THE ELECTRIC CONDUCTIVITY OF DISPERSE SYSTEMS.

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(Received for publication, May 23, 1924.)

In a paper now in press,[1] I have presented a theory for the conductivity of a suspension of homogeneous spheroids[2] in which the following formula was derived.

$$\frac{k - k_1}{k - k_2}\left(1 - \frac{k_2}{k_1}\right) = \beta \cdot \frac{\rho}{1 - \rho} \tag{1}$$

k, specific conductivity of suspension.
k_1, " " " suspending medium.
k_2, " " " suspended medium.
ρ, volume concentration " " "

$$\beta = \frac{1}{3}\left[\frac{2}{1 + \left(\frac{k_2}{k_1} - 1\right)\int_0^{\infty}\frac{d\lambda}{\left(1 + \frac{b^2}{a^2}\lambda\right)^{\frac{1}{2}}\left(1 + \lambda\right)^2}} + \frac{1}{1 + \left(\frac{k_2}{k_1} - 1\right)\left(1 - \int_0^{\infty}\frac{d\lambda}{\left(1 + \frac{b^2}{a^2}\lambda\right)^{\frac{1}{2}}\left(1 + \lambda\right)^2}\right)}\right]\left(\frac{k_2}{k_1} - 1\right)$$

a, b, axes of spheroid.

For the case of the sphere $\left(\frac{a}{b} = 1\right)$, formula (1) reads[1]

$$\frac{\frac{k}{k_1} - 1}{\frac{k}{k_1} + 2} = \rho\,\frac{\frac{k_2}{k_1} - 1}{\frac{k_2}{k_1} + 2} \tag{2}$$

[1] Fricke, H., A mathematical treatment of the electric conductivity and capacity of disperse systems, *Phys. Rev.*, 1924 (in press).

[2] The theory for the simple case of a diluted suspension of spheres was given in a recent paper. Fricke, H., *J. Gen. Physiol.*, 1923–24, vi, 375.

For a diluted solution $\frac{k}{k_1}$ is approximately equal to 1; introducing this value of $\frac{k}{k_1}$, equation (2) reduces to

$$\frac{k}{k_1} = 1 + 3\rho \frac{\frac{k_2}{k_1} - 1}{\frac{k_2}{k_1} + 2} \tag{3}$$

This is the equation, the derivation of which was given in a previous paper.[2]

β is given in graphical form in Fig. 1.

It is of interest to note that according to equation (1) and Fig. 1, for a constant volume concentration of the suspended medium the conductivity of a suspension is independent of the size of the suspended particles and also nearly independent of the form of the particles when the difference between the conductivities of the suspended and the suspending media is not very large. This is especially true for suspensions of prolated spheroids which are less conductive than the suspending medium.

Investigations in which formula (1) is being applied to different problems of practical and theoretical interest are at present being made in this laboratory. In the first of these studies which will be published shortly, the case of cream will be considered. In the present paper the formula will be applied to the case of blood.

The most exact investigations of the conductivity of blood in terms of the volume concentration of the red corpuscles have been made by Stewart[3] and Fraenckel.[4] Stewart has determined the volume concentration by two independent methods, a colorimetric method and the Hoppe-Seyler chemical method. Fraenckel has used Bleibtreu's chemical method. The agreement between the results secured by these investigations is very close. However, as has been stated also by Fraenckel, one would expect that Stewart's methods would give the most exact results and therefore we shall here consider only his measurements. In Table I is given a series of measurements

[3] Stewart, G. N., *J. Physiol.*, 1899, xxiv, 356.

[4] Fraenckel, P., *Z. klin. Med.*, 1904, lii, 470.

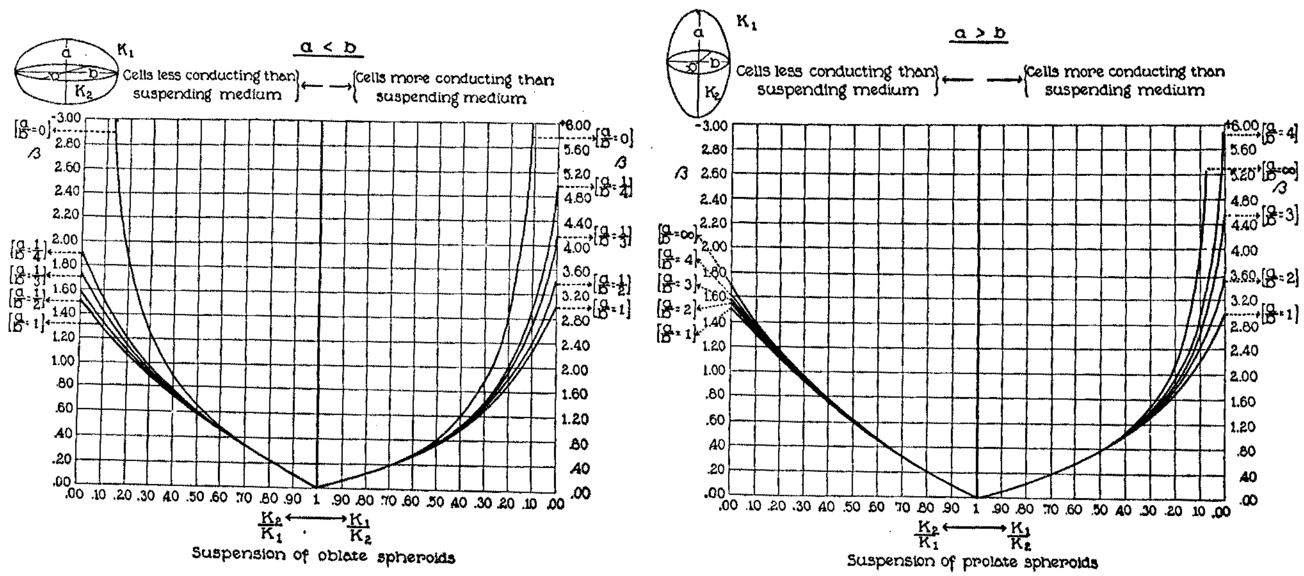

FIG. 1. Graphical representation of β.

Conductivity of suspension of homogeneous spheroids.

$$\frac{K - K_1}{K - K_2}\left(1 - \frac{K_2}{K_1}\right) = \beta \cdot \frac{\rho}{1 - \rho}$$

K, specific conductivity of suspension; K_1, specific conductivity of suspending medium; K_2, specific conductivity of spheroid; ρ, volume concentration of suspension; β, plotted.

made by Stewart[5] for the blood of a dog. The volume concentration of the normal blood was determined twice by the colorimetric method and twice by the Hoppe-Seyler method. The results were as follows: 41.16, 40.72, 41.52, and 40.99 per cent, the average being 40.98 per cent. A series of red corpuscle suspensions of varying volume concentration was made by concentration or dilution of the normal blood. The volume concentration for each of these suspensions was determined volumetrically using the value of the volume

TABLE I.

Conductivity of Blood of Dog.

$\frac{k_1}{k}$	ρ observed.	ρ calculated.	Δ
	per cent		*per cent*
15.62	90.7	88.4	+2.6
9.08	82.1	80.9	+1.5
6.56	74.5	74.4	+0.1
5.06	67.8	68.0	−0.3
4.14	61.6	62.1	−0.8
3.51	56.1	56.8	−1.2
3.063	51.0	51.9	−1.8
2.726	46.4	47.4	−2.2
2.436	42.2	42.9	−1.6
2.348	41.0	41.3	−0.7
2.225	38.4	39.0	−1.6
1.903	31.9	32.0	−0.3
1.697	26.4	26.7	−1.1
1.539	21.8	22.0	−0.9
1.428	18.1	18.3	−1.1
1.342	15.3	15.2	+0.7
1.232	11.4	10.8	+5.5

concentration of the original blood. The volume concentrations are given in Table I under ρ observed. The ratio of the observed conductivity of the serum to that of the blood is given under $\frac{k_1}{k}$. In order to employ formula (1) we consider $\frac{a}{b} = \frac{1}{4}$; consequently in Fig. 1, $\beta = -1.91$.

[5] Stewart,[3] p. 369.

Furthermore, $k_2 = 0$. The formula therefore reads

$$1 - \frac{k_1}{k} = -1.91 \cdot \frac{\rho}{1-\rho}$$

The values of the volume concentration ρ calculated from this formula using the observed values of $\frac{k_1}{k}$ are given under ρ calculated.

TABLE II.

Conductivity of Suspensions of Sand.

$\frac{k_1}{k}$ observed.	ρ observed.	β
	per cent	
2.488	40.0	2.22
2.220	36.4	2.14
2.075	33.3	2.15
1.952	30.8	2.14
1.866	28.6	2.17
1.790	26.7	2.17
1.725	25.0	2.18
1.6835	23.5	2.22

Average value of β, 2.17.

TABLE III.

Conductivity of Suspensions of Sand.

$\frac{k_1}{k}$ observed.	ρ observed.	β
	per cent	
1.190	10	1.710
1.426	20	1.705
1.734	30	1.712
2.156	40	1.732
2.715	50	1.715
3.613	60	1.74

Average value of β, 1.72.

The percentile differences between ρ observed and ρ calculated are given under Δ. The deviations throughout are within the limits of experimental errors.

Before concluding, it may perhaps be of interest to apply our formula to some measurements of sand suspensions which have been made by Oker-Blom[6] and which often are cited in connection with investigations of the conductivity of blood. Two different kinds of sand were used in these investigations. The suspensions were prepared by adding the sand to a hot salt solution containing 3 per cent gelatin and cooling this mixture to gelatination under continuous rotation. No information is given concerning the form of the sand particles except for the statement that the particles of the sand which were used in the second series of experiments (Table III) were more nearly spherical than those which were used in the first series (Table II). Our theory is therefore best applied by using Oker-Blom's observed values of $\frac{k_1}{k}$ and ρ and calculating the value of β by means of the formula $1 - \frac{k_1}{k} = -\beta \cdot \frac{\rho}{1-\rho}$ obtained from formula (1) by considering $k_2 = 0$. Our calculated results are given in Tables II and III, the sand used in the measurements in the two tables corresponding to the two kinds of sand used by Oker-Blom. The calculated values for β are constant within experimental errors. The deviation of the value of β from 1.50 is a measure of the deviation of the sand particles from the spherical form.

[6] Oker-Blom, M., *Arch. ges. Physiol.*, 1900, lxxix, 510.