Research paper

# Sample size determination and evaluation for two-stage adaptive designs of single arm clinical trials based on median event time test 

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#### Abstract

Clinical trials play a critical role in drug development which involves a series of phases and requires a significant amount of time and effort. Efficient clinical trial designs are necessary to investigate a new drug. Investigators strongly desire to use the time-to-event endpoint as the primary endpoint for Phase II studies, which evaluates the therapeutic efficacy of the new drug, with the hypothesis that the new drug improves the median survival time. The one-sample log-rank test has been used for single-arm Phase II trials, but it generally requires more samples. Recently, the median event time test was proposed to provide a simple, straightforward decision rule, which compares the observed median survival time for the new drug with the threshold, which is determined through the numerical search. We improve the computation of the method for the two-stage design of single-arm clinical trials based on the median event time test. By utilizing the large sample theory of order statistics, we provide the explicit formulas to calculate the sample size for the first and second stages and propose the testing procedure. The performance of the proposed method is evaluated through simulations and a trial example.


## 1. Introduction

Drug development is a complex process that involves a series of phases, requiring a significant time and effort. Clinical trials play a critical role investigating new drugs. The therapeutic efficacy of the new drug is assessed in Phase II and then further confirmed with usually hundreds or thousands of patients in Phase III. This confirmation is necessary to establish whether the experimental treatment provides a longer overall survival compared to a placebo or standard control. Phase II clinical trial designs have traditionally relied heavily on a binary response endpoint. For example, in cancer clinical trials, a binary indicator of having a complete response or a partial response to therapy is used. According to Response evaluation criteria in solid tumors (RECIST) forms, a complete response is defined as the removal of all target lesions, and a partial response is defined as a reduction of at least $30 \%$ in the total number of target lesions [1]. However, it may not be desirable to use the response rate as a surrogate for the desired survival time, which is a gold standard endpoint for Phase III studies. For example, the tumor shrinkage may not be measurable in rapidly lethal cancers or may not be available in uveal cancer [2]. Moreover, it is critical to avoid unexpected failures in Phase III studies after successful Phase II results. To address these issues, it is recommended to evaluate the new drugs based on the time-to-event endpoint in Phase II studies.

The one-sample log-rank test has been commonly used to compare the survival distribution of the experimental treatment with a historical control, without making parametric survival assumptions. It was first introduced by Breslow [3]. Tu and Gross [4] derives a Bartlett-type correction for the two-sided one-sample log-rank test, and Sun et al. [5] derives the correction to the one-sided one-sample log-rank test based on Edgeworth expansion. Kerschke et al. [6] improves the one-sample log-rank test based on the transformation of the underlying counting process martingale to correct the conservativeness. Several studies, including Sun et al. [5],Finkelstein et al. [7],Wu [8], and Schmidt et al. [9], provide sample size formulas to achieve the desired power for the one-sample log-rank test. The one-stage procedure and two-stage designs based on the one-sample log-rank test are proposed by Sun et al. [5],Kwak and Jung [10], and Belin et al. [11]. They compare the survival curves or hazard rates to evaluate the promising nature of the new drug. Instead of comparing entire survival curves, another approach to assess the efficacy of drugs is to compute the survival probability at a clinically meaningful time point, e.g., one-year survival probability [2,12-14].

Our previous work uses the parametric survival distributions to develop the median event time test for single-arm Phase II trials whose primary endpoint is time-to-event [15]. It extends the idea of Simon's

[^0]two-stage design [16] for a time-to-event endpoint based on the median event time test. In clinical practice, clinical investigators and practitioners report the median survivals as a measure of the time-to-event endpoints. The median information is easily accessible from historical trials, and it provides a straightforward understanding of the therapeutic efficacy of the drugs based on median survival. For clinical purposes, the median event time test is useful as it allows for a simple and straightforward interpretation. It tests the improvement of median survival and can be easily implemented by comparing the observed median survival time for the new drug with a prespecified threshold. Park [15] determines sample size and threshold of clinical trial design based on median event time test through the numerical search for the given target type I and II error rates.

This paper aims to develop a new method for the median event time test, which provides explicit formulas for determining the sample size in two-stage designs. Compared with the existing literature, we make three major contributions. First, we derive explicit sample size formulas for the two-stage design in order to attain the desired power of the one-sided median event time test with a significance level. The explicit formulas eliminate the computational burden associated with our earlier work. Second, we determine the decision rule for the median event time test with the optimal choice of design parameters, which minimizes the expected total sample size under the null hypothesis. Third, we provide a user-friendly shiny application which is freely available and easy to use.

This article is organized as follows. Section 2 describes the two-stage design of single-arm trials whose primary endpoint is time-to-event and provides explicit formulas determining the sample size based on the median event time test. We also provide the practical considerations and software which is freely available to use the proposed method for the trial implementation. The performance of the proposed method is investigated through simulations in Section 3. We illustrate the application of the method with a real trial example in Section 4 and provide the concluding remarks in Section 5.

## 2. Sample size determination

Consider a two-stage design of single-arm Phase II clinical trials whose clinical endpoint is time-to-event. For the first stage, $n_{1}$ patients are accrued, and an interim analysis is performed to determine the go/no-go of the trial when the total accrual reached $n_{1}$ patients. If the trial continues, we determine the required sample size, say $n_{2}$, for the second stage. In the second stage, the $n_{2}$ patients are enrolled while the first stage patients are being followed for the efficacy evaluations at the final analysis. The final analysis will be performed after the follow-up of all $n$ patients, where $n=n_{1}+n_{2}$.

Let $Y_{1}, \ldots, Y_{n}$ be random variables from an exponential distribution with mean $\mu$. Then, the median $\phi$ of the exponential distribution is $\mu \log 2$. We want to test the null hypothesis $H_{0}: \phi=\phi_{0}$ versus the alternative hypothesis $H_{a}: \phi=\phi_{1}$ based on the one-sided median event time test (METT) at a significance level of $\alpha$ to attain the power $\geq 1-\beta$. The null and alternative values are specified by clinicians. For example, in intervention clinical trials, the null value of $\phi_{0}$ is the median time of the standard drug and the alternative value of $\phi_{1}$ is the expected median time from the intervention (for improvement). Let $Z_{1}$ and $Z_{2}$ be test statistics for a two-stage design at the interim and final analyses, respectively. The test statistic at the interim, denoted by $Z_{1}$, is calculated by the observed median survival time of the first stage, and the test statistic at the final analysis, denoted by $Z_{2}$, is calculated by the observed median survival time of all enrolled patients after the follow-up. Let $\hat{S}_{n}(t)$ be the estimate of the survival function based on the sample of size $n$. The observed median survival time based on the sample of size $n$, denoted by $M_{n}$, is the time at which $50 \%$ of individuals are expected to have survived, i.e., $M_{n}=\min \left\{t \in T: \hat{S}_{n}(t) \leq 0.5\right\}$. Then, we have $Z_{1}=M_{n_{1}}$ and $Z_{2}=M_{n_{2}}$ based on the estimate of the survival function at interim and final analyses, respectively. The nonparametric
estimator with the Kaplan-Meier estimator can be used to calculate the test statistics $Z_{1}$ and $Z_{2}$. We assume that a censoring rate is less than $50 \%$ at the interim so that we obtain reasonably precise estimates of the median survival time $Z_{1}$. The assumption can be relaxed to handle the case when we do not observe $Z_{1}$ (See practical considerations below). Let $t_{1}$ and $t_{2}$ be the threshold to be compared with the observed median survival at the interim and final analysis, respectively. Then, METT compares the observed median survivals $Z_{1}$ and $Z_{2}$ with the corresponding thresholds $t_{1}$ and $t_{2}$ at the interim and final analysis, respectively. We stop the trial for futility at the interim analysis if $Z_{1} \leq$ $t_{1}$. Otherwise (i.e., $Z_{1}>t_{1}$ ), the trial continues to enroll the secondstage patients. At the final analysis, we compare $Z_{2}$ with $t_{2}$ to argue that the experimental treatment shows sufficient improvement in efficacy. When $Z_{2}>t_{2}$, the experimental treatment is considered promising. This framework follows Simon's two-stage design but extends Simon's two-stage design to a time-to-event endpoint based on the median event time test.

Park [15] determines the decision rule of METT by using the empirical search, which requires a lot of computation time. In this work, to save the computational cost, we use the true survival distribution to determine the decision rule for the two-stage design and show that the proposed decision rule is not sensitive to censoring information, provided the median survival time is observed at interim. Specifically, we use the theoretical results in Mosteller [17] for the observed survival time. The median of $Y_{1}, \ldots, Y_{n}$ follows asymptotical normal distribution with mean $\phi$ and variance $0.25 /\left[n\{f(\phi)\}^{2}\right]$, where $f(y)=(\log 2 / \phi) \exp (-y \log 2 / \phi)$ denotes the density function of the exponential distribution with median $\phi$.

Let $\alpha_{1}$ and $\beta_{1}$ be given such that $1-\alpha_{1}$ denotes the probability of correct decision at the interim analysis under the null hypothesis, i.e., $\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{0}\right)=1-\alpha_{1}$, and $1-\beta_{1}$ denotes the probability of correct decision at the interim analysis under the alternative hypothesis, i.e., $\operatorname{Pr}\left(Z_{1}>t_{1} \mid H_{a}\right)=1-\beta_{1}$. Since $Z_{1}$ asymptotically follows normal distribution with mean $\phi_{0}$ and variance $0.25 /\left[n_{1}\left\{f\left(\phi_{0}\right)\right\}^{2}\right]$ under the null hypothesis, we obtain
$t_{1}=\frac{0.5 z_{\alpha_{1}}}{\sqrt{n_{1}} f\left(\phi_{0}\right)}+\phi_{0}$,
where $z_{\alpha_{1}}$ denotes the critical value of the standard normal distribution at $\alpha_{1}$, i.e., $\operatorname{Pr}\left(Z \geq z_{\alpha_{1}}\right)=\alpha_{1}$, where $Z$ is a standard normal variable. Therefore, by the normality of $Z_{1}$ under $H_{a}$, we have
$n_{1}=\left(\frac{f\left(\phi_{1}\right)}{f\left(\phi_{0}\right)} z_{\alpha_{1}}+z_{\beta_{1}}\right)^{2}\left(\frac{0.5}{f\left(\phi_{1}\right)\left(\phi_{1}-\phi_{0}\right)}\right)^{2}$,
where $z_{\beta_{1}}$ denotes the critical value of the standard normal distribution at $\beta_{1}$.

Since the type I error rate is at most $\alpha$ and the expected power is at least $1-\beta$, we want to have $\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1} \mid H_{0}\right) \leq \alpha$ and $\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1} \mid H_{a}\right) \geq 1-\beta$. We notice that $\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1} \mid H_{0}\right) \geq$ $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{0}\right)-\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{0}\right)$ and $\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1} \mid H_{a}\right) \geq \operatorname{Pr}\left(Z_{2}>\right.$ $\left.t_{2} \mid H_{a}\right)-\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{a}\right)$, because
$\operatorname{Pr}\left(Z_{2}>t_{2}\right.$ or $\left.Z_{1}>t_{1}\right)=\operatorname{Pr}\left(Z_{2}>t_{2}\right)+\operatorname{Pr}\left(Z_{1}>t_{1}\right)-\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1}\right) \leq 1$.
Then, it suffices to have $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{0}\right)=\alpha$ and $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{a}\right)-$ $\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{a}\right)=1-\beta$ in order to attain $\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1} \mid H_{0}\right) \leq \alpha$ and $\operatorname{Pr}\left(Z_{2}>t_{2}, Z_{1}>t_{1} \mid H_{a}\right) \geq 1-\beta$. Setting the quantity of $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{0}\right)$ to be $\alpha$, we obtain
$t_{2}=\frac{0.5 z_{\alpha}}{\sqrt{n} f\left(\phi_{0}\right)}+\phi_{0}$.
Setting the quantity of $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{a}\right)-\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{a}\right)$ to be $1-\beta$, we obtain
$n_{2}=\left(\frac{f\left(\phi_{1}\right)}{f\left(\phi_{0}\right)} z_{\alpha}+z_{\beta_{2}}\right)^{2}\left(\frac{0.5}{f\left(\phi_{1}\right)\left(\phi_{1}-\phi_{0}\right)}\right)^{2}-n_{1}$,
where $\beta_{2}=\beta-\beta_{1}$. The details of the derivation of (1)-(4) are provided in Appendix A.


Fig. 1. Sign of the sample size for the second stage when $\alpha \leq \alpha_{1} \leq 0.5$ and $0<\beta_{1}<\beta$. The sample size $n_{2}$ is calculated from (4) when $\phi_{0}=3, \phi_{1}=6, \alpha=0.05$, and $\beta=0.2$.

Optimal choice of $\alpha_{1}$ and $\beta_{1}$ To calculate the sample size using (1)(4), $\alpha_{1}$ and $\beta_{1}$ should be specified. We choose optimal values of $\alpha_{1}$ and $\beta_{1}$ in that they minimize the expected total sample size under the null hypothesis. In the proposed two-stage design, the sample size is random. Specifically, the sample size for the first stage is fixed as $n_{1}$, but the sample size for the second stage, denoted by $\tilde{n}_{2}$, is random. If we observe at interim $Z_{1} \leq t_{1}$, the trial is terminated due to the futility and $\tilde{n}_{2}=0$. If we observe at interim $Z_{1}>t_{1}$, then the trial continues to enroll $n_{2}$ patients for the second stage and $\tilde{n}_{2}=n_{2}$. Then, the expected total sample size under the null hypothesis is $\mathrm{E}\left(n \mid H_{0}\right)=n_{1}+\mathrm{E}\left(\tilde{n}_{2} \mid H_{0}\right)=$ $n_{1}+n_{2} \alpha_{1}$, where $n_{1}$ and $n_{2}$ are functions of $\alpha_{1}$ and $\beta_{1}$. We notice that the threshold $t_{1}$ for the first stage lies between $\phi_{0}$ and $\phi_{1}$. It implies that $z_{\alpha_{1}}$ is nonnegative. Therefore, the optimal values of $\alpha_{1}$ and $\beta_{1}$ are identified by minimizing the expected total sample size $\mathrm{E}\left(n \mid H_{0}\right)$ over $\alpha \leq \alpha_{1} \leq 0.5$ and $0<\beta_{1}<\beta$. Since $n_{2}$ is nonnegative, the values of $\alpha_{1}$ and $\beta_{1}$ which lead to $n_{2}<0$, i.e., $\left\{\left(\alpha_{1}, \beta_{1}\right):\left(z_{\alpha}-z_{\alpha_{1}}\right) f\left(\phi_{1}\right) / f\left(\phi_{0}\right)<z_{\beta_{1}}-z_{\beta-\beta_{1}}\right\}$, are excluded to consider possible expected total sample size. For example, when the median progression-free survival (PFS) of the standard drug is 3 months and the investigational drug is expected to improve PFS to 6 months, we have $\phi_{0}=3$ and $\phi_{1}=6$. Using the target error rates $\alpha=0.05$ and $\beta=0.2$, we observe the sign of $n_{2}$, which is displayed in Fig. 1. The green colored region of $\left(\alpha_{1}, \beta_{1}\right)$ which yields the negative value of $n_{2}$ is not considered to obtain the optimal values of $\alpha_{1}$ and $\beta_{1}$.
Practical considerations and preservation of type I error rate The null and alternative median values could be specified by adopting clinical background and knowledge (e.g., pilot results or historical trials). For the given hypothetical median values, the proposed method determines the sample size and thresholds assuming that the test statistics $Z_{1}$ and $Z_{2}$ exist, i.e., the median survivals at interim and final analysis are observed. However, in practice, depending on the effect size and accrual rate, the median survival would not be observed because of a few events observed. In addition, some superior drugs cure patients and it could be infeasible to obtain a median survival even after sufficient follow-up. In these cases, the smallest fitted survival of the KaplanMeier curve is larger than $50 \%$. Therefore, when $Z_{1}$ does not exist, we do not want to stop the trial for futility based on $n_{1}$ patients but rather continue the trial to enroll the patients for the second stage. This allows us to have more evidence to evaluate the therapeutic efficacy at the end of the trial.

When the median $Z_{1}$ based on the first $n_{1}$ patients is not observed at the interim, we do not make any decisions for early stopping, which implies that we do not spend any errors at the interim. Thus, we suggest replacing $t_{2}$ with the threshold denoted by $t^{*}$. In other words, when $Z_{1}$
does not exist, we continue to enroll additional $n_{2}$ patients. At the final analysis, we compare the observed median $Z_{2}$ based on all $n=n_{1}+n_{2}$ enrolled patients to the threshold $t^{*}$ defined as
$t^{*}=\frac{0.5 z_{\alpha}}{\sqrt{n^{*}} f\left(\phi_{0}\right)}+\phi_{0}$,
where
$n^{*}=\left(\frac{f\left(\phi_{1}\right)}{f\left(\phi_{0}\right)} z_{\alpha}+z_{\beta}\right)^{2}\left(\frac{0.5}{f\left(\phi_{1}\right)\left(\phi_{1}-\phi_{0}\right)}\right)^{2}$.
The quantities $t^{*}$ and $n^{*}$ in (5) and (6) are obtained from a single-stage design based on METT yielding the power $1-\beta$ at the significance level of $\alpha$.

We evaluate the performance through simulations and observe that the approach using $t^{*}$ works well to control error rates for faster accrual rates. In practice, we recommend running preliminary simulations to report the operating characteristics of the designs using threshold $t_{2}$ or $t^{*}$ for the suggested accrual rate of the trial. The preliminary studies will be helpful to choose the decision rule which preserves the overall type I error rate of the trial design and avoid unnecessarily losing power from using a relatively stringent cutoff.
Software and trial implementation To facilitate the use of the twostage design based on METT, we develop a shiny application, which is freely available at https://yeonhee.shinyapps.io/METTSS/. It allows users to obtain a decision rule for the trial. Fig. 2 shows the screenshot of the application website. As seen in Fig. 2, users need to specify the following parameters

- Null median event time denoted by $\phi_{0}$ in months
- Alternative median event time denoted by $\phi_{1}$ in months
- Type I error rate denoted by $\alpha$
- Type II error rate denoted by $\beta$
- Survival distribution. Users choose one of the parametric survival distributions among exponential, uniform, and Weibull distributions based on knowledge or background of the study.
- If you choose Weibull distribution, what is the shape parameter? Users specify a positive value for the shape parameter of the Weibull distribution. The shape parameter is known as the Weibull slope. For a shape parameter being larger than 1 , the survival time follows the Weibull distribution with an increasing hazard. For a shape parameter smaller than 1 , it follows the Weibull distribution with a decreasing hazard. For a shape parameter of 1 , it follows the Weibull distribution with a constant hazard, which is equivalent to an exponential distribution. The value of the shape parameter can be specified by historical trial data or literature information.
- Increment for alpha1. It provides a sequence from $\alpha$ to 0.5 by the specified increment for searching the optimal value of $\alpha_{1}$. The default is 0.005 .
- Increment for beta1. It provides a sequence from 0 to $\beta$ by the specified increment for searching the optimal value of $\beta_{1}$. The default is 0.005 .
After hitting the button "Run" with the specifications, the application returns the decision rule in the right-side panel.


## 3. Evaluation of the two-stage design

The proposed method was conducted for the hypothesis testing $H_{0}: \phi=\phi_{0}$ versus $H_{a}: \phi=\phi_{1}$, where $\phi_{0}$ denotes the null median survival time and $\phi_{1}$ denotes the alternative median survival time. We set the type I error rate $\alpha=0.05$ and the type II error rate $\beta=0.2$ for the hypothesis testing. Design parameters such as $\alpha_{1}$ and $\beta_{1}$ were numerically searched with the increment 0.005 over the $\alpha_{1}$ between 0.05 and 0.5 and $\beta_{1}$ between 0.001 and 0.199 . Using formulas (1)-(4), the required sample sizes $n_{1}$ and $n_{2}$ for stages 1 and 2 , respectively, and

## Sample Size Determination for Two-stage Design based on Medain Event Time Test



## Decision rule:

At an interim, based on 27 subjects, if the observed median event time is smaller than or equal to 11.701 months, we stop the study for futility. Otherwise, we additionally accrue 47 patients for the second stage.

At the end of trial, based on all 74 subjects, if the observed median event time is larger than 12.759 months, we reject a null hypothesis.

Fig. 2. METTSS shiny application to determine sample size and threshold for the two-stage design based on METT.

Table 1
Decision rules for two-stage design considering hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months based on target error rates of $\alpha=0.05$ and $\beta=0.2$.

| $\phi_{0}$ | $\phi_{1}$ | Decision rule |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
|  |  | $n_{1}$ | $t_{1}$ | $n_{2}$ | $t_{2}$ | $t^{*}$ |  |  |
| 3 | 5 | 28 | 3.501 | 54 | 3.786 | 4.073 |  |  |
| 3 | 6 | 17 | 3.692 | 29 | 4.050 | 4.453 |  |  |
| 3 | 7 | 13 | 4.219 | 26 | 4.140 | 4.780 |  |  |
| 8 | 14 | 25 | 9.557 | 44 | 10.285 | 11.164 |  |  |
| 8 | 17 | 15 | 10.728 | 33 | 10.740 | 12.245 |  |  |
| 10 | 17 | 27 | 11.701 | 47 | 12.759 | 13.706 |  |  |

the thresholds $t_{1}$ and $t_{2}$ at the interim and final analysis, respectively, are provided in Table 1. For example, when the null median $\phi_{0}$ is 3 months and the alternative median $\phi_{1}$ is 6 months, the proposed method yields $n_{1}=17, t_{1}=3.692, n_{2}=29$, and $t_{2}=4.050$. It implies that the two-stage design accrues $n_{1}=17$ patients for the first stage. At the interim based on data of the first 17 patients, we stop the study for futility if the observed median $Z_{1}$ is less than or equal to the first threshold $t_{1}=3.692$ months. Otherwise, i.e., if $Z_{1}>t_{1}$, we additionally accrue $n_{2}=29$ patients for the second stage, which results in a total sample size of $n=n_{1}+n_{2}=46$ patients. At the end of the trial, based on all $n=46$ patients data, if the observed median $Z_{2}$ is larger than $t_{2}=4.050$ months, the null hypothesis is rejected and we claim that the experimental treatment is sufficiently promising. When $Z_{1}$ is not observed, we continue to the second stage enrolling $n_{2}=29$ patients and we replace the threshold $t_{2}=4.050$ with $t^{*}=4.453$ determined by the formulas (5)-(6) for final analysis.

We measured the computing time to determine the decision rules for the two-stage design and provided it in Table 2. The computing time was measured with Apple M1 Ultra and 128 GB memory. Using the proposed method, the decision rules in Table 1 were obtained very quickly with less than 0.2 s . However, using the numeric search proposed in Park [15], it took several hours to obtain the decision rules in Table 2 of Park [15], which was computationally expensive. Table 2 shows that the proposed method improves the computation to determine the decision rules for the two-stage design.

Table 2
Computing time to determine decision rules for two-stage design considering hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months based on target error rates of $\alpha=0.05$ and $\beta=0.2$.

| $\phi_{0}$ | $\phi_{1}$ | Numeric search <br> $[15]$ | Asymptotic theory <br> (Proposed method) |
| :--- | :--- | :--- | :--- |
| 3 | 5 | 13.50 h | 0.053 s |
| 3 | 6 | 4.17 h | 0.053 s |
| 3 | 7 | 69.91 min | 0.053 s |
| 8 | 14 | 8.39 h | 0.051 s |
| 8 | 17 | 2.82 h | 0.123 s |
| 10 | 17 | 11.92 h | 0.046 s |

We investigated the operating characteristics of the proposed design through simulations. We considered both null and alternative scenarios for each hypothesis testing described in Table 1. Null scenario indicates the case where there is no improvement from the experimental treatment, i.e., the true median for the experimental treatment is the same as the null hypothesized value $\phi_{0}$, and the alternative scenario indicates that experimental treatment improves the median survival time and the true median is the same as the alternative hypothesized value $\phi_{1}$. True survivals were generated from the exponential distribution whose median is the null or alternative hypothesized value described in Table 1 for the null or alternative scenario, respectively. We assumed that patients arrived according to a Poisson process with the accrual rate of 1.04 patients per month, and we continued follow-up for 24 months after the last patient was enrolled. We replicated 10000 times to obtain simulation results such as the rejection probability (denoted by $\hat{\alpha}$ or $1-\hat{\beta}$ under null or alternative scenarios, respectively), the expected sample size (denoted by $\mathrm{EN}_{0}$ or $\mathrm{EN}_{a}$ under null or alternative scenarios, respectively), and the probability of early termination (denoted by $\mathrm{PET}_{0}$ or $\mathrm{PET}_{a}$ under null or alternative scenarios, respectively).

Table 3 shows that both type I and II error rates were preserved at the target rates of $\alpha=0.05$ and $\beta=0.2$, respectively. Since the decision rules were derived by the asymptotic method, the overall error rates in Table 1 were not necessary to be the same as the exact test's results. For example, $\hat{\alpha}=0.039$ or $\hat{\beta}=1-0.869=0.131$ was acceptable because they were smaller or equal to the target error rates (or even target error

Table 3
The operating characteristics of the two-stage design considering hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months based on target error rates of $\alpha=0.05$ and $\beta=0.2$. The results of $\hat{\alpha}$ and $1-\hat{\beta}$ have two columns: the results without parenthesis are obtained by using $t_{2}$ at the final analysis while the results in the parenthesis are obtained by using the threshold $t^{*}$ at the final analysis when $Z_{1}$ does not exist at the interim.

| $\phi_{0}$ | $\phi_{1}$ | $\hat{\alpha}$ | $1-\hat{\beta}$ | $\mathrm{EN}_{0}$ | $\mathrm{EN}_{a}$ | $\mathrm{PET}_{0}$ | $\mathrm{PET}_{a}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 5 | $0.043(0.043)$ | $0.864(0.864)$ | 43.6 | 75.9 | 0.711 | 0.112 |
| 3 | 6 | $0.051(0.051)$ | $0.869(0.868)$ | 25.5 | 42.8 | 0.708 | 0.110 |
| 3 | 7 | $0.041(0.039)$ | $0.849(0.844)$ | 18.6 | 35.3 | 0.786 | 0.144 |
| 8 | 14 | $0.046(0.045)$ | $0.841(0.836)$ | 38.0 | 63.0 | 0.705 | 0.137 |
| 8 | 17 | $0.039(0.019)$ | $0.805(0.786)$ | 25.4 | 41.8 | 0.684 | 0.187 |
| 10 | 17 | $0.044(0.042)$ | $0.835(0.827)$ | 41.7 | 67.4 | 0.688 | 0.140 |



Error rate

Fig. 3. The plot of operating characteristics of the proposed design using the threshold $t^{*}$ at the final analysis when the median $Z_{1}$ does not exist at the interim for the testing $\phi_{0}=3$ versus $\phi_{1}=6$ months. The expected sample size EN is obtained under the null scenario (i.e., $E N_{0}$ ). The left panel shows the result when the follow-up time is varied, and the right panel shows the results when the accrual rate is varied.
rates plus some reasonable margin bound). We also observed that $\mathrm{EN}_{0}$ is much smaller than the total planned number of patients, i.e., $n=n_{1}+n_{2}$, and $\mathrm{EN}_{a}$ is close to $n$. Specifically, the two-stage design used on average $53.5 \%$ of the total sample size when the drug is not good enough, which saves a lot of patients treated with subtherapeutic drugs and could avoid experiencing unnecessary adverse events. This resulted from the effective futility monitoring to stop the trial earlier when the drug is not good enough. As seen in Table 3, $\mathrm{PET}_{0}$ is actually relatively higher to stop the trial earlier. On average, the probability of early stopping when the drug is not good enough is 0.714 . It implies that our futility monitoring works well across all scenarios.

As sensitivity analyses, we evaluated the sensitivity of the proposed two-stage design to follow-up time and accrual rate. Back to the setting of the second scenario in Table 1, i.e., $H_{0}: \phi_{0}=3$ and $H_{a}: \phi_{1}=6$, we varied the follow-up time $6,9,12,15,18,21,24,27,30,36$ months and accrual rate $0.5,0.7,0.9,1.04,1.1,1.3,1.5,1.7,2,2.5,3,4,5,6,7,8,9,10,11$, $13,15,17$ patients per month to reflect the different clinic situations. We replicated 10,000 times and reported the operating characteristics, such as $\mathrm{EN}_{0}$, $\hat{\alpha}$, and $\hat{\beta}$, of the two-stage design using the identified decision rule with $n_{1}=17, t_{1}=3.692, n_{2}=29, t_{2}=4.050$, and $t^{*}=4.453$. The results are summarized in Fig. 3. It shows that the proposed twostage design is robust to the follow-up time but impacted by the accrual rate. When the accrual rate is high (i.e., patients are accrued quickly), it is less likely to observe the median at interim because of few events, and we do not perform an early stopping test but continue the trial to the second stage. Specifically, in the figure, we observed that the black solid line indicating the expected sample size under the null hypothesis suddenly increases when the accrual rate is higher than 2 patients per month and then reaches the close value to the total sample size of 46. As described in Fig. 3, even though the proposed design was impacted by the accrual rate, error rates are generally controlled in most cases. Specifically, when we varied the accrual rate, the overall type I error rate ranges from 0.0166 to 0.0496 , and the overall type II error rate ranges from 0.105 to 0.219 .

## Further investigations for accrual distributions and true survival distributions

As a further investigation of the proposed method, we considered three different accrual distributions from Table 3: (1) uniform accrual which ranges from 0 to 3 (2) nonhomogeneous Poisson process with slow accrual rate (i.e., 0.5 patients per month) at the first stage and then fast accrual rate (i.e., 1.5 patients per month) at the second stage (3) nonhomogeneous Poisson process with fast accrual rate (i.e., 3 patients per month) at the first stage and slow accrual rate (i.e., 0.5 patients per month) at the second stage. Using the decision rules in Tables 1 and 4 shows the simulation results with the different accrual distributions. We included the results of Table 3 assuming the homogeneous accrual rate following the Poisson process for the comparison of the performance. We observed that the proposed method works well to preserve the error rates at the target rates regardless of the accrual distributions.

We learned several lessons from the simulation studies. As seen in Table 4, the proposed design using the decision rule ( $n_{1}, t_{1}, n_{2}, t_{2}$ ) works well once the median survival exists at the interim. However, when the median survival does not exist at the interim, the proposed design using the practical consideration, i.e., ( $n_{1}, t_{1}, n_{2}, t_{2}, t^{*}$ ), works well. For example, suppose that we consider the hypothesis test of the null median of 8 months versus the alternative median of 17 months, and the accrual rate is fast in the first stage and then low in the second stage. We could have trouble observing the median survival at interim because of the fast accrual rate, and we observed the overall type I error rate of 0.057 when $\left(n_{1}, t_{1}, n_{2}, t_{2}\right)$ was used. However, following the practical suggestion using the cutoff $t^{*}$ when the median was not observed at interim, both type I and II error rates are controlled (i.e., $\hat{\alpha}=0.012$ and $\hat{\beta}=0.135$ ). This illustrates how to use the proposed method for the practice. Using the cutoff $t^{*}$ is helpful but it might be stringent to apply to all cases. As seen in Table 3, the rejection probability decreases by replacing $t_{2}$ with $t^{*}$. Therefore, we strongly

Table 4
Simulation results considering different accrual distributions when the hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months is conducted based on target error rates of $\alpha=0.05$ and $\beta=0.2$.

| $\phi_{0}$ | $\phi_{1}$ | $\hat{\alpha}$ | $1-\hat{\beta}$ | $\mathrm{EN}_{0}$ | $\mathrm{EN}_{a}$ | $\mathrm{PET}_{0}$ | $\mathrm{PET}_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Accrual rate follows from homogeneous Poisson process |  |  |  |  |  |
| 3 | 5 | 0.043 | 0.864 | 43.6 | 75.9 | 0.711 | 0.112 |
| 3 | 6 | 0.051 | 0.869 | 25.5 | 42.8 | 0.708 | 0.110 |
| 3 | 7 | 0.041 | 0.849 | 18.6 | 35.3 | 0.786 | 0.144 |
| 8 | 14 | 0.046 | 0.841 | 38.0 | 63.0 | 0.705 | 0.137 |
| 8 | 17 | 0.039 | 0.805 | 25.4 | 41.8 | 0.684 | 0.187 |
| 10 | 17 | 0.044 | 0.835 | 41.7 | 67.4 | 0.688 | 0.140 |
|  |  | Accrual rate follows from uniform distribution |  |  |  |  |  |
| 3 | 5 | 0.046 | 0.855 | 42.7 | 75.6 | 0.728 | 0.118 |
| 3 | 6 | 0.048 | 0.879 | 25.4 | 43.1 | 0.709 | 0.099 |
| 3 | 7 | 0.039 | 0.866 | 18.1 | 35.7 | 0.802 | 0.126 |
| 8 | 14 | 0.047 | 0.857 | 37.4 | 63.8 | 0.719 | 0.118 |
| 8 | 17 | 0.035 | 0.828 | 23.0 | 42.5 | 0.757 | 0.166 |
| 10 | 17 | 0.043 | 0.851 | 40.8 | 68.2 | 0.707 | 0.124 |
|  |  | Accrual rate is slow in the first stage and then fast in the second stage |  |  |  |  |  |
| 3 | 5 | 0.043 | 0.863 | 43.5 | 75.9 | 0.712 | 0.113 |
| 3 | 6 | 0.044 | 0.878 | 25.4 | 43.0 | 0.709 | 0.103 |
| 3 | 7 | 0.040 | 0.866 | 18.3 | 35.7 | 0.794 | 0.126 |
| 8 | 14 | 0.045 | 0.855 | 37.4 | 63.7 | 0.718 | 0.121 |
| 8 | 17 | 0.039 | 0.830 | 22.9 | 42.6 | 0.759 | 0.163 |
| 10 | 17 | 0.049 | 0.854 | 41.1 | 68.3 | 0.699 | 0.121 |


|  |  | Accrual rate is fast in the first stage and then |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | slow in the second stage |  |  |  |  |
| 3 | 5 | 0.048 | 0.849 | 43.7 | 75.3 | 0.710 | 0.124 |
| 3 | 6 | 0.049 | 0.854 | 25.9 | 42.3 | 0.693 | 0.127 |
| 3 | 7 | 0.039 | 0.844 | 20.4 | 35.1 | 0.714 | 0.149 |
| 8 | 14 | 0.049 | 0.822 | 45.1 | 62.5 | 0.544 | 0.149 |
| 8 | 17 | 0.057 | 0.909 | 37.0 | 45.5 | 0.334 | 0.076 |
| 8 | $17^{\text {a }}$ | 0.012 | 0.865 | 37.5 | 45.4 | 0.318 | 0.078 |
| 10 | 17 | 0.049 | 0.838 | 52.9 | 68.0 | 0.450 | 0.129 |

${ }^{\text {a }}$ Obtained from using practical consideration with $t^{*}$ instead of $t_{2}$ when the median $Z_{1}$ does not exist at the interim.
suggest biostatisticians run simulations with the determined decision rules, $\left(n_{1}, t_{1}, n_{2}, t_{2}\right)$ and ( $\left.n_{1}, t_{1}, n_{2}, t_{2}, t^{*}\right)$, to ensure that which decision rule is appropriate for the given accrual rate, hypothetical values, and target errors.

The proposed method is applicable to any parametric survival distribution with the density function $f(y)$ in order to determine the decision rules using (1)-(4). We considered non-exponential survival assumptions such as uniform survivals or Weibull survivals with an increasing hazard function. Using the simulation setting used for Table 3, the simulation results are provided in Table 5. The proposed method works well to control error rates regardless of the parametric distribution of the survivals.

## Comparison with existing methods

Lastly, we compared the proposed two-stage design (called METTSS) with existing two-stage designs such as (1) a restricted KJ design, r-KJ, which tests for survival curves based on one-sample log-rank test [11]; (2) a two-stage design minimizing expected sample size called OES [14]; (3) a two-stage design minimizing the expected total study length called OETSL [14]. The r-KJ uses one-sample log-rank test statistics proposed by Kwak and Jung [10], and both OES and OETSL use the normalized Z-statistic to test and determine decision rules at each stage. The methods of $\mathrm{r}-\mathrm{KJ}, \mathrm{OES}$, and OETSL require specifying the clinically meaningful time point. In our simulations, we used 6 months. The null and alternative values of the 6 months survival probability were determined by the survival distribution with hypotheses $\phi_{0}$ and $\phi_{1}$, respectively. For the proposed two-stage design METTSS, we considered three designs assuming exponential, uniform, and Weibull survivals. The decision rules for each parametric survival assumption are present in Tables 1 and 5, and Appendix (See Table 7).

We assumed the accrual rate is 1.04 patients per month and the follow-up time is 24 months.

Table 6 provides the comparison results in terms of $\mathrm{EN}_{0}$ and $\mathrm{PET}_{0}$ for hypothesis tests. Note that r-KJ does not restrict to certain survival distributions but specifies the hazard ratio and 6-month survival probability from the exponential distribution. Both OES and OETSL assume the Weibull survivals. In most cases, METTSSW used a smaller expected sample size and stopped the trial early for futility when therapeutic intervention is not effective. We observed that METTSSW, OES, and OETSL yielded very similar values of $\mathrm{EN}_{0}$, but METTSSW led to smaller $\mathrm{PET}_{0}$ than OETSL. Specifically, when $\phi_{0}=3$, OETSL stopped the trial early with an average of $74.7 \%$ while METTSSW led to smaller $\mathrm{PET}_{0}$ of $55 \%$ on average. This is because METTSS designs passed the interim monitoring when the median did not exist, which could lead to smaller $\mathrm{PET}_{0}$ when the true median is large enough and the sample size is small. So, the quantitative difference in $\mathrm{PET}_{0}$ is not much meaningful to compare. We also found that the benefit of using METTSS becomes greatly increasing as the null median is large. Under METTSS assuming exponential survivals, on average 29.2, 35.2, 73.7 patients were expected to be tested for $\phi_{0}=3,6,10$, respectively; under METTSS assuming uniform survivals, on average 14.3, 18.8, 38.1 patients were expected to be tested for $\phi_{0}=3,6,10$, respectively; under METTSSW, on average 8.8, 11.7, 22.7 patients were expected to be tested for $\phi_{0}=3,6,10$, respectively; under $\mathrm{r}-\mathrm{KJ}$, on average 20.0, 39.4, 122.5 patients were expected to be tested for $\phi_{0}=3,6,10$, respectively; under OES, on average 8.4, 19.2, 51.7 patients were expected to be tested for $\phi_{0}=3,6,10$, respectively; and under OETSL, on average 8.7, 19.5, 51.8 patients were expected to be tested for $\phi_{0}=3,6,10$, respectively.

## 4. An example trial

To illustrate the application of the sample size determination for a two-stage design, we consider a study with ClinicalTrials.gov Identifier NCT00871923. The goal of the study was to determine the effect of combining whole brain radiation therapy with Tarceva (Erlotinib hydrochloride) in patients with brain metastases from Non-Small Cell Lung Cancer. The historical controls are patients who have not met criteria (1) Karnofsky Performance Status Scale (KPS) $<70$ with an average survival of 2.3 months or (2) KPS $>70$ and age $<65$ years to live approximately 7.1 months. They have an average survival of 4.2 months, which indicates a median survival of 2.9 months. Investigators assumed that survival times are exponentially distributed and the therapeutic approach will improve the mean survival by $43 \%$ to 6 months, which indicates a median survival of 4.2 months. The study started on March 26, 2009, and was completed to collect final data for the primary outcome measure on December 4, 2019. The study results in ClinicalTrials.gov show a median survival of 11.8 months with a $95 \%$ confidence interval $(7.4,19.1)$ based on 40 patients treated with Tarceva after radiation therapy completed.

We redesigned the trial with a two-stage design using METT. Using our shiny application with $\phi_{0}=2.9$ and $\phi_{1}=11.8$, we obtained $n_{1}=6$, $t_{1}=5.556, n_{2}=19$, and $t_{2}=4.276$ based on type I error rate of 0.05 and type II error rate of 0.2 . At the interim, based on 6 subjects, if the observed median event time is smaller than or equal to 5.556 months, we stop the study for futility. Otherwise, we additionally accrue 19 patients for the second stage. At the end of the trial, based on all 25 subjects, if the observed median event time is larger than 4.276 months, we reject a null hypothesis.

The study NCT00871923 was designed with a uniform accrual rate of 1.7 patients per month and a 9 -month follow-up. Using the information on the accrual rate and follow-up, we conducted simulations to evaluate the two-stage design. It led to the overall type I error rate of 0.027 and power of $82.4 \%$ to detect the improvement of median survivals from the intervention.

Table 5
Decision rules and operating characteristics of two-stage design when survivals are assumed to follow non-exponential distributions. The hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months is conducted based on target error rates of $\alpha=0.05$ and $\beta=0.2$.

|  |  | $\phi_{0}$ |  | $\phi_{1}$ |  | $\underline{\text { Decision rule }}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $n_{1}$ |  | $t_{1}$ |  | $n_{2}$ |  | $t_{2}$ |  | $t^{*}$ |
| Uniform |  | 3 |  | 5 |  | 14 |  | 3.432 |  | 22 |  | 3.822 |  | 4.077 |
|  |  | 3 |  | 6 |  | 8 |  | 3.666 |  | 14 |  | 4.052 |  | 4.424 |
|  |  | 3 |  | 7 |  | 6 |  | 4.169 |  | 13 |  | 4.132 |  | 4.745 |
|  |  | 8 |  | 14 |  | 12 |  | 9.558 |  | 21 |  | 10.291 |  | 11.102 |
|  |  | 8 |  | 17 |  | 7 |  | 10.491 |  | 15 |  | 10.805 |  | 12.161 |
|  |  | 10 |  | 17 |  | 13 |  | 11.914 |  | 25 |  | 12.668 |  | 13.678 |
| Weibull |  | 3 |  | 5 |  | 7 |  | 3.465 |  | 13 |  | 3.796 |  | 4.073 |
|  |  | 3 |  | 6 |  | 4 |  | 3.854 |  | 10 |  | 3.951 |  | 4.453 |
|  |  | 3 |  | 7 |  | 3 |  | 3.986 |  | 6 |  | 4.187 |  | 4.780 |
|  |  | 8 |  | 14 |  | 6 |  | 9.552 |  | 12 |  | 10.237 |  | 11.164 |
|  |  | 8 |  | 17 |  | 4 |  | 10.641 |  | 7 |  | 10.862 |  | 12.245 |
|  |  | 10 |  | 17 |  | 7 |  | 11.671 |  | 11 |  | 12.797 |  | 13.577 |
|  | $\phi_{0}$ |  | $\phi_{1}$ |  | Operat | ing | acteris | istics |  |  |  |  |  |  |
|  |  |  |  |  | $\hat{\alpha}$ |  | 1- $\hat{\beta}$ |  | $\mathrm{EN}_{0}$ |  | $\mathrm{EN}_{a}$ |  | $\mathrm{PET}_{0}$ | $\mathrm{PET}_{a}$ |
| Uniform | 3 |  | 5 |  | 0.033 |  | 0.821 |  | 20.7 |  | 32.9 |  | 0.693 | 0.139 |
|  | 3 |  | 6 |  | 0.029 |  | 0.803 |  | 12.4 |  | 19.6 |  | 0.688 | 0.171 |
|  | 3 |  | 7 |  | 0.024 |  | 0.787 |  | 9.8 |  | 16.4 |  | 0.71 | 0.202 |
|  | 8 |  | 14 |  | 0.031 |  | 0.786 |  | 20.5 |  | 29.2 |  | 0.595 | 0.180 |
|  | 8 |  | 17 |  | 0.032 |  | 0.851 |  | 16.8 |  | 20.1 |  | 0.350 | 0.124 |
|  | 10 |  | 17 |  | 0.030 |  | 0.788 |  | 24.5 |  | 33.5 |  | 0.541 | 0.180 |
| Weibull | 3 |  | 5 |  | 0.036 |  | 0.804 |  | 11.6 |  | 17.9 |  | 0.644 | 0.165 |
|  | 3 |  | 6 |  | 0.034 |  | 0.810 |  | 8.4 |  | 12.2 |  | 0.559 | 0.176 |
|  | 3 |  | 7 |  | 0.045 |  | 0.863 |  | 6.3 |  | 8.3 |  | 0.446 | 0.117 |
|  | 8 |  | 14 |  | 0.043 |  | 0.866 |  | 14.6 |  | 16.8 |  | 0.284 | 0.102 |
|  | 8 |  | 17 |  | 0.051 |  | 0.932 |  | 9.8 |  | 10.7 |  | 0.175 | 0.040 |
|  | 10 |  | 17 |  | 0.047 |  | 0.872 |  | 15.3 |  | 17.1 |  | 0.242 | 0.082 |

Table 6
Comparison of the expected sample size and probability of early termination under the null hypothesis with existing designs. The hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months is conducted based on target error rates of $\alpha=0.05$ and $\beta=0.2$. "Exp" indicates an exponential distribution.

| $\phi_{0}$ | $\phi_{1}$ | $\mathrm{EN}_{0}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | METTSS |  |  | r-KJ | OES | OETSL |
|  |  | Exp | Uniform | Weibull |  |  |  |
| 3 | 5 | 43.6 | 20.7 | 11.6 | 29.7 | 9.8 | 10.6 |
| 3 | 6 | 25.5 | 12.4 | 8.4 | 17.6 | 6.9 | 8.4 |
| 3 | 7 | 18.6 | 9.8 | 6.3 | 12.8 | -* | 7.0 |
| 6 | 9 | 66.5 | 32.6 | 18.1 | 74.5 | 27.6 | 27.8 |
| 6 | 11 | 32.6 | 16.7 | 11.0 | 37.5 | 18.5 | 18.8 |
| 6 | 13 | 22.5 | 14.5 | 9.8 | 25.5 | 15.8 | 16.2 |
| 6 | 15 | 19.1 | 11.4 | 7.9 | 19.9 | 14.9 | 15.3 |
| 10 | 13 | 153.8 | 74.6 | 40.5 | 250.0 | -** | -** |
| 10 | 15 | 67.3 | 33.9 | 21.2 | 114.4 | 64.7 | 64.8 |
| 10 | 17 | 41.9 | 24.7 | 15.4 | 72.4 | 48.7 | 48.8 |
| 10 | 19 | 31.8 | 19.3 | 13.5 | 53.2 | 41.6 | 41.7 |
| $\phi_{0}$ | $\phi_{1}$ | $\mathrm{PET}_{0}$ |  |  |  |  |  |
|  |  | METTSS |  |  | r-KJ | OES | OETSL |
|  |  | Exp | Uniform | Weibull |  |  |  |
| 3 | 5 | 0.71 | 0.69 | 0.64 | 0.56 | 0.40 | 0.72 |
| 3 | 6 | 0.71 | 0.69 | 0.56 | 0.54 | 0.18 | 0.76 |
| 3 | 7 | 0.79 | 0.71 | 0.45 | 0.54 | - ${ }^{\text {a }}$ | 0.76 |
| 6 | 9 | 0.66 | 0.69 | 0.55 | 0.57 | 0.63 | 0.70 |
| 6 | 11 | 0.71 | 0.58 | 0.34 | 0.57 | 0.58 | 0.70 |
| 6 | 13 | 0.70 | 0.50 | 0.28 | 0.56 | 0.58 | 0.71 |
| 6 | 15 | 0.68 | 0.36 | 0.19 | 0.55 | 0.56 | 0.71 |
| 10 | 13 | 0.65 | 0.66 | 0.66 | 0.57 | $-^{\text {b }}$ | $-^{\text {b }}$ |
| 10 | 15 | 0.65 | 0.65 | 0.38 | 0.57 | 0.66 | 0.68 |
| 10 | 17 | 0.68 | 0.53 | 0.24 | 0.57 | 0.65 | 0.68 |
| 10 | 19 | 0.70 | 0.36 | 0.15 | 0.59 | 0.65 | 0.69 |

[^1]
## 5. Discussion

The median event time test proposed by Park [15] provides a valuable tool for study developers to determine the required sample size in order to achieve sufficient power (at least $1-\beta$ ) at a significance level of $\alpha$ for hypothesized median survivals. This approach is appealing due to its simplicity in decision-making and the ease of understanding the drug's efficacy based on the median event time. However, the method determines the sample size and threshold through the numerical search, which would require much time to search over the fine grid and be a computational burden. In this article, we addressed this challenge and propose explicit formulas for calculating the sample size and determining the threshold for the decision rules. We evaluated the performance of the proposed method in various clinical scenarios, considering the presence of censoring events to reflect the characteristics of the time-to-event endpoint. Simulation studies demonstrated that the proposed method effectively maintains the overall type I and II error rates and performs favorably compared to existing methods.

The design parameters $\alpha_{1}$ and $\beta_{1}$ are searched over the grids to find the optimal pair minimizing the expected total sample size under the null hypothesis. The searching algorithm can be modified by applying the error spending function such as the O'Brien-Fleming spending function [18] to specify $\beta_{1}$. It means that the expected total sample size becomes a function of $\alpha_{1}$ for the fixed $\beta_{1}$. The idea of using the error spending function allows us to extend the two-stage design to multi-stage clinical trials by splitting the type II error rate $\beta$ into $\beta_{k}, k=1, \ldots, K$, where $K$ indicates the number of analyses and $1-\beta_{k}$ indicates the probability of correct decision at the $k$ th analysis under the alternative hypothesis, i.e., $\sum_{k=1}^{K} \beta_{k}=\beta$. In addition, the idea can also be used to extend the two-stage design used for Phase II to group sequential design for Phase III randomized controlled trials.

Since the proposed method uses the large sample theory of order statistics, it is likely to result in a large maximum sample size. To obtain a smaller maximum sample size, the minimax approach can be

Table 7
Decision rules for two-stage design and operating characteristics for hypothesis testing of median survivals $\phi_{0}$ months versus $\phi_{1}$ months at target error rates of $\alpha=0.05$ and $\beta=0.2$.

|  | $\phi_{0}$ | $\phi_{1}$ | Decision rule |  |  |  | Operating characteristics |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $n_{1}$ | $t_{1}$ | $n_{2}$ | $t_{2}$ | $\hat{\alpha}$ | $1-\hat{\beta}$ | $\mathrm{EN}_{0}$ | $\mathrm{EN}_{a}$ | $\mathrm{PET}_{0}$ | $\mathrm{PET}_{a}$ |
| Exp | 6 | 9 | 43 | 6.581 | 69 | 7.345 | 0.043 | 0.864 | 66.5 | 105.8 | 0.659 | 0.090 |
|  | 6 | 11 | 21 | 7.274 | 40 | 7.823 | 0.042 | 0.843 | 32.6 | 55.6 | 0.710 | 0.136 |
|  | 6 | 13 | 14 | 7.826 | 28 | 8.197 | 0.049 | 0.827 | 22.5 | 37.5 | 0.695 | 0.16 |
|  | 6 | 15 | 11 | 8.650 | 25 | 8.373 | 0.042 | 0.817 | 19.1 | 31.5 | 0.677 | 0.179 |
|  | 10 | 13 | 94 | 10.63 | 173 | 11.45 | 0.047 | 0.846 | 153.8 | 249.1 | 0.654 | 0.104 |
|  | 10 | 15 | 43 | 10.97 | 69 | 12.24 | 0.047 | 0.849 | 67.3 | 105.0 | 0.648 | 0.102 |
|  | 10 | 17 | 27 | 11.70 | 47 | 12.76 | 0.044 | 0.836 | 41.9 | 67.6 | 0.684 | 0.137 |
|  | 10 | 19 | 20 | 12.66 | 39 | 13.09 | 0.043 | 0.814 | 31.8 | 52.2 | 0.696 | 0.174 |
| Uniform | 6 | 9 | 20 | 6.742 | 40 | 7.274 | 0.034 | 0.806 | 32.6 | 53.6 | 0.685 | 0.159 |
|  | 6 | 11 | 10 | 6.914 | 16 | 7.935 | 0.030 | 0.789 | 16.7 | 23.2 | 0.582 | 0.175 |
|  | 6 | 13 | 7 | 8.119 | 15 | 8.104 | 0.026 | 0.815 | 14.5 | 19.5 | 0.497 | 0.166 |
|  | 6 | 15 | 5 | 8.072 | 10 | 8.548 | 0.031 | 0.854 | 11.4 | 13.8 | 0.361 | 0.117 |
|  | 10 | 13 | 44 | 10.71 | 91 | 11.42 | 0.040 | 0.813 | 74.6 | 122.2 | 0.664 | 0.141 |
|  | 10 | 15 | 20 | 11.24 | 40 | 12.12 | 0.036 | 0.780 | 33.9 | 52.6 | 0.652 | 0.185 |
|  | 10 | 17 | 13 | 11.91 | 25 | 12.67 | 0.034 | 0.793 | 24.7 | 33.6 | 0.531 | 0.177 |
|  | 10 | 19 | 9 | 11.89 | 16 | 13.29 | 0.035 | 0.825 | 19.3 | 22.9 | 0.356 | 0.131 |
| Weibull | 6 | 9 | 10 | 6.491 | 18 | 7.345 | 0.039 | 0.783 | 18.1 | 24.9 | 0.552 | 0.170 |
|  | 6 | 11 | 5 | 6.932 | 9 | 7.903 | 0.042 | 0.85 | 11.0 | 13.0 | 0.337 | 0.112 |
|  | 6 | 13 | 4 | 8.438 | 8 | 8.055 | 0.043 | 0.916 | 9.8 | 11.4 | 0.281 | 0.071 |
|  | 6 | 15 | 3 | 8.936 | 6 | 8.373 | 0.050 | 0.943 | 7.9 | 8.7 | 0.191 | 0.042 |
|  | 10 | 13 | 24 | 10.79 | 48 | 11.40 | 0.036 | 0.790 | 40.5 | 63.8 | 0.656 | 0.171 |
|  | 10 | 15 | 10 | 10.82 | 18 | 12.24 | 0.042 | 0.798 | 21.2 | 25.4 | 0.377 | 0.147 |
|  | 10 | 17 | 7 | 11.67 | 11 | 12.80 | 0.047 | 0.868 | 15.4 | 17.0 | 0.237 | 0.088 |
|  | 10 | 19 | 5 | 12.66 | 10 | 13.06 | 0.047 | 0.925 | 13.5 | 14.5 | 0.152 | 0.046 |

considered, but it would require a constraint because the maximum sample size depending on $\beta_{2}$ is attained when $\beta_{2}$ goes to $\beta$ (i.e., $\beta_{1}$ goes to 0 ).

As a future study, it is of interest to investigate the decision rule of the METT describing in terms of the number of events rather than the number of patients. This would address the practical issue of not being able to observe the median at the interim. Recently, Kundu et al. [19] provides the variance of the sample median based on a certain number of events for Weibull and exponential survivals. The idea of Kundu et al. [19] can be applied to improve the proposed monitoring rule. It is also interesting to extend the median event time test in the Bayesian framework and apply it to the two-stage adaptive clinical trials.

## Supplementary materials

R codes are provided at https://github.com/funnypyh/METTSS and a shiny application is available at https://yeonhee.shinyapps.io/METT SS/.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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## Appendix A. Derivation of sample size determination

The details of the derivation of (1)-(4) are given as follows. By the definition of $\alpha_{1}$ and $\beta_{1}$, we have $\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{0}\right)=1-\alpha_{1}$ and $\operatorname{Pr}\left(Z_{1}>t_{1} \mid H_{a}\right)=1-\beta_{1}$. Using the asymptotic normality of $Z_{1}$ under the null hypothesis, we have
$\frac{t_{1}-\phi_{0}}{\sqrt{\frac{0.25}{n_{1}\left\{f\left(\phi_{0}\right)\right\}^{2}}}}=z_{\alpha_{1}}$,
which implies Eq. (1). Similarly, using the asymptotic normality of $Z_{1}$ under the alternative hypothesis, we have
$\frac{t_{1}-\phi_{1}}{\sqrt{\frac{0.25}{n_{1}\left\{f\left(\phi_{1}\right)\right\}^{2}}}}=-z_{\beta_{1}}$,
which implies that
$t_{1}=\frac{-0.5 z_{\beta_{1}}}{\sqrt{n_{1}} f\left(\phi_{1}\right)}+\phi_{1}$.
From Eqs. (1) and (7), we have
$\frac{0.5 z_{\alpha_{1}}}{\sqrt{n_{1}} f\left(\phi_{0}\right)}+\phi_{0}=\frac{-0.5 z_{\beta_{1}}}{\sqrt{n_{1}} f\left(\phi_{1}\right)}+\phi_{1}$.
We solve the last equation for the sample size $n_{1}$. Thus, we obtain the formula in (2). The derivation of (3) and (4) is the same as the derivation of (1) and (2). It uses the fact that $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{0}\right)=\alpha$ and $\operatorname{Pr}\left(Z_{2}>t_{2} \mid H_{a}\right)-\operatorname{Pr}\left(Z_{1} \leq t_{1} \mid H_{a}\right)=1-\beta$.

## Appendix B. Decision rules of METT for Table 6

We provided in Table 7 the decision rule of the proposed two-stage design for the hypothesis tests we considered in the comparison. We also reported the operating characteristics for the different parametric assumptions of the survivals such as exponential, uniform, and Weibull distribution. Note that the decision rule and operating characteristics of the two-stage designs for the first three hypothesis testings in Table 6 are described in Tables 1 and 5 .

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[^1]:    ${ }^{\text {a }}$ Not applicable because all patients can be accrued before the event time.
    b Not applicable because sample size exceeds specified accrual rates/time.

