Developmental Dysplasia of the Hip: A Computational Biomechanical Model of the Path of Least Energy for Closed Reduction

Mohammed A. Zwawi,[1] Faissal A. Moslehy,[1] Christopher Rose,[1] Victor Huayamave,[2] Alain J. Kassab,[1] Eduardo Divo,[2] Brendan J. Jones,[1] Charles T. Price[3]

[1]Department of Mechanical and Aerospace Engineering, University of Central Florida, Orlando, Florida 32816 , [2]Department of Mechanical Engineering, Embry-Riddle Aeronautical University, Daytona Beach, Florida 32114, [3]Pediatric Orthopedic Surgery, Arnold Palmer Hospital, 1222 S. Orange Ave., Orlando, Florida 32806

ABSTRACT: This study utilized a computational biomechanical model and applied the least energy path principle to investigate two pathways for closed reduction of high grade infantile hip dislocation. The principle of least energy when applied to moving the femoral head from an initial to a final position considers all possible paths that connect them and identifies the path of least resistance. Clinical reports of severe hip dysplasia have concluded that reduction of the femoral head into the acetabulum may occur by a direct pathway over the posterior rim of the acetabulum when using the Pavlik harness, or by an indirect pathway with reduction through the acetabular notch when using the modified Hoffman–Daimler method. This computational study also compared the energy requirements for both pathways. The anatomical and muscular aspects of the model were derived using a combination of MRI and OpenSim data. Results of this study indicate that the path of least energy closely approximates the indirect pathway of the modified Hoffman–Daimler method. The direct pathway over the posterior rim of the acetabulum required more energy for reduction. This biomechanical analysis confirms the clinical observations of the two pathways for closed reduction of severe hip dysplasia. The path of least energy closely approximated the modified Hoffman–Daimler method. Further study of the modified Hoffman–Daimler method for reduction of severe hip dysplasia may be warranted based on this computational biomechanical analysis. © 2016 The Authors. *Journal of Orthopaedic Research* Published by Wiley Periodicals, Inc. on behalf of Orthopaedic Research Society. J Orthop Res 35:1799–1805, 2017.

Keywords: hip; dysplasia; DDH; model; biomechanical

Developmental dysplasia of the hip (DDH) is a common newborn condition. Complete dislocation occurs in approximately one to three per thousand live births.[1,2] Early detection and management improves outcomes, but severity remains an impediment to successful management by non-surgical methods.[3]

Grade IV hip dislocation represents the most severe type in the Graf, Tönnis, and IHDI classification systems.[4–6] In Grade IV dislocations, the femoral head is proximal and superior to the acetabulum with soft tissue and bony obstacles to reduction. Poor success with non-surgical management of Grade IV dislocations has led some physicians to proceed directly to closed reduction under anesthesia, or to recommend surgical management.[7,8] However, successful non-surgical reductions of severe dislocations have been reported by using traction with guided flexion-abduction,[9] by modified Hoffman–Daimler method,[10] and by supervised Pavlik harness treatment with hyper-flexion.[11] Proponents of these methods have theorized the path that the femoral head travels during the reduction process.

Two pathways of reduction have been identified in the clinical literature regarding reduction of infantile hip dislocation. The first pathway was identified by Iwasaki[11] during passive reduction using the Pavlik harness, or by manual reduction (Fig. 1) and is identified as the "direct path" for reduction. The second pathway was described by Papadimitriou et al.[10] during two-stage reduction of dislocated hips using the modified Hoffman–Daimler method (Fig. 2) and is identified as the "indirect path" for reduction.

Using the Pavlik harness, Iwasaki noted that the dislocated hip initially shifts to the posterior part of the acetabulum, followed by sliding of the femoral head directly over the acetabular rim during reduction. Iwasaki suggested that this is the same pathway as manual reduction except that the source of power in the Pavlik harness is the weight of the leg while the source of power during manual reduction is the operator who levers the femoral head into the reduced position. Subsequently, Suzuki[12] also evaluated reduction by the Pavlik method by applying continuous ultrasound monitoring in nine infants during the process of reduction. Less severe dislocations reduced suddenly when the femoral head was perched on the acetabular wall as proposed by Iwasaki. However, Suzuki noted that the posterior wall of the acetabulum prevented reduction of severe dislocations and these could only be reduced by manual force.

In contrast, Papadimitriou et al.[10] reported a gradual two-stage method for reducing severe dislocations in which the femoral head travels along a more indirect path: Initially proceeding caudally to a point
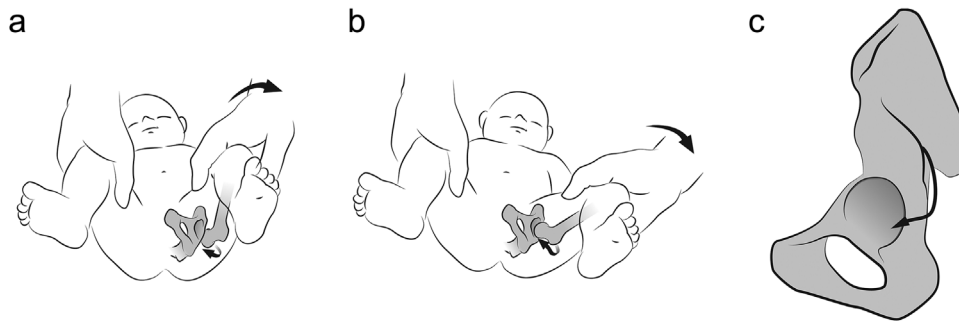
Figure 1. Direct reduction pathway identified by Iwasaki using the Pavlik harness, and for manual reduction. (a) Initial position of dislocation. (b) Clinical pathway of reduction by Pavlik harness or manual reduction. (c) Schematic of direct pathway of reduction.

proximal to the ischial tuberosity and then proceeding to enter the acetabulum from an inferior position when the hip is abducted.

The principle of least energy when applied to moving the femoral head from an initial to a final position considers all possible paths that connect them and identifies the path of least resistance. Identifying the minimum energy path that the femoral head would travel during reduction of DDH may help validate or invalidate the clinical assumptions that have been proposed. The purpose of this paper is to utilize a validated computational biomechanical model to determine the path of least energy for two pathways of reduction that have been proposed by previous authors.

A previously reported, computational biomechanical model of DDH[13] demonstrated that tension in the adductor and Pectineus muscles resists reduction for Grade IV hip dislocations. Releasing the adductor muscles to facilitate reduction is frequently advocated during closed reduction under general anesthesia. However, tension in the Pectineus muscle during abduction also exacerbates entrapment of the femoral head posterior to the acetabulum.[13] A second biomechanical study suggested that hyper-flexion and external rotation may overcome the entrapment and allow the femoral head to enter the acetabulum by following the path proposed by Papadimitriou et al.[14]

Calculation of the energy required for reduction along the two proposed pathways with and without the effect of the Pectineus muscle may also provide clinically relevant information.

MATERIALS AND METHODS

The anatomical and muscular aspects of this model were derived using MRI and OpenSim data. The least energy path was evaluated from the beginning point of a Grade IV dislocated hip to the ending point at the center of the acetabulum. The adductor muscles are frequently released prior to closed reduction, while the Pectineus remains intact. However, tension in the Pectineus during abduction also exacerbates entrapment of the femoral head posterior to the acetabulum.[13,14] In order to further study the role of the Pectineus the least energy path was determined with the effect of the Pectineus "switched off." Moreover, the indirect path of the Hoffman–Daimler method was studied with and without the effect of the Pectineus by prescribing the beginning point of a Grade IV dislocated hip, an intermediate point on the ischium near the acetabular notch, and an ending point at the center of the acetabulum. Additionally, the amount of energy required was determined for the hip to follow the direct path described by Iwasaki with all modeled muscles intact.

METHODOLOGY

The principle of stationary potential energy (SPE) was used to identify the least energy path that will vector the femoral
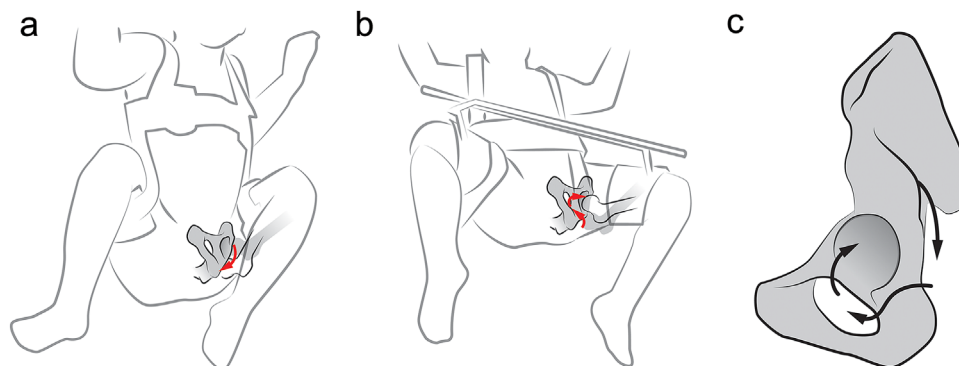

Figure 2. Two-stage reduction pathway identified by Papadimitriou using the modified Hoffman–Daimler method. (a) Movement of femoral head during first stage of indirect reduction. (b) Femoral head movement during second stage of reduction. (c) Schematic of indirect pathway of reduction.

head successfully into its correct physiological position within the acetabulum. The potential energy (Π) stems from strain energy (U) stored in the muscles and gravitational potential energy (Ω) of four rigid-body components of the lower limb. The SPE was used to find the optimal path.

A computer program was developed using the Solidworks (Dassault Systemes) Adams dynamics solver and an in-house Proper Orthogonal Decomposition-based response surface for the potential energy and an optimization algorithm based on the Dijkstra's optimal path algorithm[15] which finds the shortest paths between nodes in a network graph. In-house codes are written in MATLAB. Details of the computation model are in previous publications.[13–17] A collision detection algorithm was used to determine the location of the femoral head center coordinates while the femoral head is in contact with the pelvic surface as it travels from an original start point (computational dislocation) towards the center of the acetabulum. Femur orientation angles (flexion, abduction, and hip rotation angles) were independently varied for each femoral head location. Muscle lengths as previously determined[14] were used to find the strain energy. Center of gravity heights of the lower limb bones were measured and used in calculating gravitational potential energy. The potential energy (sum of strain energy and gravitational potential energy) was computed as a function of (x,y,z), flexion, abduction, hip rotation angles, and center of gravity height. An optimization routine was developed to determine its extremum, and thus the least-energy-path was identified by utilizing Dijkstra's optimal path algorithm[15] which finds the shortest path between nodes in a network graph.

pathogenpathogenSeveral constraints were considered. First, the femoral head was modeled as spherical[14,17] and always in contact with the pelvic surface at a distance "d," which is the femoral head radius $= 7$ mm. The distance formula was used to compute the coordinates (x,y,z) of the femoral head center.

$$d = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{1}$$

Where, (x_i, y_i, z_i) are the coordinates of contact points on the pelvis. Additionally, the harness configuration was used to constrain the femur orientation range to rotate about the flexion axis between 70° and 130°, within an abduction angle range of 0–90°, and hip rotation angle range of -30° to $+30$°.

Muscles modeled because of their effect on DDH reduction included the Pectineus, Adductor Brevis, Adductor Longus, Adductor Magnus, and Gracilis. A previous study determined that the Psoas is relaxed and may not prevent reduction when the dislocated hip is flexed and abducted.[13] For this model, the abductors, hamstrings, and iliopsoas were assumed to be relaxed because the Pavlik harness holds the knee and hip flexed with the hip abducted. To test these assumptions, the iliacus, psoas, and abductors were modeled in three positions along the indirect path. The contributions of these muscles were confirmed to be negligible relative to the muscles included in the model. Clinical observations reported by Iwasaki[11] and Suzuki[12] indicate that reduction with the Pavlik harness occurs passively in deep sleep under the combined effects of gravity in the supine position and with the constraints of the Pavlik Harness. We adopt a passive muscle model that is consistent with these clinical observations. Moreover, the muscle force behavior is hyperelastic with a line of action acting between muscle origin and insertion points. Each muscle responds according to a Fung-type hyperelastic constitutive model,[18,19] according to the following stress–strain relation.

$$F = \mathrm{PCSA}\left[a\left(e^{b(\lambda - 1)} - 1\right)\right] \tag{2}$$

Where, F is muscle force due to deformation, λ is the stretch that can be written as $\lambda = L/(pL_0)$ with L being the deformed length, and $L°$ being the initial muscle relaxed length at rest taken at 20° abduction and 120° flexion for newborns, and PCSA is the physiologic cross sectional area. The model constants a and b are calibrated to achieve equilibrium when the femur is at a flexion angle of 90°, an abduction angle of 80°, and a hip rotation angle at 0°. A prestretch of $p = 70$–80% of the reference length is applied to all muscles. Figure 3 shows the active, passive, and total muscle tension with L_o. Details of the muscle model and calibration can be found in Huayamave et al.[14] It should be noted that the calibrated muscle model exhibits an exponential behavior that becomes very pronounced after a stretch value of about 1.35 accounting for the iliofemoral, pubofemoral and ischiofemoral hip joint ligaments that prevent any further abduction of the legs past an angle of 80° and help to stabilize the hip joint. Stress is computed as $\sigma = F/\mathrm{PCSA}$ and strain is $\varepsilon = \lambda - 1$.

The second component of the potential energy is the gravitational potential energy[20] that an object possesses due to its position in a gravitational field. The gravitational potential energy is computed as the negative of the work done by the weight

$$\Omega = -m_{\mathrm{total}} g X \tag{3}$$

Where, X is the height of the center of gravity of the lower limb bones, m_{total} is the total mass of the lower limb bones, and g is the gravitational acceleration. The height "X," as shown in Figure 4, was found for all femoral head locations at all considered femur orientations. Therefore, the gravitational potential energy was computed repeatedly by changing the location of the femoral head center and the femur orientation.

Hence, the potential energy, which is the sum of strain energy (U) experienced by the muscles in the course of displacement of femoral head along the pelvis and gravitational potential energy (Ω) will be a function of femoral head center coordinate (x,y,z), flexion angle "θ," abduction angle "Φ," and hip rotation angle "φ,"

$$\Pi(x, y, z, \theta, \Phi, \varphi) = U + \Omega \tag{4}$$

A MATLAB computer program[21] was created to calculate the potential energy values for all femoral head center locations in the hip in a sampled set of discrete points.

After finding the extremum of potential energy (its local minimum) for all femoral head center locations, Dijkstra's optimization algorithm[15] was used to determine the least energy path for the hip dysplasia reduction. Dijkstra's algorithm program calculates two variables: The absolute value of the energy difference and the length between the source point and all adjacent points. This process was repeated until all points in the field were covered, and a path tree was created by the program. After this was completed, Dijkstra's algorithm selects the path of lowest energy. When paths between the source and target have the same energy difference, which is independent of the path, the code checks a second condition, which is the shortest path.
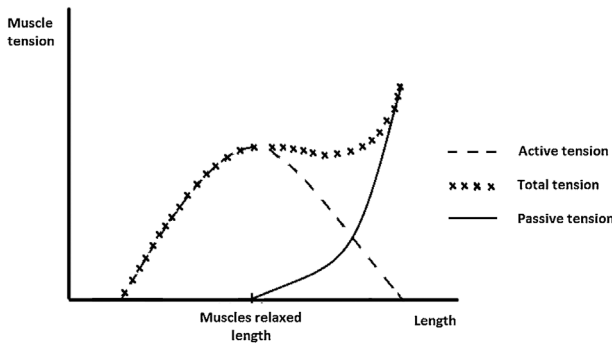
Figure 3. Muscle force behavior (active and passive behavior). Graph illustrates the active, passive, and total muscle tension with $L_\circ$, where $L_\circ$ is the initial muscle relaxed length at rest.

Following the findings of local minima of the potential energy from the MATLAB code, Dijkstra's algorithm was used to determine the least energy path. Several types of analysis were carried out for Grade IV infantile hip dislocations (Table 1): (1) determining the least energy path from the starting point of Grade IV dislocated hip to the acetabulum center with all muscles intact; and (2) with the Pectineus muscle computationally switched off; (3a) determining the energy requirement for the theoretical pathway of the modified Hoffman–Daimler method with all muscles intact and (3b) with the effect of the Pectineus muscle switched off; and (4a) determining the energy required for the direct path proposed by Iwasaki with all muscles intact and (4b) with the effects of the Pectineus muscle switched off.

RESULTS

The first analysis determined the least energy path from the starting point of Grade IV dislocated hip to the acetabulum center (Fig. 5). This path of least energy closely approximated the indirect pathway proposed by Papadimitriou et al.[10] In this computational model, the femoral head enters the acetabulum at the posterior margin of the lunate articular surface instead of through the center of the acetabular notch as described in the modified Hoffman–Daimler method. The reduction path started with a flexed femur at an angle of 70° and continued with that angle along the path. The femur was guided to be abducted at 40°, and then increased gradually to 80° while the

Figure 4. Height "X" of the center of gravity of the lower limb bones as used in calculation of potential energy. "X" was found for all femoral head locations at all considered femur orientations.

hip rotation angle changed internally from 30° to −30° with a final external rotation of 10° at the center of the acetabulum.

The second analysis computationally "switched off" the Pectineus, and the resultant pathway closely followed the direct pathway proposed by Iwasaki[11] and documented by Suzuki[12] (Fig. 6). To achieve reduction with the lowest energy, the femur flexed at 70° along the path. However, the abduction angle changed from 40°–90°, while the hip rotated internally from 0° to −30°. The energy required for this pathway with the Pectineus released was less than the energy required for the pathway with all modeled muscles intact. The energy drop was 5.2%.

The third analysis determined the energy requirement with all muscles intact and with the Pectineus muscle computationally switched off for the indirect path that was described by Papadimitriou for the modified Hoffman–Daimler method. As noted previously, the pathway proposed for the modified Hoffman–Daimler method is slightly different from the calculated path of least energy shown in Figure 5. In this study, the Modified–Hoffman Daimler pathway was calculated by defining a path that goes to the intermediate point on the ischium near the acetabular notch before entering the acetabulum through the notch. For this configuration with all muscles intact, the femur flexed to 70° during reduction, but it increased to 130° at the acetabular notch and returned to 70° as reduction continued. The femur abducted to 40° at the beginning of the path, then abduction increased to 90° until the acetabular center is reached where final abduction was 80°. Hip rotation changed from +30° to −30° at the acetabular notch, then rotated externally at the end of the reduction path. The second part of this analysis utilized the modified Hoffman–Daimler pathway and determined the energy required with the effect of the Pectineus muscle switched off. In order to achieve reduction, the femur flexed at 70° along the path, abducted from 40° to 90°, and rotated internally from 0° to −30°. The difference of the required energy with and without the effect of the Pectineus muscle was determined for the indirect path. The required energy dropped by 0.04 J which is a 9.2% change when the Pectineus is released. For reference 0.25 J is comparable to lifting 2.5 kg (5.6 lbs). over a height of 1 cm while 0.04 J is comparable to moving 0.4 kg(0.9 lbs) over the same height.

The fourth analysis compared the energy requirements for the femoral head to pass directly over the rim of the posterior acetabulum when the Pectineus muscle is intact or when the Pectineus muscle is switched off. This was determined by closely approximating the direct path proposed by Iwasaki, and determining the required energy with all modeled muscles intact. Results indicate that additional energy is required for the femoral head to pass directly over the rim of the posterior acetabulum when the Pectineus muscle is intact. The energy required increased
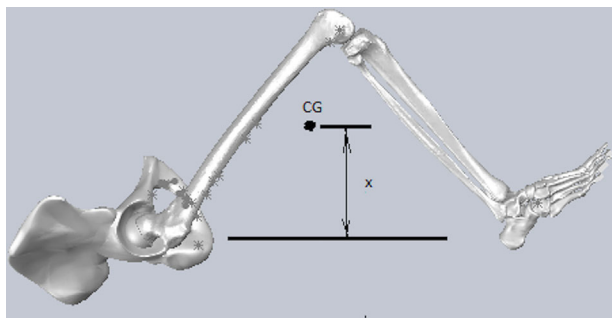
Table 1. Summary of Computational Analysis and Results

Analysis Reference Number in Text	Computational Analysis	Muscle Model	Required Energy (Joules)	Percent Difference
1	Path of least energy	All muscles intact	0.231	
2	Path of least energy	Pectineus muscle switched off	0.219	5.2% less energy than 1
3 (a)	Energy required for the indirect path (modified Hoffman–Daimler pathway)	All muscles intact	0.509	
3 (b)	Energy required for the indirect path (modified Hoffman–Daimler pathway	Pectineus muscle switched off	0.462	9.2% less energy than 3 (a)
4 (a)	Energy required for the direct path (Iwasaki pathway)	All muscles intact	0.241	
4 (b)	Energy required for the direct path (Iwasaki pathway)	Pectineus muscle switched off	0.219	9.1% less energy than 4 (a)

1. Path of least energy with all muscles intact. 2. Path of least energy with Pectineus muscle switched off. 3. Energy required for the indirect path (modified Hoffman–Daimler pathway). (a) All muscles intact (b) with Pectineus switched off. 4. Energy required for the direct path (Iwasaki pathway). (a) All muscles intact (b) with Pectineus switched off.

by 10% compared to the same pathway when the Pectineus was switched off (Table 1). This supports the findings of Suzuki that the traction or manual reduction may be needed for direct reduction of Grade IV dislocations.

Figure 5. Least energy pathway for reduction of Grade IV hip dislocations with all modeled muscles intact. The least energy pathway is similar to the pathway identified by Papadimitriou et al.[10]

The influence of the initial location of the femoral head on the final pathway was investigated using different starting points for both pathways. Regardless of starting points, the paths rapidly converge to a common final pathway.

DISCUSSION

This computational biomechanical model of the path of least energy confirms the clinical observations of two distinct pathways for reduction of a Grade IV infantile hip dislocation. Regardless of starting position, the reduction sequence rapidly assumes one of these two pathways.

The model determined that the least energy path from the dislocated position to the center of the acetabulum with all muscles intact followed the path described by Papadimitriou et al.[10] with one small exception. It appears in this model that the femoral head enters the acetabulum at the posterior margin of the lunate articular surface instead of through the center of the acetabular notch. This slight variation may represent the absence of soft tissues in this model. However, this is essentially the same path as described by Papadimitriou for the modified Hoffman–Daimler method.

Moreover, the model suggests that the structure and mechanics of the Grade IV dislocated hip may allow reduction by the modified Hoffman–Daimler pathway where the femoral head first moves to the superior rim of the acetabulum, then along the posterior rim to the inferior acetabulum and enters from the region of the acetabular notch. Papadimitriou et al. reported successful non-surgical reduction without cast immobilization at an average age of 16 months. Further study of this method may be warranted based on the findings of this computational biomechanical analysis.
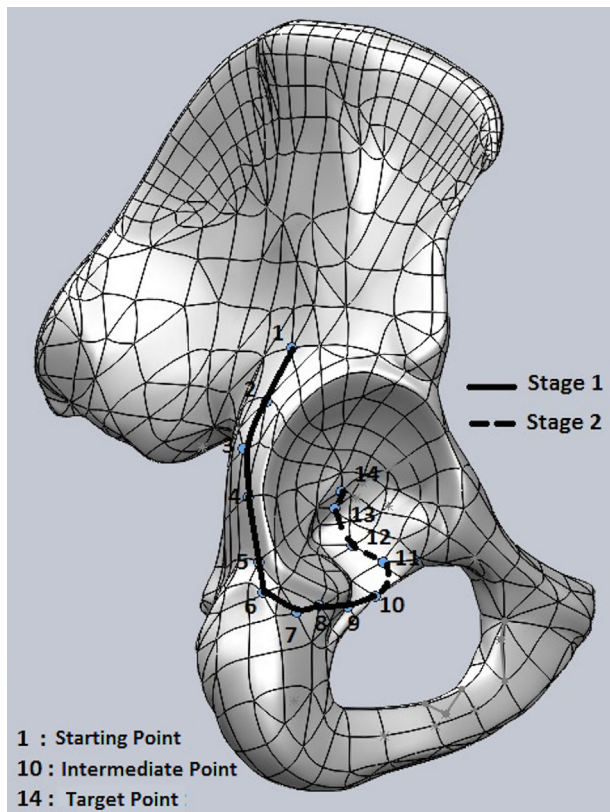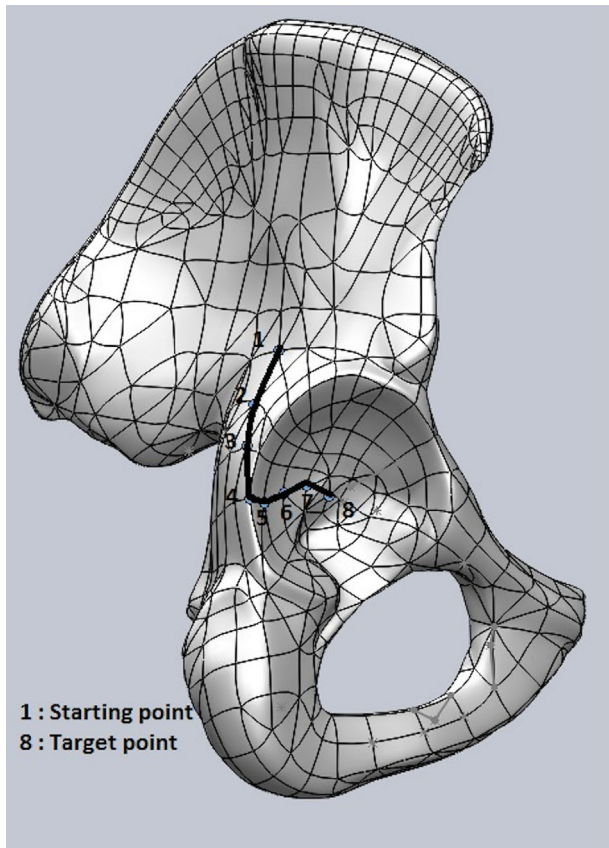
Figure 6. Least energy pathway when the Pectineus muscle is "switched off" for reduction of Grade IV hip dislocation. Without the Pectineus muscle, this pathway is similar to the pathway identified by Iwasaki.[11]

1 : Starting point
8 : Target point

Another pathway is directly over the posterior rim of the acetabulum, but this study supports Suzuki's observation that this may be clinically difficult without traction or manual force for Grade IV dislocations. The adductor muscles are known to oppose reduction and are often released during manual or surgical reduction of infantile hip dislocations. This biomechanical study suggests that the Pectineus muscle is also an obstacle to direct reduction for Grade IV dislocations when the femoral head is trapped posterior to the acetabulum as described by Suzuki. Forcing the femoral head along this same path with the Pectineus muscle intact requires more energy than when the Pectineus muscle has been removed. The calculated energy requirement for direct reduction is ~10% greater when the Pectineus muscle is intact (Table 1). This suggests that the Pectineus muscle resists reduction for high grade hip dislocations. However, the magnitude of resistance is small: ~0.022 J or comparable to lifting 0.23 kg (0.5 lbs) over a height of 1 cm. The clinical relevance of this finding remains uncertain although this finding is consistent with manual reduction when the operator provides the energy to physically reduce the femoral head over the rim of the acetabulum.

Eliminating the effect of the Pectineus muscle decreased the power required for reduction and the path of least energy proceeded more directly over the rim of the acetabulum into the reduced position. This supports the previously reported finding that the Pectineus muscle resists reduction in addition to resistance by the adductor muscles for Grade IV hip dislocations.[14] The findings of this study suggest that release of the Pectineus muscle during medial open reduction may also be considered to facilitate hip reduction in Grade IV dislocations.

A limitation of this study is that the role of the iliopsoas as a mechanical block to reduction could not be assessed. However, this biomechanical study evaluated closed reduction methods where the iliopsoas remains intact. Modeling of the Iliacus and Psoas muscles demonstrated that the force contributions of the Iliacus and Psoas were negligible with the hips flexed during closed reduction pathways. It should be noted that extreme positions of 80–90° of abduction were identified for reduction of the femoral head into the acetabulum by direct or indirect pathways, but such extreme position would not be recommended for retention of reduction. This study did not attempt to identify the range of stability following reduction.

A limitation of this biomechanical study includes assumptions about the viscoelastic nature of the hip capsule and cartilaginous labrum. As a viscoelastic substance loses energy when a load is applied and then removed, we have assumed that the capsule stretches and does not provide significant energy to either path because this is a model of gradual reduction. As such, the model does not calculate time for reduction and only calculates the energy requirements to overcome anatomical barriers and the direction of forces that would be applied by traditional braces and harnesses for reduction of dislocations. This is similar to the reason that cartilage was not modeled because cartilage deformation may be overcome with time. Incorporating cartilage deformation is a goal for our model development. However, absence of cartilage modeling is a limitation of this current study.

Although our model is frictionless, the net effect of including friction as the femoral head slides on the superior rim of the acetabulum would have increased the barrier to achieving reduction by the direct path making this even less achievable by closed reduction for Grade 4 dislocations relative to the indirect path suggested by our model.

Additional limitations include passive modeling of the hip muscles without active muscle contractions and modeling of normal acetabular morphology without secondary deformity that may be present in older infants. We consider only the most common anteversion angle of 50° encountered in DDH.[22] We plan to investigate the effect of varying degrees of anteversion in future studies. In spite of these limitations, this study provides additional support for consideration of the indirect pathway of reduction as reported for the modified Hoffman–Daimler Method.

In summary, this computational model of Grade IV infantile hip dislocations confirms the clinical observations of two different pathways for closed reduction. The direct path over the posterior rim of the acetabulum requires more energy. This model supports the observation that Grade IV dislocations may require manual reduction by the direct path. However, the indirect path described in the modified Hoffman–Daimler method requires less energy and may be an alternative to direct manual reduction of Grade IV infantile hip dislocations.

AUTHORS' CONTRIBUTIONS

MZ contributed to Implemented POD, performed coding and computations of biomechanics model, interpretation, drafted substantial portions of the manuscript. FAM contributed to supervised study design, formulation of minimum energy principle, interpretation of data, drafted, and edited manuscript revisions. CR contributed to biomechanics model for DDH, performed coding and visualization of data. VH contributed to the biomechanics model for DDH and interpretation of data. AJK contributed to study design and interpretation of data, POD formulation. EAD contributed to study design and interpretation of data, POD formulation. BJJ contributed to the biomechanical model and carried out verification with OpenSIM. CTP contributed to interpretation of data, and clinical relevance of the study, drafted substantial portions of the manuscript. All authors have read and approved the final submitted manuscript.

ACKNOWLEDGMENTS

REFERENCES

1. Sewell M, Rosendahl K, Eastwood D. 2009. Developmental dysplasia of the hip. BMJ 339:b4454.
2. Price CT, Ramo BA. 2012. Prevention of hip dysplasia in children and adults. Orthop Clin North Am 43:269–279.
3. Atalar H, Sayli U, Yavuz OY, et al. 2007. Indicators of successful use of the Pavlik harness in infants with developmental dysplasia of the hip. Intl Orthop 31:145–150.
4. Graf R. 1980. The diagnosis of congenital hip joint dislocation by ultrasonic compound treatment. Arch Orthop Trauma Surg 97:117–133.
5. Tönnis D. 1987. Arthrography of the hip joint. In: congenital dysplasia and dislocation of the hip in children and adults. Berlin, Heidelberg, New York: Springer Verlag. pp143–155.
6. Narayanan U, Mulpuri K, Sankar WN, et al. 2015. Reliability of a new radiographic classification for developmental dysplasia of the hip. J Pediatr Orthop 35:478–484.
7. Grill F, Bensahel H, Canadell J, et al. 1988. The Pavlik harness in the treatment of congenital dislocating hip: report on a multicenter study of the European paediatric orthopaedic society. J Pediatr Orthop 8:1–8.
8. Bolland B, Wahed A, Al-Hallao S, et al. 2010. Late reduction in congenital dislocation of the hip and the need for secondary surgery: radiologic predictors and confounding variables. J Pediatr Orthop 30:676–682.
9. Fukiage K, Futami T, Ogi Y, et al. 2015. Ultrasound-guided gradual reduction using flexion and abduction continuous traction for developmental dysplasia of the hip. Bone Joint J 97-B:405–411.
10. Papadimitriou NG, Papadimitriou A, Christophorides JE, et al. 2007. Late-presenting developmental dysplasia of the hip treated with the modified Hoffman–Daimler functional method. J Bone Joint Surg Am 89:1258–1268.
11. Iwasaki K. 1983. Treatment of congenital dislocation of the hip by the Pavlik harness. J Bone Joint Surg Am 65:760–767.
12. Suzuki S. 1994. Reduction of CDH by the Pavlik harness. J Bone Joint Surg Br 76:460–462.
13. Ardila O, Divo E, Moslehy FA, et al. 2013. Biomechanics of hip dysplasia reductions in infants using the Pavlik harness: a physics-based computational model. J Biomech 46:1501–1507.
14. Huayamave V, Rose C, Serra S, et al. 2015. A patient-specific model of the biomechanics of hip reduction for neonatal developmental dysplasia of the hip: investigation of strategies for low to severe grades of developmental dysplasia of the hip. J Biomech 48:2026–2033.
15. Dijkstra EW. 1959. A note on two problems in connexion with graphs. Numer Math 1:269–271.
16. Zwawi, M., "Mechanism of Hip Dysplasia and Identification of the Least Energy Path for its Treatment by using the Principle of Stationary Potential Energy" (2015). Electronic Theses and Dissertations. Paper 1418. http://stars.library.ucf.edu/etd/1418
17. Huayamave V, Rose C, Zwawi M, et al. 2014. Mechanics of hip dysplasia reduction in infants with the Pavlik harness using patient specific geometry. Proceedings of ASME-IMECE. Montreal, Canada.
18. Fung YC. 1984. Structure and stress–strain relationship of soft tissues. Am Zool 24:13–22.
19. Magid A, Law DJ. 1985. Myofibrils bear most of the resting tension in frog skeletal muscle. Science 230:1280–1282.
20. Gray GL, Costanzo F, Plesha ME. 2012. Engineering Mechanics: dynamics, 2nd ed. New York: McGraw-Hill.
21. MATLAB and Statistics Toolbox Release 2014b, The MathWorks, Inc. [computer program]. Natick, Massachusetts, United States.
22. Sankar WN, Neubuerger CO, Moseley CF. 2009. Femoral anteversion in developmental dysplasia of the hip. J Pediatr Orthop 29:885–888.