance-like property of the residuals. Second, from the empirical application, the dynamic dependency between China's and the other countries' exchange rates before and during the 2020 pandemic is evidenced by employing the ARIMA-GARCH-time-varying copula with the extended parameter functions. Lastly, the concordance-discordance-like function of the extended dynamic parameter is found to provide the optimal estimation that allows to identify the dynamic dependency among the exchange rates for all types of the time-varying copula. It is also proven that the shorter the length of past observations is, the stronger the dependence structure between marginal variables is, which also influences the calculation of the dynamic dependence.

The dynamic dependency between China's and the other countries' exchange rates before and during the 2020 pandemic is identified for the empirical application. Each state has different interdependence with China's exchange rate. Generally, the significant changes in the interdependence mostly occurred shortly after the Covid-19 pandemic broke out worldwide, i.e., around February and March 2020. India is the country whose exchange rate has been most strongly affected by the 2020 pandemic. It is evidenced by a significant drop in dependence with a reasonably strong dependency value, compared to the other countries' exchange rate. Overall, similar to the previously mentioned studies, it can be concluded that the Covid-19 pandemic influences the dynamic dependency between exchange rates.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Table A.1
Augmented-Dickey Fuller (ADF) Test for Stationarity.

Exchange Rates	D-F Statistics	$p - value$
CNY/USD	−1.590	0.752
IDR/USD	−3.044	0.137
INR/USD	−2.170	0.506
JPY/USD	−2.783	0.247
KRW/USD	−2.922	0.188

This table provide the Dickey-Fuller (D-F) Statistics and p-value of the ADF test for stationarity which result in no rejection of the unit root for all the exchange rates and means that the exchange rates data are non-stationary.

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