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An optimal control model for Covid-19 spread with impacts of vaccination and facemask

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ABSTRACT

A non-linear system of differential equations was used to explain the spread of the COVID-19 virus and a SEIQR model was developed and tested to provide insights into the spread of the pandemic. This article, which is related to the aforementioned work as well as other work covering variations of SIR models, Hermite Wavelets Transform, and also the Generalized Compartmental COVID-19 model, we develop a mathematical control model and apply it to represent optimal vaccination strategy against COVID-19 using Pontryagin's Maximum Principle and also factoring in the effect of facemasks on the spread of the virus. As background work, we analyze the mathematical epidemiology model with the facemask effect on both reproduction number and stability, we also analyze the difference between confirmed COVID-19 cases of the Quarantine class and anonymous cases of the Infectious class that is expected to recover. We also apply control theory to mine insights for effective virus spread prevention strategies. Our models are validated using Matlab mathematical model validation tools. Statistical tests against data from Jordan are used to validate our work including the modeling of the relation between the facemask effect and COVID-19 spread. Furthermore, the relation between control measure ξ, cost, and Infected cases is also studied.

1. Introduction

There are many parameters that have a bearing on the effectiveness of epidemiologic models pertaining to COVID-19 vaccine distribution and allocation strategies; these include cost, the risk level of target groups, age, and geographic scope to name but a few. [1] identified epidemiologic factors similar to the aforementioned ones in combination with active "public health policies" as well as the likely population behavior stemming from such policies as significant factors. The authors also introduced "optimization criteria" in outcome-based approaches stipulating that priorities of the allocation policies are driven by desired outcomes of those policies, for example when targeted vaccinations are applied with measures that reduce contact in specific subpopulations. One of the contentious issues that have arisen in the wake of vaccination discovery and development is the ethical distribution of vaccines, especially to developing nations that can afford neither to procure nor remanufacture vaccines and struggle with logistical details at the best of times. [2] assert that while the global community and the developed nations, generally, have committed to fair and ethical provision strategies of vaccines, this does not detract from inescapable logistical challenges, for example how to provide the "means by which a successful vaccine can be mass produced and equitably distributed", with developing nations resources stretched

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Table 1
Fully vaccinated persons and starting date of vaccination [7].

Country	Persons fully vaccinated	Starting date of vaccination
United States of America	225,043,793	14 - Dec - 2020
Brazil	167,777,151	17 - Jan - 2021
Algeria	6,481,186	30 - Jan - 2021
China	1,277,335,433	22 - Jul - 2020

to breaking point as a result of the pandemic. The authors considered four "allocation paradigms" including: i) procurement ability, ii) ability to implement, iii) reciprocity wherein developing nations help (for example with the provision of viral samples) with the development of vaccines for little or no return on investment, e.g., Indonesia and the H5N1 vaccine as well as iv) justice in distribution; they used these factors in order to develop an effective allocation model for COVID-19. The issue of prioritizing vaccines and treatment in a pandemic is not exclusive to VOCID-19; indeed, this has been the subject of many studies, strategies, and models aiming to guide authorities in response to catastrophic public health threats, for example as in the 1918 influenza pandemic. [3] highlighted the "critical" role played by awareness of the public perspective and "moral points" when ethically weighing strategies for allocating life-saving mechanical ventilators and medications. Starting with a campaign of public awareness in Maryland, USA, in combination with a score-based system for special considerations, the authors provide a blueprint for the allocation of critical care resources to be utilized by healthcare agencies and organizations in response to public health pandemics wherein demand far exceeds allocated and available resources.

Similar work has been done by the "Committee on Equitable Allocation of Vaccines for the Novel Coronavirus, Board on Health Sciences Policy, USA" to create a framework of basic principles that guide authorities in their recommended coronavirus disease 2019 (COVID-19) vaccine allocation action plans. The committee document the "goal of this framework, the risk-based allocation criteria used to apply the principles, and the resulting allocation phases". They also include an "in-depth description and discussion of the phases, including the rationale behind the inclusion of population groups listed in each phase" [4]. The World Health Organization (WHO) has also initiated a global plan for the fair provision of COVID-19 vaccines, and although nations representing the majority of the world's population joined this plan, the challenge remains to entice China and the USA to join the "COVID-19 Vaccines Global Access (COVAX)" Facility [5].

Regardless of frameworks, plans, or strategies aimed at making the provision of vaccines, hospital beds, or ventilators more accessible, classic prioritization pitfalls remain a factor, for example, age or disability discrimination, as illustrated by [6] who analyzed care provision in South Africa during the pandemic and assert that "the score-based categorical exclusions used in critical care triage guidelines disproportionately discriminate against older adults, the cognitively and physically impaired, and the disabled" especially as adults over the age of 60, who coincidentally constitute around 10% of the population in South Africa, are more likely than others to be classified with comorbidities or disabilities in triage. In summary, public health disasters create sustained challenges to governments and authorities long after the development of vaccines or treatments, these challenges include the search for a "fair/optimum" distribution mechanism for services, vaccines, or facilities and constitute a fertile ground for research and development. Table 1 shows the numbers and dates of the fully vaccinated cases. In this work, we are incorporating the impact of vaccinations on the spread rate of the virus and applying ξ as a control measure that confirms the positive impact of an increase in vaccinations on the proposed model.

1.1. Related work

There is a good amount of scholarly work on optimum allocation and distribution strategies of vaccines for COVID-19 and other epidemics. [8] developed a novel "Mixed Integer Linear Programming Model" for the (cost) optimization of vaccine supplies from a supply chain point of view and also of vaccinations from a daily service point of view. Their model parameters include logistic temperature control requirements for storage, timeline deadlines as well as limited mRNA natural shelf life. The kinds of costs their work considered include staffing and logistic costs, overheads, and waste (for example from unused vaccines). Using a combination of Divide-And-Conquer and Re-Aggregation strategies, the authors demonstrated the viability of this approach against a simulated version of the vaccination program in Greece during the 2020-21 pandemic peaks in Europe. [9] presented a study incorporating a dual-mode variation of a SIR model covering both fatality probability within a set period of time as well as the ability to model the impact of mitigation (lockdown) procedures against the spread of infections. The variation model includes novel estimation methods for the parameters corresponding to mortality rates, reproduction rates, and the time-span of the pandemic. The authors found accurate comparative results between modeled fatalities and case studies of COVID-19 in Portugal in the second quarter of 2020. In [10], the authors used Hermite wavelets transform to develop a "functional integration matrix" based method to study the digestive system via a coupled system of ODEs. For this model, various parameters were covered including eating habits, stress, and tension, sleep cycles, and medications. The authors also studied COVID-19 using fractional ODEs with variables modeling the five SEIQR states from Susceptible to Recovered. Results comparing their work with numerical methods including Runge–Kutta reveal favorable efficiency gains. The work by [10] modeling the five SEIQR states is similar to previous work [11] in which we introduced a susceptible-exposed quarantined-recovered (SEIQRS) mathematical model for COVID-19 spread prediction using a linear system of differential equations catering to a broad set of scenarios. This type of model has proven applicability in several known physics phenomena of a symmetrical nature including in fluid dynamics, bio modeling, and mathematical biology. The model's efficacy was

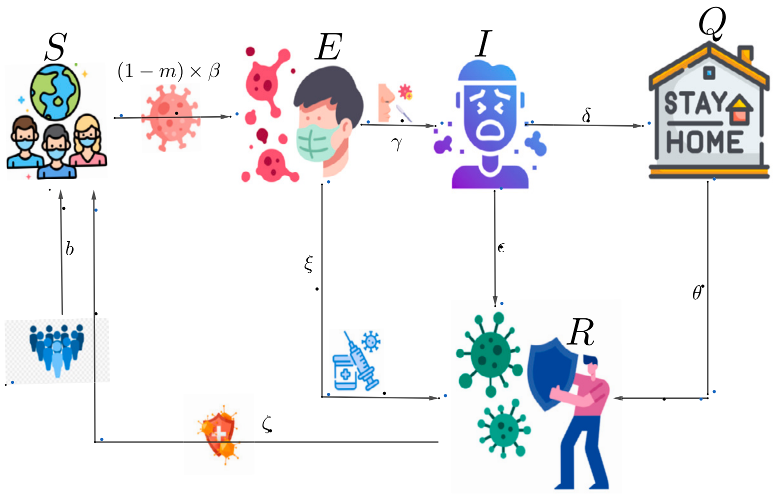


Fig. 1. Model compartments links.

tested against Jordanian Ministry of Health data pertaining to the period from March to September 2020 with at least two peaks in the number of infected cases. Simulations showed optimistic consistency between our model and actual numbers of cases with special provision for modeling COVID-19 factors including asymptomatic carriers, and recurrent infections in conjunction with the effects of the aforementioned peaks of infection. The SEIQRS model included parameters as follows: $N(t)$ for a number of nodes, this population was then divided as Susceptible nodes $S(t)$, Exposed (Infected but not yet Infectious) nodes E(t), Infectious nodes $I(t)$, Quarantined nodes $Q(t)$ and Recovered nodes $R(t)$. Additionally, the virus incubation period and disease immunity rate were modeled with constant rate parameters. Our work in [11] is also similar to the system of time-fractional equations proposed by [12] to model the complexity and non-locality of COVID-19. To resolve issues of efficient solutions to these equations the authors proposed a "generalized compartmental COVID-19 model" catering to parameters of transition and contact rates, quarantine effects, natural deaths vs disease fatalities as well as natural recovery vs. recovery from quarantine rates. Thorough analysis and detailed study of the model using Adam–Bashforth–Moulton methods allow the authors to predict 3^{rd} and 4^{th} wave case spikes as has been confirmed since the writing of the article. In [11] we highlighted the positive impact surgical masks and N95 respirator type masks have against clinical and respiratory virus infection when used stringently by the general population [13], [14] with mask effect ratings of 70% reduction of cases when applied in public places [15]. In continuation of the aforementioned SEIQRS model development and testing, this article introduces a Control Model for the Optimal Distribution of COVID-19 Vaccines including contingency for Face-mask impact on the reduction of spread rates in communities. In the proposed model, the Susceptible population (S) is increased by the recruitment of individuals into the population, at a rate of b. The population also decreases as the number of Infected cases is increased by a rate of βSI cases. Recovered cases losing their immunity are likely to be suspicious after an average of $1/\zeta$. All compartments' populations decrease by a natural death rate of d. The number of COVID-19 Infected cases increases by rate μ. The Exposed cases are likely to become infected after an average of $1/\gamma$. The typical Recovery rate for hospitalized individuals is $1/\epsilon$ days on average. The mean duration for Quarantined cases to Recover is $1/\theta$ days. The mean duration of Quarantine is $1/\delta$.

$$\begin{aligned}
\dot{S} &= b - \beta SI + \zeta R - dS, \\
\dot{E} &= \beta SI - (\gamma + \xi + d)E, \\
\dot{I} &= \gamma E - (\epsilon + \delta + \mu)I - dI, \\
\dot{Q} &= -(\theta + \mu)Q + \delta I - dQ, \\
\dot{R} &= \xi E + \epsilon I + \theta Q - (\zeta + d)R,
\end{aligned} \tag{1}$$

on $D = \{(S, E, I, Q, R) \in (\mathbb{R}_0^+)^5 : S + E + I + Q + R \leq \frac{b}{d}\}$ positively-invariant and closed. Fig. 1 illustrates the proposed model including interactions between the model's compartments; the model's variables are explained in Table 2, below.

In this article, we also introduce a function to describe the economic-financial losses attributed to the COVID-19 outbreak. [16] highlighted the strategic value of reliable cost/resource data in enabling decision-making in the management of healthcare crises; the authors studied Inpatient costs in US hospitals for COVID-19 patients using hospital data about admissions sourced from the US Premier Healthcare Database between April and December 2020 to estimate costs relating to Length of Stay, Status on Discharge and Overall Costs. After surveying nearly 250 thousand patients of whom nearly 50% were female and 75% were aged over 50 and around 35% had to be admitted to ICU, Tables 3, and 4 below summarize the data analysis in terms of costs per patient.

The figures in the Table 4 are also corroborated by an article published in November 2020 [17] in which typical costs for a COVID-19 patient range from \$51,000.00 to \$78,000.00 depending on patient age and consequently, length of stay; these figures represent analyses of the period from February to August of the same year. Other sources of cost analyses include [18] in which the

Table 2
Description of variables and parameters of the model (1).

Variable	Description
$S(t)$	Population of susceptible individuals
$E(t)$	Population of exposed individuals
$I(t)$	Population of infected individuals
$Q(t)$	Population of quarantined individuals
$R(t)$	Population of recovered individuals
Parameter	**Description**
b	Recruitment rate
d	Natural death rate
β	Contact rate
ζ	is the loss of immunity rate
μ	Disease-induced death rate of infectious individuals
γ^{-1}	The mean time of incubation period
ϵ^{-1}	The mean time from infected to recovered
ξ^{-1}	the cure rate by vaccination
θ^{-1}	The mean duration for quarantined cases for survivors
δ^{-1}	The mean duration for infected cases for quarantined

Table 3
Hospitalization costs per patient.

	Median for length of stay (days)	Cost per day (US $)	Total cost (US $)
Hospital	6	1,772.00	11,267.00
ICU	5	2,902.00	13,443.00

Table 4
Costs per patient on mechanical ventilation.

	Median for length of stay (days)	Median cost (US $)
Hospital	16	47,454.00
ICU	11	41,510.00

authors conducted a cross-section study of hospital costs and associated factors in Guandong China from January to April 2020. The authors of this study followed over 800 COVID-19 cases admitted to over 30 hospitals, collected demographic and clinical data about the patients, and applied multiple linear regression models to estimate the impact of associated factors on the total hospitalization costs. Results showed that the overall cost of hospitalization (median) (US $): $ 2,869.400 with higher costs in male patients, older patients, and also with higher case severity. The study also found a positive impact on cases that have had Chinese traditional medicine treatment, especially for non-pharmacologic therapy costs.

Table 5 shows the similarity of our work to related scholarly work; there is a clear limitation related to the study of the facemask effect on the spread of COVID-19. The relation between the vaccine effect and the number of infected cases in both differential equations and statistical analyses is insufficiently covered. Moreover, the cost and loss control by ξ variation is not covered. More details are provided in the next, objectives section.

1.2. Objectives

There are many mathematical techniques for modeling the spread of viruses aiming, mostly, to predict the number of infections over a period of time. A few of these models also assess the economic costs of virus propagation and highlight factors that play a role in these losses and costs. Our proposed model provides a summary of the following contributions.

- Our data was derived from public sources and the compartmental links reflect virus spread behavior Fig. 1.
- In addition to classical modeling shown in Table 5, most plausible scenarios are analyzed, including whether or not an infected case can become reinfected or they will build immunity as a result of vaccination and also whether or not they will require quarantine.
- Parameters that affect the spread of the virus have been highlighted including the impact of masks, vaccination on those who have been exposed and recovered, the potential for a new virus variant to reduce the efficacy of vaccines, and the role of quarantine in limiting the spread of the virus.

Table 5
Summary of selected related literature.

Ref.	Approach	Similarity with our work/limitation(weaknesses)
[8]	Mixed Integer Linear Programming Model for cost optimizing	Both studies cost /Not taking the effects of wearing a face mask into consideration.
[9]	variation of a SIR model	Mitigation measures used to control the spread of infections / Not taking the effects of wearing a face mask or of vaccines into consideration.
[10]	fractional ODEs with variables modeling the five SEIQR	Similar compartments from Susceptible to Recovered / Not taking the effects of wearing a face mask, the cost and losses associated with the spread of COVID-19 into consideration.
[16]	A strategy of reliable cost/resource data in enabling decision-making in the management of healthcare crises	Both aim to help control and management of healthcare crises/ Not taking into consideration the optimal control strategy when making statistical measurements.
[19]	Spatial model is used to predict the spread in particular locations; good for imposing targeted, focused preventive measures.	Caters for Preventive Measures/ Taking into consideration many factors that affect the modulation and require a dynamic data provider.
[20]	A modified version of the SIR model with the addition of three more parameters for Vaccinated Individual V, Symptomatic Carrier Csy, Asymptomatic Carrier CAsy.	Cover of Asymptomatic carriers similar to our original work/ Not taking the effects of wearing a face mask, the cost and losses associated with the spread of COVID-19 into consideration.
[21]	This model describes the group of infectious beings as prey and uninfectious being as predators called the predator-prey model, in which the predator population acquires infection from the prey population, during the predation process.	Covers Infectious and UnInfectious cases/ Not taking the effects of wearing a face mask or of vaccines into consideration, and using assumed parameter values instead of realistic data for model validation.
[22]	A mathematical infection model to estimate coronavirus infection in Germany with the incorporation of contact restriction into account	This model can provide an estimate of the number of infected people, new infections, and deaths in the region of study./ The cost and losses associated with the spread of COVID-19 into consideration.
[23]	people are transferred from one compartment to another, for example, susceptible to exposure, exposed to infectious, etc	SEIR is the basis of our original work (without studying Q)
[24]	Includes compartments for deceased and also for quarantined.	Caters for Deceased, Quarantined compartments in conjunction with NPI measures/ Not taking the effects of wearing a face mask into consideration.
[25]	Shows Growth rate of COVID-19 is twice that of SARS and MERS	Control and measurement based on propagation growth/ Not taking into consideration the optimal control strategy when making statistical measurements.

- To evaluate our model, we have utilized both Jordan-specific data and open-source data, as shown in Fig. 6, and the Tables 7, and 6.
- The costs and loss parameters relating to the spread of the virus and the relationship to the set of other, proposed parameters are also Incorporated.

The parameters in the main proposed model include constant and time-dynamic parameters; the model relies on control theory to produce mitigation strategies to control the spread of the pandemic. Relevant model parameters include controllable vaccination rates. We will analyze the relationship between economic losses and infection rates confirming that financial losses from the spread of the virus are proportional to the number of infected nodes, this necessitates the introduction of a special parameter that denotes economic losses caused by the infected nodes. Some of the constant parameters of the model represent the average financial loss caused by each confirmed infection case or node. This enables us to work out the sum of losses caused by all confirmed infections within a time span, we will also show this value is proportional to the infected number of nodes thus calculating losses across a predefined time interval. Because investments in vaccine development have a direct, positive correlation to the number of lives saved, our control model will also incorporate parameters representing the cost of investment in vaccine development. Finally, our model will attempt to identify the Lagrangian and Hamiltonian parameters for the optimal control problems [26]. The combination of the virus spread prediction model, the face mask impact parameter impact, and this proposed control model will serve as a basis for optimizing the provision of COVID-19 vaccines in an efficient way that inherently caters to many of the aforementioned issues relating to the effective, fair and ethical vaccine and healthcare provision in the face of limited resources and ever-increasing logistical, medical, social and political challenges.

2. Optimal control of vaccination

This section presents a study of the effect of vaccines in the prevention of the spread of COVID-19 and their role in converting the pandemic into a more controllable epidemic or endemic. We assume ξ is a controllable measure that is applied against the spread of COVID-19; hence, Exposed represents the main population affected by vaccination, and the vaccine dictates whether an Exposed case transfers to the Infected compartment or the Recovered compartment.

Some parameters in model (1) may be variable with time. Control theory will be applied to derive strategies for reducing COVID-19 spread.

Suppose the parameter ξ representing the controllable recovery rate by vaccination is variant with time $\xi(t)$. Considering a control set $\mathcal{U} = \{\xi(t) \in L^2[0,T], \xi(t) \in [0, 1], 0 \le t \le T]\}$.

Economic losses increase with the number of people infected with Corona, whether due to the health burden or the suspension of the production wheel. The economic loss caused by Corona patients is $\mathcal{F}_{\text{loss}}(I(t))$. We assumed that the losses by a corona patient with a time t are fixed Φ. Calculating the loss function:

$$\mathcal{F}_{\text{loss}}(I(t)) = \int_0^T \Phi I(t)dt \tag{2}$$

Lives saved increase by investing in vaccines; the cost investment function is

$$Ccost(\xi(t)) = \int_0^T \frac{1}{2}\phi\xi^2 dt \tag{3}$$

where ϕ is a positive trade-off factor. A proposed optimal control problem (OCP) for minimizing the objective functional $\mathcal{K}$ is shown below:

$$\mathcal{K}(I, \xi(t)) = Ccost(\xi(t)) + \mathcal{F}_{\text{loss}}(I) = \int_0^T \left[\Phi I(t) + \frac{1}{2}\phi\xi^2(t)\right] dt \tag{4}$$

subject to: model (1).

Where $\mathcal{K}(\xi(t))$ is the sum of both loss and protective vaccine costs, which are described in the equations (2), and (3). Lagrangian and Hamiltonian of the OCP [26], will be defined to solve the optimization problem (4). Lagrangian is

$$\mathcal{L}(I,\xi) = \frac{1}{2}\phi\xi^2(t) + \Phi I(t)$$

Now we find a suitable $\xi(t)$ to optimize Lagrangian integral (as it reaches a minimum). Therefore, defining Hamiltonian $\mathcal{H}$ of the control problem as:

$$\mathcal{H}(S,E,I,Q,R,\xi,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5) = \mathcal{L}(I,\xi) + \left[\lambda_1 S' + \lambda_2 E' + \lambda_3 I' + \lambda_4 Q' + \lambda_5' R\right] \tag{5}$$

for λ_i, $i = 1, ..., 5$ adjoint variables.

Theorem 1. *Consider system (1) with the objective functional (4), then $\exists$ a $\xi^\star(t) \in \mathcal{U}$ s.t. $\mathcal{K}(\xi^*(t)) = \min_{\xi(t)\in U} \mathcal{K}(\xi(t))$.*

Proof. By the non-negativity of both control parameters and the compartment variables in (1); with, both being bounded on the closed interval $[0, T]$, and using the result in [27]; we can confirm the existence of OCP to model (4).

The compartment variables and control parameters set is non-empty. The right side of the system of equations (1) is both bounded and continuous; hence they can be formulated as a linear operator of ξ with compartment variables and time function coefficients. OCP shows that the convexity of the objective functional is achieved, as seen in $\xi(t)$. The control space $\mathcal{U}$ is convex and closed, with a bounded proposed model. Hence, the compactness needed to achieve a higher level of control is determined. In addition, $\Phi I(t) + \phi\xi^2(t)/2$, is convex on $\mathcal{U}$. Also, it can be seen that a constant $\sigma > 1$ exists; this is in addition to the positive numbers τ_1 and τ_2 such that $\mathcal{K}(I, \xi(t)) \ge \tau_1(|\xi|^2)^{\sigma/2} + \tau_2$. So, the expression at the end can be assessed as $\Phi I(t) + \phi\xi^2(t)/2 \ge \tau_1|\xi|^\sigma + \tau_2$. The obtained results can be shown for the previous conditions by utilizing the work in [28]. Hence, we can conclude a solution exists.

Theorem 2. *Let a solution of OCP with control variable be $\xi^*(t)$ is $S^*(t), E^*(t), I^*(t)$ and $R^*(t)$. Then, $\exists$ adjoint variables λ_i, $i = 1, ..., 5$ satisfying*

$$\dot{\lambda_1} = (\lambda_1 - \lambda_2)\beta I^* + d\lambda_1$$

$$\dot{\lambda_2} = \lambda_2(\gamma + \xi(t) + d) - \gamma\lambda_3 - \xi(t)\lambda_5$$

$$\dot{\lambda_3} = -\Phi + (\lambda_1 - \lambda_2)\beta S^* + \lambda_3(\epsilon + \delta + \mu + d) - \delta\lambda_4 \tag{6}$$
$$\dot{\lambda_4} = \lambda_4(\theta + \mu + d) - \theta\lambda_5$$
$$\dot{\lambda_5} = \lambda_5(\zeta + d) - \zeta\lambda_1$$

with transversality conditions

$$\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = \lambda_5(T) = 0$$

with,

$$\xi^* = \min\left\{\max\left\{0, \frac{\lambda_2 - \lambda_5}{\phi}E^*\right\}, 1\right\}$$

Proof. Deriving Equation (5) w.r.t. compartment variables:

$$\dot{\lambda_1} = -\frac{\partial H}{\partial S} = (\lambda_1 - \lambda_2)\beta I^* + d\lambda_1$$
$$\dot{\lambda_2} = -\frac{\partial H}{\partial E} = \lambda_2(\gamma + \xi(t) + d) - \gamma\lambda_3 - \xi(t)\lambda_5$$
$$\dot{\lambda_3} = -\frac{\partial H}{\partial I} = -\Phi + (\lambda_1 - \lambda_2)\beta S^* + \lambda_3(\epsilon + \delta + \mu + d) - \delta\lambda_4$$
$$\dot{\lambda_4} = -\frac{\partial H}{\partial Q} = \lambda_4(\theta + \mu + d) - \theta\lambda_5$$
$$\dot{\lambda_5} = -\frac{\partial H}{\partial R} = \lambda_5(\zeta + d) - \zeta\lambda_1$$

The derived solution of OCP is free at the final state. So, the bounded conditions are $\lambda_i(T) = 0, i = 1, ..., 5$. By the optimal condition, then

$$\left.\frac{\partial H}{\partial \xi}\right|_{\xi=\xi^*} = \phi\xi^* - \lambda_2 E^* + \lambda_5 E^* = 0$$

By using measurable set U feature, then

$$\xi^*(t) = \begin{cases} 0, & \text{if } \frac{\lambda_2-\lambda_5}{\phi}E^* \le 0 \\ \frac{\lambda_2-\lambda_5}{\phi}E^* & \text{if } 0 < \frac{\lambda_2-\lambda_5}{\phi}E^* < 1 \\ 1, & \text{if } \frac{\lambda_2-\lambda_5}{\phi}E^* \ge 1 \end{cases}$$

Finally, the derived optimal model we have, is formulated as:

$$\begin{aligned}
\dot{S} &= b - \beta SI + \zeta R - dS, \\
\dot{E} &= \beta SI - (\gamma + \xi)E - dE, \\
\dot{I} &= \gamma E - (\epsilon + \delta)I - (\mu + d)I, \\
\dot{Q} &= \delta I - \theta Q - (\mu + d)Q, \\
\dot{R} &= \epsilon I + \theta Q + \xi E - \zeta R - dR, \\
\dot{\lambda_1} &= (\lambda_1 - \lambda_2)\beta I^* + d\lambda_1 \\
\dot{\lambda_2} &= \lambda_2(\gamma + \xi(t) + d) - \gamma\lambda_3 - \xi(t)\lambda_5 \\
\dot{\lambda_3} &= -\Phi + (\lambda_1 - \lambda_2)\beta S^* + \lambda_3(\epsilon + \delta + \mu + d) - \delta\lambda_4 \\
\dot{\lambda_4} &= \lambda_4(\theta + \mu + d) - \theta\lambda_5 \\
\dot{\lambda_5} &= \lambda_5(\zeta + d) - \zeta\lambda_1
\end{aligned} \tag{7}$$

with $(S(0), E(0), I(0), Q(0), R(0)) = (S_0, E_0, I_0, Q_0, R_0)$ and $\lambda_j(T) = 0,\ j = 1, ..., 5$.

2.1. Analysis and discussion of facemask efficiency

According to [29], to maximize the effectiveness of masks in fighting the spread of epidemics, they must be used as part of a comprehensive set of measures within targeted and monitored strategies to reduce virus spread and transmission. This section analyzes the effect of masks on fighting the spread of COVID-19 using the proposed model in [11]. We examine mask efficiency in reducing the epidemic spread by assuming all members of the population $N(t)$ wear masks in public venues and gatherings. It is noteworthy that homemade masks showed an efficiency between 58% – 77% when worn over three hours [30]; with efficiency rates between 70% and 85% for surgical type masks and between 98% and 99% for N95-compliant masks [31]. In the model below, the parameter m represents mask efficiency, by considering this parameter, the main system [11] if the population wears masks is:

$$\begin{aligned}
\dot{S} &= b - (1-m)\beta S I + \zeta R - dS,\\
\dot{E} &= (1-m)\beta S I - (\gamma+\xi)E - dE,\\
\dot{I} &= \gamma E - (\epsilon+\delta)I - (\mu+d)I,\\
\dot{Q} &= \delta I - \theta Q - (\mu+d)Q,\\
\dot{R} &= \epsilon I + \theta Q + \xi E - \zeta R - dR.
\end{aligned} \tag{8}$$

The Suspicious population increases by Transfers to Infected cases at a rate of $(m-1)\beta S I$, where m stands for the effect of facemask protection from COVID-19 infection. To analyze the suitability of the proposed model in representing actual COVID-19 spread we must analyze the model by finding the correspondence between mathematical parameters and obtained data. Non-negativity of model solutions indicates a stronger fit rate of the model. And analyzing R_0^m provides an indication of the virus spreading behavior to cause a pandemic over time. The impact of m over R_0^m indicates facemask effectiveness in the protection from the rapid spread of COVID-19. The model (8) with mask efficiency is free of viruses at $P_0 = (b/d, 0, 0, 0, 0)$. For the new infection terms, represented by F, and for the transition terms, represented by V, computed as follows:

$$F = \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & (1-m)\beta\,\frac{b}{d} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}, \text{ and}$$

$$V = \begin{pmatrix} d & 0 & (1-m)\beta\,\frac{b}{d} & 0\\ 0 & d+\gamma+\xi & 0 & 0\\ 0 & -\gamma & d+\delta+\epsilon+\mu & 0\\ 0 & 0 & -\delta & d+\mu+\theta \end{pmatrix}$$

Then the basic reproduction number for the model considering facemask effect R_0^m, is:

$$R_0^m = \rho(FV^{-1}) = \frac{(1-m)\beta\,\gamma\,(b/d)}{(d+\gamma+\xi)\,(d+\delta+\epsilon+\mu)} \tag{9}$$

where $\rho(FV^{-1})$ is the spectral radius. From formula (9) the reproduction number considering mask protection can be written as: $R_0^m = (1-m)R_0$.

Theorem 3. *Local and global stability of model (8) at the disease-free equilibrium* $\mathcal{P}_0 = (\frac{b}{d}, 0, 0, 0, 0)$

- $\mathcal{P}_0$ *is locally asymptotically stable if* $R_0^m \le 1$.
- $\mathcal{P}_0$ *is globally asymptotically stable if* $R_0^m < 1$*, and is unstable if* $R_0^m > 1$.

Proof. The proof is obtained in a straightforward manner from [11], by the theorems, 3.2 and 3.3. □

Model stability analysis is provided in Fig. 2, Fig. 2 (a), for different initial values the model will converge to epidemic point (i.e. COVID-19 disease will be transmitted between people with a high likelihood of an outbreak).
In Fig. 2 (b), every infected case may cause less than one new infection. COVID-19 will decline and eventually die out.

Fig. 3 shows the basic virus reproduction rate is inversely proportional to the impact factor of wearing a facemask, and this is evident according to the following derivative: $\frac{\partial R_0}{\partial m} < 0$. Moreover, the starting value with the Omicron variant has an average basic reproduction number median value $R_0^m = 10$ [32]. If we assume that there is a facemask with an ideal protection ratio, the spread will stop according to the equation (9) (i.e. $R_0 = 0$), thus implying no increase in the rate of Infected compartment and further COVID-19 spread or new cases.

Commitment to (quality) facemasks can prevent the spread of the virus, as Fig. 3 shows the virus spread is reduced by decreasing the basic reproduction value of the virus (was $R_0 > 10$ which linearly decreased by increased mask efficiency m) until it becomes zero with a highly effective type of mask.

To test mask effectiveness in reducing epidemic spread we simulate the model (8) using parameter $(\beta, \gamma, \xi, \epsilon, \mu, \delta, \theta, \zeta) = (.5, .2, .025, .14, .015, .6, .14, .01)$ [33], and we select $m = 0.95$ when using masks and $m = 0$ when not using masks. The simulation was used for comparison between the number of infected cases when using masks and when not.

From Figs. 4(b) and 4(a) the expected results show that using masks decreases the number of infected and carrier cases of the virus. Also, from Fig. 4(b) on the day 15 ($x = 15$), one can see from Fig. 4(b) that the value of $E + I + Q$ is equal to 0.048 (with masks) and 0.096 (without masks) demonstrating that using masks will decrease the number of infected and carrier cases of the virus by 50% after two weeks Fig. 4(b). And with Fig. 4(a) On the tenth day of the outbreak, it appears to us that not using the mask increases the

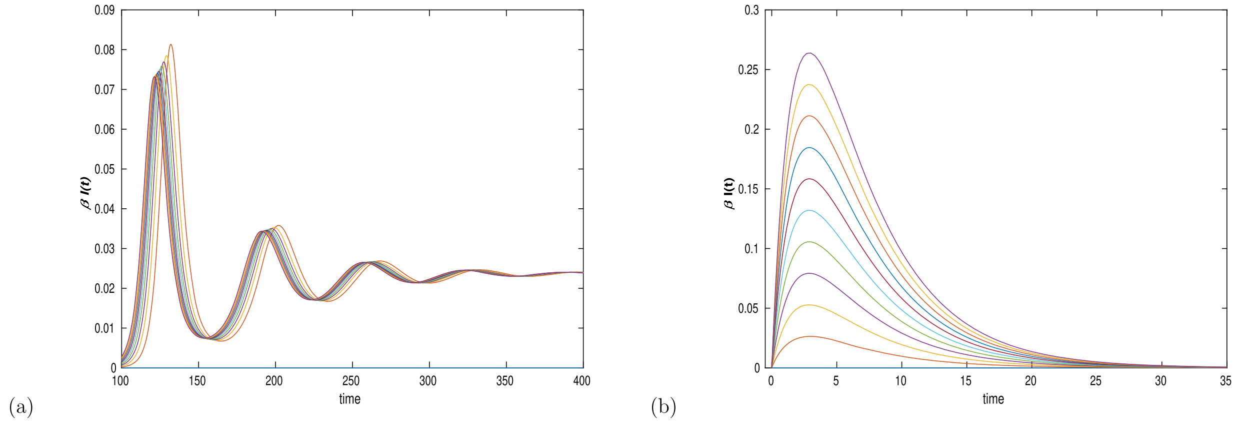


Fig. 2. Stability investigation of proposed mask model (8) for both $R_0 > 1$ and $R_0 < 1$.

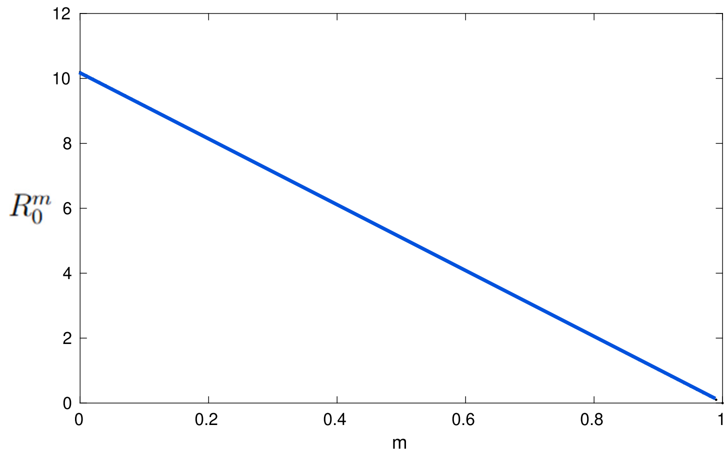


Fig. 3. Comparison between R_0^m value and m value.

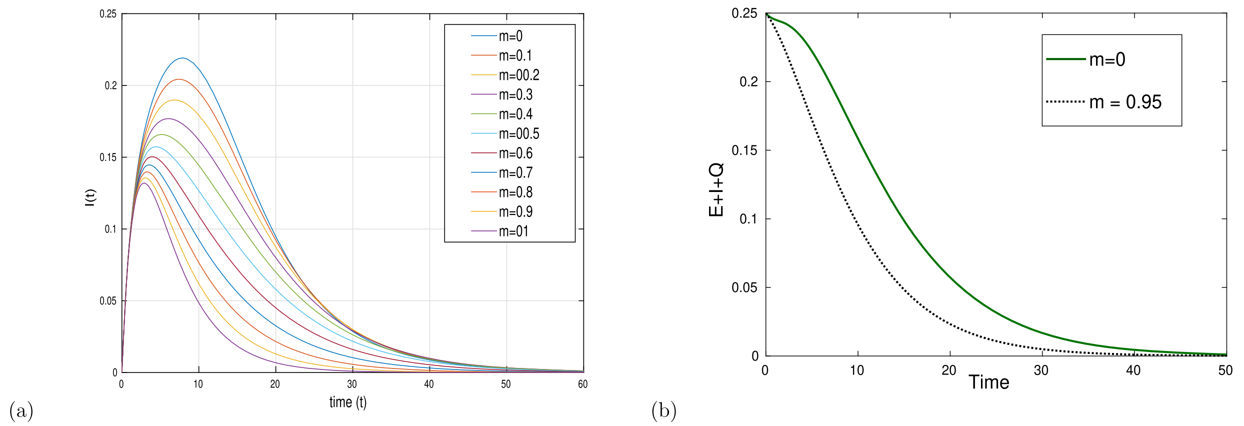
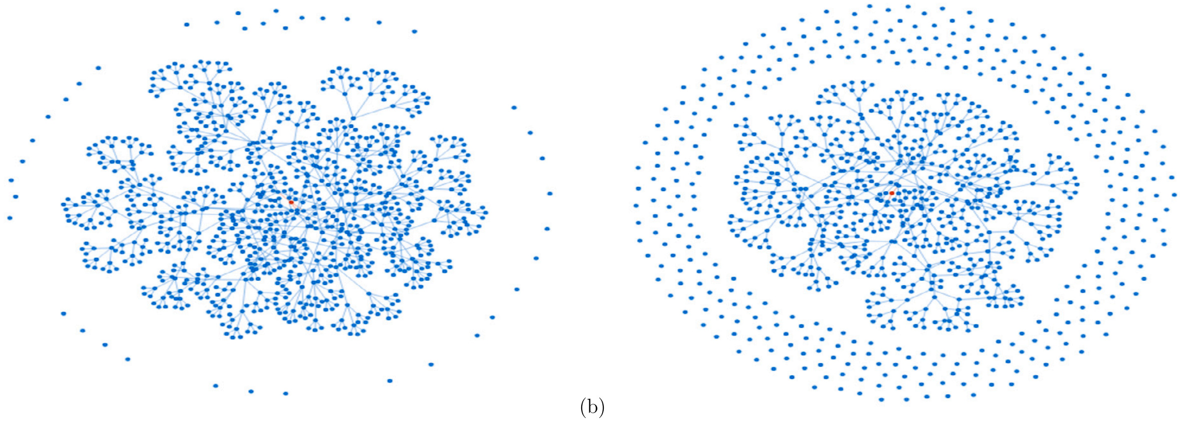


Fig. 4. (a) Comparison between $I(t)$ value and m value., (b) Comparison between the number of infected and carriers of the virus cases with and without masks.

percentage of infected cases to more than four times.

Considering the least effective mask type (cotton), according to [34] it provides an average protection rate of 30%. Applying for cotton mask protection on populations (Belgium, Netherlands, Switzerland, United Kingdom). By taking approximate value from [35] take $R_0 = 3$ of these countries. Therefore, $R_0^m = (1 - 30\%)R_0$. The spread rates are shown in Fig. 5(a) (a without facemask), for $R_0 = 3$ compared with Fig. 5(b) (b with facemask) for R_0^m, with the same populations in a fixed time period. Figs. 5(a), and 5(b) illustrate the effect of masks in countering increases in compartments S and I. Figs. 5(a), and 5(b) show how Infected cases increase



(a) (b)

Fig. 5. Spreading of Virus from the case "0" to its neighbors in time period T, (a) without facemask efficiency, (b) with facemask efficiency.

Table 6
Statistical measurements and their associated values and descriptions.

Measure	Description	Formula	Value
SSM	Sum of Squares for Model	$\sum_{i=1}^{n} \frac{1}{n}(y_i - \bar{y}_i)^2$	3.495×10^{12}
SSE	Sum of Squares for Error	$\sum_{i=1}^{n} \frac{1}{n}(y_i - \hat{y}_i)^2$	1.5242×10^{11}
dfm	Corrected Degrees of Freedom for Model	$DFM = p - 1$	4
dfe	Degrees of Freedom for Error	$DFE = n - p$	246
MSM	Mean of Squares for Model	$MSM = SSM/DFM$	1.165×10^{12}
MSE	Mean of Squares for Error (variance of the residuals)	$MSE = SSE/DFE$	6.1961×10^{8}
RMSE	standard deviation of the residuals	$\sqrt{MSE}$	2.4892×10^{4}

with and without facemasks respectively. Fig. 5(a) shows the fast decrements in S compartment (transferred) to infected, (singletons represent Non-Infected cases); 6d has more singletons - UnInfected cases. By contrast, the density of Infected cases shown in Fig. 5(a) is higher due to the low effect of facemasks.

3. Discussion

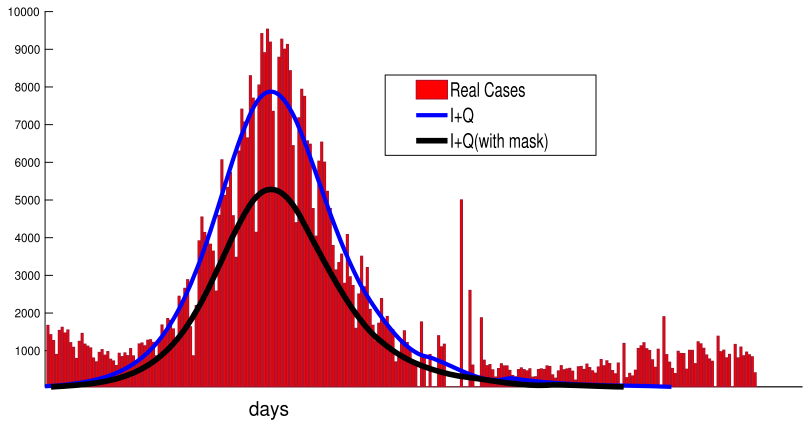
Variables (Compartments) and Parameter analysis: For fitting parameters with the proposed model we analyze Jordan data from the first wave (data available in MOH Jordan and wiki) Fig. 6(a), for further analysis, we base assumptions on a high protection effect of facemasks; The result of this assumption shows a notable reduction in assumed cases compared with original model fitting. By fitting (8) with Jordan data (some parameters are statistically fitted and others omitted due to lack of data while others are assumed by tuning).

Based on collected data from Jordan, the parameters computed $R_0 = 4.35$, $\beta = 0.104$, $\xi = 0.024$, and other parameters set to zero. Figs. 6(a) and 6(b), show data fitting with the proposed model and the effect of facemasks on the spread of the virus. Fig. 6(a) shows the model fitting and the statistical parameters as described in Table 6. Furthermore, Fig. 6(a) shows the effect of facemasks in decreasing the number of infected cases, thus contributing to an early end to the wave of infections. In addition, the F-test for linear regression tests whether any of the independent variables in a multiple linear regression model are significant. Where n is the number of observations, p is the number of regression parameters (here it is the number of compartments). This applied to the vaccinated population as described in Fig. 6(b). $F_{val} = 1.8802 \times 10^3$ and $p_{val} = 3.0242 \times 10^{-169}$. A large F-value indicates inter-compartment variation is larger. This can be interpreted to mean there is a statistically significant difference in model compartment means. Assuming also, The null hypothesis is that there is no general relationship between compartment (I), and vaccination parameter ξ and the alternative hypothesis is that such a relationship exists. Large F, with a small p-value, indicates that the null hypothesis is discredited, and we can assert that there is a general relationship between the S compartment and ξ. The general model exponentially fitting $f(x) = a \times \exp(bx) + c \times \exp(dx)$, and the values of parameters a, b, c, d are calculated in Table 7.

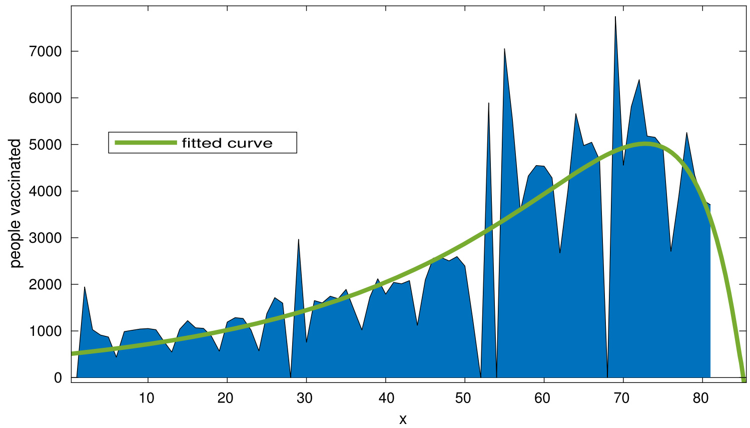
The statistical analysis of data from the Jordanian Ministry of Health is shown in Fig. 6 and Table 6.

Studying optimal strategy and mask efficiency will be described in this section, Fig. 7 shows the analysis trajectories of system (8), the convergence of the solutions containing the effective coefficient of wearing a facemask are shown, and this convergence is shown at the point P^* for different initial values.

Fig. 8(a) shows the effect of the optimal control strategy featuring a notable reduction in the number of infections during the pandemic period, and thus, vaccination awareness reduces the impact of the pandemic. The number of quarantined people during



(a)



(b)

Fig. 6. (a) Jordan first wave spreading, compared with the proposed model, (b) Vaccination people in 80 days from starting vaccine in Jordan.

Table 7
Estimated parameters values.

Coefficients	95% confidence bounds
a = 505.2	(176.3, 834.1)
b = 0.03507	(0.01988, 0.05027)
c = -0.01828	(-0.3256, 0.289)
d = 0.1552	(-0.04286, 0.3532)

the epidemic period also decreases if vaccinations are taken, and thus the cost of the quarantine decreases, and the impact of the quarantine on the drivers of the economy decreases, and this appears clearly in the Fig. 8(b).

Fig. 9(a) shows the positive impact of vaccinations on the numbers of recovered cases and hence saved lives. Furthermore, with business continuity of commerce and business, economic losses are also reduced. Fig. 9(b) confirms the necessity of higher investments in vaccine expansion strategies due to the notable, early-stage increases in the optimal control value $\xi(t)$.

Fig. 10(a) illustrates typical economic losses induced by infected cases for various Φ values (per day). As Fig. 10(b) shows with increases in the ϕ trade-off factor, vaccine investment costs also increase, as shown in the $C_{cost}(I)$ function. This indicates a direct correlation between ϕ and $C_{cost}(I)$.

Based on the results obtained in the discussion section, we have summarized the figure results in the following recommendations for helping to control the spread of the virus: testing, tracking, and tracing infected people; increasing the rate of vaccination; and implementing good hygiene practices.

- Wearing a mask is effective in reducing the spread of the virus, as there is an inverse relationship between the efficacy of the mask and the reproduction number, shown in Figs. 3, 4 and 5.
- Taking into account the effect of vaccination, this has resulted in a decrease in infection cases and a reduction in the number of people needing to be quarantined, see Fig. 8.

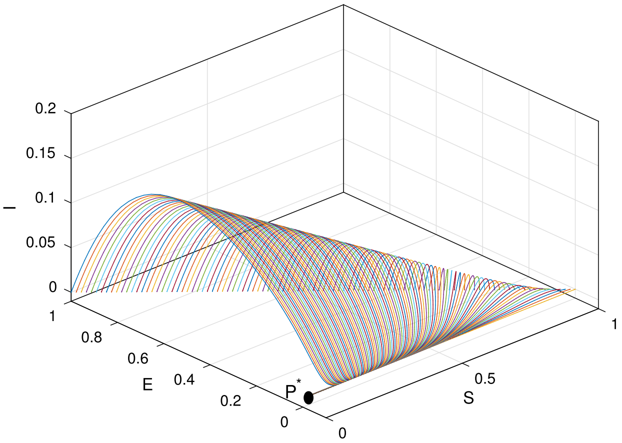


Fig. 7. Convergence of facemask model (8) in (S, E, I)-space.

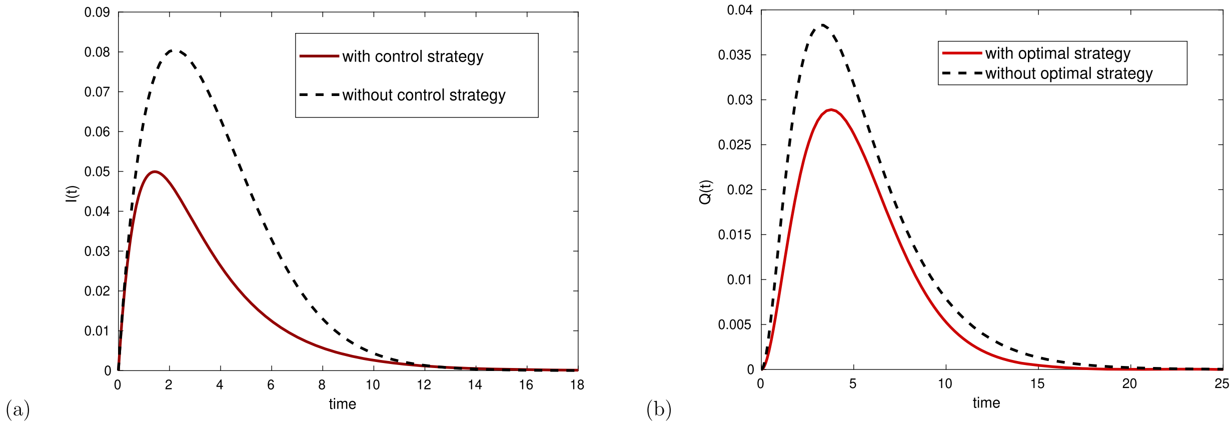


Fig. 8. (a) Number of infected patients compared with and without optimal control technique, (b) Number of quarantined patients compared with and without optimal control technique.

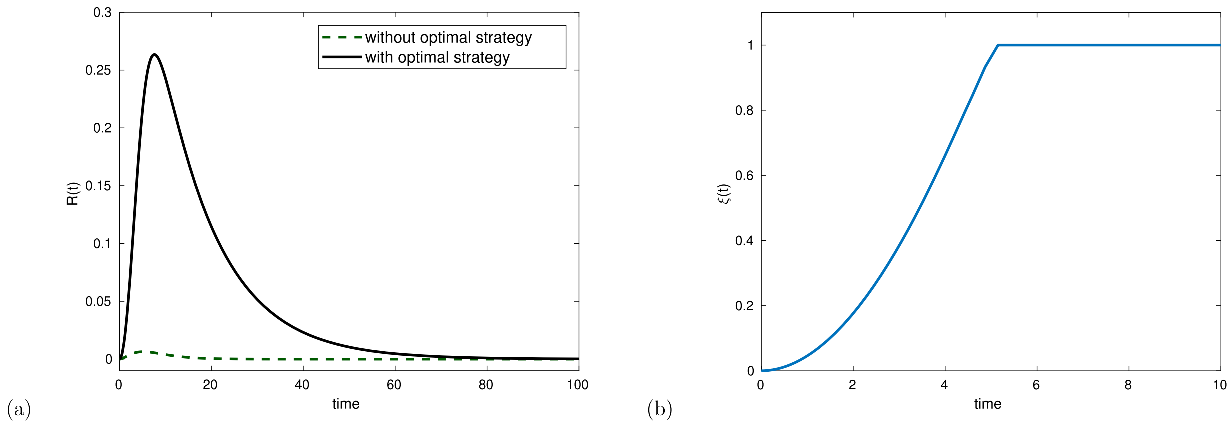


Fig. 9. (a) A significant increase in the number of recovered patients as indicated by an optimally chosen strategy compared to the number of recovered cases with an arbitrary/alternative strategy., (b) Variant values of optimal control $\xi(t)$.

- Controlling the rate of vaccination has been linked to a quicker rate of recovery, as well as higher investments in vaccine expansion strategies due to the notable, early-stage increases in the optimal control value, as in Fig. 9.
- The costs and losses associated with virus transmission can be controlled by increasing the rate of vaccination. As more people are vaccinated, the losses and costs associated with the virus will be reduced, as in Fig. 10.

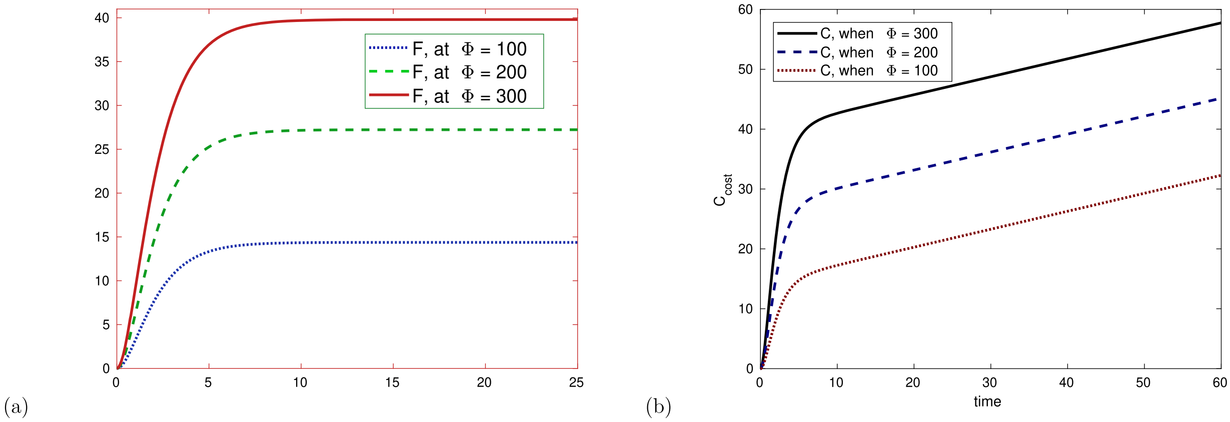


Fig. 10. (a) The effect of average loss by a single patient - for certain values- on economic loss., (b) A direct proportional between cost investment and ϕ.

The proposed modulation system is subject to the same limitations as those of related works, which researchers can use to further their studies in future work. The limitations and potential for future work of the proposed model can be summarized as follows: it is limited by the amount of data available and the accuracy of the models used, and its potential for improvement can be explored through further research into the underlying epidemiological mechanisms and the development of more effective vaccine strategies.

- The collected can be further divided into categories, such as the number of quarantined cases over time and the number of non-discovered exposed cases. These divisions can result in the omission or assumption of some parameters.
- Historical disease data of patients can be taken into account to determine whether or not a case has pre-existing medical conditions such as diabetes or lung disease.
- The age of the patient is important in determining their vulnerability to contracting COVID-19, with older individuals more at risk.
- Sensitivity analysis to examine how infection cases react to changes in parameter adjustments can be incorporated.
- The geographical location of patients is a major factor in the spread of COVID-19, as the spread of the virus varies by region.
- With a relatively small population, the data from Jordan is assumed to be homogeneous with no variations in population density or differences between urban and rural areas.

4. Conclusions

In this article, we formulated a novel model to represent an optimal vaccination strategy and distribution to minimize the number of COVID-19-infected cases over a determined period. Results indicate that higher availability of vaccines at the outset of the pandemic correlates with higher control efficacy thus reducing the spread of the pandemic. One of the significant factors in the proposed control model is the utilization of the Lagrangian and Hamiltonian functions to converge to an optimal vaccination strategy. Furthermore, the model included provision for face-mask application as a factor in reducing the spread of COVID-19; this was confirmed by the simulation results of the study. The Discussion and Analysis section with statistical F-test shows how data from Jordan was fitted to our work and the relation between m facemask effect and COVID-19 spread is clearly described, in addition, numerical solutions show the relation between vaccinations, cost, and Infected cases. In future work, it may be possible to expand this study to a fractional model and to add the sensitivity analysis for vaccination parameters.

Ethics

During the COVID-19 pandemic, the Infectious Disease Control Authority in Jordan provided daily and weekly data about the number of infected/quarantined cases; this data was made public and free of charge for research purposes.

CRediT authorship contribution statement

Ammar ElHassan: Conceived and designed the experiments; Contributed reagents, materials, analysis tools, or data; Wrote the paper. **Yousef AbuHour:** Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools, or data. **Ashraf Ahmad:** Conceived and designed the experiments; Performed the experiments; Wrote the paper.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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