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Research article

Research on cross-platform dissemination of rational and irrational information based on a two-layer network

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ABSTRACT

Numerous social media platforms have evolved into fertile grounds for the proliferation of irrational information, expanding the avenues of information dissemination. This paper initially utilized the Weibo and Bilibili social platforms as exemplars and devised a cross-platform two-layer network *SEIaIbR-FXYaYbZ* dissemination model grounded in classical infectious disease models. Subsequently, this research computed the model equilibrium point, basic reproduction number, and information entropy through dynamic equations. Finally, the model equations were fitted to real cases to determine optimal parameter solutions and conduct simulation analysis. The simulation results reveal that: (i) information entropy values on both platforms are low, with irrational information predominantly influencing public opinion; (ii) concerning various types of information, the augmentation of rational information results in a reduction of irrational information, while the quantity of rational information remains largely unaffected by changes in the quantity of irrational information; (iii) examining different platforms for information dissemination, alterations in the circulation rate and quantity of rational information on the Weibo platform impact the quantity of rational and irrational information on the Bilibili platform, while those changes on the Bilibili platform exert minimal influence on public opinion information on the Weibo platform. The results and corresponding strategies obtained from this study on the cross-platform dissemination of rational and irrational information on Weibo and Bilibili can provide a reference for relevant departments to guide the rational development of online information and enhance the effective management of public opinion in social media platforms.

1. Introduction

With the continuous evolution of new media platforms, the same network users have the ability to register accounts on various social network platforms. Consequently, during the emergence of public opinion, as information disseminators traverse different social network platforms, pertinent information disseminates across these diverse social platforms. For example, when an opinion event transpires on the Weibo platform, after being exposed to the information on Weibo, network users are likely to post related information on other social network platforms such as Facebook, TikTok, and Bilibili. This substantially expedites the circulation and permeation of information. Subsequently, the information propagated regarding the same public opinion event ceases to be a singular entity; instead,

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diverse types of information emerge within the social network platform. These may engage in a “cooperative” or “competitive” relationship with each other. Rational and irrational information, as a pair of “competitive” relationships, often appear simultaneously in online public opinion. However, when sensitive societal issues such as educational resources, healthcare, and wealth disparity erupt on the internet, forming negative public opinion events, public sentiment is prone to extreme shifts, leading to the generalization of opposing emotions and subsequently sparking confrontational opinion struggles [1]. Coupled with the anonymity of the internet, imperfect public opinion monitoring mechanisms, and the commercialization of online media, the expression of irrational information in online public opinion becomes highly prevalent, resulting in the formidable discursive pressure of irrational public opinion likely overshadowing some rational voices [2]. Irrational information encompasses outcomes of events or phenomena that cannot be articulated through logical concepts. It arises from the subconscious, illogical, unorganized feelings, imagination, and speculations of the informant [3]. Consequently, irrational information fails to get rid of the drawback of the informant’s one-sided and unfounded subjective conjectures, often manifesting as online rumors, infringement, and violence. In comparison to rational information, irrational information is more prone to contribute to group polarization in the network, possesses greater potency in dissemination, and poses a threat to social development. The dissemination of public opinion on the internet has increasingly become a major concern for social order, economic development, and even national security. Investigating the dissemination mechanisms and evolutionary patterns of rational and irrational information across various social media platforms holds significant practical importance in guiding the rational development of public opinion.

The term “irrational” information is frequently encountered in the literature, yet its definition remains somewhat fragmented, exhibiting varied connotations in different contexts. However, most scholars define irrational information in the context of its illogical and emotional characteristics. For example, Wang et al. [4] analyzed the connotation of irrational information in social media platforms, suggesting that irrational information represents opinion information expressed through an “irrational” behavior on the internet. This “irrational” behavior makes it challenging for the public to engage in in-depth critical thinking about information, making them more susceptible to control by emotional, volitional, and other irrational factors, resulting in behaviors characterized by emotionalization and lack of logic. Furthermore, other scholars have delved into the causes, effects, and governance strategies related to the dissemination of irrational information. Hao et al. [5] investigated the underlying patterns of meme generation behind the expression of irrational information on the internet, employing the four-stage theory of memes. Qu et al. [6] suggested that the irrational expression of information on the internet, moderated by a “weak force field” does not inevitably lead to social disorder. However, this “non-inevitable” social disorder remains unpredictable due to the constraints of the “weak force field”. Huang et al. [7] conducted content analysis to scrutinize the primary forms, common symbols of irrational expressions, and the types and characteristics of irrational interactions, proposing classification as a means of supervision and control. Li et al. [8], by categorizing the three evolutionary stages of the germination, outbreak, and elimination of the spread of irrational expressions, analyzed the pros and cons of different blocking approaches and explored methods and strategies for impeding the dissemination of irrational information.

The dissemination of information in the network bears similarities to the spread of infectious diseases in populations, prompting numerous scholars to investigate the dissemination pattern of network information through the lens of infectious disease dynamics theory. In 1965, Daley and Kendall [9] formulated the classic *DK* model, categorizing the population in rumor information into three groups, drawing inspiration from the Susceptible-Infected-Removed (*SIR*) infectious disease model. In 1973, Maki and Thompson [10] refined the dissemination rules of the *DK* model and introduced the *MK* model. Geng et al. [11] examined online public opinion and considered the influence of media intervention on the trend of public opinion dissemination from different perspectives. Based on the *SEIR* model, they built an online public opinion communication model under the dual role of the online and government media. Li et al. [12] constructed the *HK-SEIR* model of public opinion evolution, which combined the *HK* model of public opinion fusion and the *SEIR* model of epidemic transmission. In the *HK-SEIR* model, factors affecting public opinion dissemination were integrated, which made the model more consistent with the evolution law of real public opinion than *SIR* And *SEIR* models. Furthermore, researchers investigating information dissemination have categorized dissemination nodes based on the variety of node user states, psychology, and types of disseminated information, aligning with real network dissemination scenarios. In their exploration of a two-stage rumor reversal model for public emergencies, Jiang et al. [13] constructed the *SPNR* dissemination model, and divided disseminators into negative dissemination nodes, positive dissemination nodes that know the truth, and positive dissemination nodes that do not know the truth. Ding et al. [14] proposed a framework to describe rumor diffusion and refutation strategies and modeled rumor diffusion and refutation at the individual level from the perspective of competitive innovation diffusion. Based on an in-depth analysis of the psychological characteristics of the subjects, the disseminators were divided into three states: Rebuttal-infected, Rebuttal-forgetting, and Rumor-infected. Wang et al. [15] considered the existence of competition between original and derived topics in Weibo public opinion based on the improved *SEIR* model and classified different disseminators according to the dissemination of different topics.

As research progressed, scholars delved into the study of dissemination behavior on multilayer networks, considering the numerous interconnections and interactions between different networks in real life [16–21]. For example, Wang et al. [22] constructed a two-layer network model comprising a physical contact layer for epidemics, and a virtual contact layer for awareness diffusion, to explore the effects of coupled disease awareness dynamics in multiple networks. Liu et al. [23] proposed an opinion diffusion model to depict the opinion diffusion process in a two-layer network with a verbal dissemination layer and a social platform sharing layer. Zhang et al. [24] introduced a two-layer coupled *SEIR* network dissemination model, categorized into the original topic layer and the derived topic layer based on different topics generated by the opinion. They utilized the control variable method to analyze the effects of social conditioning time and topic derivation rate on the dissemination process.

In summary, most current studies on rational and irrational information in online public opinion are theoretical and qualitative. Few utilize dynamic models to investigate the evolution mechanism and dissemination process of rational and irrational information. Although studies on information dissemination consider the interactions between different networks, few scholars have employed different social media platforms as the basis for a hierarchical two-layer network. In this paper, we will construct a two-layer network dissemination model based on rational and irrational information for the Weibo platform layer and the Bilibili platform layer. We will calculate the model equilibrium point, basic reproduction number, and information entropy using dynamic equations, fit the model equations with real cases to find the optimal parameters, and conduct simulation experiments to investigate the dissemination mechanism and evolution process of rational and irrational information among different social media platforms.

The framework of this paper is as follows: Section 2 constructs the *SEIaIbR-FXYaYbZ* cross-platform dissemination model and dynamic equations related to rational and irrational information. Section 3 details the basic reproduction number and information entropy by combining definitions and formulas. In Section 4, an extensive numerical simulation is conducted, where model equations are fitted with real cases to determine optimal parameters. Correlation analysis and sensitivity analysis are performed based on the obtained optimal parameters and the cumulative amount of rational and irrational information released. Finally, in Section 5, we provide a summary of this work.

2. Cross-platform dissemination model of rational and irrational information

Both rational and irrational information wield a significant impact on online public opinion. In order to delve into the dissemination process and evolutionary mechanisms of these two types of information across various social networks, this paper takes Weibo and Bilibili platforms as the conduits for users' cross-platform circulation. It formulates the *SEIaIbR-FXYaYbZ* cross-platform two-layer network dissemination model grounded in rational and irrational information.

2.1. Model description

In the two-layer network dissemination model constructed in this study, one layer represents the Weibo platform network, and the other layer represents the Bilibili platform network. Nodes symbolize network users, and edges between nodes denote social relationships among users. The movement of network users between the two platforms facilitates the cross-platform dissemination of information. Building upon the foundation of epidemic models, users in social media networks are typically classified into different nodes based on differences in information infection states [25–27]. This paper, drawing from Geng's criteria for node classification [28], classifies nodes in the network into five types: the susceptible population (S or F), the exposed population (E or X), rational disseminators (Ia or Ya), irrational disseminators (Ib or Yb), and the recovered population (R or Z), considering variations in user information infection states and the psychological factors of information disseminators. Among them, the susceptible population consists of users who have not been exposed to public opinion information but are susceptible. The exposed population consists of latent people who have been exposed to public opinion information but have not disseminated it. Public opinion disseminators are divided into rational disseminators and irrational disseminators according to the dissemination of different types of information. Those who are exposed to public opinion information and disseminate rational information are rational disseminators, and users who are exposed to public opinion information and disseminate irrational information are irrational disseminators. The recovered population refers to the susceptible population that is directly immune to information, or exposers and disseminators who no longer disseminate information because of forgetfulness or disinterest. The specific node status descriptions of the Weibo and Bilibili platforms are shown in Table 1 and Table 2. Each category of nodes identified in the networks of Weibo and Bilibili platforms encompasses a substantial number of ordinary users, as well as high-influence users such as government official accounts, public opinion media, and key opinion leaders. These high-influence users have the capacity, to some extent, to influence the rational or irrational choices of information dissemination states among ordinary users.

In reality, the dissemination mechanisms of Weibo and Bilibili platforms may exhibit certain differences. However, to investigate the dissemination characteristics of rational and irrational information in cross-platform scenarios, we simplify real-world information propagation issues into a mathematical model. The rules of information dissemination are also set according to the constructed information propagation model. In the two-layer network dissemination model, with the Weibo platform and Bilibili platform as the hierarchy, public opinion information is disseminated within specific platforms and across platforms due to the cross-platform flow of internet users. The rules of information dissemination within and between platforms differ, and the specific dissemination rules are as follows:

As shown in Fig. 1, the specific dissemination rules of public opinion information on the Weibo platform are: (i) the susceptible population (S) is transformed into the exposed population (E) with a probability of α when they come into contact with information, otherwise they are transformed into the recovered population (R) with a probability of γ_1; (ii) the exposed population (E) who contacts the information is transformed into rational disseminators (Ia) with a probability of β_1 and transformed into irrational disseminators

Table 1
Weibo platform user node status illustration.

Node symbols	Node status	Illustration
S	The susceptible population	Users who have not been exposed to information but are susceptible
E	The exposed population	Latent people who have been exposed to public opinion information but have not disseminated it
Ia	Rational disseminators	Users who are exposed to information and disseminate rational information
Ib	Irrational disseminators	Users who are exposed to information and disseminate irrational information
R	The recovered population	Susceptible individuals, exposers, and disseminators who have entered immune status

Table 2
Bilibili platform user node status illustration.

Node symbols	Node status	Illustrations
F	The susceptible population	Users who have not been exposed to information but are susceptible
X	The exposed population	Latent people who have been exposed to public opinion information but have not disseminated it
Ya	Rational disseminators	Users who are exposed to information and disseminate rational information
Yb	Irrational disseminators	Users who are exposed to information and disseminate irrational information
Z	The recovered population	Susceptible individuals, exposers, and disseminators who have entered immune status

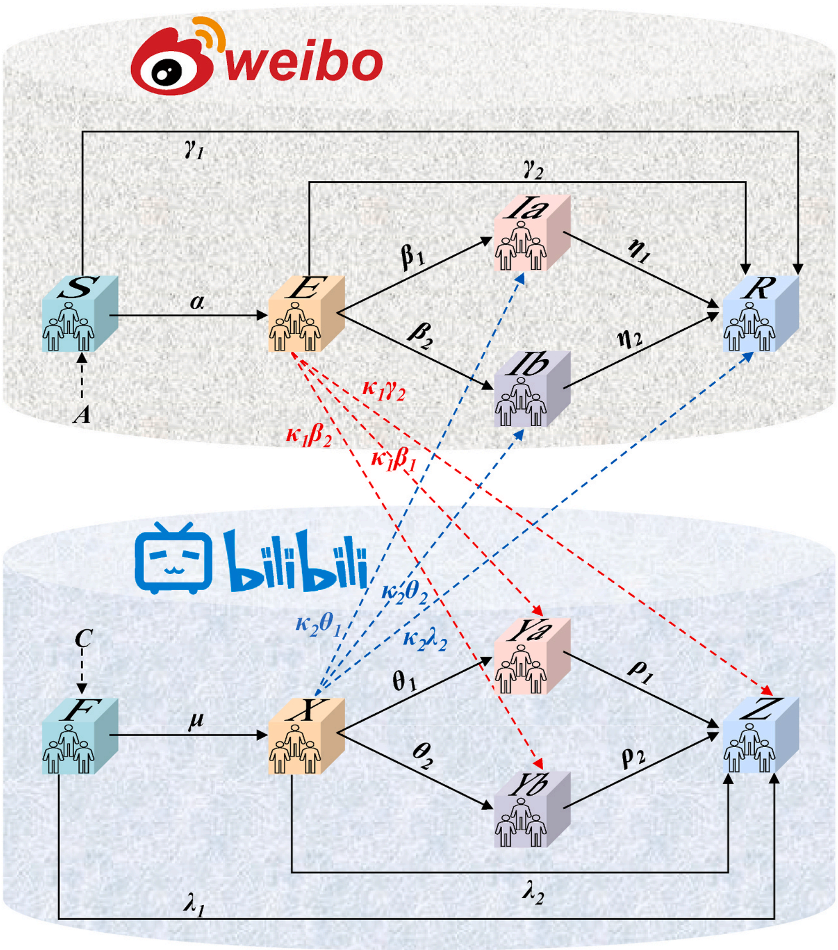

Fig. 1. Cross-platform dissemination model of rational information and irrational information based on a two-layer network. The upper network in the two-layer network is the Weibo social platform, and the lower network is the Bilibili social platform.

(*Ib*) with a probability of β_2, or directly transforms into the recovered population (*R*) with a probability of γ_2 for reasons such as forgetting or not being interested in the disseminated information; (iii) when the rational disseminators (*Ia*) and the irrational disseminators (*Ib*) stop disseminating information, they become the recovered population (*R*) with probabilities η_1 and η_2, respectively. The public opinion information dissemination rules on the Bilibili platform are: (i) the susceptible population (*F*) is transformed into the exposed population (*X*) with a probability of μ when they come into contact with information, otherwise they are transformed into the recovered population (*Z*) with a probability of λ_1; (ii) the exposed population (*X*) who contacts the information is transformed into rational disseminators (*Ya*) with a probability of θ_1 and transformed into irrational disseminators (*Yb*) with a probability of θ_2, or directly transforms into the recovered population (*Z*) with a probability of λ_2; (iii) when disseminators stop disseminating information, rational disseminators (*Ya*) become the recovered population (*Z*) with a probability of ρ_1, irrational disseminators (*Yb*) become the recovered population (*Z*) with a probability of ρ_2.

As shown in Fig. 1, the specific rules of information dissemination across platforms are: (i) social network users exposed to opinion information on the Weibo platform become the exposed population (*E*), post rational information on the Bilibili platform with a probability of $\kappa_1\beta_1$ to become rational disseminators (*Ya*) of Bilibili platform, post irrational information on the Bilibili platform with a of $\kappa_1\beta_2$ to become irrational disseminators (*Yb*) of the Bilibili platform or become the recovered population (*Z*) with a probability of $\kappa_1\gamma_2$ on the Bilibili platform; (ii) social network users exposed to opinion information on the Bilibili platform become the exposed population (*X*), post rational information on the Weibo platform with a probability of $\kappa_2\theta_1$ to become rational disseminators (*Ia*) of the Weibo platform, post irrational information on the Weibo platform with a probability of $\kappa_2\theta_2$ to become irrational disseminators (*Ib*) of the Weibo platform or become the recovered population (*R*) with a probability of $\kappa_2\lambda_2$ on the Weibo platform. κ_1 is the circulation rate of the exposed population flowing from the Weibo platform to the Bilibili platform, $\kappa_1 = \sigma M1/(M1 + M2)$, κ_2 is the circulation rate of the exposed population flowing from the Bilibili platform to the Weibo platform, $\kappa_2 = \sigma M2/(M1 + M2)$, *M1* and *M2* are the quantity of topic discussions on the Weibo platform and Bilibili platform, respectively, when the public opinion event is in a stable state, and σ is the average flow rate of users on all media platforms [29]. The meanings of the specific parameters in the model are described in Tables 3 and 4.

2.2. Dynamic equations construction

With reference to the differential equation model of system dynamics, the change of each user state in the cross-platform propagation model *SEIaIbR-FXYaYbZ* of rational and irrational information is shown in Equation (1).

$$
\begin{cases}
\frac{dS(t)}{dt} = A - \alpha S(t)(Ia(t) + Ib(t)) - \gamma_1 S(t) \\
\frac{dE(t)}{dt} = \alpha S(t)(Ia(t) + Ib(t)) - (\beta_1 + \beta_2 + \gamma_2 + \kappa_1)E(t) \\
\frac{dIa(t)}{dt} = \beta_1 E(t) + \kappa_2\theta_1 X(t) - \eta_1 Ia(t) \\
\frac{dIb(t)}{dt} = \beta_2 E(t) + \kappa_2\theta_2 X(t) - \eta_2 Ib(t) \\
\frac{dR(t)}{dt} = \gamma_1 S(t) + \gamma_2 E(t) + \eta_1 Ia(t) + \eta_2 Ib(t) + \kappa_2\lambda_2 X(t) \\
\frac{dF(t)}{dt} = C - \mu F(t)(Ya(t) + Yb(t)) - \lambda_1 F(t) \\
\frac{dX(t)}{dt} = \mu F(t)(Ya(t) + Yb(t)) - (\theta_1 + \theta_2 + \lambda_2 + \kappa_2)X(t) \\
\frac{dYa(t)}{dt} = \theta_1 X(t) + \kappa_1\beta_1 E(t) - \rho_1 Ya(t) \\
\frac{dYb(t)}{dt} = \theta_2 X(t) + \kappa_1\beta_2 E(t) - \rho_2 Yb(t) \\
\frac{dZ(t)}{dt} = \lambda_1 F(t) + \lambda_2 X(t) + \rho_1 Ya(t) + \rho_2 Yb(t) + \kappa_1\gamma_2 E(t)
\end{cases}
\tag{1}
$$

In Equation (1), $S(t)$, $E(t)$, $Ia(t)$, $Ib(t)$, $R(t)$, $F(t)$, $X(t)$, $Ya(t)$, $Yb(t)$, and $Z(t)$ denote the quantity of the susceptible population (*S*), the exposed population (*E*), rational disseminators (*Ia*), irrational disseminators (*Ib*) and the recovered population (*R*) on the Weibo platform, and the quantity of the susceptible population (*F*), the exposed population (*X*), rational disseminators (*Ya*), irrational disseminators (*Yb*) and the recovered population (*Z*) on the Bilibili platform at time *t*. For any *t*, $S(t) \geq 0$, $E(t) \geq 0$, $Ia(t) \geq 0$, $Ib(t) \geq 0$, $R(t) \geq 0$, $F(t) \geq 0$, $X(t) \geq 0$, $Ya(t) \geq 0$, $Yb(t) \geq 0$, $Z(t) \geq 0$. Except for parameters *A* and *C*, the values of other parameters are between [0,1].

In addition, during the dissemination of public opinion information, besides examining the changes in the status of each user, it is essential to examine the cumulative amount of information related to the current public opinion on the two social network platforms. The formulas for the cumulative amount of the two types of information released on the two platforms are presented in Equation (2) and **(3)**, where $W_1(t)$ is the cumulative amount of rational information released on the Weibo platform, $W_2(t)$ is the cumulative amount of irrational information released on the Weibo platform, $W_3(t)$ is the cumulative amount of rational information released on the

Table 3
Weibo platform parameter symbols and illustrations.

Parameter Symbols	Illustrations
α	The rate at which the susceptible population is converted into the exposed population
β_1	The rate at which the exposed population is converted into rational disseminators
β_2	The rate at which the exposed population is converted into irrational disseminators
η_1	The rate at which rational disseminators are converted into the recovered population
η_2	The rate at which irrational disseminators are converted into the recovered population
γ_1	The rate at which the susceptible population is converted into the recovered population
γ_2	The rate at which the exposed population is converted into the recovered population
κ_1	Circulation rate of users flowing from the Weibo platform to the Bilibili platform
A	The number of newly increased susceptible users within the system

Table 4
Bilibili platform parameter symbols and illustrations.

Parameter Symbols	Illustrations
μ	The rate at which the susceptible population is converted into the exposed population
θ_1	The rate at which the exposed population is converted into rational disseminators
θ_2	The rate at which the exposed population is converted into irrational disseminators
ρ_1	The rate at which rational disseminators are converted into the recovered population
ρ_2	The rate at which irrational disseminators are converted into the recovered population
λ_1	The rate at which the susceptible population is converted into the recovered population
λ_2	The rate at which the exposed population is converted into the recovered population
κ_2	Circulation rate of users flowing from the Bilibili platform to the Weibo platform
C	The number of newly increased susceptible users within the system

Bilibili platform, and $W_4(t)$ is the cumulative amount of irrational information released on the Bilibili platform.

(i) Cumulative amount of rational and irrational information released on the Weibo platform:

$$\begin{cases} W_1(t) = \int_0^t [\beta_1 E(t) + \kappa_2\theta_1 X(t) - \eta_1 Ia(t)]dt \\ W_2(t) = \int_0^t [\beta_2 E(t) + \kappa_2\theta_2 X(t) - \eta_2 Ib(t)]dt \end{cases} \tag{2}$$

(ii) Cumulative amount of rational and irrational information released on the Bilibili platform:

$$\begin{cases} W_3(t) = \int_0^t [\theta_1 X(t) + \kappa_1\beta_1 E(t) - \rho_1 Ya(t)]dt \\ W_4(t) = \int_0^t [\theta_2 X(t) + \kappa_1\beta_2 E(t) - \rho_2 Yb(t)]dt \end{cases} \tag{3}$$

Equation (2) and **(3)** primarily examine cumulative amount of rational and irrational information released on the Weibo and Bilibili platforms from time 0 to t. Taking $W_1(t)$ as an example, $W_1(t)$ represents the cumulative amount of rational information released on the Weibo platform from 0 to t. The amount of rational information released on the Weibo platform primarily arises from the increase in rational information, denoted as $\beta_1 E(t)$, when the exposed population on the Weibo platform transform into rational disseminators on the Weibo platform. Additionally, it is influenced by the increase in rational information, denoted as $\kappa_2\theta_1 X(t)$, when the exposed population on the Bilibili platform transform into rational disseminators on the Weibo platform. Furthermore, the decrease in rational information quantity, represented as $\eta_1 Ia(t)$, when rational disseminators on the Weibo platform transform into the recovered population also impacts the amount of rational information on the Weibo platform.

3. Basic reproduction number and information entropy

3.1. Basic reproduction number

According to the study of Gonalez-Parra et al. [30] regarding dynamic equations in infectious disease models, for computational convenience, the set of **Equation (1)** is simplified to the set of **Equation (4)**. Only the system consisting of the remaining eight equations that do not include equations $\frac{dR(t)}{dt}$ and $\frac{dZ(t)}{dt}$ needs to be considered.

$$\begin{cases} \dfrac{dS(t)}{dt} = A - \alpha S(t)(Ia(t) + Ib(t)) - \gamma_1 S(t) \\ \dfrac{dE(t)}{dt} = \alpha S(t)(Ia(t) + Ib(t)) - (\beta_1 + \beta_2 + \gamma_2 + \kappa_1)E(t) \\ \dfrac{dIa(t)}{dt} = \beta_1 E(t) + \kappa_2\theta_1 X(t) - \eta_1 Ia(t) \\ \dfrac{dIb(t)}{dt} = \beta_2 E(t) + \kappa_2\theta_2 X(t) - \eta_2 Ib(t) \\ \dfrac{dF(t)}{dt} = C - \mu F(t)(Ya(t) + Yb(t)) - \lambda_1 F(t) \\ \dfrac{dX(t)}{dt} = \mu F(t)(Ya(t) + Yb(t)) - (\theta_1 + \theta_2 + \lambda_2 + \kappa_2)X(t) \\ \dfrac{dYa(t)}{dt} = \theta_1 X(t) + \kappa_1\beta_1 E(t) - \rho_1 Ya(t) \\ \dfrac{dYb(t)}{dt} = \theta_2 X(t) + \kappa_1\beta_2 E(t) - \rho_2 Yb(t) \end{cases} \tag{4}$$

Setting each equation in the system of Equation (4) with the left side as zero, solving for unknown variables yields a set of disease-free equilibrium points, denoted as $E_0\left(\frac{A}{\gamma_1}, 0, 0, 0, \frac{C}{\lambda_1}, 0, 0, 0\right)$, representing zero dissemination equilibrium points.

Referring to the definition of basic reproduction number in communication science, the basic reproduction number of the information dissemination model refers to the number of susceptible individuals transformed into disseminators by an information disseminator during the process of information dissemination. It is a crucial parameter for measuring the infectious ability of information dissemination. The threshold value of information dissemination is $R_0 = 1$. When $R_0 < 1$, the quantity of disseminated public opinion information steadily declines until it reaches zero, preventing the spread of public opinion information. When $R_0 > 1$, the quantity of public opinion information exhibits exponential growth, and such information spreads within a certain range. According to the spectral radius method of reproduction matrix, the basic reproduction number R_0 is calculated. Let x=($E(t)$, $X(t)$, $Ia(t)$, $Ib(t)$, $Ya(t)$, $Yb(t)$, $S(t)$, $F(t)$) $^{\mathrm{T}}$, construct equations $G(x)$ and $V(x)$, where $V(x) = V^-(x) - V^+(x)$. Dividing exposed users E, X and disseminators Ia, Ib, Ya, Yb into an infected compartment, where $G(x)$ represents the newly infected individuals added to the infected compartment, $V^-(x)$ is the individuals who move out of the infected compartment, and $V^+(x)$ is the individuals who move into the infected compartment in other ways. Rewriting Equation (4) of the public opinion dissemination model as Equation (5), **(6)**, **(7)**.

$$\frac{dx}{dt} = G(x) - V(x) \tag{5}$$

$$G(x) = \begin{bmatrix} \alpha S(t)(Ia(t) + Ib(t)) \\ \mu F(t)(Ya(t) + Yb(t)) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

$$V(x) = \begin{bmatrix} (\beta_1 + \beta_2 + \gamma_2 + \kappa_1)E(t) \\ (\theta_1 + \theta_2 + \lambda_2 + \kappa_2)X(t) \\ -\beta_1 E(t) - \kappa_2\theta_1 X(t) + \eta_1 Ia(t) \\ -\beta_2 E(t) - \kappa_2\theta_2 X(t) + \eta_2 Ib(t) \\ -\theta_1 X(t) - \kappa_1\beta_1 E(t) + \rho_1 Ya(t) \\ -\theta_2 X(t) - \kappa_1\beta_2 E(t) + \rho_2 Yb(t) \\ -A + \alpha S(t)(Ia(t) + Ib(t)) + \gamma_1 S(t) \\ -C + \mu F(t)(Ya(t) + Yb(t)) + \lambda_1 F(t) \end{bmatrix} \tag{7}$$

When there is no public opinion in the system, and only susceptible users are present in the network, a set of zero dissemination equilibrium points E_0 exists. At this point, the Jacobian matrices of matrix $G(x)$ and matrix $V(x)$ at E_0 are denoted as G_0 and V_0, respectively. The detailed process of solving matrix eigenvalues using the classical Jacobi method is presented in Equation (8), **(9)**, **(10)** and **(11)**.

$$G_0 = \begin{bmatrix} 0 & 0 & \frac{A\alpha}{\gamma_1} & \frac{A\alpha}{\gamma_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{C\mu}{\lambda_1} & \frac{C\mu}{\lambda_1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$V_0 = \begin{bmatrix} \beta_1+\beta_2+\gamma_2+\kappa_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta_1+\theta_2+\lambda_2+\kappa_2 & 0 & 0 & 0 & 0 \\ -\beta_1 & -\kappa_2\theta_1 & \eta_1 & 0 & 0 & 0 \\ -\beta_2 & -\kappa_2\theta_2 & 0 & \eta_2 & 0 & 0 \\ -\kappa_1\beta_1 & -\theta_1 & 0 & 0 & \rho_1 & 0 \\ -\kappa_1\beta_2 & -\theta_2 & 0 & 0 & 0 & \rho_2 \end{bmatrix} \tag{9}$$

$$G_0{V_0}^{-1} = \begin{bmatrix} \frac{A\alpha(\beta_1\eta_2+\beta_2\eta_1)}{(\beta_1+\beta_2+\gamma_2+\kappa_1)\eta_1\eta_2\gamma_1} & \frac{A\alpha\kappa_2(\theta_1\eta_2+\theta_2\eta_1)}{(\theta_1+\theta_2+\lambda_2+\kappa_2)\eta_1\eta_2\gamma_1} & \frac{A\alpha}{\eta_1\gamma_1} & \frac{A\alpha}{\eta_2\gamma_1} & 0 & 0 \\ \frac{C\mu\kappa_1(\beta_1\rho_2+\beta_2\rho_1)}{(\beta_1+\beta_2+\gamma_2+\kappa_1)\rho_1\rho_2\lambda_1} & \frac{C\mu(\theta_1\rho_2+\theta_2\rho_1)}{(\theta_1+\theta_2+\lambda_2+\kappa_2)\rho_1\rho_2\lambda_1} & 0 & 0 & \frac{C\mu}{\rho_1\lambda_1} & \frac{C\mu}{\rho_2\lambda_1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$\left|\varepsilon E - G_0{V_0}^{-1}\right| = 0 = \begin{bmatrix} \varepsilon_1 - \frac{A\alpha(\beta_1\eta_2+\beta_2\eta_1)}{(\beta_1+\beta_2+\gamma_2+\kappa_1)\eta_1\eta_2\gamma_1} & -\frac{A\alpha\kappa_2(\theta_1\eta_2+\theta_2\eta_1)}{(\theta_1+\theta_2+\lambda_2+\kappa_2)\eta_1\eta_2\gamma_1} & -\frac{A\alpha}{\eta_1\gamma_1} & -\frac{A\alpha}{\eta_2\gamma_1} & 0 & 0 \\ -\frac{C\mu\kappa_1(\beta_1\rho_2+\beta_2\rho_1)}{(\beta_1+\beta_2+\gamma_2+\kappa_1)\rho_1\rho_2\lambda_1} & \varepsilon_2 - \frac{C\mu(\theta_1\rho_2+\theta_2\rho_1)}{(\theta_1+\theta_2+\lambda_2+\kappa_2)\rho_1\rho_2\lambda_1} & 0 & 0 & -\frac{C\mu}{\rho_1\lambda_1} & -\frac{C\mu}{\rho_2\lambda_1} \\ 0 & 0 & \varepsilon_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_6 \end{bmatrix} \tag{11}$$

In Equation (11), where $\varepsilon = \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, the symbols $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5$ and ε_6 represent the six eigenvalues of matrix $G_0V_0^{-1}$, respectively.

The solution of the eigenvalue ε has been solved by Matlab software, but since the form of the solution is too complicated, the basic reproduction number is expressed as: $R_0 = \max\{\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \varepsilon_4 \quad \varepsilon_5 \quad \varepsilon_6\}$.

3.2. Information entropy

Information entropy is a crucial concept in data mining and machine learning. In machine learning, entropy calculations are grounded in sample data, often termed empirical entropy. Information entropy is an abstract notion utilized to gauge the level of certainty, orderliness or chaos within information, or to assess the value of information. In this paper, we employ the concept of information entropy to assess the degree of chaos and certainty in the dissemination of rational and irrational information within the system. Greater information entropy indicates higher chaos within the system, signifying the presence of a substantial amount of both rational and irrational information, with roughly equal probabilities of occurrence. Conversely, lower information entropy implies greater orderliness within the system, where either rational or irrational information predominates, leading to reduced information uncertainty.

In accordance with the defining equation of information entropy, $P\{X = x_i\} = p_i (i = 1, 2, 3, \ldots, m), p_i \geq 0, \sum_{i=1}^{m} p_i = 1$ represents the probability distribution of the discrete random variable X, and the formula to quantify information entropy is presented in Equation (12):

$$H(X) = H(p_1, p_2, p_3, \ldots, p_m) = -\sum_{i=1}^{m} p_i \log p_i \tag{12}$$

Referring to the formulas defined above and combining them with the requirements of this paper, the information entropy is defined in Equation (13) and **(14)**:

$$H^1(X) = H^1(p_1, p_2) = -(p_1 \log p_1 + p_2 \log p_2), p_1 = \frac{W_1(t)}{W_1(t) + W_2(t)}, p_2 = \frac{W_2(t)}{W_1(t) + W_2(t)} \tag{13}$$

$$H^2(X) = H^2(p_3, p_4) = -(p_3 \log p_3 + p_4 \log p_4), p_3 = \frac{W_3(t)}{W_3(t) + W_4(t)}, p_4 = \frac{W_4(t)}{W_3(t) + W_4(t)} \tag{14}$$

In Equation (13) and **(14)**, p_1 and p_2 denote the probability of the appearance of rational and irrational information on the Weibo platform, respectively; p_3 and p_4 denote the probability of the appearance of rational and irrational information on the Bilibili platform, respectively. $W_1(t)$ and $W_2(t)$ denote the cumulative amount of rational information and irrational information released on the Weibo platform, respectively; $W_3(t)$ and $W_4(t)$ denote the cumulative amount of rational information and irrational information released on the Bilibili platform, respectively. The expressions for $W_1(t)$, $W_2(t)$, $W_3(t)$, $W_4(t)$ are given in Equation (2) and **(3)**. $H^1(X)$ denotes the information entropy of the Weibo platform and $H^2(X)$ denotes the information entropy of the Bilibili platform.

4. Numerical simulations

This paper utilized the trending hashtag "Boy died after raising his hand 7 times when he was unwell in class" as a case study. The topic falls within the category of social news. Following the boy's mother's post, multiple media outlets covered the incident, triggering a wave of discussions on various online media platforms. But, in this incident, the parents lost the first trial. There was a lot of missing information and evidence relying solely on the parents' narration. The determination of whether the school and teachers bear responsibility and the specific details of the incident await the outcome of the second trial. Additionally, some media reports exhibited bias, deliberately using phrases such as "Boy died after raising his hand" and "Child died after being sent to the hospital" to imply sudden death and obscure the fact that the boy died 470 days after the incident. This directed public opinion towards the school and teachers, leading to numerous netizens posting irrational comments on online platforms, engaging in online violence against the school and teachers, and even using offensive language to criticize the education industry and society as a whole. This incident holds significant representativeness amid numerous instances of rampant irrational information in public opinion events.

4.1. Dataset description

The Octopus Collector was used to collect the quantity, time, and text content of comments on the trending hashtag "Boy died after raising his hand 7 times when he was unwell in class" from the Weibo and Bilibili platforms. We primarily collected comments from popular posts on the Weibo platform and popular videos on the Bilibili platform. Many first-level comments are initial comments, and as time elapses, the average comment depth increases [31]. However, it is particularly noteworthy that even with subsequent comments, the initial response from the user who posted the initial comment directly addresses the initial topic or content [32]. Therefore, the first-level comments on social platforms can more comprehensively and widely reflect users' initial perspectives, spark broader discussions, and better represent popular topics on the social platform. Second-level comments are generally discussed for the first-level comments themselves, and they easily deviate from the original topic content [31,33], so they are not included in the study. This paper selects first-level comments as the dataset, which is representative for studying the cross-platform dissemination process

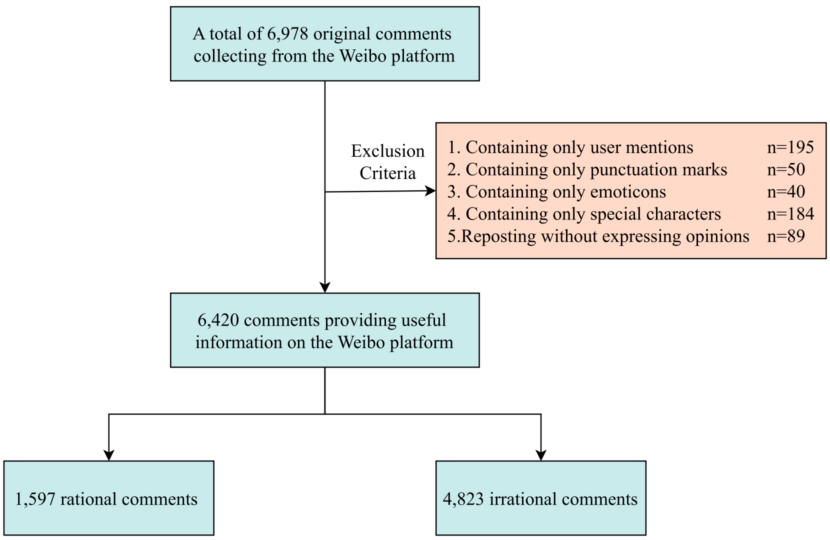

Fig. 2. Data Processing Flowchart on the Weibo platform.

and evolutionary mechanisms of rational and irrational information. Based on the collected first-level comments, we excluded some invalid comments involving special characters, emoticons, and other useless content resulting in a total of 6420 data pieces being collected on the Weibo platform and 2549 data pieces being collected on the Bilibili platform. The data processing flowcharts for the Weibo platform and Bilibili platform are illustrated in Figs. 2 and 3, respectively. In terms of temporal processing, taking into account that the “Boy died after raising his hand 7 times when he was unwell in class” topic became a trending hashtag on social media platforms from February 10th to February 13th, we selected this timeframe for our study. Data were collected during the 90 h with the highest event heat from February 10th to February 13th, with 1 h as a time unit. In terms of information text content processing, comments that merely raised questions about the incident, expressed condolences for the boy’s passing, claimed maintaining a neutral stance, and awaited the outcome of the second trial, as well as those that criticized news media for potentially misleading public opinion, were intentionally categorized as rational information. On the other hand, comments lacking a comprehensive understanding of the situation that engaged in verbal denigration, spread rumors, targeted teachers and schools with abuse, or even condemned the entire education sector and society were intentionally classified as irrational information. The word cloud diagrams illustrating rational and irrational information based on frequently occurring words in the comments are presented in Fig. 4A and B. In addition, we have counted the number of characters in all comments. The total number of characters in all comments is 327,216, and the average length of comments is 36 characters.

4.2. *Equation fitting*

To verify the rationality and validity of the model constructed in this paper, the model equations were fitted with the information data collected from rational and irrational disseminators posted on the two platforms. The least squares method, one of the most commonly used algorithms in machine learning, was employed to find the optimal parameters of the *SEIaIbR-FXYaYbZ* cross-platform information dissemination model constructed above. The least squares method is a mathematical optimization technique that seeks the best function fit for data by minimizing the sum of squared errors. Set the initial values of the model as follows: $S(0) = 8000$, $F(0) = 5000$, $Ia(0) = 3$, $Ib(0) = 6$, $Ya(0) = 1$, $Yb(0) = 3$, $E(0) = 1$, $X(0) = 1$, $R(0) = 2$, $Z(0) = 2$, and set the parameter vector as $L = (\alpha, \beta_1, \beta_2, \eta_1, \eta_2, \gamma_1, \gamma_2, A, \mu, \theta_1, \theta_2, \rho_1, \rho_2, \lambda_1, \lambda_2, C)$. Let $f_{W_1}(t,L)$, $f_{W_2}(t,L)$, $f_{W_3}(t,L)$, $f_{W_4}(t,L)$ be the numerical solutions of $W_1(t)$, $W_2(t)$, $W_3(t)$, $W_4(t)$, respectively.

Equation (15) to calculate the optimal solution of the parameters using the least squares method is:

$$LS = \min \sum_{k=1}^{4} \left[\sum_{t=0}^{90} \left| f_{W_k}(t,L) - W_k(t) \right|^2 \right] \tag{15}$$

According to the least squares Equation (15), the optimal parameters were obtained by utilizing the Ode45 function and the Lsqcurvefit function in the Matlab software to fit the equations. The specific numerical results of the optimal parameters are presented in Tables 5 and 6.

Based on the real information data and the constructed model equations in the given case, the Matlab software generated fitting figures for the cumulative amount of rational information and irrational information, as shown in Fig. 5. Fig. 5A illustrates the real data points and fitting curves of the model equations for the cumulative amount of both types of information released on the Weibo platform, while Fig. 5B displays the real data points and fitting curves of the model equations for the cumulative amount of both types of information released on the Bilibili platform. Dotted lines represent the actual cumulative amount of information, and solid lines

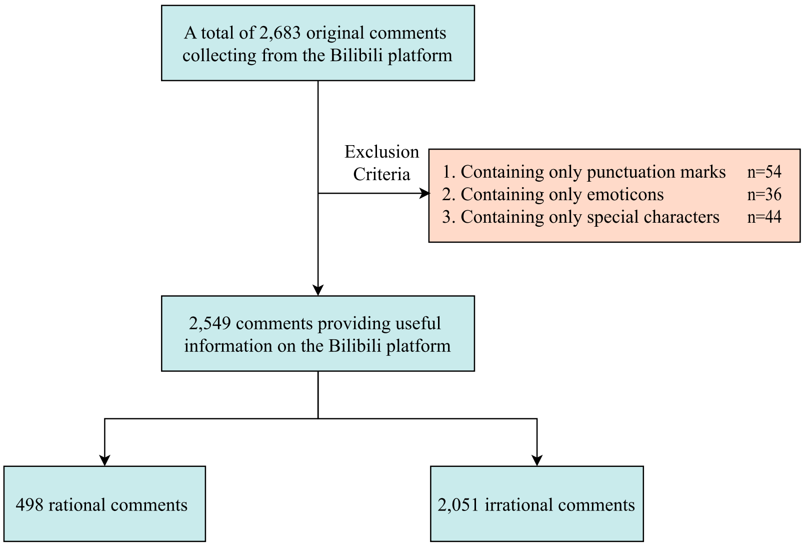

Fig. 3. Data Processing Flowchart on the Bilibili platform.

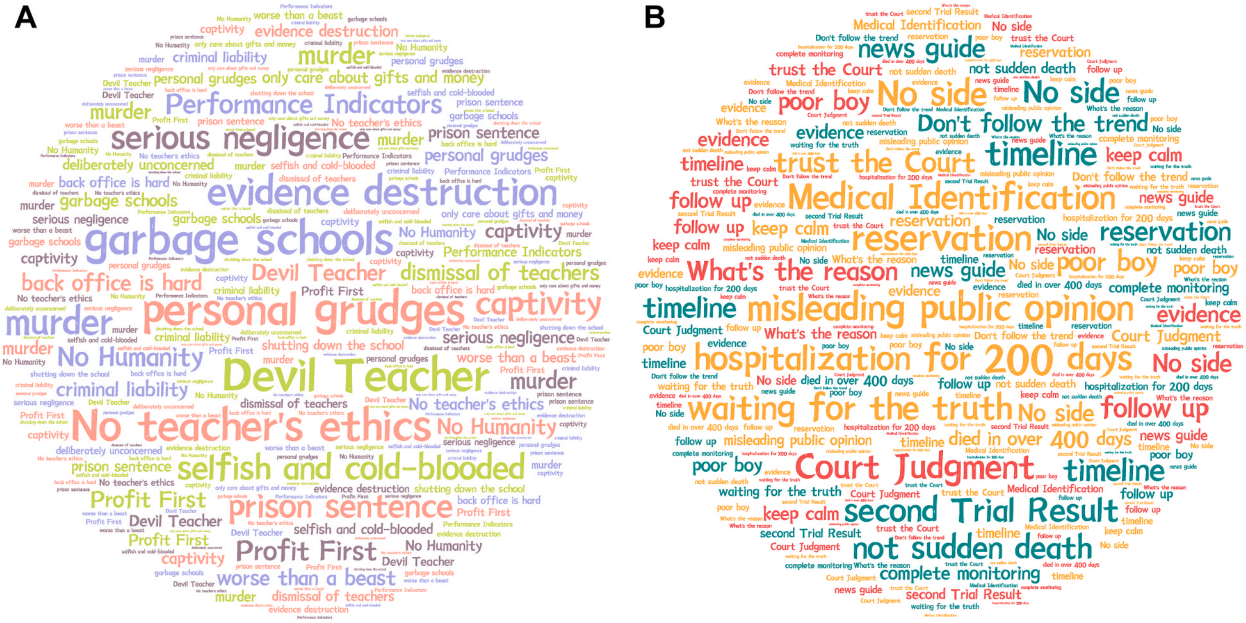

Fig. 4. Rational and irrational information word cloud. Panel A represents the irrational information word cloud. Panel B represents the rational information word cloud.

Table 5
Optimal parameters on the Weibo platform.

parameters	α	β_1	β_2	η_1	η_2	γ_1	γ_2	κ_1	A
value	0.0411	0.0066	0.0340	0.1115	0.1889	0.0010	0.1295	0.1400	2.0000

Table 6
Optimal parameters on the Bilibili platform.

parameters	μ	θ_1	θ_2	ρ_1	ρ_2	λ_1	λ_2	κ_2	C
value	0.0014	0.0010	0.0052	0.1105	0.1368	0.1165	0.0049	0.0600	1.0000

represent the simulated cumulative amount of information. As depicted in Fig. 5, the four fitting curves generally align with the real trend of information dissemination, with the best-fitted curve observed for the cumulative amount of rational information released on the Bilibili platform. In addition, all four fitting curves share a common characteristic: the smoother the cumulative amount of information tends to be, the better the fitting curves match the real data points.

From the perspective of different platforms, the scale and growth rate of information disseminated on the Weibo platform are superior to those on the Bilibili platform, and it enters a stable period relatively quickly. This is due to the fact that the parties involved in public opinion were the first to post on the Weibo platform, which triggers public opinion with the help of a large number of key opinion leaders and public opinion media. The majority of online users tend to focus their attention on the Weibo platform where the involved party posted and are inclined to engage in discussions on the Weibo platform. Despite the presence of numerous key opinion leaders and public opinion media on the Bilibili platform, the larger user base on the Weibo platform results in significantly higher follower counts for key opinion leaders and public opinion media on Weibo compared to Bilibili. This phenomenon leads to a more prominent role played by key opinion leaders and public opinion media on Weibo in public opinion, which makes public opinion information disseminated on the Weibo platform more influential than that on the Bilibili platform. In terms of different types of information, the amount of irrational information is larger and grows faster than the number of rational information on the two platforms. This is because irrational emotional expressions are widespread in current online media, particularly prominent in comments, and are more likely to inspire similar irrational emotions in others. Ultimately, irrational information has a greater influence in spreading across the network.

4.3. Basic reproduction number and information entropy analysis

By substituting the parameter data derived from the equation fitting into Equation (11) of the basic reproduction number, the basic

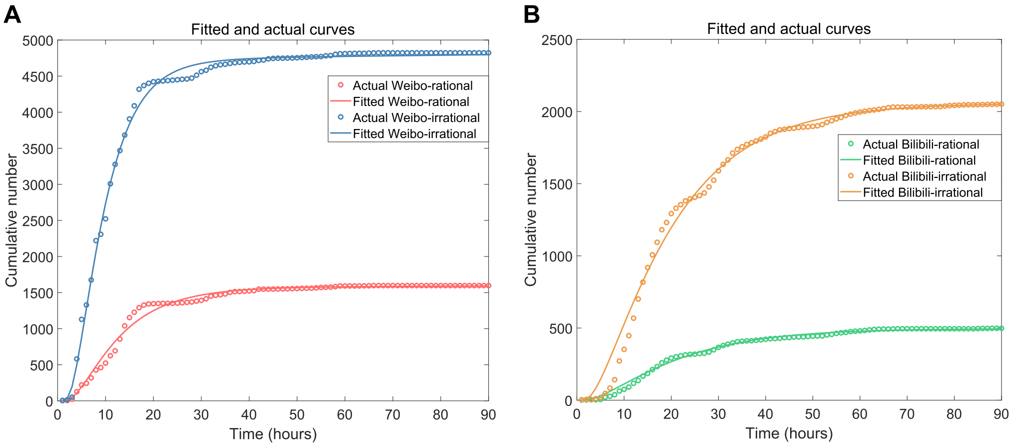

Fig. 5. Fitting curves of the cumulative amount of rational and irrational information released on two social media platforms. Dotted lines indicate the real cumulative amount of information and solid lines indicate the simulated cumulative amount of information, respectively. Panel A represents the fitting results of the Weibo platform, where the red lines represent the cumulative amount of rational information, and the blue lines represent the cumulative amount of irrational information. Panel B represents fitting results of the Bilibili platform, where the green lines represent the cumulative amount of rational information, and the orange lines represent the cumulative amount of irrational information. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

reproduction number R_0 can be determined as 63. This implies that, on average, one information disseminator influences 63 people to become disseminators in the process of dissemination, indicating the spread of public opinion within a certain range. Fig. 6 shows the Pearson correlation coefficients between the optimal parameters and the basic reproduction numbers, obtained using SPSS software. The correlation coefficients range from −1 to 1, where positive coefficients signify positive correlations, negative coefficients indicate negative correlations, and the proximity of the coefficient to −1 or 1 signifies a stronger correlation, while proximity to 0 suggests a weaker correlation. As shown in Fig. 6, the number of new susceptible users (A and C), the probability (α and μ) of converting susceptible users into exposed users, the circulation rate (κ_2) of users flowing from the Bilibili platform to the Weibo platform, and the dissemination rates (β_1, β_2, θ_1, θ_2) are positively correlated with the basic reproduction number. Therefore, controlling the dissemination of public opinion can be achieved by decreasing these parameters. Conversely, the probabilities (γ_1, γ_2, λ_1, λ_2) of direct conversion of susceptible and exposed users into recovered users, as well as the probabilities (η_1, η_2, ρ_1, ρ_2) of conversion of disseminators

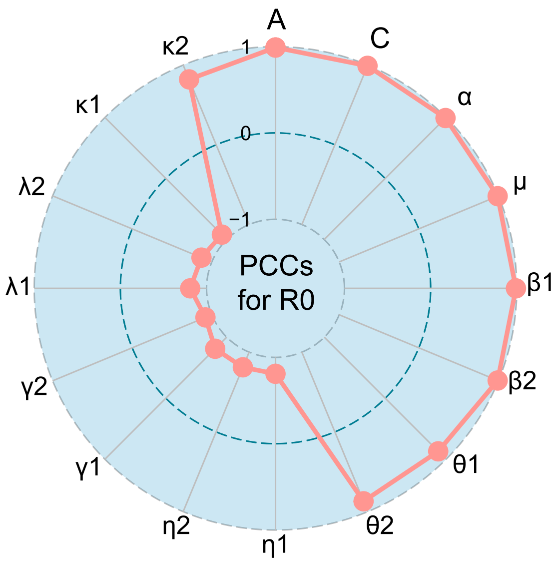

Fig. 6. Correlation coefficients of optimal parameters and basic regeneration number. There are three circular dashed lines in the figure, which represent correlation coefficients of −1, 0, and 1 respectively. The position of the red dot determines the correlation coefficient of the parameter and basic reproduction number. PCC, Pearson correlation coefficient. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

into recovered users, including the circulation rate (κ_1) of users flowing from the Weibo platform to the Bilibili platform, are negatively correlated with the basic reproduction number. Increasing these parameters can facilitate the dissipation of public opinion. Furthermore, the correlation between κ_1 and the basic reproduction number, as well as the correlation between κ_2 and the basic reproduction number, further substantiates that the impact of the Weibo platform on public opinion is greater than that of the Bilibili platform. When a larger population migrates to the Weibo platform for information dissemination, it results in a broader dissemination scope of public opinion.

Fig. 7A and B shows the numerical curves of information entropy within the two platforms, generated using Matlab software based on the information entropy **Equation (13)** and **(14)**. The information entropy of the Weibo platform reaches a maximum value of 0.276 at $t = 0$ and a minimum value of 0.205 at $t = 3$. When $t \in (25,90)$, the information entropy tends to stabilize and maintains a value generally fixed around 0.244. On the other hand, the information entropy of the Bilibili platform peaks at 0.244 at $t = 0$ and drops to a minimum of 0.201 at $t = 6$ and is stable around 0.214 when $t \in (20,90)$. In summary, the information entropy values on both platforms exhibit a lower level. A smaller information entropy implies reduced uncertainty in information, indicating that a specific type of information dominates the overall public opinion within the system. Consequently, irrational information predominantly influences public opinion on social platforms, as individuals with the same emotions, opinions, and perceptions quickly converge and reach a consensus, leading to a group polarization in the irrational dissemination.

4.4. Correlation and sensitivity analysis of parameters

To investigate the impact of model parameters on the dissemination of public opinion, Pearson correlation coefficients were computed using R language software to determine the correlation between each optimal parameter and the cumulative amount of rational and irrational information released on the two online platforms. As depicted in Fig. 8A and B, the green points represent the correlation between the parameters and the cumulative amount of rational information, while the red points represent the correlation between the parameters and the cumulative amount of irrational information. On the Weibo platform, the cumulative amount of rational information demonstrates a positive correlation with parameters A, C, β_1, μ, θ_1, α, and κ_2, while exhibiting a strong negative correlation with the remaining parameters. The cumulative amount of irrational information on the Weibo platform is positively correlated with parameters A, C, β_2, μ, θ_2, α, and κ_2, and displays a strong negative correlation with the remaining parameters. The correlations of most parameters with the cumulative amount of both types of information released on the Bilibili platform are similar to those observed on the Weibo platform. However, a few parameters exhibit distinct characteristics. For instance, the cumulative amount of rational information on the Bilibili platform is positively correlated with parameters κ_1, β_2, and has a strong negative correlation with parameter κ_2. The cumulative amount of irrational information on the Bilibili platform is positively correlated with parameters κ_1, β_1, and has a strong negative correlation with parameter κ_2. In general, the cumulative amount of information exhibits a robust correlation with changes in the values of the 18 model parameters. Increasing the values of parameters that positively correlate with the cumulative amount of information facilitates the spread of public opinion, while increasing the values of parameters that negatively correlate with the cumulative amount of information exerts a certain inhibiting effect on the dissemination of public opinion.

To further investigate the influence of some key parameters in the model on cross-platform public opinion dissemination, this section used R language software to conduct simulations and utilized the control variable method to study the influence of parameter values (β_1, β_2, θ_1, θ_2, κ_1, κ_2) on the cumulative amounts of rational and irrational information, respectively. The results are presented in Figs. 9–11. The solid lines in the figures represent the change curves of the cumulative amount of irrational information with varying

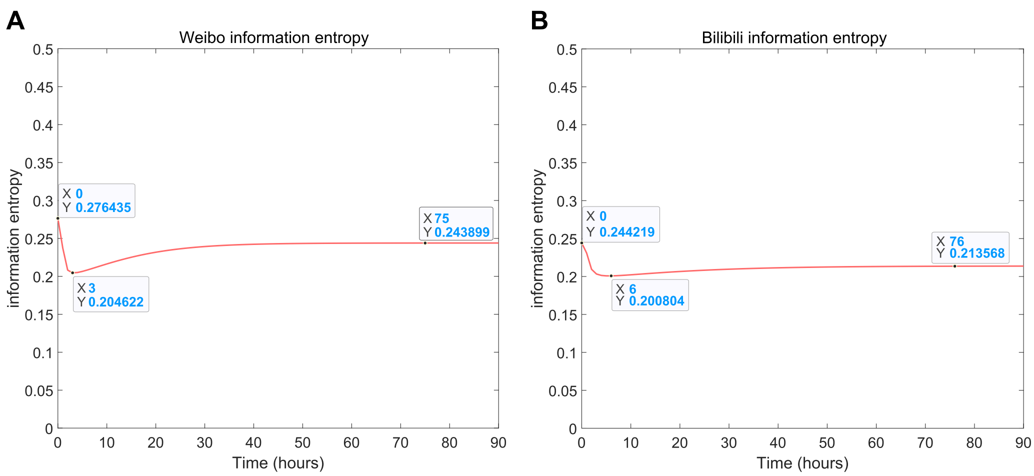

Fig. 7. Information entropy curves over time on the Weibo and Bilibili platforms. Panel A represents Information entropy curves over time on the Weibo platforms. Panel B represents Information entropy curves over time on the Bilibili platforms.

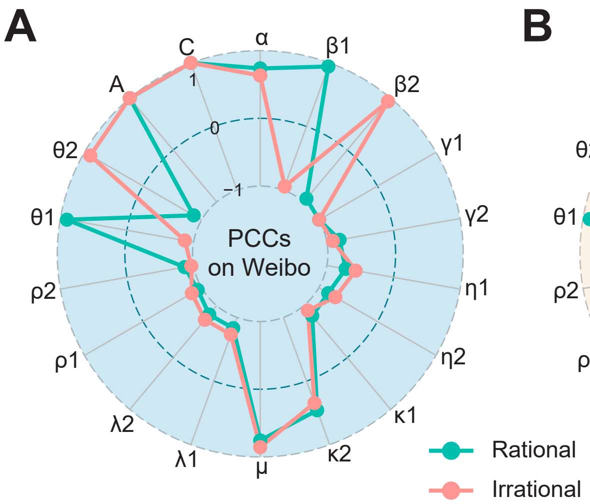

Fig. 8. Correlation coefficients of the cumulative amount of two types of information and optimal parameters on the Weibo platform (A) and the Bilibili platform (B). The green points represent the correlation between the cumulative amount of rational information and the optimal parameters, and the red points represent the correlation between the cumulative amount of irrational information and the optimal parameters. PCC, Pearson correlation coefficient. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

parameters, while the dashed lines depict the change curves of the cumulative amount of rational information with varying parameters.

Fig. 9A illustrates the impact of variations in the rational dissemination rate (β_1) on the cumulative amounts of both types of information released on the Weibo platform, while Fig. 9B displays the effect of changes in the rational dissemination rate (β_1) on the cumulative amounts of both types of information released on the Bilibili platform. In these figures, the values of β_1 are set to 0.0006, 0.0036, 0.0066, 0.0096, and 0.0126, respectively. From Fig. 9A, it is observed that an increase in β_1 leads to an elevation in the cumulative amount of rational information released on the Weibo platform, accompanied by a decrease in the cumulative amount of irrational information. This phenomenon may be attributed to the positive guidance provided by key opinion leaders and the media on the Weibo platform, coupled with the improvement in the quality of internet users themselves. Consequently, a growing number of internet users exposed to public opinion information on Weibo tend to become rational disseminators. On the Bilibili platform, as depicted in Fig. 9B, the increase of β_1 not only results in an increase in the cumulative amount of rational information but also contributes to an elevation in the cumulative amount of irrational information on the platform. This implies that the increase of rational information on Weibo, a platform with a large user base and extensive public opinion dissemination, leads to an increase in both rational and irrational information on Bilibili, a platform with fewer users and a smaller scale of public opinion dissemination. This suggests that the increase in rational disseminators on Weibo prompts a portion of disseminators on Bilibili to adopt a rational stance. However, simultaneously, a fraction of the reduced irrational disseminators on Weibo shifts their focus to Bilibili, initiating the expression of irrational discourse on the Bilibili platform.

Fig. 9C and D depict the impact of variations in the irrational dissemination rate (β_2) on the cumulative amounts of rational and irrational information released on the Weibo platform and the Bilibili platform, respectively. In these figures, the values of β_2 are set to 0.0320, 0.0330, 0.0340, 0.0350, and 0.0360. The influence of parameter β_2 on public opinion dissemination is similar to that of parameter β_1. Despite β_2 exhibiting a strong correlation with the cumulative amount of rational information, the cumulative amount of rational information on both platforms does not demonstrate high sensitivity to changes in the irrational dissemination rate (β_2). Moreover, the cumulative amount of rational information undergoes minimal changes compared to irrational information. This suggests that a limited number of rational disseminators exhibit strong determination, possess the ability to discern information, and are less susceptible to the influence of irrational disseminators.

Based on the aforementioned analysis, it is evident that the Weibo platform, characterized by its large user base and the presence of numerous key opinion leaders and mainstream media with substantial follower counts, facilitates the diffusion of public opinion information in a manner akin to nuclear fission. This phenomenon significantly impacts the trends of public opinion on the Bilibili media platform, which has a relatively smaller user base and primarily offers video content related to gaming, beauty makeup, and the anime. In light of these findings, government departments should prioritize attention to the Weibo platform, given its substantial influence on public opinion. Official media outlets should, therefore, ensure the timely and accurate disclosure of real information. Simultaneously, there is a need to guide opinion leaders and mainstream media on the Weibo platform to actively embrace social responsibility and propagate rational information to platform users through interpersonal channels, thereby fostering a two-layer communication approach.

Fig. 10A and B illustrate the effects on the cumulative amounts of rational and irrational information released on the two platforms when varying the rational dissemination rate (θ_1) on the Bilibili platform. In these figures, the values of θ_1 are set to 0, 0.0005, 0.0010,

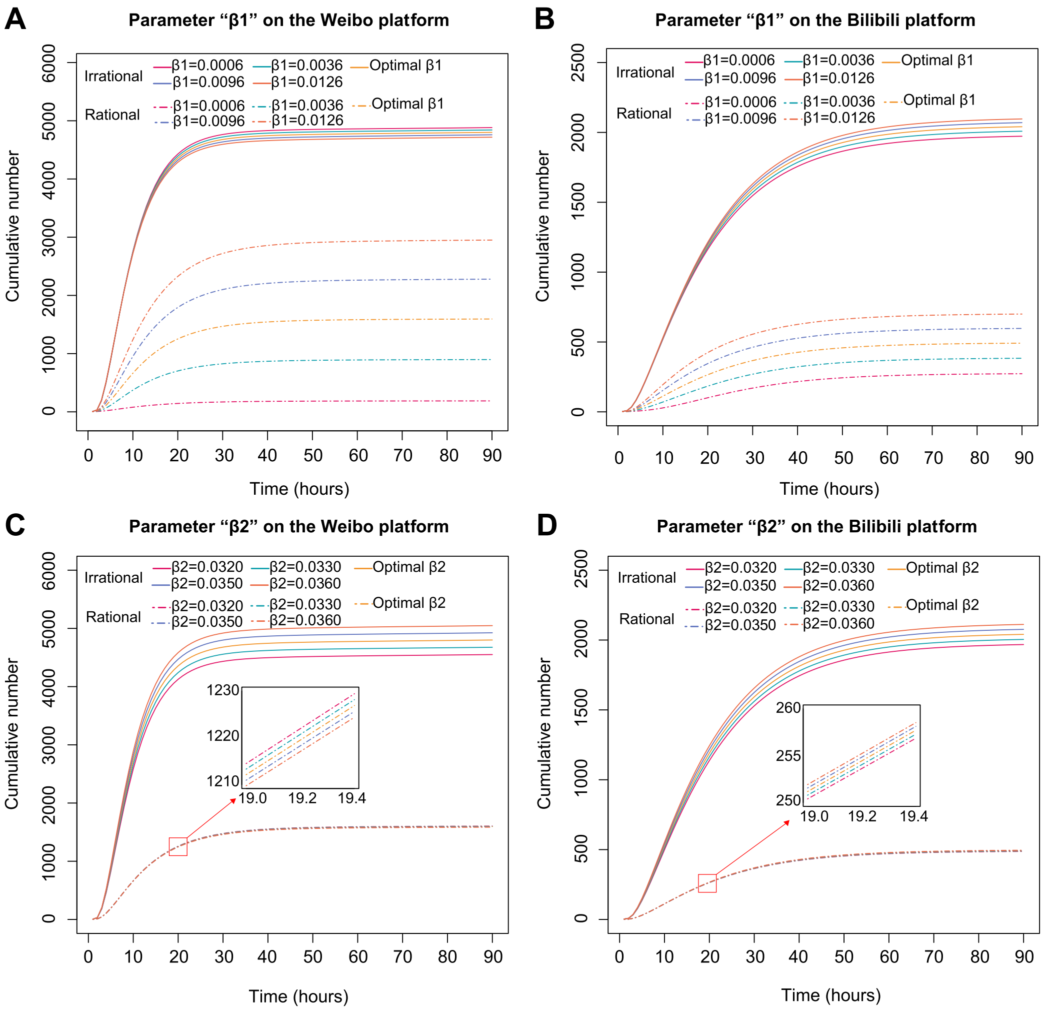

Fig. 9. The change curves of the cumulative amount of rational and irrational information released on the two platforms when the rational dissemination rate β_1 and the irrational dissemination rate β_2 change. The solid lines in the figure are the change curves of the cumulative amount of irrational information with the change of parameters, and the dashed lines are the change curves of the cumulative amount of rational information with the change of parameters. Panel A represents the change curve of two types of information on the Weibo platform when β_1 changes. Panel B represents the change curve of two types of information on the Bilibili platform when β_1 changes. Panel C represents the change curve of two types of information on the Weibo platform when β_2 changes. Panel D represents the change curve of two types of information on the Bilibili platform when β_2 changes.

0.0015, and 0.0020, respectively. As θ_1 increases, it not only results in an augmentation of rational information on the Bilibili platform but also contributes to a slight increase in rational information on the Weibo platform due to the movement of individuals between platforms. However, within the specified parameter range of 0–0.02, the change in θ_1 primarily influences the cumulative amount of rational information released on the Bilibili platform, with the numbers of both types of information on the Weibo platform not exhibiting significant sensitivity to variations in the parameter θ_1. This suggests that, under the parameter settings in this study, changes in the dissemination rate on the Bilibili platform do not significantly impact the Weibo platform in terms of public opinion spread. Fig. 10C and D illustrate the impact of variations in the irrational dissemination rate (θ_2) on the Bilibili platform on the cumulative amounts of rational and irrational information released on the two platforms. The values of θ_2 are set to 0.0032, 0.0042, 0.0052, 0.0062, and 0.0072, respectively. The evolution of public opinion information depicted in these figures is analogous to that presented in Fig. 10A and B.

While the Bilibili platform may not wield significant influence over public opinion on the Weibo platform, the control of irrational public opinion on Bilibili cannot be overlooked. In recent years, Bilibili has evolved from an immature video media platform to a comprehensive content video website, expanding its reach into diverse fields such as economy, law, society, culture, education, and entertainment. The platform has witnessed a growing user base, leading to an expansion in the scale of information dissemination. Moreover, irrespective of the platform’s size, the inherent contingency in information growth may trigger an explosion of public opinion information. Therefore, the focus of public opinion regulation should extend beyond the Weibo platform to include the Bilibili

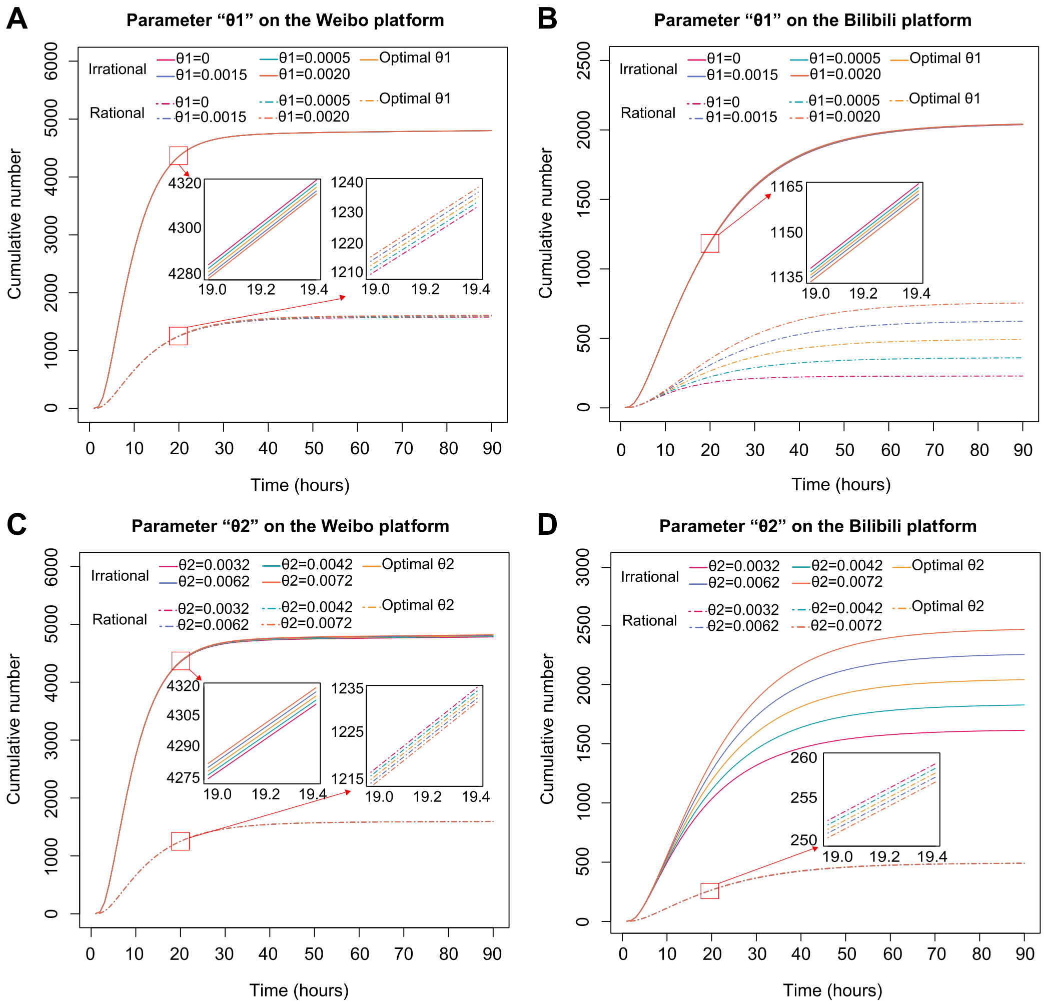

Fig. 10. The change curves of the cumulative amount of rational and irrational information released on the two platforms when the rational dissemination rate θ_1 and the irrational dissemination rate θ_2 change. The solid lines in the figure are the change curves of the cumulative amount of irrational information with the change of parameters, and the dashed lines are the change curves of the cumulative amount of rational information with the change of parameters. Panel A represents the change curve of two types of information on the Weibo platform when θ_1 changes. Panel B represents the change curve of two types of information on the Bilibili platform when θ_1 changes. Panel C represents the change curve of two types of information on the Weibo platform when θ_2 changes. Panel D represents the change curve of two types of information on the Bilibili platform when θ_2 changes.

platform. The management of irrational public opinion information requires both targeted and comprehensive control strategies. Additionally, the influence of higher-profile user nodes on the Bilibili platform should be leveraged to encourage users to maintain "rationality", thereby curbing the impulsiveness and violence prevalent in netizens' public opinion.

Fig. 11A and B depict the influence of variations in the user circulation rate (κ_1) from the Weibo platform to the Bilibili platform on the cumulative amounts of rational and irrational information released on both platforms. The values of parameter κ_1 are set to 0.1200, 0.1300, 0.1400, and 0.1500, respectively. As κ_1 increases, exposed users from the Weibo platform transition to the Bilibili platform to disseminate information with a certain probability. This results in a decrease in the cumulative amount of both types of information released on the Weibo platform and an increase in the cumulative amount on the Bilibili platform. Fig. 11C and D illustrate the impact of variations in κ_2 on the cumulative amounts of rational and irrational information released on both platforms, with the values of κ_2 set to 0.0400, 0.0500, 0.0600, and 0.0700, respectively. As κ_2 increases, exposed users from the Bilibili platform migrate to the Weibo platform to disseminate information with a certain probability. This leads to a decrease in the cumulative amount of both types of information on the Bilibili platform and a marginal increase in the cumulative amount on the Weibo platform. However, within the specified κ_2 parameter range in this study, the change in information on the Weibo platform is not highly sensitive to variations in κ_2, indicating that the circulation rate of users from the Bilibili platform to the Weibo platform does not significantly impact the spread of public opinion on the Weibo platform. In summary, amidst variations in user circulation rates across social media platforms, the Weibo platform exerts a more substantial influence on the dissemination of public opinion information on the Bilibili platform.

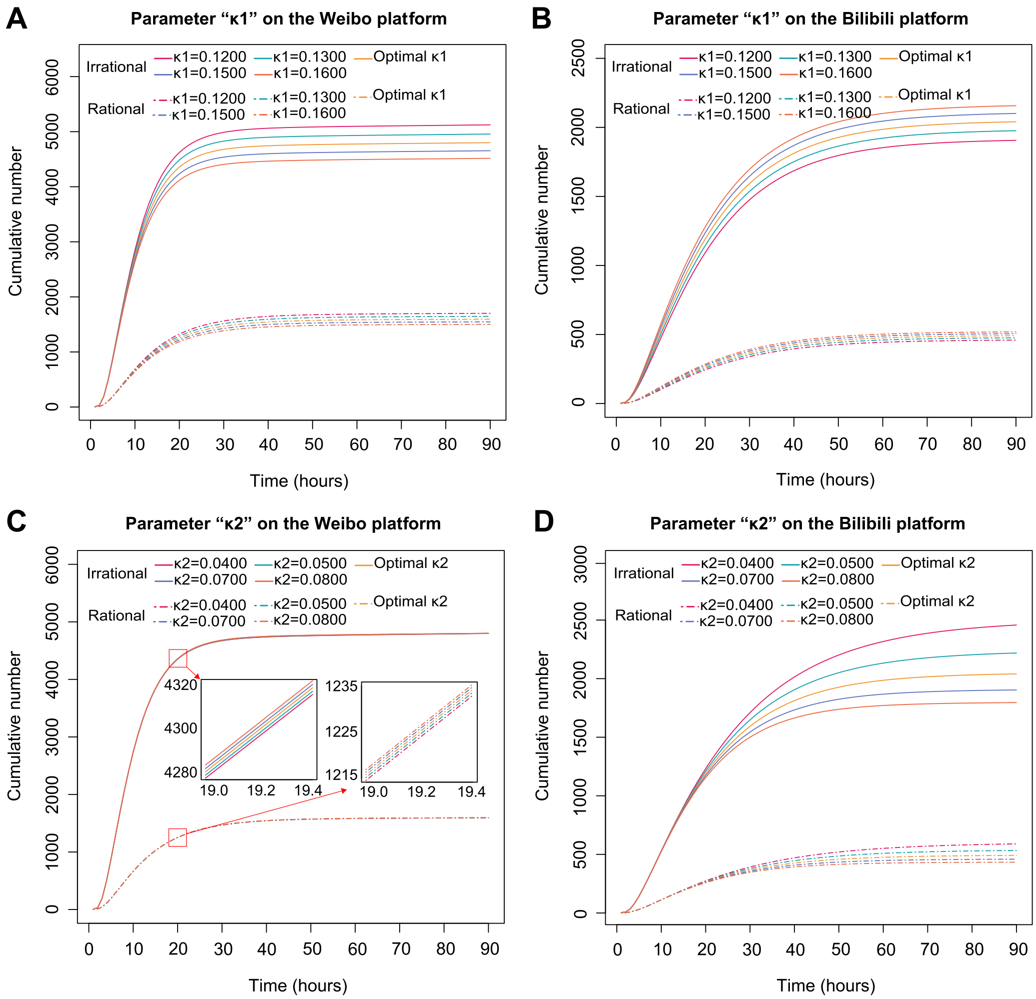

Fig. 11. The change curves of the cumulative amount of rational and irrational information released on the two platforms when the user circulation rate κ_1 and κ_2 change. The solid lines in the figure are the change curves of the cumulative amount of irrational information with the change of parameters, and the dashed lines are the change curves of the cumulative amount of rational information with the change of parameters. Panel A represents the change curve of two types of information on the Weibo platform when κ_1 changes. Panel B represents the change curve of two types of information on the Bilibili platform when κ_1 changes. Panel C represents the change curve of two types of information on the Weibo platform when κ_2 changes. Panel D represents the change curve of two types of information on the Bilibili platform when κ_2 changes.

Because of the mobility of users between the Weibo and Bilibili platforms, information between the platforms penetrates each other, thus making the dissemination of public opinion information take on new characteristics. While the mobility of users within the online environment parallels the cross-regional mobility of potential carriers during real-life epidemics, both public opinion and viruses spread at an accelerated pace due to human mobility. However, unlike potential carriers of viruses who can be controlled through isolation measures, curtailing the flow of users between media platforms cannot be achieved through simplistic and stringent restrictions. This necessitates the collective efforts of internet users to enhance their self-awareness, strengthen dialectical thinking abilities, and the government's support through means such as widespread education to foster the development of analytical thinking skills. This would encourage the public to express rational opinions and contribute to the creation of a healthy online environment.

5. Conclusion

In this study, a cross-platform two-layer network dissemination model, incorporating rational and irrational information, was developed based on the classical infectious disease dynamics model. The Weibo and Bilibili platforms were used as examples for constructing the model. Dynamic equations and the spectral radius method of the reproduction matrix were employed to calculate the equilibrium point and the basic reproduction number. Additionally, information entropy formulas were defined to quantify information entropy. The *SEIaIbR-FXYaYbZ* dissemination model was fitted to real cases using the least squares method to determine optimal parameters, validating the effectiveness and rationality of the model. Correlation analysis and sensitivity analysis were conducted based on the optimal parameters and the cumulative amounts of rational and irrational information to explore the trends of rational and irrational information on the two media social platforms. Finally, we come to the following conclusions. (i) The Weibo platform exhibits superior scale and growth rates of public opinion information compared to the Bilibili platform, entering the dissemination plateau relatively quickly. Moreover, irrational information demonstrates a larger scale and faster growth rate than rational information. (ii) Both platforms maintain a low level of information entropy, with irrational information predominantly controlling opinion information. (iii) The increase of rational information leads to the decrease of irrational information, while the quantity of rational information remains largely unaffected by changes in irrational information. (iv) An increase in rational information on the Weibo platform leads to simultaneous increases in both rational and irrational information on the Bilibili platform. (v) Changes in the dissemination rate and user circulation rate among platforms reveal that the Weibo platform exerts a greater influence on the dissemination of public opinion information on the Bilibili platform. The research findings and coping strategies offer valuable insights for relevant departments managing online public opinion, emphasizing the importance of focusing on Weibo platforms with substantial user bases and extensive public opinion dissemination. Simultaneously, attention should be directed towards public opinion prevention and control on the Bilibili platform, leveraging the positive influence of key opinion leaders and mainstream media on both platforms to foster an orderly and healthy development of the online society. In real-life scenarios, cross-platform public opinion dissemination across social media platforms involves more than just Weibo and Bilibili. The movement of users across multiple platforms, leading to a broader range of public opinion dissemination, is a focal point for future research.

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Data availability statement

Original data in the current study are not publicly available but are available from the corresponding authors upon reasonable request.

CRediT authorship contribution statement

Qingqing Ji: Writing – original draft, Software, Methodology, Data curation, Conceptualization. **Xiaoli Wang:** Writing – review & editing, Supervision, Conceptualization. **Jing Zhang:** Writing – review & editing. **Rui Gong:** Writing – review & editing. **Yifan Zhong:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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